01

When a glass rod is rubbed with silk, it acquires a power to attract light bodies such as, small pieces of paper. The objects which acquire the attracting power are said to be electrified or charged. Benjamin Franklin demonstrated that lighting was related to static electricity. The branch of Physics which deals with static electricity is called **electrostatics**.

ELECTRIC CHARGES AND FIELDS

Concept Physics classes, for:-11,12,NEET & JEE
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All directly experienced forces except the gravitational force are manifestations of electromagnetic force.

Electrostatics deals with study of forces, fields and potentials arising from static charges or charges at rest. In this particular chapter, we will discuss all the above mentioned topics in a detailed form in order to understand them very thoroughly.

CHAPTER CHECKLIST

- Electric Charges
- Coulomb's Law and Electrostatic Field
- Electric Dipole
- Electric Flux

|TOPIC 1|
Electric Charges

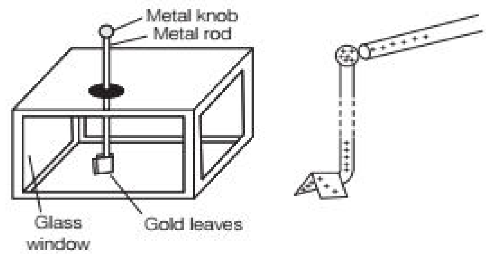
The physical property of matter that causes it to experience a force when placed in an electromagnetic field is called electric charge. Electric charge is a characteristic that accompanies fundamental particles, wherever they exist.

According to William Gilbert, charge is something possessed by material objects that makes it possible for them to exert electrical force and respond to the electrical force.

Electric charge is a scalar quantity.

Gold Leaf Electroscope

It is an instrument which detects the electric charges by means of electrostatic forces. It consists of two gold leaves which are suspended side by side from a conducting rod which is held by an insulated support and placed in a grounded enclosure, such as a glass jar. When a charge is applied to a plate to which the rod is connected, the leaves separate due to their mutual repulsion. One variation involves having one fixed plate along with a single leaf.



There are two kinds of charges such as positive charge and negative charge.

An object can attain positive charge by loosing electrons while other can attain negative charge by gaining electrons. Charges with same sign, i.e. like charges repel each other while charges with opposite sign, i.e. unlike charges attract each other.

Charges always reside on the surface of the charged conducting object. An object can be charged by different methods like friction, conduction and induction.

Charges can be added and subtracted as a number.

Conductors and Insulators

Conductors are those substances which can be used to carry or conduct electric charge/electron from one point to other. They allow electricity to pass through them easily.

e.g. Silver, copper, iron, aluminium, etc.

Insulators are those substances which cannot conduct electricity. They are also called **dielectrics**. They offer high resistance to the passage of electricity through them,

e.g. Glass, rubber, plastic, ebonite, mica, etc.

Difference between Dielectrics and Conductors

Dielectrics are non-conductors and do not have free electrons at all, while conductors have free electrons in their any volume which makes them able to pass the electricity through them.

Charging by Induction

The process of charging a neutral body by bringing a charged body nearby it without making contact between the two bodies is known as **charging by induction**.

Figures given below are showing the sequential steps of charging a conductor permanently by using the process of charging by induction.

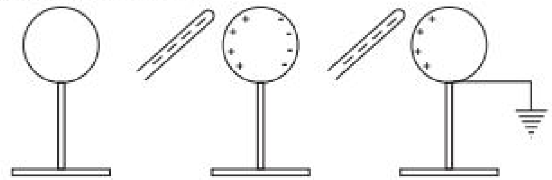


Fig. (a) Fig. (b) Fig. (c)

Using the process of charging by induction, a conductor may be charged permanently.

EXAMPLE |1| A comb run through one's hair attracts small bits of paper. What happens, if the hairs are wet or it is a rainy day?

Sol. If the hairs are wet or it is a rainy day, then the friction between the hair and the comb reduces. The comb does not get charged and it will not attract small bits of paper.

BASIC PROPERTIES OF ELECTRIC CHARGE

Some basic properties of the electric charge are discussed below

Additive Nature of Electric Charge

Electric charge is additive in nature. In general, if a system consists of n charges $q_1, q_2, q_3, \ldots, q_n$, then the total charge of the system will be $q_1 + q_2 + q_3 + \ldots + q_n$.

In order to calculate the net charge on a system, we have to just add algebraically, all the charges present in the system. This is known as the **principle of superposition of charge.**

If the sizes of charged bodies are very small as compared to distance between them, then they can be considered as point charges.

Conservation of Electric Charge

During any process, the net electric charge of an isolated system remains constant (i.e. conserved). In simple words, charge can neither be created nor be destroyed.

In any physical process, the charge may get transferred from one part of the system to another, but the net charge will always remain the same.

Quantisation of Electric Charge

The charge on any body can be expressed as an integral multiple of basic unit of charge, i.e. charge on one electron. This phenomena is called **quantisation of electric charge**.

It can be written as $\boxed{q = \pm ne}$

where, $n = 1, 2, 3, \ldots$ is any integer, positive or negative and e is the basic unit of charge.

The SI unit of charge is called coulomb and denoted by C and its value is $e = 1.602192 \times 10^{-19}$ C or 1.6×10^{-19} C.

EXAMPLE |2| A polythene piece rubbed with wool is found to have a negative charge of 3×10^{-7} C.

(i) Estimate the number of electrons transferred from which to which?

(ii) Is there a transfer of mass from wool to polythene?

NCERT

Sol. (i) Here, $q = -3 \times 10^{-7}$ C

Charge on one electron, $e = -1.6 \times 10^{-19}$ C

$\therefore$ Number of electrons transferred from wool to polythene piece,

$$n = \frac{q}{e} = \frac{-3 \times 10^{-7}\text{ C}}{-1.6 \times 10^{-19}\text{ C}} = 1.875 \times 10^{12}$$

(ii) Yes, there is a transfer of mass from wool to polythene piece.

As, mass of each electron, $m_e = 9 \times 10^{-31}$ kg

$\therefore$ Mass transferred from wool to polythene,

$$m = n \times m_e = 1.875 \times 10^{12} \times 9 \times 10^{-31}\text{kg}$$
$$= 1.687 \times 10^{-18}\text{ kg}$$

EXAMPLE |3| A copper slab of mass 2 g contains 2×10^{22} atoms. The charge on the nucleus of each atom is 29 e. What fraction of the electrons must be removed from the sphere to give it a charge of $+2\ \mu$C?

Sol. Total number of electrons in the slab

$$= 29 \times 2 \times 10^{22}$$

Number of electrons removed

$$= \frac{q}{e} = \frac{2 \times 10^{-6}}{1.6 \times 10^{-19}} = 1.25 \times 10^{13}$$

$\therefore$ Fraction of electrons removed

$$= \frac{1.25 \times 10^{13}}{29 \times 2 \times 10^{22}} = 2.16 \times 10^{-11}$$

Difference between Charge and Mass

The difference between charge and mass is given in the following table

Charge	Mass
Electric charge on a body may be positive, negative or zero.	Mass of a body is a positive quantity.
Charge carried by a body does not depend upon velocity of the body.	Mass of a body increases with its velocity as $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$, where c is velocity of light in vacuum, m is the mass of the body moving with velocity v and m_0 is rest mass of the body.
Charge is quantised.	The quantisation of mass is yet to be established.
Electric charge is always conserved.	Mass is not conserved as it can be changed into energy and *vice-versa*.
Force between charges can be either attractive or repulsive, as charges are unlike or like charges.	The gravitational force between two masses is always attractive.

| TOPIC PRACTICE 1 |

OBJECTIVE Type Questions

1. One metallic sphere A is given positive charge whereas another identical metallic sphere B of exactly same mass as of A is given equal amount of negative charge. Then,

(a) mass of A and mass of B still remain equal
(b) mass of A increases
(c) mass of B decreases
(d) mass of B increases

2. In general, metallic ropes are suspended from the carriers to the ground which take inflammable material. The reason is

(a) their speed is controlled
(b) to keep the gravity of the carrier nearer to the earth
(c) to keep the body of the carrier in contact with the earth
(d) nothing should be placed under the carrier

3. In charging by induction,
(a) body to be charged must be an insulator
(b) body to be charged must be a semiconductor
(c) body to be charged must be a conductor
(d) any type of body can be charged by induction

4. Charge on a body is q_1 and it is used to charge another body by induction. Charge on second body is found to be q_2 after charging. Then,
(a) $\frac{q_1}{q_2}=1$ (b) $\frac{q_1}{q_2}<1$
(c) $\frac{q_1}{q_2}\leq 1$ (d) $\frac{q_1}{q_2}\geq 1$

5. An object of mass 1 kg contains 4×10^{20} atoms. If one electron is removed from every atom of the solid, the charge gained by the solid of 1 g is
(a) 2.8 C (b) 6.4×10^{-2} C
(c) 3.6×10^{-3} C (d) 9.2×10^{-4} C

6. Number of electrons present in a negative charge of 8 C is
(a) 5×10^{19} (b) 2.5×10^{19}
(c) 12.8×10^{19} (d) 1.6×10^{19}

VERY SHORT ANSWER Type Questions

7. A glass rod when rubbed with silk cloth acquires a charge 1.6×10^{-13} C. What is the charge on the silk cloth?

8. Consider three charged bodies A, B and C. If A and B repel each other and A attracts C, then what is nature of the force between B and C ?

9. What does $q_1 + q_2 = 0$ signify in electrostatics?

10. Which property of dielectrics make them different from conductors?

11. Two insulated charged copper spheres A and B of identical size have charges q_A and q_B, respectively. A third sphere C of the same size but uncharged is brought in contact with the first and then in contact with the second and finally removed from both. What are the new charges on A and B ?

12. What is the basic cause of quantisation of charge?

13. Can a body has charge 1.5 e, where e is the electronic charge?

14. Which is bigger, a coulomb of charge or a charge on an electron?

15. "An object becomes positively charged through the removal of negatively charged electrons rather than through the addition of positively charged protons". Explain, why?

16. A glass object is charged to +3 nC by rubbing it with a silk cloth. In this rubbing process, have protons been added to the object or have electrons been removed from it?

SHORT ANSWER Type Questions

17. In filling the gasoline tank of an aeroplane, the metal nozzle of hose from the gasoline truck is always carefully connected to the metal body of the aeroplane by a wire, before the nozzle is inserted in the tank. Explain, why?

18. Automobile ignition failure occurs in damp weather. Explain, why?

19. A bird perches on a bare high power line and nothing happens to the bird. A man standing on the ground touches the same line and gets a fatal shock. Why?

20. An ebonite rod held in hand can be charged by rubbing with flannel but a copper rod cannot be charged like this, why?

21. Ordinary rubber is an insulator. But the special rubber tyres of aircrafts are made slightly conducting. Why is this necessary?

22. Why does a charged glass rod attract a piece of paper?

23. Can a charged body attract another uncharged body? Explain.

24. Can two balls having same kind of charge on them attract each other? Explain.

25. Can ever the whole excess charge of a body P be transferred to the other body Q ? If yes, how and if not, why?

26. When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain, how this observation is consistent with the law of conservation of charge? **NCERT**

27. Give any two points of difference between charge and mass.

28. A balloon gets negatively charged by rubbing ceilings of a wall. Does this mean that the wall is positively charged? Why does the balloon eventually fall?

29. A paisa coin is made up of Al-Mg alloys and weighs 0.75 g. It has a square shape and its diagonal measures 17 mm. It is electrically neutral and contains equal amount of positive and negative charges. Treating the paisa coins made up of only Al, find the magnitude of equal number of positive and negative charges. What conclusion do you draw from this magnitude? **NCERT Exemplar**

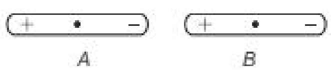
LONG ANSWER Type I Questions

30. Describe some of the differences between charging by induction and charging by contact.

31. (i) Explain the meaning of the statement "electric charge of a body is quantised".

(ii) Why can one ignore quantisation of electric charge, when dealing with macroscopic, i.e. large scale charges? **NCERT**

32. Two insulated rods A and B are oppositely charged on their ends. They are mounted at the centres, so that they are free to rotate and then held in the position shown in the figure, in a view from above. The rods rotate in the plane of the paper. Will the rods stay in those positions when released? If not, then what position(s) will they move? Will their final configuration(s) be stable?



33. It is now believed that protons and neutrons (which constitute nuclei of ordinary matter) are themselves built out of more elementary units called quarks. A proton and a neutron consist of three quarks each. Two types of quarks, so called 'up' quark (denoted by u) of charge $+\left(\frac{2}{3}\right)e$ and the 'down' quark (denoted by d) of charge $\left(-\frac{1}{3}\right)e$, together with electrons build up ordinary matter. (Other types of quark have also been found which give rise to different unusual varieties of matter). Suggest a possible quark composition of a proton and a neutron. **NCERT**

NUMERICAL PROBLEMS

34. What is the total charge of a system containing five charges $+1, +2, -3, +4$ and -5 in some arbitrary unit?

35. How many electrons are there in one coulomb of negative charge?

36. A metal sphere has a charge of $-6\,\mu C$. When 5×10^{12} electrons are removed from the sphere, what would be net charge on it?

37. A sphere of lead of mass 10 g has net charge -2.5×10^{-9} C.

(i) Find the number of excess electrons on the sphere.

(ii) How many excess electrons are per lead atom? Atomic number of lead is 82 and its atomic mass is 207 g/mol.

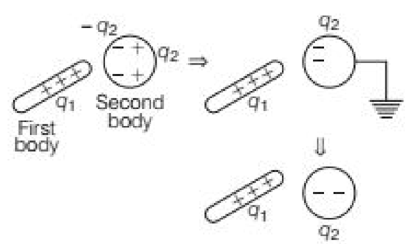
HINTS AND SOLUTIONS

1. (d) When a body is negatively charged more electrons are given to it, so its mass increases.

2. (c) During its motion, body of carrier is charged due to rubbing with dry air and dust. If spark occurs near container, then inflammable material may catch fire. So, metallic ropes are suspended so that excess charge flows away from carrier, to ground (for earthing).

3. (c) Induction requires shifting of free charge carrier which are present only in conductors.

4. (d)



Numerically, $q_1 \geq q_2 \Rightarrow \frac{q_1}{q_2} \geq 1$

5. (b) Here, number of electrons removed in 1 g = number of atoms in 1 g

or $n = \frac{4\times10^{20}}{10^3} = 4\times10^{17}$

$\therefore$ Charge, $q = ne = 4\times10^{17}\times1.6\times10^{-19} = 6.4\times10^{-2}$ C

6. (a) Given, charge, $q = 8$ C and

charge of electron, $e = 1.6\times10^{-19}$ C

$\therefore$ Charge, $q = ne$

$\therefore$ Number of electrons, $n = \frac{q}{e} \Rightarrow n = \frac{8}{1.6\times10^{-19}} = 5\times10^{19}$

7. Silk cloth will also acquire a charge 1.6×10^{-13} C. However, it will be negative in nature.

8. It is also attractive in nature.

9. The charges q_1 and q_2 are equal and opposite.

10. Dielectrics do not have free electrons at all. They offer high resistance to passage of electricity through them. e.g. Glass, rubber, plastic, etc.

11. When sphere C is brought in contact with A, then charge on sphere C,

$$q_C = \frac{q_A + 0}{2} = \frac{q_A}{2}$$

and new charge on sphere A, $q'_A = \frac{q_A}{2}$

When sphere C is brought in contact with B, then charge on sphere C,

$$q'_C = \frac{q_C + q_B}{2} = \frac{\frac{q_A}{2} + q_B}{2}$$

$$= \frac{q_A + 2q_B}{4}$$

$\therefore$ New charge on sphere B, $q'_B = \frac{q_A + 2q_B}{4}$

12. The basic cause of quantisation of charge is only the integral number of electrons which is transferred from one body to another, i.e. $\pm ne$.

13. No, a body cannot have charge 1.5 e. It is because the physically existing charge is always an integral multiple of e, i.e. 1.6×10^{-19} C.

14. We know that, $q = ne$

$\Rightarrow \quad 1 = n \times 1.6 \times 10^{-19}$ [given, $q = 1$C]

i.e. $\quad n = \frac{1}{1.6 \times 10^{-19}} = 6 \times 10^{18}$

$\therefore$ 1C is the charge of 6×10^{18} electrons.

So, a coulomb of charge is bigger than the charge on an electron.

15. In ordinary matter, a positive charge is much less mobile than a negative charge. For this reason, an object becomes positively charged through the removal of negatively charged electrons rather than through the addition of positively charged protons.

16. Electrons have been removed from the object.

17. Since, the aeroplane and the gasoline truck usually have wheels with rubber tyres, they are insulated from the ground. Further, the service ramps are usually made of concrete and are not necessarily good conductors to the earth. Therefore, inspite of grounding metallic ropes, the aeroplane and the truck could remain charged.

18. The insulating porcelain of the spark plugs accumulates a film of dirt.

The surface dirt is hygroscopic and picks up moisture from the air. Therefore, in humid weather, the insulating porcelain of the plugs becomes quasi-conductor.

This allows an appreciable proportion of the spark to leak across the surface of the plug instead of discharging across the gap.

19. When a bird is perched on a bare high power line, the circuit does not get completed between the bird and the earth, therefore nothing happens to the bird.

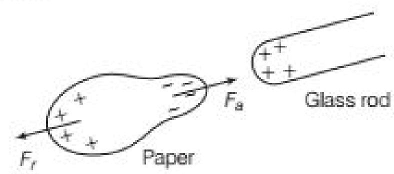
When a man standing on ground touches the same line, the circuit between the man and the earth gets completed. As a result, he gets a fatal shock.

20. Both the human body and the copper rod conduct electricity. When it is attempted to charge a copper rod by rubbing, the charge flows from the rod to the earth through the hand. However, when ebonite rod is charged by rubbing, the charges so produced stay on the ebonite rod as it is a bad conductor of electricity.

21. During landing or take off, the tyres of aircrafts get charged due to the friction between tyres and ground. In case, the tyres are slightly conducting, the charge developed on the tyres will not stay on them and it finds its way to the earth.

its way to the earth.

22. Paper is a dielectric, so when a positively charged glass rod is brought near it, atoms of paper get polarised, with centre of negative charge of atoms coming closer to the glass rod.



Therefore, force of attraction F_a between glass rod and piece of paper becomes greater than the force of repulsion F_r between the glass rod and the piece of paper. This results in attraction of the piece of paper towards the glass rod.

23. Yes, because when a charged body is brought near to uncharged body, opposite kind of induced charge is produced on an uncharged body. Therefore, the charged body attracts the uncharged body.

24. Yes, two balls having same kind of charge can attract each other. If any one of them has more charge as compared to the other, then due to the induction, they induce opposite kind of charges on the faces of each other when they are brought nearer. Therefore, they behave as oppositely charged balls and hence they attract each other.

25. Yes, the whole charge of a body P can be transferred to a conducting body Q, when P is enclosed by Q and is connected to it. This is because the charge always resides on the outer surface of the conductor.

26. When a glass rod is rubbed with a silk cloth, charges appear on both, these charges are equal in magnitude and opposite in sign, so that algebraic sum of the charges produced on both is zero. The net charge on the two bodies was zero even before rubbing them. Thus, we find that charges can be created only in equal and unlike pairs. This is consistent with the law of conservation of charge.

27. Refer to text on page 3.

28. No, this does not imply that the wall is positively charged. The balloon induces a charge of opposite sign in the ceiling of the wall, causing the balloon and the ceiling to be attracted to each other. The balloon eventually falls because its charge slowly diminishes as it leaks to ground. Some of the charge on the balloon could also be lost due to the presence of positive ions in the surrounding atmosphere, which would tend to neutralise the negative charges on the balloon.

29. Given, mass of a paisa coin, $m = 0.75$ g

Atomic mass of aluminium, $M = 26.9815$ g

Length of the diagonal of square shaped paisa coin= 17 mm

Avogadro's number, $N_A = 6.023 \times 10^{23}$

$$\Rightarrow \quad n = \frac{N_A}{M} \times m = \frac{6.023 \times 10^{23}}{26.9815 \text{ g}} \times 0.75 \text{ g} = 1.6742 \times 10^{22}$$

Since, atomic number (Z) of Al is 13, therefore each atom of Al contains 13 protons and 13 electrons. Now, find out the magnitude of positive and negative charges present in one paisa coin.

$$nZe = 1.6742 \times 10^{22} \times 13 \times 1.6 \times 10^{-19} \text{ C} = 34.8 \text{ kC}$$

Now, write the conclusion drawn from this magnitude of charge.

34.8 kC is a very large amount of charge. This concludes that ordinary neutral matter contains an enormously large amount of positive and negative charges.

30. (i) When an object is charged by induction, there is no physical contact between the object being charged and the object used to do the charging. In contrast, charging by contact, as the name implies, involves the direct physical contact to transfer charge from one object to the another.

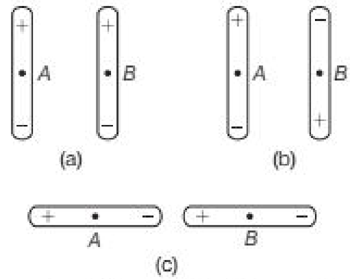
(ii) When an object is charged by induction, the sign of the charge that the object acquires is opposite to that of the object used to do the charging. Charging by contact gives the object being charged the same sign of charge as the original charged object.

31. (i) Refer to text on page 3.

(ii) In practice, the charge on a charged body is very large. On the other hand, the charge on an electron is very small. When electrons are added to a body or removed from a body, the change taking place in the total charge on the body is so small that the charge seems to be varying in a continuous manner. Therefore, quantisation of electric charge can be ignored, when dealing with a large scale charged body.

32. Initially, the configuration shown is unstable. The negative charges repel each other. If there is any slight rotation of one of the rods, the repulsion can result in further rotation away from this configuration.

There are three possible final configurations as shown below.



Configuration (A) is stable. If the positive upper ends of both the rods are pushed towards each other, then their mutual repulsion will move the system back to the original configuration. Configuration (B) is an equilibrium configuration, but it is unstable. If the lower ends of both the rods are moved towards each other, then their mutual attraction will be larger than that of the upper ends and thus, the configuration will shift to (c), another possible stable configuration.

33. For the protons, the charge on proton is $+e$.

If the number of up quarks are a, then the number of down quarks are $(3 - a)$ as the total number of quarks are 3. So, $a \times$ up quark charge + $(3 - a)$ down quark charge) = $+e$

$$a \times \left(\frac{2}{3}e\right) + (3 - a)\left(-\frac{e}{3}\right) = e$$

$$\Rightarrow \quad \frac{2ae}{3} - \frac{(3-a)e}{3} = e \Rightarrow 2a - 3 + a = 3$$

$$\Rightarrow \quad 3a = 6$$

$$\Rightarrow \quad a = 2$$

Thus, in the proton, there are two up quarks and one down quark.

$\therefore$ Possible quark composition for proton = uud.

For the neutron, the charge on neutron is 0.

Let the number of up quarks be b and the number of down quarks be $3 - b$.

So, $b \times$ up quark charge + $(3 - b)$ down quark charge

$= 0$

$$\Rightarrow \quad b\left(\frac{2e}{3}\right) + (3 - b)\left(-\frac{e}{3}\right) = 0$$

$$\Rightarrow \quad 2b - 3 + b = 0$$

$$\Rightarrow \quad 3b = 3$$

$$\Rightarrow \quad b = 1$$

Thus, in neutron, there is one up quark and two down quarks.

$\therefore$ Possible quark composition for neutron = udd.

34. As charges are additive in nature, i.e. the total charge of a system is the algebraic sum of all the individual charges located at different points inside the system, i.e.

$$q_{net} = q_1 + q_2 + q_3 + q_4 + q_5$$

$\therefore$ Total charge $= +1 + 2 - 3 + 4 - 5 = -1$ in the same unit.

35. The negative charge is due to the presence of excess electrons. Because an electron has a charge whose magnitude is $e = 1.6 \times 10^{-19}$ C, the number of electrons is equal to the charge q divided by the charge e on each electron.

Therefore, the number of electrons is

$$n = \frac{q}{e} = \frac{1.0}{1.6 \times 10^{-19}} = 6.25 \times 10^{18} \text{ electrons}$$

36. Here, $q_1 = -6\,\mu C$

and $q_2 = ne = 5 \times 10^{12} \times (1.6 \times 10^{-19})$

$= 8.0 \times 10^{-7}$ C

$= 0.8 \times 10^{-6}$ C $= 0.8\mu$C

Since, electrons are removed from the sphere, q_2 is positive.

Therefore, net charge on the sphere,

$$q = q_1 + q_2$$
$$= (-6.0 + 0.8)\,\mu C$$
$$= -5.2\mu C$$

37. (i) The charge of an electron $= -1.6 \times 10^{-19}$C

Net charge on sphere $= -2.5 \times 10^{-9}$C

So, the number of excess electrons

$$= \frac{-2.5 \times 10^{-9}\text{ C}}{-1.6 \times 10^{-19}\text{ C}} = 1.5 \times 10^{10} \text{ electrons}$$

(ii) Atomic number of lead is 82.

Atomic mass of lead is 207 g/mol.

$\therefore$ 10 g of lead will have

$$\frac{10\text{ g}}{207\text{ g / mol}} \times 6.02 \times 10^{23} \text{ atoms/mol}$$

$$= 2.91 \times 10^{22} \text{ atoms}$$

$\therefore$ The number of excess electrons per atom

$$= \frac{1.56 \times 10^{10}}{2.91 \times 10^{22}}$$

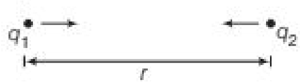
$$= 5.36 \times 10^{-13} \text{ electrons}$$

|TOPIC 2|

Coulomb's Law and Electrostatic Field

COULOMB'S LAW

The force of interaction (attraction or repulsion) between two stationary point charges in vacuum is directly proportional to the product of the charges and inversely proportional to the square of distance between them. Mathematically, electrostatic force between two stationary charges is given by



$$\boxed{F = \frac{k|q_1 q_2|}{r^2}}$$

where, k is a proportionality constant.

In SI unit, k is given by

$$k = \frac{1}{4\pi\varepsilon_0}$$

$$= 9 \times 10^9 \text{ N-m}^2\text{C}^{-2}$$

where, $\varepsilon_0 = 8.85 \times 10^{-12}\text{ C}^2\text{N}^{-1}\text{m}^{-2}$ and is called the **permittivity of free space.**

i.e. $$F = 9 \times 10^9 \frac{|q_1 q_2|}{r^2}$$

The Coulomb force acts along the straight line connecting the points of location of the charges. It is central and spherically symmetric.

If $q_1 = q_2 = 1$C

and $r = 1$ m

Then, $$F = 9 \times 10^9 \frac{1 \times 1}{(1)^2}$$

$$\boxed{F = 9 \times 10^9 \text{ N}}$$

i.e. **One coulomb** is the charge, that when placed at distance of 1m from another charge of same magnitude in vacuum, experiences an electric force of repulsion of magnitude 9×10^9N. Coulomb is a bigger unit, in practice we use smaller units like mC or μC.

Absolute Permittivity of Medium (Dielectric Constant)

The force between two charges q_1 and q_2 located at a distance r apart in a medium may be expressed as,

$$F_{\text{medium}} = \frac{1}{4\pi\varepsilon}\frac{|q_1 q_2|}{r^2}$$

where, ε is **absolute permittivity of the medium**.

Now, $$\frac{F_{\text{vacuum}}}{F_{\text{medium}}} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{|q_1q_2|}{r^2}}{\frac{1}{4\pi\varepsilon}\frac{|q_1q_2|}{r^2}} = \frac{\varepsilon}{\varepsilon_0}$$

The ratio $\frac{\varepsilon}{\varepsilon_0}$ is denoted by ε_r, which is called **relative permittivity** of the medium with respect to vacuum. It is also denoted by K called **dielectric constant** of the medium. It has no unit being a ratio.

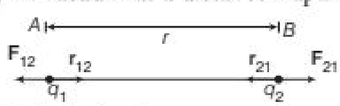
$$\therefore \quad K \text{ (or } \varepsilon_r) = \frac{\varepsilon}{\varepsilon_0} = \frac{F_{\text{vacuum}}}{F_{\text{medium}}}$$

$$\Rightarrow \quad \varepsilon = K\varepsilon_0$$

$$\therefore \quad \boxed{F_{\text{medium}} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1q_2|}{Kr^2}}$$

Coulomb's Law in Vector Form

Consider two like charges q_1 and q_2 present at points A and B respectively in vacuum at a distance r apart.



Coulomb force between two charges

According to Coulomb's law, the magnitude of force on charge q_1 due to q_2 (or on charge q_2 due to q_1) is given by

$$F_{12} = F_{21} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2} \quad \text{...(i)}$$

Let $\mathbf{r}_{12}$ be the unit vector pointing from charge q_1 to q_2.

$$\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2}\hat{\mathbf{r}}_{21} \quad \text{...(ii)}$$

[$\because$ $\mathbf{F}_{12}$ is along the direction of unit vector $\mathbf{r}_{21}$]

Also, $\mathbf{r}_{21}$ be the unit vector pointing from charge q_2 to q_1.

$$\therefore \quad \mathbf{F}_{21} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2}\hat{\mathbf{r}}_{12} \quad \text{...(iii)}$$

[$\because$ $\mathbf{F}_{21}$ is along the direction of unit vector $\mathbf{r}_{12}$]

$$\therefore \quad \hat{\mathbf{r}}_{21} = -\hat{\mathbf{r}}_{12}$$

Eq. (ii) becomes,

$$\mathbf{F}_{12} = \frac{-1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2}\hat{\mathbf{r}}_{12} \quad \text{...(iv)}$$

On comparing Eq. (iii) with Eq. (iv), we get

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

i.e. Coulomb's law agrees with Newton's third law.

Comparison of Coulomb's Law with Gravitational Law

Both the Coulomb's and Newton's law follow inverse square law. According to Newton's universal law of gravitation, "every body in the universe attracts every other body with a force which is directly proportional to the product of the masses of two bodies and inversely proportional to the square of distance between them." i.e.

$$F = \frac{Gm_1m_2}{r^2}$$

As discussed earlier, according to Coulomb's law

$$F = \frac{kq_1q_2}{r^2}$$

The electric force is much stronger than the gravitational force between two electrons.

i.e. $$F_E = 10^{39} F_G$$

EXAMPLE |1| What is the force between two small charged spheres having charges of 2×10^{-7} C and 3×10^{-7} C placed 30 cm apart in air? **NCERT**

Sol. Given, $q_1 = 2\times10^{-7}$ C

$q_2 = 3\times10^{-7}$ C

$r = 30$ cm

$= 30\times10^{-2}$ m

$= 0.3$ m

$k = 9\times10^9$ N - m^{2}C^{-2}

$F = ?$

We have, $$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1q_2|}{r^2}$$

$$= \frac{k|q_1q_2|}{r^2} \quad \left[\because \frac{1}{4\pi\varepsilon_0} = k\right] \quad \text{...(i)}$$

Substituting the given values in Eq. (i), we get

$$F = \frac{(9\times10^9\,\text{N-m}^2\text{C}^{-2})(2\times10^{-7}\,\text{C})\times(3\times10^{-7}\,\text{C})}{(0.3\,\text{m})^2}$$

$$\therefore \quad F = 6\times10^{-3}\,\text{N}$$

This force is repulsive, since the spheres have same charges.

EXAMPLE |2| The sum of two point charges is $7\mu C$. They repel each other with a force of 1 N when kept 30 cm apart in free space. Calculate the value of each charge.

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Sol. Let one of two charges be $x\mu C$. Therefore, other charge will be $(7-x)\mu C$.

By Coulomb's law,

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$$\Rightarrow \quad 1 = 9\times 10^9 \times \frac{(x\times 10^{-6})(7-x)\times 10^{-6}}{(0.3)^2}$$

$$\Rightarrow \quad 9\times 10^{-2} = 9\times 10^{9-12} x(7-x)$$

$$\Rightarrow \quad 10 = x(7-x)$$

$$\Rightarrow \quad x^2 - 7x + 10 = 0$$

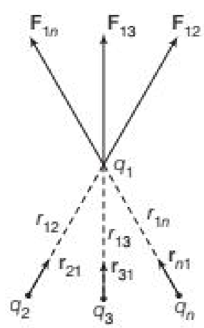
$$\Rightarrow \quad (x-2)(x-5) = 0$$

$$\therefore \quad x = 2\mu C \text{ or } 5\mu C$$

Therefore, charges are $2\mu C$ and $5\mu C$.

FORCES BETWEEN MULTIPLE CHARGES: SUPERPOSITION PRINCIPLE

According to the superposition principle, forces on any charge due to number of other charges is the vector sum of all the forces on that charge due to other charges, taken one at a time. The individual forces are unaffected due to the presence of other charges.



Consider a system of n point charges $q_1, q_2, q_3, \ldots, q_n$ be distributed in space in a discrete manner. The charges are interacting with each other. Let the charges be $q_2, q_3, \ldots, q_n$ exert forces $\mathbf{F}_{12}, \mathbf{F}_{13}, \ldots, \mathbf{F}_{1n}$, respectively on charge q_1.

Then, according to the principle of superposition, the total force on charge q_1 is given by

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \ldots + \mathbf{F}_{1n} \quad \ldots\text{(i)}$$

If the distance between the charges q_1 and q_2 is denoted as r_{12} and $\hat{\mathbf{r}}_{21}$ is unit vector from charge q_2 to q_1, then

$$\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{21}^2} \hat{\mathbf{r}}_{21}$$

Similarly, the force on charge q_1 due to other charges is given by

$$\mathbf{F}_{13} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_3}{r_{31}^2} \hat{\mathbf{r}}_{31}, \quad \mathbf{F}_{1n} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_n}{r_{n1}^2} \hat{\mathbf{r}}_{n1}$$

Substituting these values in Eq.(i), we get

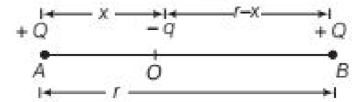
$$\mathbf{F}_1 = \frac{1}{4\pi\varepsilon_0} \left(\frac{q_1 q_2}{r_{21}^2} \hat{\mathbf{r}}_{21} + \frac{q_1 q_3}{r_{31}^2} \hat{\mathbf{r}}_{31} + \cdots + \frac{q_1 q_n}{r_{n1}^2} \hat{\mathbf{r}}_{n1} \right)$$

$$\Rightarrow \quad \mathbf{F}_{1i} = \frac{q_1}{4\pi\varepsilon_0} \sum_{i=2}^{n} \frac{q_i}{r_{i1}^2} \hat{\mathbf{r}}_{i1}$$

Note This force is on the charge which is to be studied due to other charges.

EXAMPLE |3| Two charges each of $+q$ Coulomb are placed along a line. A third charge $-q$ is placed between them. At what position will the system be in equilibrium?

Sol.



For charge $-q$ to be in equilibrium, force on the charge $-q$ at point O due to the charge $+Q$ at point A should be equal and opposite to that due to the charge $+Q$ at the point B, i.e.

$$\frac{1}{4\pi\varepsilon_0} \frac{Qq}{x^2} = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{(r-x)^2}$$

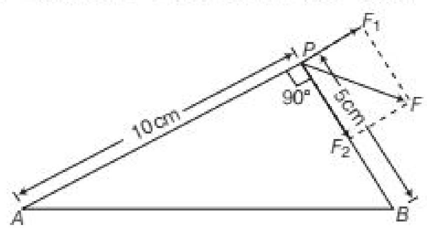
$$\Rightarrow \quad x^2 = (r-x)^2$$

$$\text{or} \quad x = r - x$$

$$\therefore \quad x = \frac{r}{2}$$

Hence, for equilibrium the charge $-q$ should be kept at the middle of the line joining the points A and B.

EXAMPLE |4| Find the magnitude of the resultant force on a charge of $1\mu C$ held at P due to two charges of $+2\times 10^{-8}$ C and -10^{-8} C at A and B, respectively. Given, $AP = 10$ cm, $BP = 5$ cm and $\angle APB = 90°$.



Sol. Here, charge at P, $q = 1\mu C = 10^{-6}$ C

Charge at A, $q_1 = 2\times 10^{-8}$ C

Charge at B, $q_2 = -10^{-8}$ C

$AP = 10$ cm $= 0.1$ m, $BP = 5$ cm $= 0.05$ m,

$\angle APB = 90°$, $F = ?$

Force at P due to q_1 charge at A, $F_1 = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q}{AP^2}$, along

$$AP \text{ produced} = \frac{9\times 10^9 \times 2\times 10^{-8} \times 10^{-6}}{(0.1)^2} = 18\times 10^{-3}\text{ N}$$

Force at P due to q_2 charge at B, $F_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2 q}{BP^2}$, along PB produced $= \frac{9\times10^9\times-10^{-8}\times10^{-6}}{(0.05)^2} = -36\times10^{-3}$ N

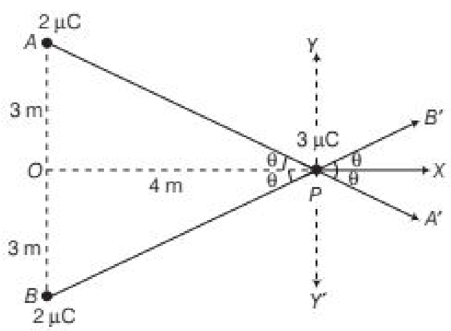
As, angle between F_1 and F_2 is 90°.

∴ Resultant force,

$$F = \sqrt{F_1^2 + F_2^2} = \sqrt{(18\times10^{-3})^2 + (-36\times10^{-3})^2}$$
$$= \sqrt{(324+1296)\times10^{-6}}$$
$$= \sqrt{1620\times10^{-6}}$$
$$= 40.2\times10^{-3}$$
$$= 4.0\times10^{-2}\,\text{N}$$

EXAMPLE |5| Two equal positive charges, each of 2μC interact with a third positive charge of 3μC situated as shown in figure. Calculate the magnitude and direction of the force on the 3μC charge. **NCERT**

Sol. In the figure, $OA = OB = 3$ m, $OP = 4$ m



∴ $AP = BP = \sqrt{3^2 + 4^2} = 5$ m

According to Coulomb's law,

force on charge at P due to charge at A,

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{AP^2}$$
$$= \frac{9\times10^9\times(2\times10^{-6})\times(3\times10^{-6})}{5^2} = \frac{54}{25}\times10^{-3}$$
$$= 2.16\times10^{-3}\ \text{N, along } PA'.$$

It has two rectangular components $F_1\cos\theta$ along PX and $F_1\sin\theta$ along PY'.

Similarly, force on charge at P due to charge at B, $F_2 = F_1$ (in magnitude). It is along PB'. It also has two rectangular component $F_2\cos\theta$ along PX and $F_2\sin\theta$ along PY.

The components along PY and PY' cancel. The components along PX add up.

∴ Total force on 3 μC charge is

$$F = 2F_1\cos\theta$$
$$= 2\times2.16\times10^{-3}\times\frac{4}{5}$$
$$= 3.5\times10^{-3}\ \text{N, along } PX.$$

ELECTROSTATIC FORCE DUE TO CONTINUOUS CHARGE DISTRIBUTION

The region in which charges are closely spaced is said to have continuous distribution of charge. Continuous charge distribution is of three types; linear charge distribution (one dimensional), surface charge distribution (two dimensional) and volume charge distribution (three dimensional).

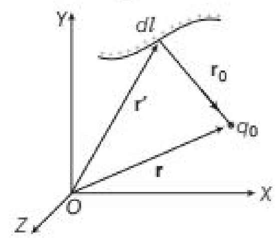
Linear Charge Density

Linear charge density is defined as the charge per unit length of linear charge distribution.

i.e. $\lambda = \frac{dq}{dl}$

Its SI unit is coulomb/metre.

Electric force at a point due to a linear charge distribution is given by $\mathbf{F} = \frac{q_0}{4\pi\varepsilon_0}\int_l \frac{\lambda\, dl}{r_0^2}\hat{\mathbf{r}}_0$



where, $\mathbf{r}_0 = \mathbf{r} - \mathbf{r}'$, $\mathbf{r}'$ is the position vector of length element dl with respect to origin and $\mathbf{r}$ is the position vector of charge q_0 with respect to origin.

Surface Charge Density

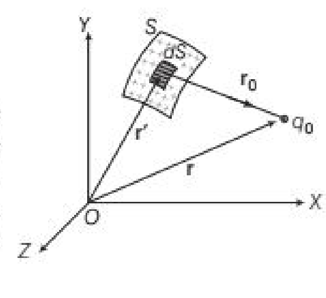
Surface charge density is defined as the charge per unit surface area of surface charge distribution.

i.e. $\sigma = \frac{dq}{dS}$

Its SI unit is coulomb/metre^2.

Electric force at a point due to a surface charge distribution is given by

$$\mathbf{F} = \frac{q_0}{4\pi\varepsilon_0}\int_S \frac{\sigma\, dS}{r_0^2}\hat{\mathbf{r}}_0$$



where, $\mathbf{r}_0 = \mathbf{r} - \mathbf{r}'$, $\mathbf{r}'$ is the position vector of surface element dS with respect to origin and $\mathbf{r}$ is the position vector of charge q_0 with respect to origin.

Volume Charge Density

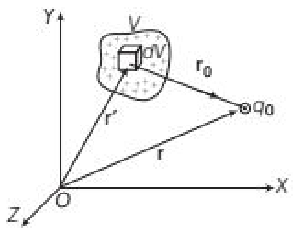
Volume charge density is defined as the charge per unit volume of volume charge distribution.

i.e. $$\delta = \frac{dq}{dV}$$

Its SI unit is coulomb/metre^3.

Electric force at a point due to volume charge distribution is given by

$$\mathbf{F} = \frac{q_0}{4\pi\varepsilon_0}\int_V \frac{\rho\, dV}{r_0^2}\,\hat{\mathbf{r}}_0$$



where, $\mathbf{r}_0 = \mathbf{r} - \mathbf{r}'$, $\mathbf{r}'$ is the position vector of volume element dV with respect to origin and $\mathbf{r}$ is the position vector of charge q_0 with respect to origin.

EXAMPLE |6| What charge would be required to electrify a sphere of radius 25 cm, so as to get a surface charge density of $\frac{3}{\pi}$ Cm^{-2}?

Sol. Here, $r = 25\text{ cm} = 0.25\text{ m}, \sigma = \frac{3}{\pi}\text{ Cm}^{-2}$

As, $$\sigma = \frac{q}{4\pi r^2}$$

$$\therefore \quad q = 4\pi r^2 \sigma = 4\pi \times (0.25)^2 \times \frac{3}{\pi}\text{C} = 0.75\text{C}$$

EXAMPLE |7| The radius of gold nucleus ($Z = 79$) is about 7.0×10^{-15}m. Assuming that the positive charge is distributed uniformly throughout the nuclear volume, find the volume charge density.

Sol. The total positive charge in the nucleus is given by

$$q = +Ze = 79 \times 1.6 \times 10^{-19}\text{C}$$

$$\therefore \text{ Volume charge density, } \delta = \frac{q}{\frac{4}{3}\pi R^3}$$

$$= \frac{79 \times 1.6 \times 10^{-19}}{4/3 \times 3.14 \times (7.0 \times 10^{-15})^3}$$

$$= 0.088 \times 10^{26}$$

$$= 8.8 \times 10^{24}\text{ Cm}^{-3}$$

ELECTRIC FIELD

The electric field due to a charge Q at a point in space may be defined as the force that a unit positive charge would experience if placed at that point.

The charge Q which produces the electric field is called **source charge** and the charge q which experiences the effect of source charge is called **test charge**.

Electric Field Intensity

The electric field intensity at any point due to source charge is defined as the force experienced per unit positive test charge placed at that point without disturbing the source charge.

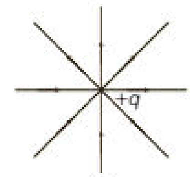
It is expressed as, $\mathbf{E} = \frac{\mathbf{F}}{q_0}$

where, $\mathbf{E}$ = electric field intensity and

and $\mathbf{F}$ = force experienced by the test charge q_0.

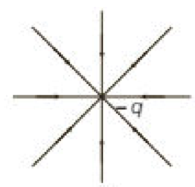
It is a vector quantity and its SI unit is N C^{-1}.

The figure (a) is representing the electric field due to charge $+q$. In this, it can be seen that for a positive charge, the electric field vector is directed radially outwards, i.e. away from **positive charge.**



(a)

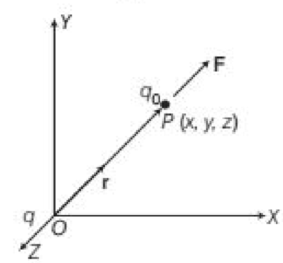
The figure (b) is representing the electric field due to charge $-q$. In this, it can be seen that for a negative charge, the electric field vector is directed radially inwards, i.e. towards **negative charge.**



(b)

Electric Field due to a Point Charge

We have to find the electric field at a point P due to a point charge $+q$ placed at the origin such that $\mathbf{OP} = \mathbf{r}$.



Electric field due to a point charge in coordinate frame

To find the electric field at point P, we have to find the electric force on a test charge q_0 placed at point P, due to source charge q.

According to Coulomb's law, force on the test charge q_0 due to charge q is given by

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\hat{\mathbf{r}}$$

If **E** is the electric field at a point P, then

$$\mathbf{E} = \lim_{q_0 \to 0}\frac{\mathbf{F}}{q_0} = \lim_{q_0 \to 0}\left(\frac{1}{q_0}\cdot\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\hat{\mathbf{r}}\right)$$

$$\Rightarrow \quad \mathbf{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\hat{\mathbf{r}} \quad \ldots(\text{i})$$

The magnitude of the electric field at a point P is given by

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}}$$

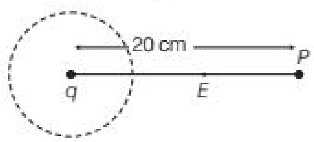
From the above formula, it is clear that electric field at any point in space due to a charge depends only on the distance. That means, the magnitude of electric field due to point charge is same at all the points of sphere, i.e. it has spherical symmetry.

EXAMPLE |8| A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of the sphere is 1.5×10^3 N/C and points radially inwards, then what is the net charge on the sphere?

NCERT

Sol. Let the value of unknown charge be q.

Electric field at 20 cm away, $E = 1.5 \times 10^3$ N/C



From the formula, electric field,

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

$$\Rightarrow 1.5 \times 10^3 = \frac{9 \times 10^9 \times q}{(20 \times 10^{-2})^2}$$

$$\therefore \quad q = \frac{1.5 \times 10^3 \times 20 \times 20 \times 10^{-4}}{9 \times 10^9} = 6.67 \times 10^{-9}\text{C}$$

As the electric field is radially inwards which shows that the nature of unknown charge q is negative.

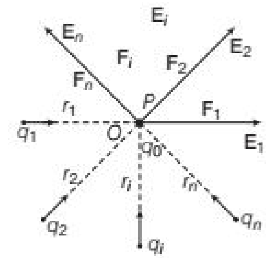
Electric Field due to System of Charges

Consider that n point charges $q_1, q_2, q_3, \ldots, q_n$ exert forces $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3, \ldots, \mathbf{F}_n$ on a test charge q_0 placed at origin O.

Let $\mathbf{F}_i$ be the force due to ith charge q_i on q_0, then

$$\mathbf{F}_i = \frac{1}{4\pi\varepsilon_0}\frac{q_i q_0}{r_i^2}\hat{\mathbf{r}}_i$$

where, r_i is the distance of the test charge q_0 from q_i.



The electric field at the observation point P is given by

$$\mathbf{E}_i = \lim_{q_0 \to 0}\frac{\mathbf{F}_i}{q_0} = \lim_{q_0 \to 0}\left[\frac{1}{q_0}\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_i q_0}{r_i^2}\hat{\mathbf{r}}_i\right)\right]$$

$$\mathbf{E}_i = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_i}{r_i^2}\hat{\mathbf{r}}_i \quad \ldots(\text{i})$$

If **E** is electric field at point P due to the system of charges, then by principle of superposition of electric fields,

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \cdots + \mathbf{E}_n = \sum_{i=1}^{n}\mathbf{E}_i$$

Using Eq. (i), we get

$$\mathbf{E} = \sum_{i=1}^{n}\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_i}{r_i^2}\hat{\mathbf{r}}_i$$

or

$$\boxed{\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{\mathbf{r}}_i}$$

$\therefore$ **E** is a vector quantity.

EXAMPLE |9| Two charges $+Q$ and $-Q$ are kept at points $(-x_2, 0)$ and $(x_1, 0)$ respectively, in the XY-plane. Find the magnitude and direction of the net electric field at the origin $(0, 0)$.

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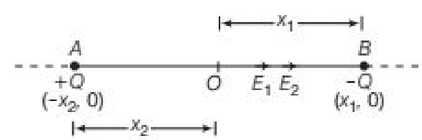
> **Hints:** To find the electric field intensity at a point due to two charges, first of all find the individual electric field due to both charges and then find the resultant field by using vector addition.

Sol.



Electric field intensity at point O due to $+Q$ charge,

$$E_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{(x_2)^2} \text{ (towards } B) \quad \ldots(\text{i})$$

Electric field intensity at point O due to $-Q$ charge,

$$E_2 = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{(x_1)^2} \text{ (towards } B) \quad \ldots(\text{ii})$$

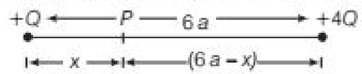
$\because E_1$ and E_2 act along the same direction.

$\therefore$ Net electric field intensity at point O is given by

$$E = E_1 + E_2$$
$$= \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{(x_2)^2} + \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{(x_1)^2} \text{ (towards} B)$$
$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{x_2^2} + \frac{1}{x_1^2}\right]$$

EXAMPLE |10| Two point charges $+Q$ and $+4Q$ are separated by a distance of $6a$. Find the point on the line joining the two charges, where the electric field is zero.

Sol. The electric field is zero at point P only, if the field due to charge $+Q$ balances the field due to charge $+4Q$.



$$\therefore \quad \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{x^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4Q}{(6a - x)^2} \Rightarrow \frac{1}{x^2} = \frac{4}{(6a - x)^2}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{2}{(6a - x)}$$

$$\Rightarrow \quad 2x = 6a - x$$

$$\Rightarrow \quad x = 2a$$

$\therefore$ The required point is at a distance of $2a$ from $+Q$.

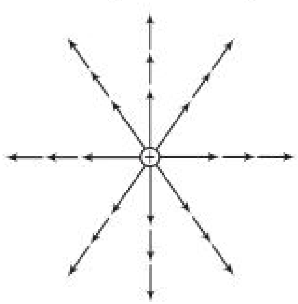
Physical Significance of Electric Field

The physical significance of electric field is that we can readily calculate the magnitude and direction of force experienced by any charge q_0 placed at a point by knowing the electric field intensity at that point.

ELECTRIC FIELD LINES

An electric field line in general is a curve drawn in such a way that the tangent to it at each point is in the direction of the electric field at that point. A field line is a space curve, i.e. a curve in three dimensions.

Electric field lines are thus used to pictorially map the electric field around a charge or a configuration of charges.



Field lines showing electric field of a point charge

The density of field lines is more near the charge. Away from the charge, the field is weak, so the density of field lines is less.

Properties of Electric Field Lines

Electric field lines follow some important properties which are discussed below

(i) Electric field lines start from positive charges and end at negative charges. In the case of a single charge, they may start or end at infinity.

(ii) Tangent to any point on electric field lines shows the direction of electric field at that point.

(iii) Two field lines can never intersect each other because if they intersect, then two tangents drawn at that point will represent two directions of field at that point, which is not possible.

(iv) In a charge free region, electric field lines can be taken to be continuous curves without any breaks.

(v) Electric field lines do not form closed loops (because of conservative nature of electric field).

(vi) Electric field lines are perpendicular to the surface of a charged conductor.

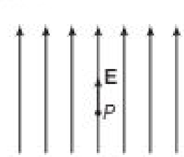
(vii) Electric field lines contract lengthwise to represent attraction between two unlike charges.

(viii) Electric field lines exert sideways pressure to represent repulsion between two like charges.

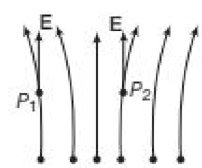
Note Electric field lines and its properties have been generally asked in the form of questions in previous years. All India 2014, 2011, Delhi 2012.

Representations of Electric Field

For different types of electric field, lines are represented as shown below

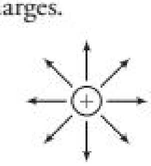


Electric field lines for a uniform field

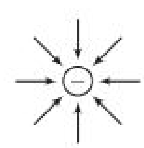


Electric field lines for a non-uniform field

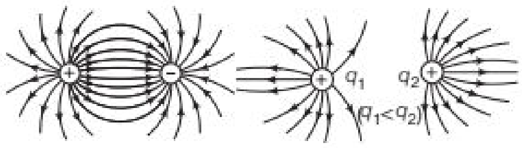
The electric field lines start from positive charges and end at negative charges.



Direction is away from the positive charge



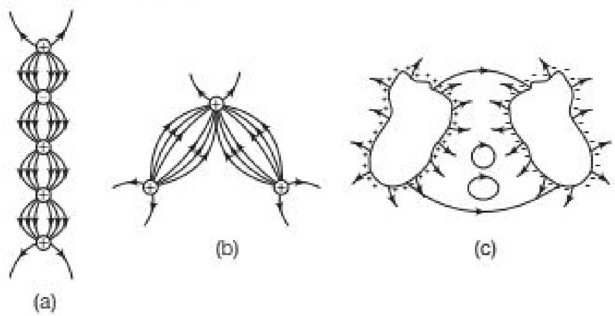
Direction is towards the negative charge



(a) (b)

It is a common misconception that the path traced by a positive charge is a field line. The path traced by a unit positive test charge represents a field line only when it moves along a straight line.

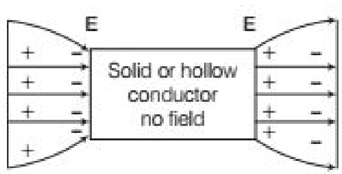
EXAMPLE |11| Explain, why the following curves cannot possibly represent electrostatic field lines? **NCERT**



Sol. (a) Electrostatic field lines cannot start from a negative charge.
(b) Electrostatic field lines cannot end at positive charge.
(c) Electrostatic field lines cannot form closed loops.

Conductors in an Electrostatic Field

(i) Electric field lines do not pass through a conductor. Hence, the interior of the conductor is free from the influence of the electric field.



Electrostatic shielding

(ii) Total charge of a charged conductor lies at the outer surface of the conductor.

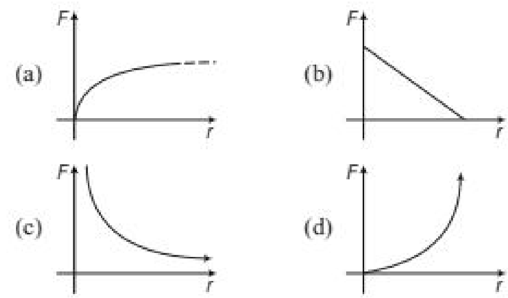
(iii) The magnitude of field strength at any point on the surface of the conductor is proportional to surface charge density at that point.

TOPIC PRACTICE 2

OBJECTIVE Type Questions

1. SI unit of electrical permittivity is
(a) $N\text{-}m^2C^{-2}$ (b) Am^{-2} (c) NC^{-1} (d) $C^2N^{-1}m^{-2}$

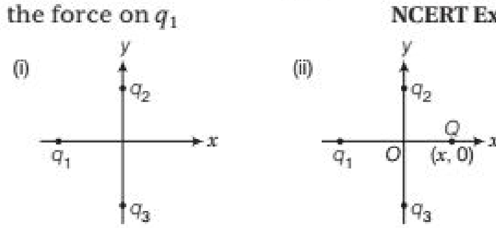
2. Force between two charges varies with distance between them as



3. Two charges $+1\mu C$ and $+4\mu C$ are situated at a distance in air. The ratio of the forces acting on them is
(a) 1 : 4 (b) 4 : 1 (c) 1 : 1 (d) 1 : 16

4. A charge q is placed at the centre of the line joining two equal charges Q and Q. The system of the three charges will be in equilibrium, if q is equal to
(a) $-Q/2$ (b) $-Q/4$ (c) $+Q/4$ (d) $+Q/2$

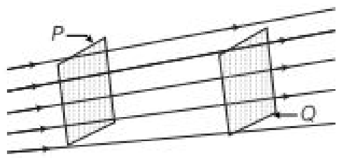
5. In figure two positive charges q_2 and q_3 fixed along the y-axis, exert a net electric force in the $+x$-direction on a charge q_1 fixed along the x-axis. If a positive charge Q is added at $(x, 0)$, the force on q_1 **NCERT Exemplar**



(a) shall increase along the positive x-axis
(b) shall decrease along the positive x-axis
(c) shall point along the negative x-axis
(d) shall increase but the direction changes because of the intersection of Q with q_2 and q_3

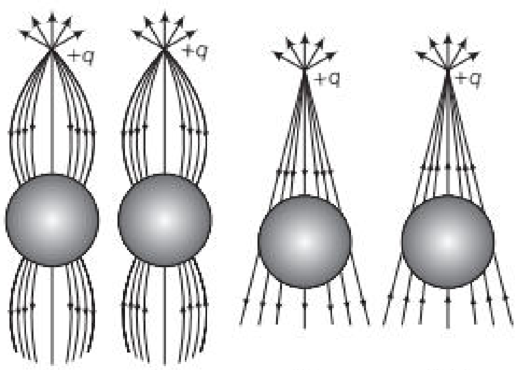
6. A force of 2.25 N acts on a chrage of 15×10^{-4} C. The intensity of electric field at that point is
(a) 150 NC^{-1} (b) 15 NC^{-1}
(c) 1500 NC^{-1} (d) 1.5 NC^{-1}

7. In the diagram shown below,



(a) field strength at P is less than field strength at Q
(b) field strength at P and Q are equal
(c) field is more strong at P and less strong at Q
(d) cannot be tell from the figure

8. A point positive charge is brought near an isolated conducting sphere (figure). The electric field is best given by **NCERT Exemplar**



(a) (b) (c) (d)

9. A hemisphere is uniformly charged. The electric field at a point on a diameter away from the centre is directed **NCERT Exemplar**
(a) perpendicular to the diameter
(b) parallel to the diameter
(c) at an angle tilted towards the diameter
(d) at an angle tilted away from the diameter

10. A point charge $+q$ is placed at a distance d from an isolated conducting plane. The field at a point P on the other side of the plane is **NCERT Exemplar**
(a) directed perpendicular to the plane and away from the plane
(b) directed perpendicular to the plane but towards the plane
(c) directed radially away from the point charge
(d) directed radially towards the point charge

VERY SHORT ANSWER Type Questions

11. Does the Coulomb force that one charge exerts on another charge changes, if other charge is brought nearby?

12. In Coulomb's law, $F = \frac{k_e q_1 q_2}{r^2}$, what are the factors on which the proportionality constant k_e depends?

13. If the distance between two equal point charges is doubled and their individual charges are also doubled, then what would happen to the force between them?

14. A metallic spherical shell has an inner radius R_1 and outer radius R_2. A charge Q is placed at the centre of the spherical cavity. What will be surface charge density on (i) the inner surface and (ii) the outer surface? **NCERT Exemplar**

15. The test charge used to measure electric field at a point should be vanishingly small. Why?

16. A point charge q is placed at the origin. How does the electric field due to the charge vary with the distance r from the origin?

17. Force experienced by an electron in an electric field is F newton. What will be the force experienced by a proton in the same field? Take, mass of a proton is 1836 times the mass of an electron.

18. Two point charges of $+ 3 \mu C$ each are 100 cm apart. At what point on the line joining the charges will the electric field intensity be zero?

19. A proton is placed in a uniform electric field directed along a positive X-axis. In which direction will it tend to move?

20. Why electrostatic field be normal to the surface at every point of a charged conductor?

21. An electrostatic field line is continuous curve, i.e. a field line cannot have sudden breaks. Why not?

22. Why should electrostatic field be zero inside a conductor?

23. Why do the electric field lines not form closed loops? **Delhi 2015**

24. The dimensions of an atom are of the order of an angstrom. Thus, there must be large electric fields between the protons and electrons. Why, then is the electrostatic field inside a conductor zero? **NCERT Exemplar**

SHORT ANSWER Type Questions

25. In the given statement, point out the correct or incorrect word or phrase with a proper explanation.

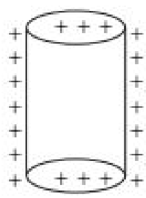
"The mutual forces between two charges do not get affected by the presence of other charges."

26. Plot a graph showing the variation of Coulomb's force (F) *versus* $1/r^2$, where r is the distance between the two charges of each pair of charges ($1\,\mu$C, $2\,\mu$C) and ($1\,\mu$C, $-3\,\mu$C). Interpret the graphs obtained.

27. A charge q is placed at the centre of the line joining two equal charges (Q). Show that the system of three charges will be in equilibrium, if $q = -\frac{Q}{4}$.

28. An uncharged metallic ball is suspended in the region between two vertical metal plates. If the two plates are charged, one positively and one negatively, then describe the motion of the ball after it is brought into contact with one of the plates.

29. Sketch the electric field lines for a uniformly charged hollow cylinder as shown in the figure. **NCERT Exemplar**



LONG ANSWER Type I Questions

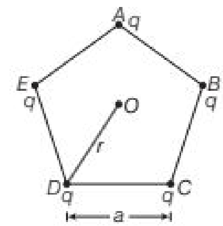
30. Check that the ratio ke^2/Gm_em_p is dimensionless.

Look up a table of physical constants and determine the value of this ratio. What does the ratio signify? **NCERT**

31. Consider three charges Q_1, Q_2, Q_3 each equal to Q at the vertices of an equilateral triangle of side a. What is the force on a charge q (with the same sign as q) placed at the centroid of the triangle? **NCERT**

32. An oil drop of 12 excess electrons is held stationary under a constant electric field of 2.55×10^4 N/C in Millikan's oil drop experiment. The density of the oil is $1.26\,\text{g/cm}^3$. Estimate the radius of the drop.
(Take, $g = 9.81\,\text{m/s}^2$, $e = 1.6 \times 10^{-19}$C). **NCERT**

33. Five charges, q each are placed at the corners of regular pentagon of side a as shown in the figure.



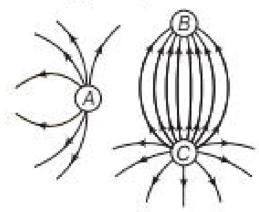
(i) (a) What will be the electric field at O, the centre of the pentagon?

(b) What will be the electric field at O, if the charge from one of the corners (say A) is removed?

(c) What will be the electric field at O, if the charge q at A is replaced by $-q$?

(ii) How would your answer be affected, if pentagon is replaced by n-sided regular polygon with charge q at each of its corners? **NCERT Exemplar**

34. Figure shows the electric field lines around three point charges A, B and C.



(i) Which charges are positive?

(ii) Which charge has the largest magnitude? Why?

(iii) In which region or regions of the picture could the electric field be zero? Justify your answer. **NCERT Exemplar**

(a) Near A (b) Near B

(c) Near C (d) Nowhere

LONG ANSWER Type II Questions

35. Four point charges $q_A = 2\,\mu$C, $q_B = -5\,\mu$C, $q_C = 2\,\mu$C and $q_D = -5\,\mu$C are located at the corners of a square $ABCD$ of side 10 cm. What is the force on a charge of $1\,\mu$C placed at the centre of the square? **NCERT**

36. A free pith-ball of 8 g carries a positive charge of 5×10^{-8}C. What must be the nature and magnitude of charge that should be given to a second pith-ball fixed 5 cm vertically below the former pith-ball, so that the upper pith-ball is stationary? **All India 2011**

NUMERICAL PROBLEMS

37. The dielectric constant of water is 80. What is its permittivity?

38. Two equal balls having equal positive charge q coulombs are suspended by two insulating strings of equal length. What would be the effect on the force when a plastic sheet is inserted between the two? **All India 2014**

39. Two point charges having equal charges separated by 1 m distance experience a force of 8 N. What will be the force experienced by them, if they are held in water, at the same distance? (Given, $K_{water} = 80$) **All India 2011**

40. A charge $q = 1\mu$C is placed at point (1 m, 2 m, 4 m). Find the electric field at point P (0 m, −4 m, 3 m).

41. An infinite number of charges each equal to q are placed along X-axis at $x = 1, x = 2, x = 4, x = 8$ and so on. Find the electric field at the point $x = 0$ due to this set up of charges.

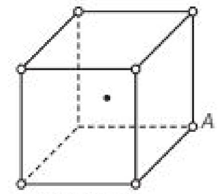
42. The opposite corners of a square carry Q charge each and the other two opposite corners of the same square carry q charge each. If the resultant force on q is zero, how are Q and q related?

43. (i) Two insulated charged copper spheres A and B have their centres separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion, if the charge on each is 6.5×10^{-7} C and the radii of A and B are negligible compared to the distance of separation?

(ii) What is the force of repulsion, if each sphere is charged double the above amount and the distance between them is halved? **NCERT**

44. Suppose the spheres A and B in Q. 43 have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second and finally removed from both. What is the new force of repulsion between A and B? **NCERT**

45. Figure represents a crystal unit of caesium chloride CsCl. The caesium atoms, represented by open circles are situated at the corners of a cube of side 0.40 nm, whereas a Cl atom is situated at the centre of the cube. The Cs atoms are deficient in one electron while the Cl atom carries an excess electron.



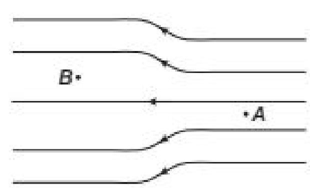
(i) What is the net electric field on the Cl atom due to eight Cs atoms?

(ii) Suppose that the Cs atom at the corner A is missing. What is the net force now on the Cl atom due to seven remaining Cs atoms? **NCERT Exemplar**

46. In the figure below, the electric field lines on the left have twice the separation of those on the right.

(i) If the magnitude of the field of A is 40 N/C, then what force acts on a proton at A?

(ii) What is the magnitude of the field at B?



HINTS AND SOLUTIONS

1. (d) From Coulomb's law, $F = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q_1 q_2}{r^2}$

$\therefore$ Electrical permittivity, $\varepsilon_0 = \dfrac{q_1 q_2}{4\pi \times F \times r^2} = \dfrac{C\times C}{N\times m^2}$

$\therefore$ Unit of electrical permittivity $= C^2N^{-1}m^{-2}$

2. (c) According to Coulomb's law, force between two point charges, *i.e.*, $F \propto \dfrac{1}{r^2}$. Therefore, the graph between F and r will be as shown in Fig.(c).

3. (c) According to the Coulomb's law,

Force, $F = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q_1 q_2}{r^2}$

where, $q_1 = +1\mu C = 1\times10^{-6}$ C

$q_2 = +4\mu C = 4\times10^{-6}$ C

r = distance between the charges.

Force on first charge due to second charge,

$$F_{12} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{10^{-6}\times4\times10^{-6}}{r_{12}^2}$$

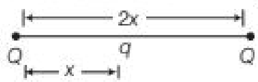
Force on second charge due to first charge,

$$F_{21} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{10^{-6}\times4\times10^{-6}}{r_{21}^2}$$

$\because \quad r_{12} = r_{21} \therefore F_{12} = F_{21}$

$\therefore \quad F_{12} : F_{21} = 1:1$

4. (b) Consider the situation shown in figure.

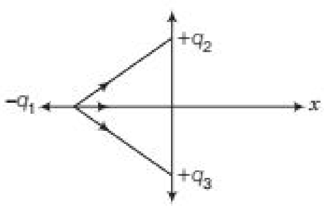


For the system to be in equilibrium,

$$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{x^2} = \frac{-1}{4\pi\varepsilon_0}\frac{Q^2}{(2x)^2}$$

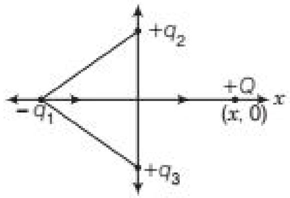
$\Rightarrow \quad q = -Q/4$

5. (a) The net force on q_1 by q_2 and q_3 is along the $+x$-direction, so nature of force between q_1, q_2 and q_1, q_3 is attractive. This can be represent by the figure given below



The attractive force between these charges states that q_1 is a negative charge (since, q_2 and q_3 are positive).

Thus, nature of force between q_1 and newly introduced charge Q (positive) is attractive and net force on q_1 by q_2, q_3 and Q are along the same direction as given in the diagram below



The figure given above clearly shows that the force on q_1 shall increase along the positive x-axis due to the positive charge Q.

6. (c) Electric field, $E = \frac{F}{q} = \frac{2.25 \text{ N}}{15\times10^{-4} \text{ C}}$

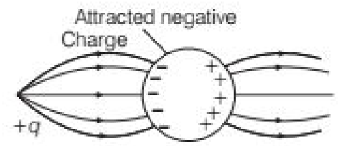
$= 1500 \text{ NC}^{-1}$

7. (c) Areas of P-Q are equal but more lines pass through area at P. So, field is stronger at P as compared to Q.

8. (a) The free electrons in the sphere are attracted towards the positive charge. This leaves an excess of positive charge on the rear (right) surface of sphere.

Electric field lines enter or leave perpendicular to the surface of charged conductor.

Thus, the left surface of sphere has an excess of negative charge and the right surface of sphere has an excess of positive charge as given in the figure below



An electric field lines start from positive charge and ends at negative charge (in this case from point positive charge to negative charge created inside the sphere).

Here, all these conditions are fulfilled in Fig. (a).

9. (a) When the point is situated at a point on diameter away from the centre of hemisphere charged uniformly, the electric field is perpendicular to the diameter. The component of electric intensity parallel to the diameter cancel out.

10. (a) When a point positive charge brought near an isolated conducting plane, some negative charge developes on the surface of the plane towards the charge and an equal positive charge developes on opposite side of the plane, so field lines are directed perpendicular and away from the plane.

11. Yes, it changes as the distance becomes less.

12. Here, k_e is also called dielectric constant, whose value depends essentially on the type of substance and on the external conditions like temperature, pressure and so on.

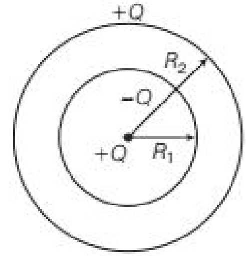
13. We know that by Coulomb's law, $F = k\frac{q_1q_2}{r^2}$

According to question, $q_1' = 2q_1, q_2' = 2q_2, r' = 2r$

$$\therefore \quad F' = \frac{k(2q_1)(2q_2)}{(2r)^2} = \frac{kq_1q_2}{r^2} = F$$

Hence, force between these charges remains same.

14. When a charge $+Q$ is placed at the centre of spherical cavity as shown in the figure, then charge induced on the inner surface of a shell is $-Q$ and charge induced on the outer surface of a shell is $+Q$.



Therefore, surface charge density on inner surface of shell is $\frac{-Q}{4\pi R_1^2}$ and on outer surface of shell is $\frac{+Q}{4\pi R_2^2}$.

15. In case, test charge is not vanishingly small, it will produce its own electric field and the measured value of electric field will be different from the actual value of an electric field at that point.

16. The electric field varies inversely as the square of the distance from the point charge.

17. The proton will experience the same force F newton, but in the opposite direction.

18. At the centre, since the electric field due to two charges is equal and opposite at this point.

19. Proton will tend to move along the positive X-axis in the direction of a uniform electric field.

20. For the condition of electrostatics, the electric field lines must be normal to the surface of the conductor, otherwise there would be a non-zero component of electric field along the surface of conductor and charges could not be at rest.

21. An electrostatic field line cannot be a discontinuous curve, i.e. it cannot have breaks.If it has breaks, then it will indicate absence of electric field at the break points. But the electric field vanishes only at infinity.

22. Electric field lines do not pass through a conductor. Hence, the interior of the conductor is free from the influence of the electric field.

23. Electric field lines do not form closed loops because they are always directed from positive charge to negative charge.

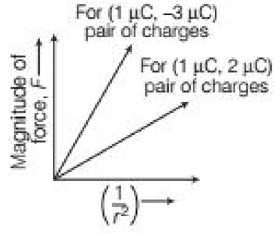
24. The electric fields find the atoms to neutral entity. As it is known that, electrostatic fields are caused by excess charges. However, there is no excess charge on the inner surface of an isolated conductor. Therefore, electrostatic field inside a conductor is zero.

25. Correct, because mutual force acting between two point charges is proportional to the product of magnitude of charges and inversely proportional to the square of the distance between them, i.e. independent of the other charges.

26. According to Coulomb's law, the magnitude of force acting between two stationary point charges is given by

$$F = \left(\frac{q_1 q_2}{4\pi\varepsilon_0}\right)\left(\frac{1}{r^2}\right)$$

For given $q_1 q_2$, $F \propto \left(\frac{1}{r^2}\right)$

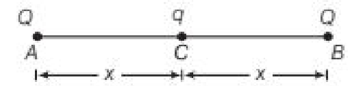


The slope of F - $\frac{1}{r^2}$, graph depends on q_1 and q_2.

Magnitude of $q_1 q_2$ is higher for second pair.

$\therefore$ Slope of F - $\frac{1}{r^2}$ graph, corresponding to second pair (1 μC, −3 μC) is greater. Higher the magnitude of product of charges q_1 and q_2, higher will be the slope.

27. Suppose the three charges be placed as shown in the figure.



As the net force on q is zero, so it is already in equilibrium. For equilibrium of other two charges, the net force on each charge must be zero.

Total force on charge Q at B is

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{x^2} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q\cdot Q}{(2x)^2} = 0$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{x^2} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{4x^2}$$

$$\therefore \quad q = -\frac{Q}{4}$$

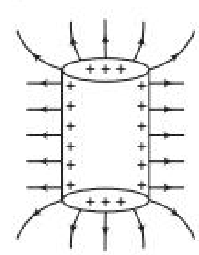
28. The two charged plates create a region with a uniform electric field between them, directed from the positive towards the negative plate.

Once the ball is disturbed so as to touch one plate (say, the negative one), some negative charge will be transferred to the ball and an electric force will act on the ball, that will accelerate it to the positive plate.

Once the ball touches the positive plate, it will release its negative charge, acquire a positive charge and accelerate back to the negative plate. The metallic ball will continue to move back and forth between the plates until it has transferred all their net charges, thereby making both the plates neutral.

29. Here, the hollow cylinder is positively charged.

We know that, the electric field lines appear to come out from the conductor. Thus, the field lines for a uniformly positive charged hollow cylinder is shown in the figure.



30. In the ratio $\frac{ke^2}{Gm_e m_p}$, $k = 4\pi\varepsilon_0$ (constant)

where, G = gravitational constant,

m_e = mass of an electron and m = mass of a proton.

From Coulomb's law, $F = k\frac{q_1 q_2}{r^2}$

$$\Rightarrow \quad k = \frac{Fr^2}{q_1 q_2}$$

$$\text{or} \quad k = \frac{Fr^2}{q^2}$$

The dimension of $k = \left(\frac{1}{4\pi\varepsilon_0}\right) = \frac{[MLT^{-2}][L^2]}{[AT]\,[AT]}$

$= [ML^3T^{-4}A^{-2}]$

The dimension of e (electronic charge) = [AT]

The dimension of G (universal gravitational constant)

$= \frac{[MLT^{-2}]\,[L^2]}{[M^2]} = [M^{-1}L^3T^{-2}]$

The dimension of m_e or m_p (mass of electron or mass of proton) = [M]

The dimension of $\frac{ke^2}{Gm_e m_p} = \frac{[ML^3T^{-4}A^{-2}]\,[A^2T^2]}{[M^{-1}L^3T^{-2}]\,[M^2]}$

$= [M^0L^0T^0A^0]$

Thus, the given ratio is dimensionless.

The value of $k = \left(\frac{1}{4\pi\varepsilon_0}\right) = 9\times 10^9$ N-m^2/C^2

The value of e (charge of an electron)

$= 1.6\times 10^{-19}$ C

The value of G (universal gravitational constant)

$= 6.67\times 10^{-11}$ N-m^2/kg^2

The value of m_e (mass of electron) $= 9.1\times 10^{-31}$ kg

The value of m_p (mass of proton) $= 1.67\times 10^{-27}$ kg

The value of $\frac{ke^2}{Gm_e m_p}$

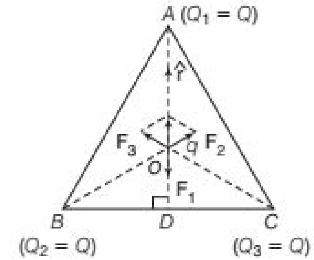
$$= \frac{9\times 10^9\times (1.6\times 10^{-19})^2}{6.67\times 10^{-11}\times 9.1\times 10^{-31}\times 1.67\times 10^{-27}}$$

$= 2.29\times 10^{39}$

The ratio signifies that the ratio of electrostatic force to the gravitational force is 2.29×10^{39}. This means the electrostatic force between an electron and a proton is 2.29×10^{39} times the gravitational force between an electron and a proton.

31. As shown in the figure, draw $AD \perp BC$.

$\therefore \quad AD = AB\cos 30° = \frac{a\sqrt{3}}{2}$ $\quad \left[\because \cos 30° = \frac{\sqrt{3}}{2}\right]$



Distance AO of the centroid O from A

$$= \frac{2}{3}AD = \frac{2}{3}\cdot\frac{a\sqrt{3}}{2} = \frac{a}{\sqrt{3}}$$

$\therefore$ Force F_1 on q at O due to charge $(Q_1 = Q)$ at A,

$$F_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{\left(\frac{a}{\sqrt{3}}\right)^2} = \frac{3Qq}{4\pi\varepsilon_0 a^2}, \text{ along } AO$$

Similarly, force F_2 on q due to charge $(Q_2 = Q)$ at B,

$$F_2 = \frac{3Qq}{4\pi\varepsilon_0 a^2}, \text{ along } BO$$

Force F_3 on q due to charge $(Q_3 = Q)$ at C,

$$F_3 = \frac{3Qq}{4\pi\varepsilon_0 a^2}, \text{ along } CO$$

The resultant of forces F_2 and F_3 is $\left(\frac{3}{4\pi\varepsilon_0}Q\frac{q}{a^2}\right)$, along OA by the parallelogram law.

Therefore, the total force on

$$q = \frac{3}{4\pi\varepsilon_0}Q\frac{q}{a^2}(\hat{\mathbf{r}} - \hat{\mathbf{r}}) = 0$$

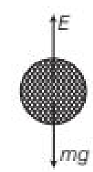
where, $\hat{\mathbf{r}}$ is the unit vector along OA. It is also clear by symmetry that the sum of three forces will zero.

32.

> **Hints:** Here, oil drop is held stationary under electric field that means the weight of the drop is balanced by the electrostatic force applied on it.

Given, the number of excess electrons, $n = 12$

Electric field, $E = 2.55\times 10^4$ N/C



Density of oil, $\rho = 1.26$ g/cm^3

$= 1.26\times 10^3$ kg/m^3

Electronic charge, $e = 1.6\times 10^{-19}$ C

$\Rightarrow \quad g = 9.81$ m/s^2

Let the radius of drop be r.

The electrostatic force on drop $= qE = neE$ $\quad [\because q = ne]$

The gravitational force on the drop $= mg$

[where, m = mass of the drop]

$= \text{Volume}\times\text{Density}\times g$ $\quad [\because \text{mass} = \text{volume}\times\text{density}]$

$= \frac{4}{3}\pi r^3\times\rho\times g$

As the drop is held stationary. So, the net force on the drop is zero.

$\therefore$ Electrostatic force = Gravitational force

$$\Rightarrow \quad neE = \frac{4}{3}\pi r^3 \rho g$$

$$\Rightarrow \quad r^3 = \frac{3neE}{4\pi\rho g}$$

$$= \frac{3 \times 12 \times 1.6 \times 10^{-19} \times 2.55 \times 10^4}{4 \times 3.14 \times 1.26 \times 10^3 \times 9.81}$$

$$\Rightarrow \quad r^3 = 0.94 \times 10^{-18}$$

$$\Rightarrow \quad r = (0.94 \times 10^{-18})^{1/3} = 9.81 \times 10^{-7}$$

Thus, the radius of the drop is 9.81×10^{-7} m.

33. (i) (a) The point O is equidistant from all the charges at the end points of pentagon. Thus, due to symmetry, the forces due to all the charges are cancelled out. As a result, electric field at O is zero.

(b) When charge q is removed from A, electric field at O would become

$$E = \frac{q \times 1}{4\pi\varepsilon_0 r^2} \text{(along } OA\text{)}$$

(c) If charge q at A is replaced by $-q$, then it is equivalent to adding charge $-2q$. Thus, the electric field at O would become

$$E = \frac{2q}{4\pi\varepsilon_0 r^2} \text{(along } OA\text{)}$$

(ii) When pentagon is replaced by n-sided regular polygon with charge q at each of its corners, the electric field at O would continue to be zero as symmetricity of the charges is due to the regularity of the polygon. It does not depend on the number of sides or the number of charges.

34. Electric field lines always start from a positive charge and end at a negative charge. In case of a single charge, electric lines of force start from positive charge and end at infinity.

The magnitude of a charge depends on the number of lines of force emanating from a charge, i.e. higher the number of lines of force, higher the magnitude of charge and *vice-versa*.

(i) In the given figure, the electric lines of force emanate from A and C. Therefore, charges A and C must be positive.

(ii) The number of electric lines of force emanating is maximum from charge C here, so C must have the largest magnitude.

(iii) Point between two like charges, where electrostatic force is zero, is called neutral point. So, the neutral point lies between A and C only.

Now, the position of neutral point depends on the strength of the forces of charges. Here, more number of electric lines of force show higher strength of charge C than A. So, neutral point lies near A.

35. **Hints:** Charge placed at the centre is in the influence field of four charges located at the corners of the square. Therefore, we can find force acting on charge placed at the centre using superposition principle. Use the law of vectors to find the net resultant force because force is a vector quantity.

Let the centre of the square be at O. The charge placed on the centre is $1\,\mu$C.

$$AB = BC = CD = DA = 10 \text{ cm}$$

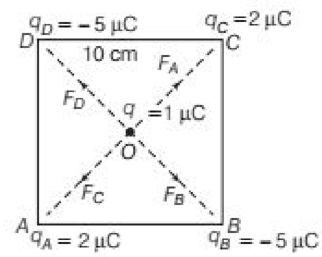
$$AC = \sqrt{2} \times 10 = 10\sqrt{2} \text{ cm}$$

$$AC = BD = 10\sqrt{2} \text{ cm}$$

$$AO = BO = CO = DO = \frac{10\sqrt{2}}{2} = 5\sqrt{2} \text{ cm}$$

Let the force on charge 1μC due to q_A be F_A which is directed away from both charges q_A and q (because both charges are positive in nature, so they will repel each other).

The force on charge 1μC due to q_B is F_B which is towards q_B (because q_B is negatively charged and q is positively charged, so they will attract each other).



The force on charge 1μC due to q_C is F_C which is directed away from both q_C and q (as they both are positive in nature, so will repel each other).

The force on charge 1μC due to q_D is F_D which is towards q_D (because q_D is negatively charged and q is positively charged, so they will attract each other).

Force between q and q_A

$$F_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|qq_A|}{(OA)^2} = \frac{9 \times 10^9 \times 1 \times 10^{-6} \times 2 \times 10^{-6}}{(5\sqrt{2} \times 10^{-2})^2}$$

$$= \frac{9 \times 2 \times 10^{-3}}{25 \times 2 \times 10^{-4}} = \frac{90}{25} = \frac{18}{5}$$

$$= 3.6 \text{ N} \qquad \text{[direction towards } O \text{ to } C\text{]}$$

Force between q and q_B

$$F_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|qq_B|}{(OB)^2}$$

$$= 9.0 \text{ N} \qquad \text{[direction towards } O \text{ to } B\text{]}$$

Force between q and q_C

$$F_C = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|qq_C|}{(OC)^2} = \frac{9 \times 10^9 \times 1 \times 10^{-6} \times 2 \times 10^{-6}}{(5\sqrt{2} \times 10^{-2})^2}$$

$= \frac{90}{25} = \frac{18}{5} = 3.6\,N$ [direction towards O to A]

Here, we observe that F_A and F_C are of same magnitude and opposite in direction. So, the resultant force of F_A and F_C is zero.

Force between q and q_D

$$F_D = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|qq_D|}{(OD)^2} = \frac{9\times 10^9 \times 1\times 10^{-6} \times 5\times 10^{-6}}{(5\sqrt{2}\times 10^{-2})^2}$$

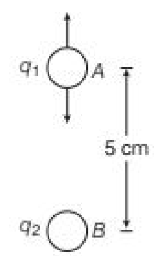
$= 2.25$ N [direction towards O to D]

Here, we observe that F_B and F_D are of same magnitude and opposite in direction. So, the resultant force of F_D and F_B is zero.

Thus, the net resultant force on $1\,\mu C$ (placed at O) is zero, as all the forces balance each other.

36. Here, charge on the pith-ball A, $q_1 = 5\times 10^{-8}$C

Mass of the pith-ball A, $m_1 = 8\,g = 8\times 10^{-3}$ kg



The weight $m_1 g$ of the pith-ball A acts vertically downwards.

Let q_2 be charge on the pith-ball B held 5 cm below the pith ball A, so that the pith-ball A remains stationary.

It can be possible only, if the charges on two pith-balls are of same signs, i.e. if charge on the pith-ball A is positive, the charge on B should also be positive. As such the force on the pith-ball A due to B, i.e. F_{AB} will act vertically upwards.

For charge q_1 to remain stationary,

$$F_{AB} = m_1 g$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{AB^2} = m_1 g$$

Here, $AB = 5$ cm $= 0.05$ m

$$\Rightarrow \quad 9\times 10^9 \times \frac{5\times 10^{-8} \times q_2}{(0.05)^2} = 8\times 10^{-3} \times 9.8$$

$$\therefore \quad q_2 = 4.36 \times 10^{-7}\text{ C (positive)}$$

37. Given, $K = 80$

We have, $K = \frac{\varepsilon_m}{\varepsilon_0}$

$\therefore \quad \varepsilon_m = K\varepsilon_0 \quad [\because \varepsilon_0 = 8.85\times 10^{-12}\,C^2N^{-1}m^{-2}]$

$= 80\times 8.85\times 10^{-12}$

$= 708\times 10^{-12}$

$= 7.08\times 10^{-10}\,C^2N^{-1}m^{-2}$

38. From Coulomb's law, electric force between the two charged bodies, in a medium,

$$F = \frac{1}{4\pi\varepsilon_0 K}\frac{|q_1 q_2|}{r^2}$$

where, K = dielectric constant of the medium.

For vacuum, $K = 1$

For plastic, $K > 1$

Therefore, after insertion of plastic sheet, the force between the two balls will reduce.

39. Two point charges system is taken from air to water keeping other variables (e.g. distance, magnitude of charge) unchanged. So, the only factor which may affect the interacting force is dielectric constant of medium.

Force acting between two point charges,

$$F = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{r^2} \text{ or } F \propto \frac{1}{K} \Rightarrow \frac{F_{air}}{F_{medium}} = K$$

$$\Rightarrow \frac{8}{F_{water}} = 80 \quad \Rightarrow F_{water} = \frac{8}{80} = \frac{1}{10}\text{ N}$$

40. Here, $\mathbf{r}_q = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and $\mathbf{r}_p = -4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

$\therefore \quad \mathbf{r}_p - \mathbf{r}_q = -\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - \hat{\mathbf{k}}$

or $|\mathbf{r}_p - \mathbf{r}_q| = \sqrt{(-1)^2 + (-6)^2 + (-1)^2} = \sqrt{38}$ m

Now, electric field, $\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{|\mathbf{r}_p - \mathbf{r}_q|^3}(\mathbf{r}_p - \mathbf{r}_q)$

Substituting the values, we get

$$\mathbf{E} = \frac{(9.0\times 10^9)(1.0\times 10^{-6})}{(38)^{3/2}}(-\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - \hat{\mathbf{k}})$$

$$= (-38.42\,\hat{\mathbf{i}} - 230.52\,\hat{\mathbf{j}} - 38.42\,\hat{\mathbf{k}})\text{N/C}$$

41. At the point $x = 0$, the electric field due to all the charges are in the same negative x-direction and hence get added up, i.e.

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1^2} + \frac{q}{2^2} + \frac{q}{4^2} + \frac{q}{8^2} + \dots\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{1}{[1-1/4]} = \frac{q}{3\pi\varepsilon_0}$$

This electric field is along negative X-axis.

42. Let each side of square be x.

Diagonal $= \sqrt{x^2 + x^2} = x\sqrt{2}$

$$F_1 = F_2 = \frac{Qq}{4\pi\varepsilon_0 x^2}$$

and $F_3 = \frac{qq}{4\pi\varepsilon_0 (x\sqrt{2})^2} = \frac{q^2}{2\times 4\pi\varepsilon_0 x^2}$

As, F_1 and F_2 are perpendicular to each other, their resultant force,

$$F = \sqrt{F_1^2 + F_2^2}$$

$$= \sqrt{F_1^2 + F_1^2}$$

$$\Rightarrow \quad F = F_1\sqrt{2}$$

As, net force on q is zero, therefore

$$F_1\sqrt{2} = -F_3$$

$$\Rightarrow \quad \frac{Qq\sqrt{2}}{4\pi\varepsilon_0 x^2} = \frac{-q^2}{2 \times 4\pi\varepsilon_0 x^2}$$

$$\Rightarrow \quad q = -2\sqrt{2}\,Q$$

43. (i) Here, $q_1 = q_2 = 6.5 \times 10^{-7} C$, $r = 50$ cm $= 0.5$ m

Electrostatic force of repulsion,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = \frac{9 \times 10^9 \times (6.5 \times 10^{-7})^2}{(0.5)^2}$$

$$= 1.521 \times 10^{-2}\,\text{N}$$

(ii) Now, q_1, q_2 both are doubled and r is halved in

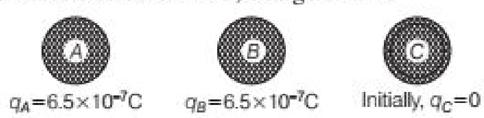
$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}, \text{ then}$$

F becomes 16 times, i.e. $F' = 16$ F.

$$F' = 16 \times 1.521 \times 10^{-2}\,\text{N} \text{ or } F' = 0.24 \text{ N}$$

44. **Hints:** It is based on the distribution of charges when the two identical bodies come into contact, charge is distributed equally on identical bodies.

Now, the sphere C comes in contact with A, the charges will be divided equally on both spheres as they have same mass and size. Now, charge on A is



$$q'_A = \frac{q_A + q_C}{2} = \frac{6.5 \times 10^{-7} + 0}{2}$$

$$= 3.25 \times 10^{-7}\,\text{C}$$

Now, the charge on C will also be 3.25×10^{-7} C.

$$\therefore \quad q'_C = 3.25 \times 10^{-7}\,\text{C}$$

Now, the sphere C comes in contact with B, the charges are shared again.

Then, charge on B is

$$q'_B = \frac{q_B + q'_C}{2}$$

$$= \frac{6.5 \times 10^{-7} + 3.25 \times 10^{-7}}{2}$$

$$= 4.875 \times 10^{-7}\,\text{C}$$

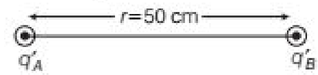
Finally, the charge on C is $q'_C = 4.875 \times 10^{-7}$ C

Finally, the charge on A is $q'_A = 3.25 \times 10^{-7}$ C

The charge on B is $q'_B = 4.875 \times 10^{-7}$ C

From the Coulomb's law, the force between two spheres,

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q'_A \cdot q'_B}{r^2}$$



$$= \frac{9 \times 10^9 \times 3.25 \times 10^{-7} \times 4.875 \times 10^{-7}}{(50 \times 10^{-2})^2}$$

$$= \frac{9 \times 3.25 \times 4.875 \times 10^{-5}}{50 \times 50 \times 10^{-4}} = 5.7 \times 10^{-3}\ \text{N}$$

This force will be repulsive in nature because both spheres have like charges.

45. **Hints:** Net force on a charge due to two equal and opposite charges will be zero. Also, electric field on a charge is given by

$$E = \frac{F}{q}$$

where, E = electric field, F = force on charge q due to electric field and q = magnitude of charge.

If a Cs atom is removed from the corner A, then a singly charged negative Cs ion at A will appear.

(i) From the given figure, we can analyse that the chlorine atom is at the centre of the cube, i.e. at equal distance from all the eight corners of cube, where caesium atoms are placed.

Thus, due to symmetry, the force due to all Cs atoms, on Cl atom will cancel out.

Hence, $\quad E = \dfrac{F}{q'}$

where, $\quad F = 0$

$\therefore \quad E = 0$

(ii) Thus, net force on Cl atom at A would be given by

$$F = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

where, r = distance between Cl ion and Cs ion.

Applying Pythagoras theorem, we get

$$r = \sqrt{(0.20)^2 + (0.20)^2 + (0.20)^2} \times 10^{-9}\,\text{m} = 0.346 \times 10^{-9}\,\text{m}$$

$$\therefore \quad F = \frac{q^2}{4\pi\varepsilon_0 r^2} = \frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{9 \times 10^9 (1.6 \times 10^{-19})^2}{(0.346 \times 10^{-9})^2}$$

$$= 1.92 \times 10^{-9}\ \text{N}$$

46. (i) Charge of proton, $q = 1.6 \times 10^{-19}$ C

Force on proton at A is $F = qE_A$

$$= (1.6 \times 10^{-19}\,\text{C})(40\ \text{N/C}) = 6.4 \times 10^{-18}\,\text{N}$$

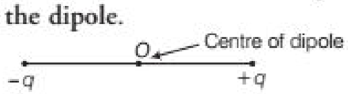
(ii) Since, electric field,

$$E \propto \frac{\text{Number of electric field lines}}{\text{Area}}$$

Hence, $E_B = \dfrac{1}{2} E_A = \dfrac{1}{2}(40\ \text{N/C}) = 20\text{N/C}$

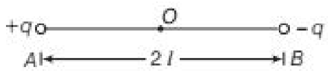
|TOPIC 3|
Electric Dipole

An electric dipole is a pair of point charges with equal magnitude and opposite in sign separated by a very small distance. The mid-point of locations of $-q$ and q is called the **centre of the dipole**.



Dipole Moment of an Electric Dipole

The strength of an electric dipole is measured by a vector quantity known as **electric dipole moment** (**p**) which is the product of the charge (q) and separation between the charges ($2l$).



Electric dipole

i.e. $p = q \times 2l$

or $\boxed{|\mathbf{p}| = q(2l)}$

It is a vector quantity and its direction is always from negative charge to positive charge. The SI unit of dipole moment is coulomb-metre (C-m).

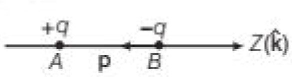
If charge q gets larger and the distance $2l$ gets smaller and smaller, keeping the product $|\mathbf{p}| = q \times 2l$ = constant, we get what is called an **ideal dipole** or **point dipole**. Thus, an ideal dipole is the smallest dipole having almost no size.

Physical Significance of Dipoles

In most molecules, the centres of positive charges and of negative charges lie at the same place, hence their dipole moment is zero, e.g. CO_2, CH_4. However, they develop a dipole moment when an electric field is applied. But some molecules have permanent dipole moment, e.g. H_2O which are called **polar molecules.** If the centre of mass of positive charges coincides with the centre of mass of negative charges, the molecule behaves as a **non-polar molecule**.

EXAMPLE |1| A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A (0, 0, −15 cm) and B (0, 0, +15 cm), respectively. What are the total charge and electric dipole moment of the system? **NCERT**

Sol. Two charges q_A and q_B are located at points $A(0, 0, -15\text{cm})$ and $B(0, 0, 15\text{cm})$ on Z-axis. They form an electric dipole.



Total charge, $q = q_A + q_B = 2.5 \times 10^{-7} - 2.5 \times 10^{-7}$

$\Rightarrow \quad q = 0$

Also, $AB = 15 + 15 = 30$ cm

or $AB = 30 \times 10^{-2}$ m

Electric dipole moment,

p = Either charge × **BA**

$$= 2.5 \times 10^{-7} \times (30 \times 10^{-2})(-\hat{\mathbf{k}})$$

$$= -7.5 \times 10^{-8} \hat{\mathbf{k}} \text{ C-m}$$

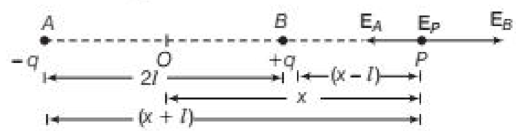
ELECTRIC FIELD INTENSITY DUE TO AN ELECTRIC DIPOLE

Electric field of an electric dipole is the space around the dipole in which the electric effect of the dipole can be experienced.

An electric dipole consists of two charges $+q$ and $-q$, therefore according to the superposition principle, the electric field due to an electric dipole at a point will be equal to the vector sum of the electric fields due to the two individual charges.

At a Point on the Axial Line

We have to calculate the field intensity (E) at a point P on the axial line of the dipole and at a distance $OP = x$ from the centre O of the dipole.



Electric field on axial line of an electric dipole

Resultant electric field intensity at the point P,

$$\mathbf{E}_P = \mathbf{E}_A + \mathbf{E}_B$$

The vectors $\mathbf{E}_A$ and $\mathbf{E}_B$ are collinear and opposite.

$$\therefore \quad E_P = E_B - E_A$$

Here, $E_A = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{(x+l)^2}$ and $E_B = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{(x-l)^2}$

$$\therefore \quad E_P = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(x-l)^2} - \frac{q}{(x+l)^2}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{4qlx}{(x^2 - l^2)^2}$$

Hence, $E_P = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{2px}{(x^2 - l^2)^2}$ $\quad [\because p = q \times 2l]$

In vector form, $\mathbf{E}_P = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\mathbf{p}x}{(x^2 - l^2)^2}$

If dipole is short, i.e. $2l << x$, then

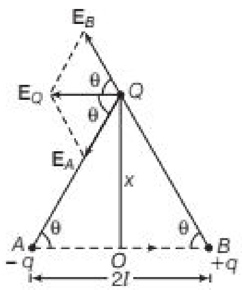
$$E_P = \frac{2|\mathbf{p}|}{4\pi\varepsilon_0 x^3} \quad \text{...(i)}$$

The direction of $\mathbf{E}_P$ is along BP produced.

Clearly, $E_P \propto \frac{1}{x^3}$

At a Point on the Equatorial Line

Consider an electric dipole consisting of two point charges $+q$ and $-q$ separated by a small distance $AB = 2l$ with centre at O and dipole moment, $p = q(2l)$ as shown in the figure.



Resultant electric field intensity at the point Q,

$$\mathbf{E}_Q = \mathbf{E}_A + \mathbf{E}_B$$

The vectors $\mathbf{E}_A$ and $\mathbf{E}_B$ are acting at an angle 2θ.

Here, $E_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(x^2 + l^2)}$ and $E_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(x^2 + l^2)}$

On resolving $\mathbf{E}_A$ and $\mathbf{E}_B$ into two rectangular components, the vectors $E_A \sin\theta$ and $E_B \sin\theta$ are equal in magnitude and opposite to each other and hence cancel out.

The vectors $E_A \cos\theta$ and $E_B \cos\theta$ are acting along the same direction and hence add up.

$$\therefore E_Q = E_A \cos\theta + E_B \cos\theta = 2E_A \cos\theta \qquad [\because E_A = E_B]$$

$$= \frac{2}{4\pi\varepsilon_0} \cdot \frac{q}{(x^2 + l^2)} \cdot \frac{l}{(x^2 + l^2)^{1/2}} \left[\because \cos\theta = \frac{l}{(x^2 + l^2)^{1/2}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{2ql}{(x^2 + l^2)^{3/2}}$$

But $q \times 2l = |\mathbf{p}|$, the dipole moment

$$E_Q = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|\mathbf{p}|}{(x^2 + l^2)^{3/2}}$$

The direction of $\mathbf{E}$ is along $\mathbf{QE} \parallel \mathbf{BA}$, i.e. opposite to $\mathbf{AB}$. In vector form, we can rewrite as

$$\mathbf{E}_Q = \frac{-\mathbf{p}}{4\pi\varepsilon_0 (x^2 + l^2)^{3/2}}$$

Obviously, $\mathbf{E}_Q$ is in a direction opposite to the direction of $\mathbf{p}$. If the dipole is short, i.e. $2l << x$, then

$$\therefore \quad E_Q = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|\mathbf{p}|}{x^3} \quad \text{...(ii)}$$

Clearly, $E_Q \propto \frac{1}{x^3}$

From Eqs. (i) and (ii), we get

$$\frac{E_{\text{axial}}}{E_{\text{equatorial}}} = 2$$

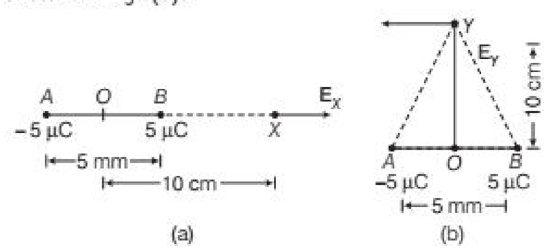
Both the magnitude and the direction of dipole field depend not only on the distance r, but also on the angle between the position vector $\mathbf{r}$ and dipole moment $\mathbf{p}$.

The electric field due to a dipole falls off at large distances, at a much faster rate $\left(\propto \frac{1}{r^3}\right)$ than the electric field due to a single charge $\left(\propto \frac{1}{r^2}\right)$.

EXAMPLE |2| Two charges $\pm 5\,\mu C$ are placed 5 mm apart. Determine the electric field at

(i) a point X on the axis of dipole 10 cm away from its centre O on the side of the positive charge as shown in Fig. (a).

(ii) a point Y, 10 cm away from centre O on a line passing through O and normal to the axis of the dipole as shown in Fig. (b).



Sol. Given, $q = \pm 5\,\mu C = \pm 5 \times 10^{-6}$ C,

$2l = 5\,\text{mm} = 5 \times 10^{-3}$ m

$x = OX = OY = 10$ cm

$= 10 \times 10^{-2}$ m

$E_X = ?$

and $E_Y = ?$

Dipole moment, $\mathbf{p} = q \times 2l$

$= 5 \times 10^{-6}\,\text{C} \times 5 \times 10^{-3}\,\text{m}$

$= 25 \times 10^{-9}$ C-m

(i) Now, find out the electric field at point X on the axial line of dipole.

$$E_X = \frac{2px}{4\pi\varepsilon_0(x^2 - l^2)^2}, \text{ along } BX \text{ produced}$$

Since $l << x$, therefore

$$E_X = \frac{2p}{4\pi\varepsilon_0 x^3}$$

$$= \frac{2 \times 25 \times 10^{-9} \times 9 \times 10^9}{(10 \times 10^{-2})^3}$$

$$= 4.5 \times 10^5 \text{ NC}^{-1}, \text{ along } BX \text{ produced.}$$

(ii) Now, find out the electric field at point Y on equatorial line of dipole.

$$E_Y = \frac{p}{4\pi\varepsilon_0(x^2 + l^2)^{3/2}}, \text{ along a line parallel to } BA$$

Since, $l << x$, therefore $E_Y = \dfrac{p}{4\pi\varepsilon_0 x^3}$

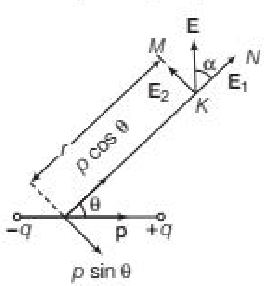
$$\therefore \quad E_Y = \frac{25 \times 10^{-9} \times 9 \times 10^9}{(10 \times 10^{-2})^3}$$

$$= 2.25 \times 10^5 \text{ NC}^{-1}, \text{ along a line parallel to } BA.$$

ELECTRIC FIELD INTENSITY AT ANY POINT DUE TO A SHORT ELECTRIC DIPOLE

Let K be any point which is neither on the axial line nor on the equatorial line.

Let **P** be the dipole moment of the short electric dipole and O be the mid-point of the dipole. Let the line OK make an angle θ with **P**. Resolving **P** along OK and perpendicular to OK, we get $p\cos\theta$ and $p\sin\theta$, respectively.



The electric field at K due to dipole moment $p\cos\theta$ is given by

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3} \quad [\because K \text{ is on the axial line of the dipole } p\cos\theta]$$

The electric field at K due to dipole moment $p\sin\theta$ is given by

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3} \text{ [along } KM \text{ which is} \perp p\cos\theta]$$

$\therefore$ Resultant electric field,

$$E^2 = E_1^2 + E_2^2 = \left(\frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}\right)^2 + \left(\frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}\right)^2$$

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}(4\cos^2\theta + \sin^2\theta)^{1/2}$$

i.e. $\boxed{E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{p}{r^3}\sqrt{3\cos^2\theta + 1}}$

If α is the angle between **E** and $\mathbf{E}_1$, then

$$\tan\alpha = \frac{E_2}{E_1} = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3} \times \frac{4\pi\varepsilon_0 r^3}{2p\cos\theta}$$

i.e. $\boxed{\tan\alpha = \dfrac{1}{2}\tan\theta}$

Special cases

***Case* I** K lies on the axial line of dipole, then $\theta = 0°$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{3\cos^2 0° + 1} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

$$\Rightarrow \tan\alpha = \frac{\tan 0°}{2} = 0 \Rightarrow \alpha = 0$$

***Case* II** K lies on the equitorial line of dipole, then $\theta = 90°$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{3\cos^2 90° + 1} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$

$$\Rightarrow \tan\alpha = \frac{\tan 90°}{2} = \infty$$

$$\Rightarrow \quad \alpha = \tan^{-1}\infty \quad \Rightarrow \quad \alpha = 90°$$

EXAMPLE |3| Find the magnitude of electric field intensity due to a dipole of dipole moment 3×10^{-8} C-m at a point distance 1 m from the centre of dipole, when line joining the point to the centre of dipole makes an angle of 60° with the dipole axis.

Sol. Here, $p = 3 \times 10^{-8}$ C-m, $r = 1\text{ m}$, $\theta = 60°$ and $E = ?$

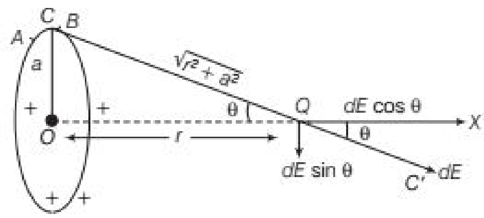
$$\text{As, } |\mathbf{E}| = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{3\cos^2\theta + 1}$$

$$\therefore \quad E = \frac{3 \times 10^{-8} \times 9 \times 10^9}{(1)^3}\sqrt{3(\cos 60°)^2 + 1} = 357.17 \text{ N/C}$$

ELECTRIC FIELD INTENSITY AT ANY POINT ON THE AXIS OF UNIFORMLY CHARGED RING

The electric field intensity at any point Q on the axis is given by

$$\boxed{E = \frac{qr}{4\pi\varepsilon_0 (r^2 + a^2)^{3/2}}}$$



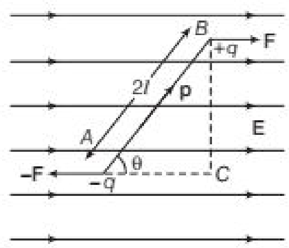
where, q = total charge, a = radius of the ring and
r = distance of the point Q from the centre of the ring.

Note The direction of **E** is along QX, the axis of the loop.

DIPOLE IN A UNIFORM EXTERNAL FIELD

Torque on an Electric Dipole in a Uniform Electric Field

Consider an electric dipole consisting of two charges $-q$ and $+q$ placed in a uniform external electric field of intensity **E**.



The length of the electric dipole is $2l$. The dipole moment **p** makes an angle θ with the direction of the electric field. Two forces **F** and $-$**F** which are equal in magnitude and opposite in directions act on the dipole.

$$|\mathbf{F}| = |-\mathbf{F}| = qE$$

The net force is zero. Since, the two forces are equal in magnitude and opposite in direction and act at different points, therefore they constitute a couple. A net torque τ acts on the dipole about an axis passing through the mid-point of the dipole.

Now, τ = Either force × Perpendicular distance BC between the parallel forces $= qE(2l\sin\theta)$

$$\tau = (q \times 2l) E \sin\theta$$

or $\boxed{\tau = pE\sin\theta}$

In vector notation, $\boxed{\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}}$

SI unit of torque is newton-metre (N-m) and its dimensional formula is $[ML^2T^{-2}]$.

Case I If $\theta = 0°$, then $\tau = 0$

The dipole is in stable equilibrium.

Case II If $\theta = 90°$, then $\tau = pE$ (maximum value)

The torque acting on dipole will be maximum.

Case III If $\theta = 180°$, then $\tau = 0$

The dipole is in unstable equilibrium.

EXAMPLE |4| An electric dipole consists of two charges of 0.1 μC separated by a distance of 2.0 cm. The dipole is placed in an external field of 10^5 N/C. What maximum torque does the field exert on the dipole?

Sol. Here, $q = 0.1\ \mu C = 10^{-7}$ C, $2l = 2.0$ cm $= 2 \times 10^{-2}$ m,

$$E = 10^5 \text{ N/C} \Rightarrow \tau = pE\sin\theta = q \times 2l \times E\sin\theta$$

$$\therefore \quad \tau_{max} = 10^{-7} \times 2 \times 10^{-2} \times 10^5 \times 1 \qquad [\because \sin 90° = 1]$$

$$= 2 \times 10^{-4} \text{ N-m}$$

Work Done on a Dipole in a Uniform Electric Field

When an electric dipole is placed in a uniform electric field, it experiences torque and tends to align it in such a way to attain stable equilibrium. Small amount of work done in rotating the dipole through a small angle $d\theta$ against the torque is given by

$$dW = \tau\, d\theta = pE\sin\theta\, d\theta$$

$\therefore$ Total work done in rotating the dipole from orientation θ_1 to θ_2, $W = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta = pE(\cos\theta_1 - \cos\theta_2)$

$$\Rightarrow \quad \boxed{W = pE(\cos\theta_1 - \cos\theta_2)}$$

Similarly, potential energy of electric dipole, when it rotates from θ_1 to θ_2, $\boxed{U = W = pE(\cos\theta_1 - \cos\theta_2)}$

Let us assume that the dipole is initially oriented perpendicular to the direction of electric field and brought to the orientation making an angle θ with the field direction, then the work done in rotating the dipole from $\theta_1 = 90°$ to $\theta_2 = \theta$,

$$W = pE(\cos 90° - \cos\theta) = -pE\cos\theta = -\mathbf{p}\cdot\mathbf{E}$$

$$\boxed{W = -\mathbf{p}\cdot\mathbf{E}}$$

EXAMPLE |5| An electric dipole of moment 2×10^{-8} C-m is aligned in a uniform electric field of 2×10^{4} N/C. Calculate the work done in rotating the dipole from 30° to 60°.

Sol. Here, $p = 2\times10^{-8}$ C-m, $E = 2\times10^{4}$ N/C,

$\theta_1 = 30°, \theta_2 = 60°, W = ?$

$$\therefore \quad W = pE(\cos\theta_1 - \cos\theta_2)$$
$$= (2\times10^{-8})(2\times10^{4})(\cos 30° - \cos 60°)$$
$$= (2\times10^{-8})(2\times10^{4})(0.366)$$
$$= 1.464\times10^{-4}\text{ J}$$

EXAMPLE |6| An electric dipole of length 2 cm, when placed with its axis making an angle of 60° with a uniform electric field, experiences a torque of $8\sqrt{3}$ N-m. Calculate the potential energy of the dipole, if it has a charge of $\pm$ 4 nC. **Delhi 2014**

Sol. Here, length, $2a = 2\text{ cm} = 2\times10^{-2}$ m,

$\theta = 60°, \tau = 8\sqrt{3}$ N-m

Charge, $Q = 4\text{nC} = 4\times10^{-9}$ C, $U = ?$

As we know that, $\tau = Q(2a)E\sin\theta$

$\Rightarrow$ Electric field,

$$E = \frac{\tau}{Q(2a)\sin\theta} = \frac{8\sqrt{3}}{4\times10^{-9}\times2\times10^{-2}\times\sin 60°}\text{ N/C}$$

$\therefore$ Potential energy, $U = -pE\cos\theta = -Q(2a)E\cos\theta$

$$= -4\times10^{-9}\times2\times10^{-2}\times\frac{8\sqrt{3}\times\cos 60°}{4\times10^{-9}\times2\times10^{-2}\times\sin 60°}$$
$$= -\frac{8\sqrt{3}}{\sqrt{3}} = -8\text{ J}$$

Note Electric dipole and its properties have been frequently asked in previous years 2014, 2012, 2011, 2010.

TOPIC PRACTICE 3

OBJECTIVE Type Questions

1. Two equal and opposite charges each of 2C are placed at a distance of 0.04 m. Dipole moment of the system will be

(a) 6×10^{-8} C-m (b) 8×10^{-2} C-m

(c) 1.5×10^{2} C-m (d) 8×10^{-6} C-m

2. What is the angle between the electric dipole moment and the electric field strength due to it on the equatorial line?

(a) 0° (b) 90°

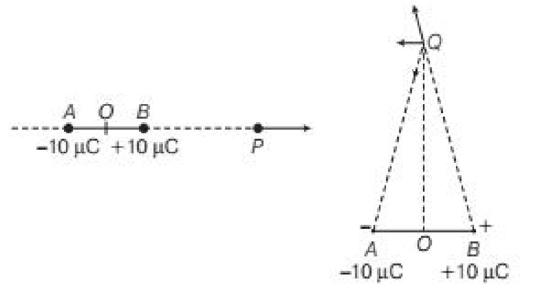
(c) 180° (d) None of these

3. Electric charges $q, q, -2q$ are placed at the corners of an equilateral ΔABC of side l. The magnitude of electric dipole moment of the system is

(a) ql (b) $2ql$

(c) $\sqrt{3}ql$ (d) $4ql$

4.



In given figures, $OP = OQ = 15$ cm, $OA = OB = 2.5$ mm

Magnitudes of electric field at P and Q are respectively

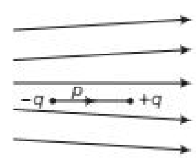
(a) $2.6\times10^{5}\text{ NC}^{-1}, 2.6\times10^{5}\text{ NC}^{-1}$

(b) $1.3\times10^{5}\text{ NC}^{-1}, 1.3\times10^{5}\text{ NC}^{-1}$

(c) $2.6\times10^{5}\text{ NC}^{-1}, 1.3\times10^{5}\text{ NC}^{-1}$

(d) $1.3\times10^{5}\text{ NC}^{-1}, 2.6\text{ NC}^{-1}$

5. Figure shows electric field lines in which an electric dipole P is placed as shown. Which of the following statements is correct? **NCERT Exemplar**



(a) The dipole will not experience any force

(b) The dipole will experience a force towards right

(c) The dipole will experience a force towards left

(d) The dipole will experience a force upwards

6. In an electric field **E**, the torque acting on a dipole moment **p** is

(a) $\mathbf{p}\cdot\mathbf{E}$ (b) $\mathbf{p}\times\mathbf{E}$

(c) zero (d) $\mathbf{E}\times\mathbf{p}$

7. When an electric dipole **p** is placed in a uniform electric field **E**, then at what angle between **p** and **E** the value of torque will be maximum?

(a) 90° (b) 0°

(c) 180° (d) 45°

VERY SHORT ANSWER Type Questions

8. Is it correct to write the unit of electric dipole moment as mC?

9. What do you mean by an "ideal electric dipole"?

10. At what points dipole field intensity is parallel to the line joining the charges?

11. If an electric dipole is placed in a uniform electric field, then state whether it always experiences a torque or not?

12. What happens when an electric dipole is placed in a non-uniform electric field?

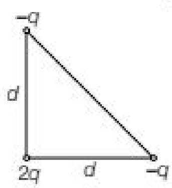
13. A dipole of dipole moment $\mathbf{p}$ is present in a uniform electric field $\mathbf{E}$. Write the value of the angle between $\mathbf{p}$ and $\mathbf{E}$ for which the torque, experienced by the dipole is minimum. **All India 2010**

14. A ring of radius R carries a uniformly distributed charge $+Q$. A point charge $-q$ is placed on the axis of the ring at a distance $2R$ from the centre of the ring and released from rest. Will the particle execute simple harmonic motion along the axis of the ring?

SHORT ANSWER Type Questions

15. What is meant by the statement, "the electric field of a point charge has spherical symmetry, whereas that of an electric dipole is cylindrically symmetric"?

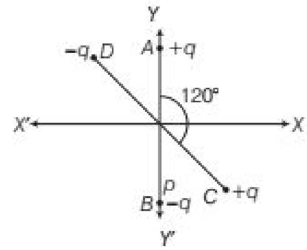
16. Three charges are placed as shown. Find dipole moment of the arrangements.



17. Prove that when an electric dipole is placed in a uniform electric field, potential energy U is given by $U = -\mathbf{p} \cdot \mathbf{E}$.

18. Two small identical dipoles AB and CD, each of dipole moment $\mathbf{p}$ are kept at an angle of $120°$ as shown in the figure.

What is the resultant dipole moment of this combination? If this system is subjected to electric field ($\mathbf{E}$) directed along positive x-direction, what will be the magnitude and direction of the torque acting on this? **Delhi 2011**



19. A dipole, with a dipole moment of magnitude p, is in stable equilibrium in an electrostatic field of magnitude E. Find the work done in rotating this dipole to its position of unstable equilibrium.

LONG ANSWER Type I Questions

20. An electric dipole of dipole moment $\mathbf{p}$ consists of point charges $+q$ and $-q$ separated by a distance $2a$ apart. Deduce an expression for the electric field $\mathbf{E}$ due to the dipole at a distance x from the centre of the dipole on its axial line in terms of the dipole moment $\mathbf{p}$. Hence, show that in the limit $x >> a$, $\mathbf{E} \longrightarrow 2\,\mathbf{p} / 4\pi\varepsilon_0 x^3$.

21. (i) Derive an expression for electrical field at a point on the equatorial line of an electric dipole.

(ii) Depict the orientation of the dipole in (a) stable, (b) unstable equilibrium in a uniform electric field. **Delhi 2017**

22. A charge is distributed uniformly over a ring of radius a. Obtain the expression for the electric field intensity E at a point on the axis of the ring. Hence, show that for points at large distances from the ring, it behaves like a point charge. **Delhi 2016**

23. (i) Obtain the expression for the torque τ experienced by an electric dipole of dipole moment $\mathbf{p}$ in a uniform electric field $\mathbf{E}$.

(ii) What will happen, if the field were not uniform? **Delhi 2017**

24. An electric dipole is held at any angle θ in a uniform electric field E. Will there be any

(i) net translating force and

(ii) torque acting on it?

Explain, what happens to dipole on being released?

LONG ANSWER Type II Questions

25. Define electric dipole moment. Is it a scalar or a vector? Derive the expression for the electric field of a dipole at a point on the equatorial plane of the dipole.

For a short dipole, what is the ratio of electric field intensities at two equidistant points from the centre of the dipole? One along the axial line and other on the equatorial line.

26. Deduce the expression for the torque acting on a dipole of dipole moment **p** in the presence of a uniform electric field E. **All India 2014**

27. In a certain region of space, electric field is along the z-direction throughout. The magnitude of electric field is, however not constant but increases uniformly along the positive z-direction, at the rate of 10^5 N $C^{-1}m^{-1}$. What are the force and torque experienced by a system having a total dipole moment equal to 10^{-7} C-m in the negative z-direction? **NCERT**

28. (i) Derive the expression for the electric field E due to a dipole of length $2l$ at a point distant r from the centre of the dipole on the axial line.

(ii) Draw a graph of E *versus* r for $r >> a$.

(iii) If this dipole is kept in a uniform external electric field E_0, diagrammatically represent the position of the dipole in stable and unstable equilibrium and write the expressions for the torque acting on the dipole in both the cases. **All India 2017**

29. Derive the expression for the work done in rotating an electric dipole from angle θ_1 to θ_2 in a uniform electric field (**E**). Hence, find the work done when the dipole is

(i) initially parallel to the field and

(ii) initially perpendicular to the field. **All India 2009**

NUMERICAL PROBLEMS

30. Find the electric dipole moment electron and a proton which distance is 4.3 nm apart.

31. Two charges of $-9\,\mu C$ and $+9\,\mu C$ are placed at the points $P(1, 0, 4)$ and $Q(2, -1, 5)$ located in an electric field $E = 0.20\,\hat{i}$ V/cm. Calculate the torque acting on the dipole.

32. Two charges of $+25\times10^{-9}$ C and -25×10^{-9} C are placed 6 m apart. Find the electric field at a point 4 m from the centre of the electric dipole (i) on axial line (ii) on equatorial line. **Delhi 2011**

33. An electric dipole with dipole moment 4×10^{-9} C-m is aligned at 30° with the direction of a uniform electric field of magnitude 5×10^4 N/C. Calculate the magnitude of the torque acting on the dipole.

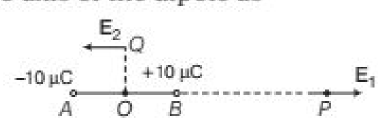
34. A system has two charges $q_A = 3.5\times10^{-7}$ C and $q_B = -3.5\times10^{-7}$ C located at points A (0,0, −10cm) and B (0, 0, +10 cm), respectively. What are the total charge and electric dipole moment of the system?

35. Two charges q_1 and q_2 of $0.1\,\mu\mu C$ and $-\,0.1\,\mu\mu C$ respectively are 10Å apart. What is the electric field at a point on the line joining them at a distance of 10 cm from their mid-point?

36. Two charges $\pm10\,\mu C$ are placed 5 mm apart. Determine the electric field at

(i) a point P on the axis of dipole 15 cm away from its centre O on the side of the positive charge

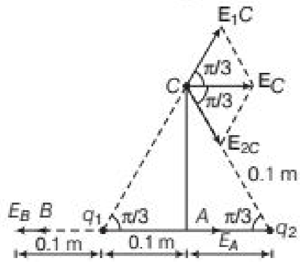
(ii) a point Q, 15 cm away from centre O on a line passing through centre O and normal to axis of the dipole as **NCERT**



37. An electric dipole consists of two opposite charges each of magnitude 1.0×10^{-6} C separated by 2 cm. The dipole is placed in an external uniform field of 1×10^5 N/C. Find (i) the maximum torque exerted by the field on the dipole, (ii) the work which an external agent will have to do in turning the dipole through 180° starting from the position, $\theta = 0°$.

38. The electric field at a point on the axial line at a distance of 10 cm from the centre of an electric dipole is 3.75×10^5 N/C in air, while at a distance of 20 cm, the electric field is 3×10^4 N/C. Calculate the length of an electric dipole.

39. A two point charges q_1 and q_2 of magnitude 10^{-7} C and -10^{-7} C, respectively are placed 0.2 m apart. Calculate the electric fields at points *A*, *B* and *C* as shown in the figure. **NCERT**



40. (i) Calculate the maximum torque experienced by a water molecule whose electric dipole moment is 6.2×10^{-30} C-m, when it is placed in an electric field of intensity 10^6 N/C.

(ii) Determine the work that must be done to take a water molecule aligned with the above field and set it anti-parallel to the field.

HINTS AND SOLUTIONS

1. (b) Electric dipole moment, $p = q \times d$

Here, q = value of one charge on dipole = 2 C

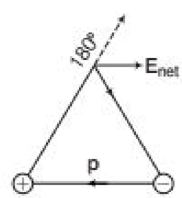
d = distance between the dipoles = 0.04 m

$\therefore$ Electric dipole moment, $p = 2 \times 0.04$

$= 0.08$ C-m

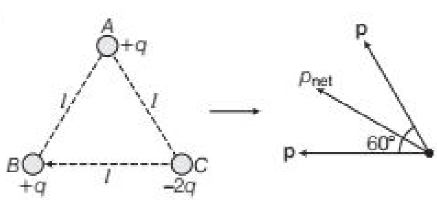
$= 8 \times 10^{-2}$ C-m

2. (c)



Observing $\mathbf{E}_{net}$ and $\mathbf{p}$ are in opposite directions, so angle between them is 180°.

3. (c)



Net dipole moment, *i.e.*,

$$p_{net} = \sqrt{p^2 + p^2 + 2pp\cos 60^\circ} = \sqrt{3}p$$

$$= \sqrt{3}\, ql \qquad (\because p = ql)$$

4. (c) Here, $a = 2.5$ mm, $r = 15$ cm $= 150$ mm

As, $r >> a$

$$E_{axis} = \frac{2p}{4\pi\varepsilon_0 r^3} = \frac{2(5\times10^{-3}\times10\times10^{-6})\times(9\times10^9)}{(15\times10^{-2})^3}$$

$$= 2.6\times10^5\ \text{NC}^{-1}$$

$$E_{\text{equatorial plane}} = \frac{p}{4\pi\varepsilon_0 r^3} = \frac{1}{2}E_{axis} = \frac{1}{2}\times2.6\times10^5$$

$$= 1.3\times10^5\ \text{NC}^{-1}$$

5. (c) The space between the electric field lines is increasing, here from left to right and its characteristics states that, strength of electric field decreases with the increase in the space between electric field lines. As a result force on charges also decreases from left to right. Thus, the force on charge $-q$ is greater than force on charge $+q$ in turn dipole will experience a force towards left.

6. (b) In electric field $(\mathbf{E})$, torque acting on a dipole moment $(\mathbf{p})$ is $\tau = \mathbf{pE}\sin\theta$

where, θ = angle between $\mathbf{p}$ and $\mathbf{E}$, $\Rightarrow \tau = \mathbf{p}\times\mathbf{E}$

7. (a) Torque, $\tau = pE\sin\theta\hat{\mathbf{n}}$

$|\tau| = pE\sin\theta$

$\therefore$ Torque is maximum, when $\theta = 90°$

8. No, it is not correct to write the unit of electric dipole moment as mC. The symbol mC represents milli-coulomb, i.e. unit of electric charge. In SI system, unit symbols are written in alphabetical order.

$\therefore$ Unit of dipole moment is C-m.

9. If charge q gets larger and distance $2l$ gets smaller; and smaller keeping the product $|p| = q\times2l$ = constant. The dipole is called an ideal electric dipole.

10. At any point on axial line or equatorial line of dipole.

11. No, it does not experience a torque, when it is placed along the direction of electric field.

12. It experiences some net force and some net torque.

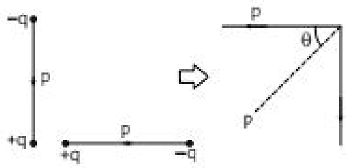
13. $\tau = pE\sin\theta$, τ is minimum when $\theta = 0°$.

14. Yes, but motion is simple harmonic only when charge $-q$ is not very far from the centre of ring on its axis. Otherwise motion is periodic, but not simple harmonic in nature.

15. The electric field due to a point charge q at a distance r is given by $E = \frac{q}{4\pi\varepsilon_0 r^2}$. Clearly, the magnitude of field **E** will be the at all points on the surface of a sphere of radius r drawn around the point charge and does not depend on the direction **r**. Hence, the field line due to a point charge is spherically symmetric. Electric field at distance r on the equatorial line of a dipole moment p is given by $E = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2+a^2)^{3/2}}$.

The electric field E is same at all points which lie on a cylinder of radius r with its axis on the dipole axis and the field pattern looks same in all planes passing through the dipole axis. We say that the electric field of an electric dipole is cylindrical symmetric.

16. Here, two dipoles are formed. These are shown in diagram below.

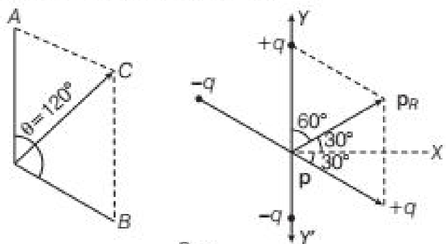


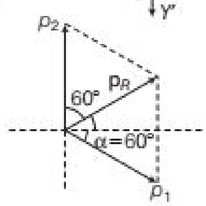
Resultant dipole moment,

$$P = \sqrt{2}p = \sqrt{2}qd,\ \theta = 45°$$

17. Refer to text on page 28.

18. Consider the figure, $|\mathbf{p}_A| = \mathbf{p}_C = p$





The magnitude of resultant $\mathbf{p}_R$,

$$\mathbf{p}_R = \sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos\theta}$$

$$= \sqrt{p^2 + p^2 + 2p^2\cos\theta}$$

$$= \sqrt{2p^2(1+\cos\theta)}$$

$$= \sqrt{2p^2 \times 2\cos^2\frac{\theta}{2}} = 2p\cos\frac{\theta}{2}$$

$$\tan\alpha = \frac{p_2\sin\theta}{p_1 + p_2\cos\theta} = \frac{p\sin 120°}{p + p\cos 120°}$$

$$= \frac{p\sqrt{3}/2}{p - \frac{p}{2}} = \sqrt{3}$$

$$\Rightarrow\ |\mathbf{p}_R| = 2p\cos\frac{\theta}{2} = 2p\cos\frac{120°}{2} = 2p\times\frac{1}{2} = p$$

$\mathbf{p}_R$ will subtend an angle of 30° with X-axis.

Now, torque acting on the system,

$$\tau = \mathbf{p}_R \times \mathbf{E} = p_R E\sin\theta = \frac{1}{2}pE$$

Torque will work to align the dipole in the direction of electric field E .

19. The position of stable equilibrium corresponds to $\theta = 0°$.

The position of unstable equilibrium corresponds to $\theta = 180°$.

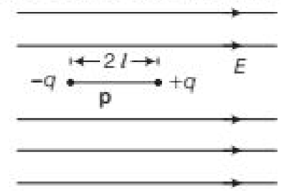
$$\therefore \text{Work done} = \int_{\theta=0°}^{\theta=180°} pE\sin\theta\,d\theta = pE[-\cos\theta]_{0°}^{180°} = 2pE$$

20. Refer to pages 25 and 26 (replacing $2l$ by $2a$)

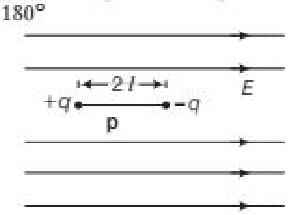
21. (i) Refer to text on page 26.

(ii) The orientation of the dipole

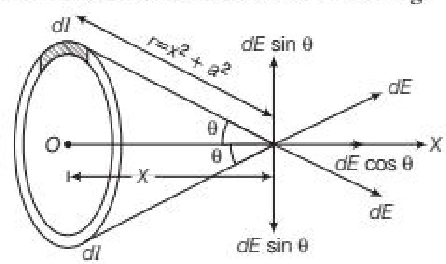
(a) In stable equilibrium, **p**, is parallel to **E**, i.e. $\theta = 0°$



(b) In unstable equilibrium, **p** is anti-parallel to **E,** i.e. $\theta = 180°$



22. According to question, suppose that the ring is placed with its plane perpendicular to the X-axis as shown in figure. Consider small element dl of the ring.



As the total charge q is uniformly distributed, so the charge dq on element dl is $dq = \frac{q}{2\pi a}\cdot dl$

$$\Rightarrow dq = \frac{q}{2\pi a}\frac{dl}{r^2}\cos\theta = dE\cos\theta \qquad \left[\text{where } \cos\theta = \frac{x}{2}\right]$$

Since, only the axial component gives the net **E** at point P due to charge on ring.

$$\text{So, } \int_0^E dE = \int_0^{2\pi a} dE\cos\theta = \int_0^{2\pi a} \frac{kq}{2\pi a}\cdot\frac{dl}{r^2}\times\frac{x}{r}$$

$$= \frac{kqx}{2\pi a}\frac{1}{r^3}\int_0^{2\pi a} dl = \frac{kq}{2\pi a}\cdot\frac{1}{r^3}\int_0^{2\pi a}[l]_0^{2\pi a}$$

$$= \frac{kqx}{2\pi a}\cdot\frac{1}{(x^2+a^2)^{3/2}}\cdot 2\pi a \qquad [\because r^2 = x^2 + a^2]$$

$$\therefore \qquad E = \frac{kqx}{(x^2+a^2)^{3/2}}$$

Now, for points at large distances from the ring $x >> a$.

$\therefore \quad E = \dfrac{kq}{x^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{x^2}$

This is same as the field due to a point charge indicating that for far-off axial point, the charged ring behaves as a point charge.

23. (i) Refer to text on page 28.

(ii) If the field is non-uniform, the net force will be non-zero.

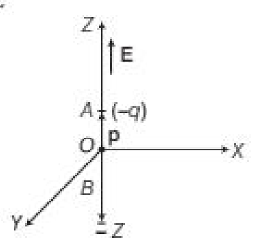
24. Refer to text on page 28.

25. Refer to text on pages 25 and 26.

26. Refer to text on page 28.

27. Consider an electric dipole with charge $-q$ at A and charge $+q$ at B, placed along Z-axis, such that its dipole moment is in negative z-direction, i.e. $p_Z = -10^{-7}$ C-m, as shown in the figure.

The electric field is along positive direction of Z-axis, such that $\dfrac{dE}{dZ} = 10^5\,\text{N}\,\text{C}^{-1}\text{m}^{-1}$.



Using, $\quad F = qdE = q \times \dfrac{dE}{dZ} \times dZ$

$= (q \times dZ) \times \dfrac{dE}{dZ} = p\dfrac{dE}{dZ}$

$= 10^{-7} \times 10^5 = -\,10^{-2}$ N

Thus, the force on the dipole is along negative direction of Z-axis.

As, $\quad \theta = 180°$

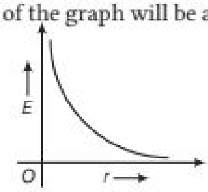
$\therefore$ Torque on dipole,

$\tau = pE \sin 180° = 0$

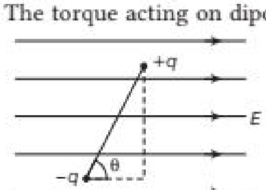
28. (i) Refer to text on pages 25 and 26.

(ii) $E \propto \dfrac{1}{r^3}$. As r will increase, E will sharply decreases.

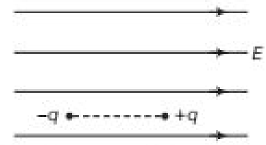
The shape of the graph will be as given in the figure.



(iii) When the dipole were kept in a uniform electric field E_0. The torque acting on dipole, $\tau = p \times E$

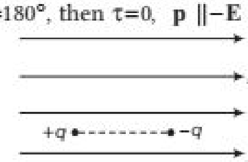


(a) If $\theta = 0°$, then $\tau = 0$, $\mathbf{p} \| \mathbf{E}$



The dipole is in stable equilibrium.

(b) If $\theta = 180°$, then $\tau = 0$, $\mathbf{p} \| -\mathbf{E}$



The dipole is in unstable equilibrium.

29. Refer to text on page 28.

(i) If the dipole is initially parallel to the field, $\theta_1 = 0°$

$W = pE(1 - \cos\theta_2)$

(ii) If the dipole is initially perpendicular to the field, $\theta_1 = 90°$

$W = -\,pE\cos\theta_2$

30. Dipole moment,

$p = q \times r = 1.6 \times 10^{-19} \times 4.3 \times 10^{-9} = 6.8 \times 10^{-28}$ C-m

31. As, $P(1, 0, 4)$ and $Q(2, -1, 5)$

$\therefore \quad 2l = \text{PQ} = [(2-1)\hat{\text{i}} + (-1-0)\hat{\text{j}} + (5-4)\hat{\text{k}}] = (\hat{\text{i}} - \hat{\text{j}} + \hat{\text{k}})$

and $\quad q = \pm 9 \times 10^{-6}$ C, $E = 0.20\,\hat{\text{i}}$ V/cm, $\tau = ?$

Since, $\tau = p \times E = q(2l) \times E$

$\therefore \quad \tau = 9 \times 10^{-6}\,(\hat{\text{i}} - \hat{\text{j}} + \hat{\text{k}}) \times 0.20\hat{\text{i}} = 18 \times 10^{-7}(\hat{\text{k}} - \hat{\text{j}})$

$\therefore$ Magnitude of torque,

$\tau = 18 \times 10^{-7}\,[\sqrt{(1)^2 + (1)^2}] = 25.45 \times 10^{-7}$ N-m

32. Here, $q = 25 \times 10^{-9}$C, $2a = 6$ m, $r = 4$ m,

$p = q(2a) = 25 \times 10^{-9} \times 6 = 1.5 \times 10^{-7}$ C-m

Now, $E_{\text{axial}} = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{2pr}{(r^2 - a^2)^2}$

$= \dfrac{9 \times 10^9 \times 2 \times 1.5 \times 10^{-7} \times 4}{(4^2 - 3^2)^2} = \dfrac{2700 \times 4}{49}$

$= 220.4\ \text{NC}^{-1}$

$\therefore \quad E_{\text{equatorial}} = \dfrac{1}{4\pi\,\varepsilon_0} \cdot \dfrac{p}{(r^2 + a^2)^{3/2}}$

$= \dfrac{9 \times 10^9 \times 1.5 \times 10^{-7}}{(4^2 + 3^2)^{3/2}} = \dfrac{1350}{125} = 10.8\ \text{N}\,\text{C}^{-1}$

33. Given, $p = 4 \times 10^{-9}$ C-m, $E = 5 \times 10^4$, $\theta = 30°$

$\therefore \quad \tau = pE\sin\theta$

$= 4 \times 10^{-9} \times 5 \times 10^4 \times \sin 30°$

$= 4 \times 10^{-9} \times 5 \times 10^4 \times \dfrac{1}{2} \qquad \left[\because \sin 30° = \dfrac{1}{2}\right]$

$= 10 \times 10^{-5} = 10^{-4}$ N-m

34. Refer to Example 1 on page 25.

35. Here, $q_1 = q_2 = q = 0.1\mu\mu C = 10^{-13}$ C [in magnitude]

Length of the electric dipole formed by these charges,

$$2a = 10 \text{Å} = 10^{-9} \text{ m}$$

Thus, electric dipole moment,

$$p = 2aq = 10^{-13} \times 10^{-9} = 10^{-22} \text{ C-m}$$

Distance of the point under consideration on the axial line from the mid-point, $r = 10$ cm $= 0.1$ m

Since, $a << r$, electric field at a point on the axial line,

$$E = k_c \frac{2p}{r^3}$$

$$= (9 \times 10^9) \frac{2 \times 10^{-22}}{(0.1)^3}$$

$$= 18 \times 10^{-10} \text{ N/C}$$

36. Refer to Example 2 on pages 26 and 27.

37. Here, $q = 1 \times 10^{-6}$ C, $2a = 2$ cm $= 0.02$ m

$\therefore p = q \times 2a = (1 \times 10^{-6}) \times 0.02 = 2 \times 10^{-8}$cm

Intensity of the external electric field, $E = 1.0 \times 10^5$ N/C

(i) $\tau_{max} = pE = (2 \times 10^{-8})(1.0 \times 10^5) = 2 \times 10^{-3}$N-m

(ii) Net work done in turning the dipole from 0° to 180°,

i.e. $W = \int_{0°}^{180°} \tau d\theta = \int_{0°}^{180°} pE \sin\theta d\theta = pE[-\cos\theta]_{0°}^{180°}$

$$= -pE(\cos 180° - \cos 0°) = 2pE$$

$$= 2 \times (2 \times 10^{-8})(1 \times 10^5) \text{ J} = 4 \times 10^{-3} \text{ J}$$

38. We know that,

$$E_{\text{axial}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2pr}{(r^2 - a^2)^2}$$

Case I When $r = 10$ cm $= 0.1$ m and

$E_{\text{axial}} = 3.75 \times 10^5$ N/C

$$\Rightarrow \quad 3.75 \times 10^5 = 9 \times 10^9 \times \frac{2p \times 0.1}{[(0.1)^2 - a^2]^2} \quad \text{...(i)}$$

Case II When $r = 20$ cm $= 0.2$ m and

$E_{\text{axial}} = 3 \times 10^4$ N/C

$$\Rightarrow \quad 3 \times 10^4 = 9 \times 10^9 \times \frac{2p \times 0.2}{[(0.2)^2 - a^2]^2} \quad \text{... (ii)}$$

Solving the Eqs. (i) and (ii), we get

$$a = 0.05 \text{ m}$$

Therefore, length of the dipole is $2a$.

So, $2a = 2 \times 0.05 = 0.1$ m

39. The electric field vector E_{1A} at A due to the positive charge q_1 points towards the right. Its magnitude,

$$E_{1A} = \frac{(9 \times 10^9 \text{N-m}^2\text{C}^{-2})(10^{-7}\text{C})}{(0.1 \text{ m})^2} = 9 \times 10^4 \text{N C}^{-1}$$

The electric field vector E_{2A} due to q_2 points to the right and has the same magnitude.

Hence, the magnitude of total electric field E_A at A,

$$E_A = E_{1A} + E_{2A} = 18 \times 10^4 \text{NC}^{-1}$$

i.e. E_A is directed towards right.

The electric field E_{1B} at B due to q_1 points towards the left and has a magnitude,

$$E_{1B} = \frac{(9 \times 10^9 \text{N-m}^2\text{C}^{-2})(10^{-7}\text{C})}{(0.1 \text{ m})^2} = 9 \times 10^4 \text{NC}^{-1}$$

The electric field E_{2B} at B due to the negative charge q_2 points towards the right and has a magnitude

$$E_{2B} = \frac{(9 \times 10^9 \text{ N-m}^2\text{C}^{-2})(10^{-7}\text{C})}{(0.3 \text{ m})^2} = 1 \times 10^4 \text{NC}^{-1}$$

The magnitude of the total electric field at B,

$$E_B = E_{1B} - E_{2B}$$

$$= 8 \times 10^4 \text{NC}^{-1}$$

i.e. E_B is directed towards the left.

The magnitude of each electric field vector at point C due to charge q_1 and q_2,

$$E_{1C} = E_{2C}$$

$$= \frac{(9 \times 10^9 \text{N-m}^2\text{C}^{-2})(10^{-7}\text{C})}{(0.2 \text{ m})^2}$$

$$= 2.25 \times 10^4 \text{ NC}^{-1}$$

The resultant of these two vectors,

$$\mathbf{E}_C = E_{1C} \cos\frac{\pi}{3} + E_{2C} \cos\frac{\pi}{3}$$

$$= 2.25 \times 10^4 \text{NC}^{-1}$$

i.e. E_C points towards the right.

40. (i) Here, $p = 6.2 \times 10^{-30}$ C-m and $E = 10^6$ N/C

$\therefore \tau = pE \sin\theta$ [for maximum value $\theta = 90°$]

$$= pE \sin 90° = 6.2 \times 10^{-30} \times 10^6 \times 1$$

$$= 6.2 \times 10^{-24} \text{ N-m}$$

(ii) When dipole is aligned anti-parallel to the field, $\theta = 180°$.

$\therefore W = pE(1 - \cos\theta)$

$= 6.2 \times 10^{-30} \times 10^6(1 - \cos 180°)$ [$\because \cos 180° = -1$]

$$= 6.2 \times 10^{-30} \times 10^6(1 - (-1))$$

$$= 6.2 \times 10^{-30} \times 10^6 \times 2$$

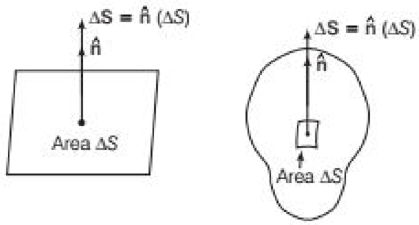
$$= 1.24 \times 10^{-23} \text{ J}$$

|TOPIC 4|
Electric Flux

AREA VECTOR

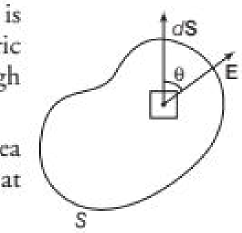
The vector associated with every area element of a closed surface is taken to be in the direction of the outward normal. Thus, the area element vector $\Delta \mathbf{S}$ at a point on a closed surface is equal to $\Delta S\ \hat{\mathbf{n}}$, where ΔS is the magnitude of the area element and $\hat{\mathbf{n}}$ is a unit vector in the direction of outward normal at the point.

$$\Delta \mathbf{S} = \hat{\mathbf{n}}(\Delta S)$$



ELECTRIC FLUX

Electric flux linked with any surface is defined as the total number of electric field lines that normally pass through that surface.



Electric flux $d\phi$ through a small area element $d\mathbf{S}$ due to an electric field $\mathbf{E}$ at an angle θ with $d\mathbf{S}$ is

$$d\phi = \mathbf{E} \cdot d\mathbf{S} = E\ dS\ \cos\theta$$

which is proportional to the number of field lines cutting the area element. Total electric flux ϕ over the whole surface S due to an electric field $\mathbf{E}$,

$$\phi = \oint_S \mathbf{E} \cdot d\mathbf{S} = \oint_S E\ dS\ \cos\theta$$

Electric flux is a scalar quantity. But it is a property of vector field.

SI unit of electric flux is N-m^{2}C^{-1}

and dimensional formula of electric flux is expressed as

$$\phi = [MLT^{-2}][L^2][AT]^{-1}$$
$$= [ML^3T^{-3}A^{-1}]$$

If $\oint \mathbf{E} \cdot d\mathbf{S}$ over a closed surface is negative, then the surface encloses a net negative charge.

Special cases

(i) For $0° < \theta < 90°$, ϕ is positive.

(ii) For $\theta = 90°$, ϕ is zero.

(iii) For $90° < \theta < 180°$, ϕ is negative.

> ***Analogy Between Electric Flux And Liquid Flux***
>
> It should be known that electric flux is analogous to flux of a liquid flowing across a plane, which is equal to $\mathbf{v} \cdot \Delta \mathbf{S}$, where $\mathbf{v}$ is the velocity of flow of liquid. In electric flux, there is no flow of a physically observable quantity like liquid.

EXAMPLE |1| A box encloses an electrical dipole consisting of charge $5\ \mu$C and $-5\ \mu$C and of length 10 cm. What is the total electric flux through the box?

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Sol. Since, an electric dipole consists of two equal and opposite charges, the net charge on the dipole is zero.

Hence, the net electric flux coming out of the closed surface of the box or through the box is zero.

GAUSS' THEOREM

Statement

The surface integral of the electric field intensity over any closed surface (called Gaussian surface) in free space is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed within the surface.

$$\phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\varepsilon_0} \sum_{i=1}^{n} q_i = \frac{q}{\varepsilon_0}$$

where, $q = \sum_{i=1}^{n} q_i$ is the algebraic sum of all the charges inside the closed surface.

Hence, total electric flux over a closed surface in vacuum is $\frac{1}{\varepsilon_0}$ times the total charge within the surface, regardless of how the charges may be distributed.

Proof of Gauss' Theorem for Spherically Symmetric Surface

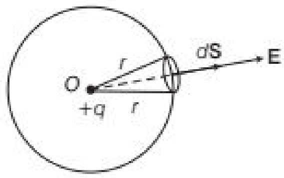
Electric flux through a surface element $d\mathbf{S}$ is given by

$$d\phi_E = \mathbf{E} \cdot d\mathbf{S} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \hat{\mathbf{r}} \cdot (dS\,\hat{\mathbf{n}})$$

$$\Rightarrow \quad d\phi_E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q\ dS}{r^2} \hat{\mathbf{r}} \cdot \hat{\mathbf{n}}$$

Here, $\hat{\mathbf{r}} \cdot \hat{\mathbf{n}} = 1 \cdot 1 \cos 0° = 1$

$$\therefore \quad d\phi_E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q\,dS}{r^2}$$



Total electric flux through the spherical surface,

$$\phi_E = \oint_S d\phi_E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \oint_S dS$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad \boxed{\phi_E = \frac{q}{\varepsilon_0}}$$

If the medium surrounding the charge has a dielectric constant K, then

$$\phi_E = \frac{q}{K\varepsilon_0} = \frac{q\varepsilon_0}{\varepsilon\varepsilon_0} = \frac{q}{\varepsilon}, \text{ where } K = \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

$$\therefore \quad \phi_E = \frac{q}{\varepsilon}$$

If there is no net charge within the closed surface, i.e. when $q = 0$, then $\phi_E = 0$.

$\therefore$ The total electric flux through a closed surface is zero, if no charge is enclosed by the surface.

Some Features of Gauss' Law

(i) Gauss' law is true for any closed surface, no matter what its shape or size be.

(ii) In the situation, when the surface is so chosen that there are some charges inside and some outside, the electric field is due to all the charges, both inside and outside the closed surface.

(iii) Gauss' law is often useful when the system has some symmetry. This is facilitated by the choice of a suitable Gaussian surface.

EXAMPLE |2| A charge q is placed at the centre of a cube of side l. What is the electric flux passing through each face of the cube? **All India 2012; Foreign 2010**

Sol. By Gauss' theorem, total electric flux linked with a closed surface is given by

$$\phi = \frac{q}{\varepsilon_0}$$

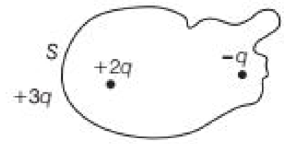
where, q is the total charge enclosed by the closed surface.

$\therefore$ Total electric flux linked with cube, $\phi = \frac{q}{\varepsilon_0}$.

As charge is at centre, therefore electric flux is symmetrically distributed through all 6 faces.

$\therefore$ Flux linked with each face $= \frac{1}{6}\phi = \frac{1}{6} \times \frac{q}{\varepsilon_0} = \frac{q}{6\varepsilon_0}$

EXAMPLE |3| Figure shows three point charges, $+2q$, $-q$ and $+3q$. Two charges $+2q$ and $-q$ are enclosed within a surface S. What is the electric flux due to this configuration through the surface S? **Delhi 2010**



Sol. Electric flux through the closed surface S is

$$\phi_S = \frac{\Sigma q}{\varepsilon_0} = \frac{+2q - q}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

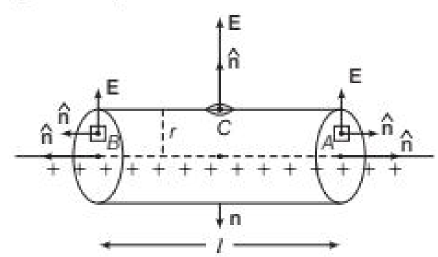
Charge $+3q$ is outside the closed surface S, therefore it would not be taken into consideration in applying Gauss' theorem.

Applications of Gauss' Theorem

The electric field due to some symmetric charge configurations can be obtained using Gauss' law.

Field due to an Infinitely Long Thin Straight Charged Wire

Consider an infinitely long thin straight wire with uniform linear charge density (λ).



From symmetry, the electric field is everywhere radial in the plane cutting the wire normally and its magnitude only depends on the radial distance (r).

From Gauss' law, $\phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$

Now, $\phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \oint_S \mathbf{E} \cdot \hat{\mathbf{n}}\, dS$

$$= \oint_A \mathbf{E} \cdot \hat{\mathbf{n}}\, dS + \oint_B \mathbf{E} \cdot \hat{\mathbf{n}}\, dS + \oint_C \mathbf{E} \cdot \hat{\mathbf{n}}\, dS$$

$$\therefore \quad \oint_S \mathbf{E} \cdot d\mathbf{S} = \oint_A \mathbf{E}\, d\mathbf{S} \cos 90° + \oint_B \mathbf{E}\, d\mathbf{S} \cos 90° + \oint_C \mathbf{E}\, d\mathbf{S} \cos 0°$$

$$= \oint_C E\, dS = E(2\pi r l)$$

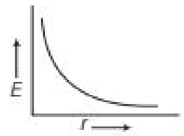
Charge enclosed in the cylinder, $q = \lambda l$

$$\therefore \quad E(2\pi r l) = \frac{\lambda l}{\varepsilon_0} \quad \text{or} \quad \boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

Vectorially, $\mathbf{E} = \dfrac{\lambda}{2\pi\varepsilon_0 r}\hat{\mathbf{r}}$

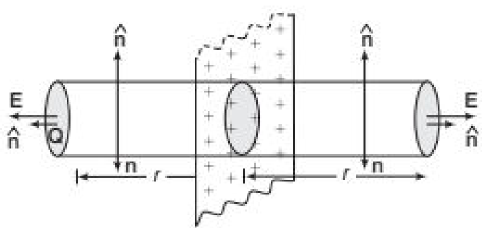
The direction of the electric field is radially outward from the positive line charge. For negative line charge, it will be radially inward.

Thus, electric field (E) due to the linear charge is inversely proportional to the distance (r) from the linear charge. The variation of electric field (E) with distance (r) is shown in figure.



Field due to a Thin Infinite Plane Sheet of Charge

Let σ be the surface charge density of the sheet. From symmetry, E on either side of the sheet must be perpendicular to the plane of the sheet, having same magnitude at all points equidistant from the sheet.



We take a cylindrical cross-sectional area A and length $2r$ as the Gaussian surface.

On the curved surface of the cylinder, E and $\hat{\mathbf{n}}$ are perpendicular to each other. Therefore, flux through curved surface = 0.

Flux through the flat surfaces = $EA + EA = 2EA$

$\therefore$ Total electric flux over the entire surface of cylinder,

$$\phi_E = 2EA$$

Total charge enclosed by the cylinder, $q = \sigma A$

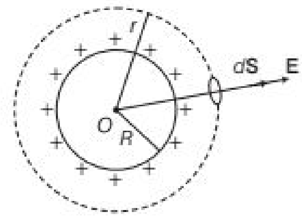
According to Gauss' law, $\phi_E = \dfrac{q}{\varepsilon_0}$

$$\therefore \quad 2EA = \frac{\sigma A}{\varepsilon_0} \quad \text{or} \quad \boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

E is independent of r, the distance of the point from the plane charged sheet.

Field due to a Uniformly Charged Thin Spherical Shell

Let σ be the uniform surface charge density of a thin spherical shell of radius (R). The Gaussian surface will be a spherical surface centered at the centre of shell.



(i) **At a point outside the shell** ($r > R$)

Since, E and $d\mathbf{S}$ are in the same direction.

$$\therefore \quad \phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0} \text{ or } E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$\therefore \quad \boxed{E = \frac{q}{4\pi\varepsilon_0 r^2}}$$

Since, $\quad q = \sigma \times 4\pi R^2$

$$\therefore \quad E = \frac{\sigma R^2}{\varepsilon_0 r^2}$$

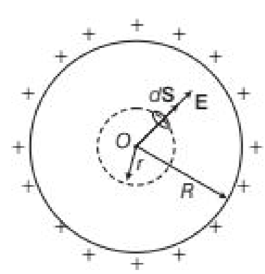
Vectorially, $\quad \mathbf{E} = \dfrac{\sigma R^2}{\varepsilon_0 r^2}\hat{\mathbf{r}}$

(ii) **At a point on the surface of the shell** ($r = R$)

$$\boxed{E = \frac{q}{4\pi\varepsilon_0 R^2}}$$

and $\quad E = \dfrac{\sigma}{\varepsilon_0}$

(iii) **At a point inside the shell** ($r < R$)



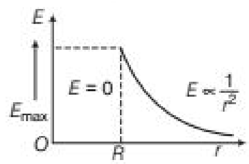
Here, the charge inside the Gaussian surface shell.

$$q = 0$$

$$\therefore \quad E(4\pi r^2) = 0$$

$\therefore \qquad E = 0$

This important result is a direct consequence of Gauss' law which follows from Coulomb's law. The experimental verification of this result confirms $1/r^2$ dependence in Coulomb's law. The variation of electric field intensity (E) with distance from the centre of a uniformly charged spherical shell is shown in figure.

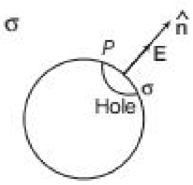


EXAMPLE |4| A hollow charged conductor has a tiny hole cut into its surface. Show that the electric field in the hole is $\left(\frac{\sigma}{2\varepsilon_0}\right)\hat{n}$, where $\hat{n}$ is the unit vector in the outward normal direction and σ is the surface charge density near the hole. **NCERT**

Sol. Surface charge density near the hole = σ

Unit vector = $\hat{n}$ (normal directed outwards)

Let P be the point on the hole. The electric field at point P closed to the surface to conductor, according to Gauss' theorem,



$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

where, q is the charge near the hole.

$$EdS \cos\theta = \frac{\sigma dS}{\varepsilon_0}$$

$[\because \sigma = q/dS \Rightarrow q = \sigma dS$, where dS = area$]$

$\therefore$ Angle between electric field and area vector is 0°.

$$\therefore \quad E\,dS = \frac{\sigma\,dS}{\varepsilon_0} \qquad [\because \cos 0° = 1]$$

$$\Rightarrow \quad E = \frac{\sigma}{\varepsilon_0} \Rightarrow \mathbf{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

This electric field is due to the filled up hole and the field due to the rest of the charged conductor. The two fields inside the conductor are equal and opposite.

So, there is no electric field inside the conductor. Outside the conductor, the electric fields are equal in the same direction.

So, the electric field at point P due to each part

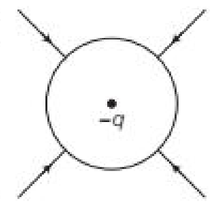
$$= \frac{1}{2}\mathbf{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$$

EXAMPLE |5| A point charge causes an electric flux -3×10^{-14} N - m^2/C to pass through a spherical Gaussian surface.

(i) Calculate the value of the point charge.

(ii) If the radius of the Gaussian surface is doubled, how much flux would pass through the surface? **Foreign 2008**

Sol. (i) By Gauss' theorem, total electric flux through closed Gaussian surface is given by



$$\phi = \frac{q}{\varepsilon_0}$$

$$\therefore \quad q = \phi\varepsilon_0$$

But electric flux passing through the surface,

$$\phi = -3 \times 10^{-14} \text{ N-m}^2/\text{C}$$

$$\therefore \quad q = (-3 \times 10^{-14}) \times 8.85 \times 10^{-12} = -26.55 \times 10^{-26} \text{ C}$$

$$= -2.655 \times 10^{-25} \text{ C}$$

(ii) Electric flux passing through the surface remains unchanged because it depends only on charge enclosed by the surface and is independent of its size.

Note Electric flux, Gauss's law and numericals based on them have been frequently asked in previous years 2015, 2014, 2013, 2012, 2011, 2010.

| TOPIC PRACTICE 4 |

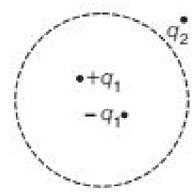
OBJECTIVE Type Questions

1. The SI unit of electric flux is

(a) $\frac{\text{volt}}{\text{metre}}$ (b) $\frac{\text{newton}}{\text{coulomb}}$

(c) $\frac{\text{newton} \times \text{metre}^2}{\text{coulomb}}$ (d) volt × metre^2

2. Consider the charge configuration and spherical Gaussian surface as shown in the figure. When calculating the flux of the electric field over the spherical surface, the electric field will be due to



(a) q_2 (b) only the positive charges

(c) all the charges (d) $+q_1$ and $-q_1$

3. Total electric flux coming out of a unit positive charge put in air is

(a) ε_0 (b) ε_0^{-1}

(c) $(4p\varepsilon_0)^{-1}$ (d) $4\pi\varepsilon_0$

4. In a system, 'n' electric dipole are placed in a closed surface. The value of emergent electric flux from enclosed surface is

(a) $\frac{q}{\varepsilon_0}$ (b) $\frac{2q}{\varepsilon_0}$ (c) $-\frac{2q}{\varepsilon_0}$ (d) zero

5. The intensity of electric field at the surface of conducting hollow sphere is $10\,NC^{-1}$ and its radius is 10 cm. The value of electric field at the centre of sphere is

(a) zero (b) $10\ NC^{-1}$ (c) $1\ NC^{-1}$ (d) $100\ NC^{-1}$

6. The surface densities on the surfaces of two charged spherical conductors of radii R_1 and R_2 are equal. The ratio of electric intensities on the surfaces are

(a) $R_1^2\ /R_2^2$ (b) $R_2^2\ /R_1^2$ (c) $R_1\ /R_2$ (d) $1:1$

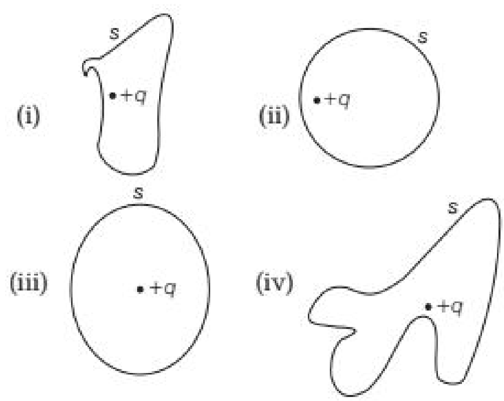
7. The electric flux in a charged spherical conductor is

(a) zero inside and outside the sphere
(b) maximum inside the sphere and zero outside the sphere
(c) zero inside the sphere and decreases outside the sphere with increase of square of distance
(d) maximum inside the sphere and decreases outside the sphere with increase of distance.

8. Radius of a hollow sphere is R and a charge q is placed at the centre of hollow sphere. If the radius of sphere becomes half and charge also becomes half, then the value of emergent total flux from the surface of sphere is

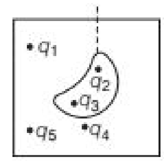
(a) $4\,q/\varepsilon_0$ (b) $2\,q/\varepsilon_0$ (c) $q/2\varepsilon_0$ (d) q/ε_0

9. The electric flux through the surface



(a) in Fig. (iv) is the largest
(b) in Fig. (iii) is the least
(c) in Fig. (ii) is same as Fig. (iii) but is smaller than Fig. (iv)
(d) is the same for all the figures

10. Five charges q_1, q_2, q_3, q_4, and q_5 are fixed at their positions as shown in Figure, S is a Gaussian surface. The Gauss' law is given by $\int_s E \cdot dS = \frac{q}{\varepsilon_0}$. Which of the following statements is correct?



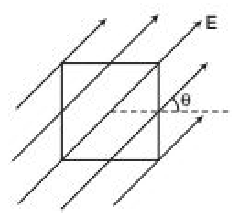
(a) **E** on the LHS of the above equation will have a contribution from q_1, q_5 and q_1, q_5 and q_3 while q on the RHS will have a contribution from q_2 and q_4 only
(b) **E** on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_2 and q_3 only
(c) **E** on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_1, q_3 and q_5 only
(d) Both **E** on the LHS and q on the RHS will have contributions from q_2 and q_4 only

VERY SHORT ANSWER Type Questions

11. Can Gauss' law in electrostatics tell us exactly, where the charge is located within the Gaussian surface?

12. An arbitrary surface encloses a dipole. What is the electric flux through this surface? **NCERT Exemplar**

13. A square surface of side l metres in the plane of paper is placed in a uniform electric field **E** acting along the same plane at an angle θ with the horizontal side of square as shown in the figure. What is the electric flux linked to the surface?



14. What is the net flux of the uniform electric field through a cube of side 20 cm oriented, so that its faces are parallel to the coordinate planes? **NCERT**

15. What is the number of electric field lines that radiate outward from one coulomb of charge in vacuum?

16. Does the strength of electric field due to an infinite long line charge depend upon the distance of the observation point from the line charge?

17. How does electric field at a point charge vary with distance r from an infinitely long charged wire?

18. Does the strength of electric field due to an infinite plane sheet of charge depend upon the distance of the observation point from the sheet of charge? **Delhi 2010**

19. How does the electric flux due to a point charge enclosed by a spherical Gaussian surface get affected when its radius is increased? **Delhi 2016**

20. Two charges of magnitudes $-2Q$ and $+Q$ are located at points $(a, 0)$ and $(4a, 0)$, respectively. What is the electric flux due to these charges through a sphere of radius $3a$ with its centre at the origin? **All India 2013**

21. What is the electric flux through a cube of side 1 cm which encloses an electric dipole? **All India 2015**

22. (i) A charge q is placed at the centre of a cube. What is the electric flux passing through each face of cube?

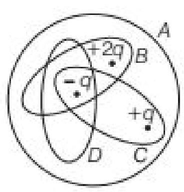
(ii) If radius of Gaussian surface enclosing some charge q is halved, then how does electric flux through Gaussian surface change?

SHORT ANSWER Type Questions

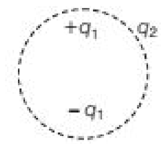
23. If the total charge enclosed by a surface is zero, does it imply that the electric field everywhere on the surface is zero, conversely, if the electric field everywhere on the surface is zero? Does it imply the net charge inside is zero? **NCERT Exemplar**

24. A charge q is enclosed by a spherical surface of radius R. If the radius is reduced to half, how would the electric flux through the surface change?

25. Rank the Gaussian surfaces as shown in the figure. In order of increasing electric flux, starting with the most negative.



26. Consider the charge configuration and a spherical Gaussian surface as shown in the figure.



Which one of the three charges will be the cause of electric field while calculating the flux of the field over the spherical surface?

27. Deduce Coulomb's law from Gauss' law.

28. What will be the electric field intensity at the centre of a uniformly charged circular wire of linear charge density?

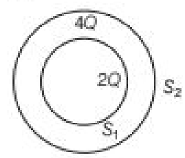
29. A thin straight infinitely long conducting wire having charge density λ is enclosed by a cylindrical surface of radius r and length l, its axis coinciding with the length of the wire. Find the expression for the electric flux through the surface of the cylinder. **All India 2011**

30. A hemispherical body is placed in a uniform electric field E. What is the flux associated with the curved surface, if field is

(i) parallel to base?

(ii) perpendicular to base?

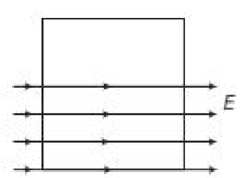
31. Consider two hollow concentric spheres S_1 and S_2 enclosing charges $2Q$ and $4Q$ respectively, as shown in the figure.



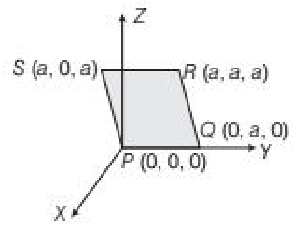
(i) Find out the ratio of the electric flux through them.

(ii) How will the electric flux through the sphere S_1 change, if a medium of dielectric constant ε_r is introduced in the space inside S_1 in place of air? Deduce the necessary expression.

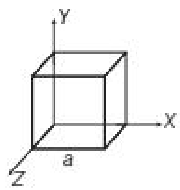
32. A square surface of side l metre is in the plane of paper. A uniform electric field E (volt/metre), also in the plane of the paper, is limited only to the lower half of the square surface, (see figure). What is the electric flux associated with this surface?



33. Consider an electric field $\mathbf{E} = E_0\,\hat{x}$, where E_0 is a constant. What is the flux through the shaded area (as shown in figure) due to this field?

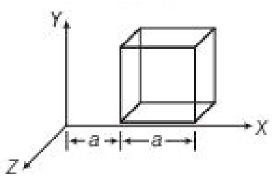


34. Given the electric field in the region $\mathbf{E} = 2x\hat{\mathbf{i}}$, find the net electric flux through the cube and the charge enclosed by it. **All India 2015**



LONG ANSWER Type I Questions

35. State Gauss' law in electrostatics. A cube with each side a is kept in an electric field given by $\mathbf{E} = Cx\hat{\mathbf{i}}$ as shown in the figure, where C is a positive dimensionless constant.



Find out

(i) the electric flux through the cube and

(ii) the net charge inside the cube. **Foreign 2012**

36. Use Gauss' law to derive the expression for the electric field between two uniformly charge parallel, sheets with surface charge densities σ and $-\sigma$, respectively. **All India 2009**

37. Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is 8.0×10^3 N-m^{2}C^{-1}.

(i) What is the net charge inside the box?

(ii) If the net outward flux through the surface of the box was zero, could you conclude that there were no charges inside the box. Why or why not? **NCERT**

LONG ANSWER Type II Questions

38. (i) Define electric flux. Write its SI unit. Gauss' law in electrostatics is true for any closed surface, no matter what its shape or size is. Justify this statement with the help of a suitable example.

(ii) Use Gauss' law to prove that the electric field inside a uniformly charged spherical shell is zero. **Delhi 2015**

39. (i) State Gauss' theorem.

(ii) Using Gauss' law, prove that the electric field at a point due to a uniformly charged infinite plane sheet is independent of the distance from it.

(iii) How is the field directed, if

(a) the sheet is positively charged,

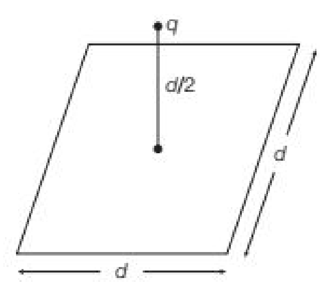
(b) negatively charged? **Delhi 2012**

40. (i) Use Gauss' theorem to find the electric field due to a uniformly charged infinitely large plane thin sheet with surface charge density σ.

(ii) An infinitely large thin plane sheet has a uniform surface charge density $+\sigma$. Obtain the expression for the amount of work done in bringing a point charge q from infinity to a point, distant r, in front of the charged plane sheet. **All India 2017**

41. (a) Define electric flux. Is it a scalar or a vector quantity?

A point charge q is at a distance of $d/2$ directly above the centre of a square of side d, as shown in the figure. Use Gauss' law to obtain the expression for the electric flux through the square. **CBSE 2018**

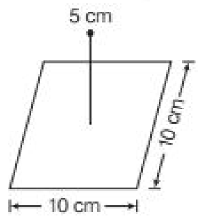


(b) If the point charge is now moved to a distance d from the centre of the square and the side of the square is doubled, explain how the electric flux will be affected.

NUMERICAL PROBLEMS

42. An infinite line charge produces a field of 9×10^4 N/C at a distance of 2 cm. Calculate the linear charge density. **NCERT**

43. A point charge $+10\,\mu C$ is at a distance 5 cm directly above the centre of a square of side 10 cm, as shown in figure. What is the magnitude of the electric flux through the square? **NCERT**



44. A point charge of 2.0 μC is at the centre of a cubic Gaussian surface 9.0 cm on edge. What is the net electric flux through the surface? **NCERT**

45. Consider a uniform electric field $\mathbf{E} = 3 \times 10^3 \,\hat{\mathbf{i}}$ N/C.

(i) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the YZ-plane?

(ii) What is the flux through the same square, if the normal to its plane makes an angle 60° with the X-axis? **NCERT**

46. Given a uniform electric field $\mathbf{E} = 5 \times 10^3 \,\mathbf{i}$ N/C, find the flux of this field through a square of 10 cm on a side whose plane is parallel to the YZ-plane. What would be the flux through the same square, if the plane makes an angle of 30° with the X-axis? **Delhi 2014**

47. A uniformly charged conducting sphere of diameter 2.4 m has a surface charge density of $80.0\,\mu C/m^2$.

(i) Find the charge on the sphere.

(ii) What is the total electric flux leaving the surface of the sphere? **NCERT**

48. A hollow cylindrical box of length 1 m and area of cross-section 25 cm^2 is placed in a three dimensional coordinate system as shown in the figure. The electric field in the region is given by $\mathbf{E} = 50\,x\,\hat{\mathbf{i}}$, where E is in NC^{-1} and x is in metre.

Find

(i) net flux through the cylinder and

(ii) charge enclosed by the cylinder.

Delhi 2014

49. A uniform electric field is given as $\mathbf{E} = 100\,\hat{\mathbf{i}}$ N/C for $x > 0$ and $\mathbf{E} = 100\,\hat{\mathbf{i}}$ N/C for $x < 0$. A right circular cylinder of length 20 cm and radius 5 cm has its centre at the origin and its axis along the X-axis, so that one face is at $x = +10$ cm and other is at $x = -10$ cm.

(i) What is the net outward flux through each flat face?

(ii) What is the flux through the side of cylinder?

(iii) What is the net outward flux through the cylinder?

(iv) What is the net charge inside the cylinder?

50. Two large thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude $17.0 \times 10^{-22} Cm^{-2}$. What is E

(i) to the left of the plates,

(ii) to the right of the plates and

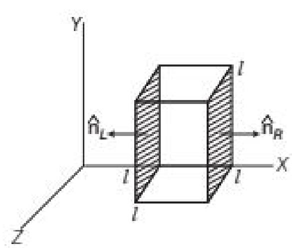
(iii) in between the plates? **NCERT**

51. A point charge causes an electric flux of -1.0×10^3 N-m^2/C to pass through a spherical Gaussian surface of 10.0 cm, radius centred on the charge.

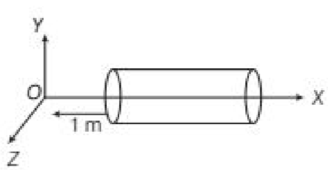
(i) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface?

(ii) What is the value of point charge? **NCERT**

52. The electric field components in given figure are $E_X = \alpha x^{1/2}$, $E_Y = E_Z = 0$, in which $\alpha = 600$ N/ C-$m^{1/2}$.



Calculate (i) the flux through the cube and (ii) the charge within the cube. Assume that



Calculate (i) the flux through the cube and (ii) the charge within the cube. Assume that
$l = 0.1$ m.

HINTS AND SOLUTIONS

1. (c) The SI unit of electric flux is $N{\cdot}m^2\,C^{-1}$.

2. (c) As flux $= \frac{q_{\text{enclosed}}}{\varepsilon_0}$. So, flux is due to only charges $+q_1$ and $-q_1$ that makes a sum zero. But q_2 produces its own flux and net flux linked with sphere is zero. Electric field will be due to all the charges.

3. (b) By Gauss' law, ϕ = Electric flux through closed surface area

$$= \frac{q_{\text{enclosed}}}{\varepsilon_0}, \text{ if } q_{\text{enclosed}} = 1 \text{ unit}$$

$$\Rightarrow \qquad \phi = \frac{1}{\varepsilon_0} = \varepsilon_0^{-1}$$

4. (d) According to the definition of an electric dipole, net charge in enclosed surface $= +q - q = 0$
Hence, electric flux,

$$\phi_E = \frac{q}{\varepsilon_0} = 0$$

5. (a) Inside the conducting sphere, electric field at every point is zero.

6. (d) Intensity of electric field on the surface of conducting sphere, $(E = \sigma/\varepsilon_0)$.
Since, both charged spheres have same surface charge density, so according to Gauss' theorem, these have same electric intensity *i.e.*, the ratio is $1:1$.

7. (c) Electric flux is zero inside of spherical conductor and outside is $E \propto (1/r^2)$ and decreases outside the sphere with increase of distance.

8. (c) According to Gauss' law, emergent flux, $\phi = \frac{q}{\varepsilon_0}$.

If charge becomes half, then the value of charge in the surface is $q' = \frac{q}{2}$, so $\phi = \frac{q}{2\varepsilon_0}$.

9. (d) Gauss' law of electrostatics state that the total of the electric flux out of a closed surface is equal to the charge enclosed decided by the permittivity *i.e.*, Q electric $= \frac{Q}{\varepsilon_0}$.

Thus, electric flux through a surface doesn't depend on the shape, size or area of a surface but it depends on the number of charges enclosed by the surface.
So, here in this question, all the figures have same electric flux as all of them has single positive charge.

10. (b) According to Gauss' law, the term q on the right side of the equation $\int_s \mathbf{E}\cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$ includes the sum of all charges enclosed by the surface.

The charges may be located anywhere inside the surface, if the surface is so chosen that there are some charges inside and some outside, the electric field on the left side of equation is due to all the charges, both inside and outside S. So, E on LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_2 and q_3 only.

11. No, it tells us only about the magnitude of charge enclosed by the Gaussian surface.

12. If any arbitrary surface encloses a dipole, then the net charge is zero because the total charge on the dipole is zero (dipole consists of two equal and opposite charges).
According to Gauss' law,

total flux $= \frac{1}{\varepsilon_0} \times$ charge enclosed $= \frac{1}{\varepsilon_0} \times (0) = 0 \Rightarrow \phi = 0$

13. Since, E is acting along the same plane at an angle θ as shown in the figure, therefore electric flux,
$\phi = EA\cos\theta$, where θ is angle between E and normal to surface, i.e.

$$\theta = 90^\circ \Rightarrow \phi = EA\cos 90^\circ = 0$$

14. As we know that, the number of field lines entering in the cube is the same as that the number of field lines leaving the cube. So, no flux is remained on the cube and hence, the net flux over the cube is zero.

15. Number of electric field lines $= \frac{1}{\varepsilon_0} = \frac{1}{8.85\times 10^{-12}}$
$= 1.13\times 10^{11}$ $\quad [\because q = 1\text{ C}]$

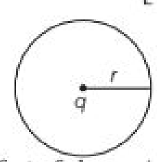
16. Yes, the electric field due to an infinitely long line charge depends upon the distance of the observation point from the line charge.

17. The electric field due to a line charge falls of with distance as $1/r$.

18. No, the electric field due to an infinite plane sheet of charge does not depend upon the distance of the observation point from the plane sheet of charge.

19. According to question, electric flux (ϕ) due to a point charge enclosed by a spherical Gaussian surface is given by

$$\phi = \mathbf{E}\cdot\mathbf{A}$$
$$= \frac{kq}{r^2}\cdot 4\pi r^2 = kq\cdot 4\pi \qquad \left[\because E = \frac{kq}{r^2} \text{ and } A = 4\pi r^2\right]$$

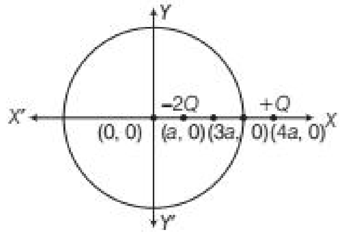


So, there is no effect of change in radius on the electric flux.

20. Gauss' theorem states that the total electric flux linked with closed surface S,

$$\phi_E = \mathbf{E}\cdot d\mathbf{S} = \frac{q}{E_0}$$

where, q is the total charge enclosed by the closed Gaussian (imaginary) surface.



The sphere enclosed charge = $-2Q$

Therefore, $\phi = \frac{2Q}{\varepsilon_0}$ (inwards)

21. Since, according to the Gauss' law of electrostatics, electric flux through any closed surface is given by

$$\phi_E = \oint E \cdot dS = \frac{q}{\varepsilon_0} \quad ...(i)$$

where, E = electrostatic field,

q = total charge enclosed by the surface

and ε_0 = absolute electric permittivity of free space.

So, in the given case, cube encloses an electric dipole.

Therefore, the total charge enclosed by the cube is zero.

i.e. $q = 0$

Therefore, from Eq. (i), we get

$$\phi_E = \frac{q}{\varepsilon_0} = 0$$

i.e. Electric flux is zero.

22. (i) Flux through each face

$$= \frac{1}{6}\left(\frac{q}{\varepsilon_0}\right) = \frac{q}{6\varepsilon_0}$$

(ii) Electric flux, $\phi = \frac{q}{6\varepsilon_0}$, since q does not change, ϕ will remain same.

23. No, since $\oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0} = 0$, therefore the field may be normal to the surface.

However, the reverse is true, i.e. when $E = 0$ everywhere on the surface, the net charge inside is zero.

24. We know that, $\phi_E = \frac{Q_{enclosed}}{\varepsilon_0}$

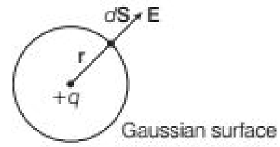
Here, $Q_{enclosed}$ remains unchanged. Hence, electric flux through the surface remains same.

25. Since, surface D enclosed negative charge, hence it has least flux negative.

In parts C and A, there is zero net charge, hence flux is zero, surface B has most flux, which is positive in nature, since it consist positive charge, i.e. $+2q$.

26. Refer to Example 3 on page 37.

27. According to Gauss' theorem,



$$\int_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0} \Rightarrow E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\therefore \quad E = \frac{q}{4\pi\varepsilon_0 r^2}$$

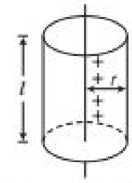
If a charge q_0 is kept on the surface, then

$F = E \times q_0 = \frac{qq_0}{4\pi\varepsilon_0 r^2}$, which is Coulomb's law.

28. A uniformly charged circular wire can be considered to be subdivided into pairs of diametrically opposite elements. The electric field intensity at the centre of wire due to each of the pairs is zero, therefore the electric field intensity due to the entire circular wire will be zero.

29. **Hints:** A thin straight conducting wire will have a uniform linear charge distribution.

Let q charge be enclosed by the cylindrical surface.



$\therefore$ Linear charge density, $\lambda = \frac{q}{l} \Rightarrow q = \lambda l \quad ...(i)$

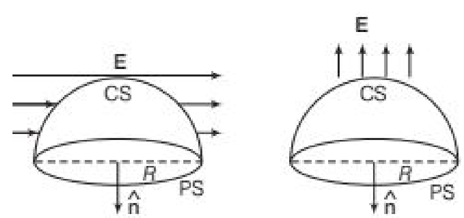
By Gauss' theorem,

total electric flux through the surface of cylinder,

$$\phi = \frac{q}{\varepsilon_0}$$

$$\therefore \quad \phi = \frac{\lambda l}{\varepsilon_0} \quad \text{[from Eq. (i)]}$$

30. Considering the hemispherical body as a closed body with a Curved Surface (CS) and a Plane Surface (PS), the total flux (ϕ) linked with the body will be zero, as no charge is enclosed by the body.



$$\therefore \quad \phi = \phi_{CS} + \phi_{PS} = 0 \quad ...(i)$$

(i) When field is parallel to the base,

$$\phi_{PS} = E \times \pi R^2 \cos 90° = 0$$

From Eq. (i), we get

$$\phi = \phi_{cs} = 0$$

(ii) When field is perpendicular to the base,

$$\phi_{PS} = E \times \pi R^2 \cos 180°$$
$$= -E\pi R^2$$

From Eq. (i), we get

$$\phi_{cs} - E\pi R^2 = 0$$

$$\Rightarrow \quad \phi_{CS} = E\pi R^2$$

31. (i) According to Gauss' theorem,

$$\phi = \frac{\Sigma q}{\varepsilon_0 \varepsilon_r} \propto \Sigma q$$

$$\therefore \quad \frac{\phi_{S_1}}{\phi_{S_2}} = \frac{2Q}{2Q + 4Q} = \frac{2Q}{6Q} = \frac{1}{3}$$

(ii) If a medium of dielectric constant ε_r is introduced in the space inside S_1 in place of air, then

$$\phi_{S_1} = \frac{\Sigma q}{\varepsilon_0 \varepsilon_r} = \frac{2Q}{\varepsilon_0 \varepsilon_r}$$

32. Electric flux ϕ is a measure of number of field lines crossing a surface. The number of field lines passing through unit area (N/S) will be proportional to the electric field, i.e. $N/S \propto E$

$$\Rightarrow \quad N \propto ES$$

The quantity ES is the electric flux through surface S. As in the given question, the field lines that enter the closed surface leave the surface immediately, so the net electric flux is bound to the system. Thus, electric flux is zero.

33. We have, $\mathbf{E} = E_0 \hat{\mathbf{x}}$

Consider $\hat{\mathbf{x}}, \hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ be the unit vectors along X, Y and Z-axes, respectively.

In the figure, shaded area, $\mathbf{A} = \mathbf{PQ} \times \mathbf{PS}$

$$\therefore \quad \mathbf{A} = (0\hat{\mathbf{x}} + a\hat{\mathbf{y}} + 0\hat{\mathbf{z}}) \times (a\hat{\mathbf{x}} + 0\hat{\mathbf{y}} + a\hat{\mathbf{z}}) = a^2\hat{\mathbf{x}} - a^2\hat{\mathbf{z}}$$

$\therefore$ Electric flux through the shaded area is given by

$$\phi = \mathbf{E} \cdot \mathbf{A} = (E_0\hat{\mathbf{x}}) \cdot (a^2\hat{\mathbf{x}} - a^2\hat{\mathbf{z}}) = E_0 a^2$$

34. Since, the electric field has only x-component, for faces normal to x-direction, the angle between E and ΔS is $\pm \frac{\pi}{2}$. Therefore, the flux is separately zero for each of the cube except the shaded ones.

The magnitude of the electric field at the left face is

$E_L = 0$ [as $x = 0$ at the left face]

The magnitude of the electric field at the right face is

$E_R = 3a$ [as, $x = a$ at the right face]

The corresponding fluxes are

$$\phi_L = \mathbf{E}_L \cdot \Delta \mathbf{S} = 0$$

$$\phi_L = \mathbf{E}_L \cdot \Delta \mathbf{S} = 0$$

and $\phi_R = \mathbf{E}_L \cdot \Delta \mathbf{S}$

$$= E_R \Delta S \cos\theta = E_R \Delta S \qquad [\because \theta = 0°]$$

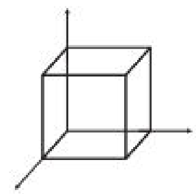
$$\Rightarrow \phi_R = E_R a^2$$

Net flux (ϕ) through the cube

$$= \phi_L + \phi_R = 0 + E_R a^2 = E_R a^2$$

$$\Rightarrow \quad q = 2a\,(a)^2 = 2a^3$$

We can use Gauss' law to find the total charge q inside the cube.

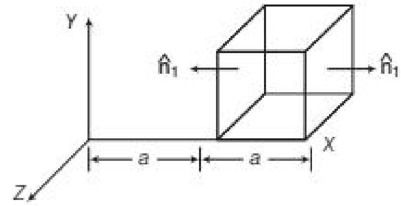


$$\phi = \frac{q}{\varepsilon_0} \Rightarrow \phi = \frac{2a^3}{\varepsilon_0}$$

35. Refer to text on page 36.

(i) Now, the electric field, $\mathbf{E} = Cx\,\hat{\mathbf{i}}$ is in x-direction only.

So, face with surface normal vector perpendicular to this field would give zero electric flux, i.e. $\phi = E\,dS \cos 90° = 0$, through it.



So, flux would be across only two surfaces.

Magnitude of E at left face,

$E_L = Cx = Ca$ [$x = a$ at left face]

Magnitude of E at right face,

$E_R = Cx = C\,2a = 2aC$ [$x = 2a$ at right face]

Thus, corresponding fluxes are

$$\phi_L = \mathbf{E}_L \cdot d\mathbf{S} = E_L dS \cos\theta$$
$$= -aC \times a^2 = -a^3C \qquad [\because \theta = 180°]$$

and

$$\phi_R = \mathbf{E}_R \cdot d\mathbf{S}$$
$$= 2aC\,dS\cos\theta \qquad [\because \theta = 0°]$$
$$= 2aCa^2 = 2a^3C$$

Now, net flux through cube

$$= \phi_L + \phi_R$$
$$= -a^3C + 2a^3C$$
$$= a^3C \text{ N-m}^2\text{C}^{-1}$$

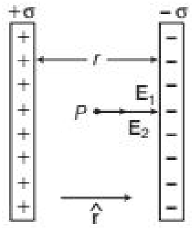
(ii) Net charge inside the cube, again we can use Gauss' law to find total charge q inside the cube.

We have, $\phi = \frac{q}{\varepsilon_0}$ or $q = \phi\varepsilon_0$

$$\Rightarrow \quad q = a^3 C \varepsilon_0$$

36. Let us consider two uniformly charge parallel sheets carrying surface charge densities $+\sigma$ and $-\sigma$ respectively and are separated by a small distance from each other.

By Gauss' law, it can be proved that, electric field intensity due to a uniformly charged infinite plane sheet as nearby is given by, $E = \dfrac{\sigma}{2\varepsilon_0}$



The electric field is directed normally outward from the plane sheet, if nature of charge on sheet is positive and normally inward, if charge is of negative nature. Let $\hat{\mathbf{r}}$ represents unit vector directed from positive plate to negative plate.

Now, electric field intensity (EFI) at any point P between the two plates is given by

(i) $\mathbf{E}_1 = +\dfrac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}}$ [due to positive plate]

(ii) $\mathbf{E}_2 = +\dfrac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}}$ [due to negative plate]

$\therefore$ New EFI at point P,

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}} + \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{r}}$$

Thus, uniform electric field is produced between the two infinite parallel plane sheet of charge which is directed from positive plate to negative plate.

37. Here, $\phi = 8.0 \times 10^3$ N-m^{2}C^{-1}

(i) Suppose that the net charge inside the box is q, then according to Gauss' theorem,

$$\phi = \frac{q}{\varepsilon_0} \text{ or } q = \varepsilon_0 \phi$$

$\because$ $\varepsilon_0 = 8.854 \times 10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2}$

$\therefore$ $q = 8.854 \times 10^{-12} \times 8.0 \times 10^3$

$= 70.832 \times 10^{-9}$C

(ii) If the net outward flux through the surface of the box is zero, then it cannot be concluded that there is no charge inside the box. There may be equal amount of positive and negative charges inside the box. Therefore, if the net outward flux is zero, we cannot conclude that the charge inside the box is zero.

One can only say that the net charge inside the box is zero.

38. (i) Electric flux over an area in an electric field represents the total number of electric field lines crossing the area. The SI unit of electric flux is N-m^{2}C^{-1}.

According to Gauss' law in electrostatics, the surface integral of electrostatic field **E** produced by any source over any closed surface S enclosing a volume V in vacuum, i.e. total electric flux over the closed surface S in vacuum, is $1/\varepsilon_0$ times the total charge (q) contained inside S, i.e.

$$\phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

Gauss' law in electrostatics is true for a closed surface, no matter what its shape or size is.

So, in order to justify the above statement, suppose in isolated positive charge q is situated at the centre O of a sphere of radius r.

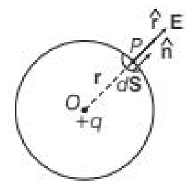
According to Coulomb's law, electric field intensity at any point P on the surface of the sphere is

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0} \cdot \frac{\hat{\mathbf{r}}}{r^2}$$

where, $\hat{\mathbf{r}}$ is unit vector directed from O to P.

Consider a small area element $d\mathbf{S}$ of the sphere around P. Let it be represented by the vector $d\mathbf{S} + \hat{\mathbf{n}} \cdot d\mathbf{S}$.

where, $\hat{\mathbf{n}}$ is unit vector along out drawn normal to the area element.



$\therefore$ Electric flux over the area element,

$$d\phi_E = \mathbf{E} \cdot d\mathbf{S}$$

$$= \left(\frac{q}{4\pi\varepsilon_0} \cdot \frac{\hat{\mathbf{r}}}{r^2}\right) \cdot (\hat{\mathbf{n}} \cdot dS)$$

$$\mathbf{E} \cdot d\mathbf{S} = \frac{q}{4\pi\varepsilon_0} \cdot \frac{dS}{r^2} \cdot \hat{\mathbf{r}} \cdot \hat{\mathbf{n}}$$

As normal to a surface of every point is along the radius vector at that point, therefore $\hat{\mathbf{r}} \cdot \hat{\mathbf{n}} = 1$

$$\mathbf{E} \cdot d\mathbf{S} = \frac{q}{4\pi\varepsilon_0} \cdot \frac{dS}{r^2}$$

Integrating over the closed surface area of the sphere, we get total normal electric flux over the entire sphere,

$$\phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{4\pi\varepsilon_0 r^2} \oint_S dS$$

$$= \frac{q}{4\pi\varepsilon_0 r^2} \times \text{total area of surface of sphere.}$$

$$= \frac{q}{4\pi\varepsilon_0 r^2}(4\pi r^2) = \frac{q}{\varepsilon_0}$$

Hence, $\oint_S \mathbf{E} \cdot d\mathbf{S} = \dfrac{q}{\varepsilon_0}$, which proves Gauss' theorem.

(ii) **Electric field inside a uniformly charged spherical shell**

Refer to text on pages 38 and 39.

39. (i) Refer to text on page 36.

(ii) and (iii) Refer to text on page 38.

40. (i) Refer to text on page 38.

(ii) Surface charge density of the uniform plane sheet which is infinitely large $=+\sigma$. The electric potential (V) due to infinite sheet of a uniform charge density $+\sigma$,

$$V=\frac{-\sigma r}{2\varepsilon_0}$$

The amount of work done in bringing a point charge q from infinite to point, at distance r in front of the charged plane sheet, is

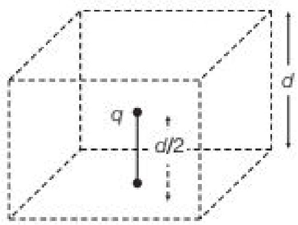
$$W=q'\times V=q'\cdot\frac{-\sigma r}{2\varepsilon_0}=-\frac{\sigma r\cdot q'}{2\varepsilon_0}\text{ joule}$$

41. (a) **Electric flux** It is defined as the total number of electric field lines that are normally pass through that surface.

Total electric flux ϕ over the whole surface S due to an electric field E is given as

$$\phi=\oint_S \mathbf{E}\cdot d\mathbf{S}=\oint EdS\cos\theta$$

It is a scalar quantity.



From the given problem, q is the point charge at a distance of $\frac{d}{2}$ directly above the centre of the square side.

Now, construct a Gaussian surface in form of a cube of side d to evaluate the amount of electric flux.

$\therefore$ We can calculate the amount of electric flux for six surfaces by using Gauss'S law,

$$\phi_E=\int_s \mathbf{E}\cdot d\mathbf{S}=\frac{q}{\varepsilon_0}$$

$\therefore$ For one surface of the cube, amount of electric flux is given as $\phi_E'=\frac{q}{6\varepsilon_0}$

(b) Even if the point charge is moved to a distance d from the centre of the square and side of the square is doubled, but amount of charge enclosed into the Gaussian surface does not changes.

$\therefore$ The amount of electric flux remains same.

42. Here, $E=9\times10^4$ N/C, $r=2$ cm $=2\times10^{-2}$ m

and $\lambda=?$

As, $E=\frac{\lambda}{2\pi\varepsilon_0 r}\Rightarrow\lambda=2\pi\varepsilon_0 rE$

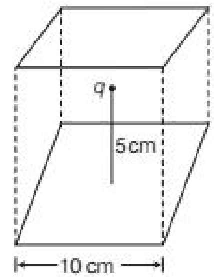
$$=\frac{1}{2\times9\times10^9}\times2\times10^{-2}\times9\times10^4=10^{-7}\text{Cm}^{-1}$$

43. **Hints:** Think of the square as one face of a cube with edge 10 cm.

Electric flux linked with a surface can be calculated using Gauss' theorem, according to which total electric flux linked with a closed surface is $\frac{q}{\varepsilon_0}$.

Now, we imagine an enclosed cubical surface and the given square be one side of this cubical surface. Let a charge q be placed at the centre of cube.

Now, the figure looks like as shown below.



The total flux enclosed through the centre of cube is given by

$$\phi=\frac{q}{\varepsilon_0}\qquad\text{...(i)}$$

[according to Gauss' theorem]

Here, $q=10\,\mu$C

The flux enclosed by one face, i.e. square is 1/6 of total flux (because the cube has six square shaped faces), so the flux linked with each square,

$$\phi'=\frac{\phi}{6}=\frac{1}{6}\cdot\frac{q}{\varepsilon_0}\qquad\text{[from Eq. (i)]}$$

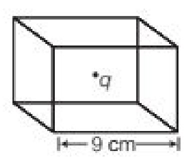
$$\Rightarrow\quad\phi'=\frac{1}{6}\times\frac{10\times10^{-6}}{8.854\times10^{-12}}=1.88\times10^5\text{ N-m}^2/\text{C}$$

Thus, the flux linked with the square is 1.88×10^5 N-m^2/C.

44. Let us consider a charge q is placed at the centre of a cubic Gaussian surface. As per the question,

$$q=2\,\mu\text{C}=2\times10^{-6}\text{ C}$$

Length of edge = 9 cm



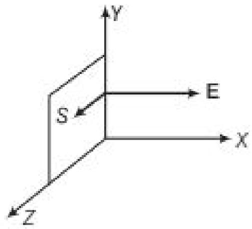
According to Gauss' theorem, the net electric flux (ϕ) through the surface is given by

$$\phi=\frac{q}{\varepsilon_0}=\frac{2\times10^{-6}}{8.854\times10^{-12}}=2.26\times10^5\text{ N-m}^2/\text{C}$$

Thus, the net electric flux through the surface is 2.26×10^5 N-m^2/C.

45. Electric field, $\mathbf{E} = 3 \times 10^3 \hat{\mathbf{i}}$ N/C, i.e. electric field is directed towards X-axis (due to involvement of $\hat{\mathbf{i}}$).

(i) As the surface is in YZ-plane, so the area vector (normal to the square) is along X-axis.



Area, $S = 10 \times 10 = 100 \text{ cm}^2 = 10^{-2} \text{m}^2$

Area vector, $\mathbf{S} = 10^{-2} \hat{\mathbf{i}} \text{ m}^2$

Using the formula of electric flux,

$$\phi = \mathbf{E} \cdot \mathbf{S} = ES \cos\theta$$
$$= ES \qquad [\because \text{angle between E and S is } 0°]$$
$$= 3 \times 10^3 \times 10^{-2} = 30 \text{ N-m}^2/\text{C}$$

(ii) Now, the area vector makes an angle of 60° with X-axis.

$$\mathbf{E} = 3 \times 10^3 \hat{\mathbf{i}} \text{ N/C}$$
$$\mathbf{S} = 100 \text{ cm}^2 = 10^{-2} \text{m}^2, \theta = 60°$$

Using the formula of electric flux, $\phi = \mathbf{E} \cdot \mathbf{S}$

$$\Rightarrow \quad \phi = ES \cos\theta = 3 \times 10^3 \times 10^{-2} \cos 60°$$
$$= 3 \times 10 \times \frac{1}{2}$$
$$= 15 \text{ N-m}^2/\text{C}$$

46. Given, electric field intensity,

$$\mathbf{E} = 5 \times 10^3 \hat{\mathbf{i}} \text{ N/C}$$

Magnitude of electric field intensity,

$$|\mathbf{E}| = 5 \times 10^3 \text{ N/C}$$

Side of square, $S = 10$ cm $= 0.1$ m

Area of square, $A = (0.1)^2 = 0.01 \text{ m}^2$

The plane of the square is parallel to the yz-plane.

Hence, the angle between the unit vector normal to the plane and electric field is zero, i.e. $\theta = 0°$.

$\therefore$ Flux through the plane,

$$\phi = |\mathbf{E}| \times A \cos\theta$$
$$= 5 \times 10^3 \times 0.01 \cos 0°$$
$$= 50 \text{ N-m}^2/\text{C}$$

If the plane makes an angle of 30° with the X-axis, then $\theta = 60°$.

$\therefore$ Flux through the plane,

$$\phi = |\mathbf{E}| \times A \times \cos 60° = 5 \times 10^3 \times 0.01 \times \cos 60°$$
$$= 25 \text{ N-m}^2/\text{C}$$

47. Given, $R = \dfrac{D}{2} = \dfrac{2.4}{2} = 1.2$ m

and $\sigma = 80.0 \ \mu\text{C/m}^2$

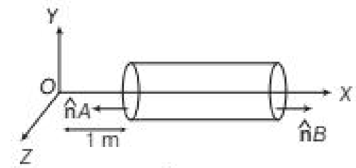
(i) Charge on sphere, $q = 4\pi R^2 \cdot \sigma$

$$= 4 \times 3.14 \times (1.2)^2 \times 80$$
$$= 1446.912 \ \mu\text{C}$$
$$= 1.45 \times 10^{-3} \text{C}$$

(ii) Electric flux, $\phi = \dfrac{q}{\varepsilon_0}$

$$= \frac{1.45 \times 10^{-3}}{8.854 \times 10^{-12}}$$
$$= 0.1637 \times 10^9$$
$$= 1.637 \times 10^8 \text{ N-m}^2/\text{C}$$

48. (i)



Given, $\quad \mathbf{E} = 50x \,\hat{\mathbf{i}}$

and $\quad \Delta S = 25 \text{ cm}^2$

$$= 25 \times 10^{-4} \text{m}^2$$

As the electric field is only along the X-axis, so flux will pass only through the cross-section of cylinder.

Magnitude of electric field at cross-section A,

$$E_A = 50 \times 1 = 50 \text{ N C}^{-1}$$

Magnitude of electric field at cross-section B,

$$E_B = 50 \times 2 = 100 \text{ NC}^{-1}$$

The corresponding electric fluxes are

$$\phi_A = \mathbf{E}_A \cdot \Delta\mathbf{S}$$
$$= 50 \times 25 \times 10^{-4} \times \cos 180°$$
$$= -0.125 \text{ N-m}^2 \text{ C}^{-1}$$

and

$$\phi_B = \mathbf{E}_B \cdot \Delta\mathbf{S}$$
$$= 100 \times 25 \times 10^{-4} \times \cos 0°$$
$$= 0.25 \text{ N-m}^2\text{C}^{-1}$$

So, the net flux through the cylinder,

$$\phi = \phi_A + \phi_B = -0.125 + 0.25$$
$$= 0.125 \text{ N-m}^2 \text{ C}^{-1}$$

(ii) Using Gauss' law,

$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0} \qquad [\because \oint \mathbf{E} \cdot d\mathbf{S} = \phi]$$

$$\Rightarrow \quad 0.125 = \frac{q}{8.85 \times 10^{-12}}$$

$$\Rightarrow \quad q = 8.85 \times 0.125 \times 10^{-12} = 1.1 \times 10^{-12} \text{ C}$$

49. (i) On the left face, the outward flux,

$$\phi_L = \mathbf{E} \cdot \Delta\mathbf{S} = -100\hat{\mathbf{i}} \cdot \Delta\mathbf{S} = 100\Delta S,$$

since $\quad \hat{\mathbf{i}} \cdot \Delta\mathbf{S} = -\Delta S$

$$= 100 \times \pi(0.05)^2 = 0.785 \text{ N-m}^2\text{C}^{-1}$$

On the right face, $\mathbf{E}$ and $\Delta\mathbf{S}$ are parallel and therefore,

$$\phi_R = \mathbf{E} \cdot \Delta\mathbf{S} = 0.785 \text{ N-m}^2\text{C}^{-1}$$

(ii) For any point on the side of the cylinder, $\mathbf{E} \perp \Delta\mathbf{S}$ and hence $\mathbf{E} \cdot \Delta\mathbf{S} = 0$

$\therefore$ Flux out of the side of the cylinder $= 0$

(iii) Net outward flux through the cylinder,

$$\phi = 0.785 + 0.785 + 0$$
$$= 1.57 \text{ N-m}^2\text{C}^{-1}$$

(iv) From Gauss' law, the net charge within the cylinder

$$= 1.57 \times 8.854 \times 10^{-12}\text{C}$$
$$= 1.39 \times 10^{-11}\text{C}$$

50. We know that,

$$\sigma = 17.0 \times 10^{-22}\ \text{Cm}^{-2}$$

(i) **E** on the left of the plates is zero.

(ii) **E** on the right of the plates is zero.

(iii) In between the plates, $E = \dfrac{\sigma}{\varepsilon_0}$

$$E = \frac{17.0 \times 10^{-22}}{8.85 \times 10^{-12}}$$
$$= 1.9 \times 10^{-10}\ \text{NC}^{-1}$$

51. Refer to Example 5 on page 39.

52. Given, $E_X = \alpha x^{1/2}$, $E_Y = 0$, $E_Z = 0$

$$\alpha = 600\ \text{N/C-m}^{1/2},\ l = 0.1\ \text{m},\ \phi = ?,\ q = ?$$

As the electric field has only x-component, therefore $\text{E} \cdot \Delta\,\text{S} = \phi_E = 0$ for each of four faces of cube perpendicular to Y-axis and Z-axis.

Flux is there only for left face L and right face R of the cube.

At the left face, $x = l$

$$\therefore \quad E_L = \alpha l^{1/2}$$

The flux for left face of cube,

$$\phi_L = \text{E}_L \cdot \Delta S = \alpha l^{1/2}\,(l^2)\cos 180° = -\alpha l^{5/2}$$

Similarly, at right face, $x = l + l = 2l$

The flux for right face of cube,

$$E_R = \alpha(2l)^{1/2}$$
$$\phi_R = \text{E}_R \cdot \Delta\text{S}$$
$$= \alpha(2l)^{1/2}(l^2)\cos 0°$$
$$= \alpha l^{5/2}\sqrt{2}$$

The net flux through the cube,

$$\phi = \phi_R + \phi_L$$
$$= \alpha l^{5/2}\sqrt{2} - \alpha l^{5/2}$$
$$= \alpha l^{5/2}\,(\sqrt{2} - 1)$$
$$= 600\,(0.1)^{5/2}\,(\sqrt{2} - 1)$$
$$= 0.785\ \text{N-m}^2\ \text{C}^{-1}$$

Apply Gauss' theorem and the charge within the cube,

$$q = \varepsilon_0 \phi = 8.85 \times 10^{-12} \times 0.785$$
$$= 6.95 \times 10^{-12}\ \text{C}$$

SUMMARY

- **Electric Charge** It is the intrinsic property of the material which is responsible to exert the electric force.
- **Conductors and Insulators** Conductors are those substances which conduct the electricity, whereas insulators are those substance which cannot conductor electricity.
- **Charging by Induction** The process of charging a neutral body by bringing a charged body nearby it without making contact between the two bodies is called charging by induction.
- **Quantisation of Electric Charge** The charge on any body can be expressed as an integral multiple of basic unit of charge, i.e. charge on one electron. It can be expressed as
  $$q = \pm ne, \text{ where } n = 1, 2, \ldots$$
- **Coulomb's Law** The force of interaction (attraction or repulsion) between two stationary point charges in vacuum is directly proportional to the product of the charges and inversely proportional to the square of distances between them,

  i.e. $$F = \frac{kq_1q_2}{r^2}$$
- **Superposition Principle** Force on any charge due to number of charges is the vector sum of all the forces on that charge due to other charges, taken one at a time.
- **Electrostatic Force due to Continuous Charge Distribution**

  $\lambda = \frac{q}{l}$, where λ is a linear charge density.

  $\sigma = \frac{q}{A}$, where σ is a surface charge density.

  $\delta = \frac{q}{V}$, where δ is a volume charge density.
- **Electric Field** It is the space around the given charge in which another charge experiences an electrostatic force of repulsion or attraction.
- **Electric Field Lines** It is a path traversed by a test charge around the given charge.
- **Properties of Electric Field Lines**

  Electric lines of force start from positive charges and end at negative charges.

  Two field lines never intersect each other.

  These are perpendicular to the surface of charged conductor.

  These do not pass through a conductor.
- **Electric Field Intensity** At a point is the force experienced per unit positive test charge placed at that point without disturbing the source charge, i.e. $E = \left(\frac{F}{q_0}\right)$
- **Electric Dipole** It is a pair of point charges with equal magnitude and opposite in sign separated by a very small distance.
- **Dipole Moment** It is the product of the charge and separation between the charges, i.e. $p = 2a \cdot q$
- **Electric Field Intensity due to an Electric Dipole**

  At a point on axial line, $E = \frac{2kpx}{(x^2 - l^2)^2}$.

  At a point on equatorial line, $E = \frac{2kp}{(x^2 + l^2)^{3/2}}$.

  At any point, $|E| = \frac{p\sqrt{3\cos^2\theta + 1}}{4\pi\varepsilon_0 r^3}$.
- **Torque on an electric Dipole in a Uniform Electric Field** It is given by, $\tau = pE\sin\theta$
- **Work done on Dipole in a Uniform Electric Field** Total work done in rotating the dipole from orientation θ_1 to θ_2 is
  $$W = pE(\cos\theta_1 - \cos\theta_2)$$
- **Electric Flux** It is defined as the total number of electric lines of force passing normally through the surface.
- **Gauss' Theorem** The surface integral of the electric field intensity over any closed surface in free space is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed with in the surface.
  $$\phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$
- **Applications of Gauss' Theorem**

  Field due to an infinitely long straight charged wire, $E = \frac{\lambda}{2\pi\varepsilon_0 r}$.

  Field due to thin infinite sheet of charge, $E = \frac{\sigma}{2\varepsilon_0}$.

  Field due to a uniformly charged thin spherical shell

  Outside the shell, $E = \frac{\sigma R^2}{\varepsilon_0 r^2}$

  On the surface of shell, $E = \frac{\sigma}{\varepsilon_0}$

  Inside the shell, $E = 0$

CHAPTER PRACTICE

OBJECTIVE Type Questions

1. A negatively charged object X is repelled by another charged object Y. However, an object Z is attracted to object Y. Which of the following is the most possibility for the object Z? **CBSE SQP (Term-I)**

(a) Positively charged only
(b) Negatively charged only
(c) Neutral or positively charged
(d) Neutral or negatively charged

2. In an experiment, three microscopic latex spheres are sprayed into a chamber and became charged with charges $+3e$, $+5e$ and $-3e$, respectively. All the three spheres came in contact simultaneously for a moment and got separated. Which one of the following are possible values for the final charge on the spheres? **CBSE SQP (Term-I)**

(a) $+5e, -4e, +5e$
(b) $+6e, +6e, -7e$
(c) $+4e, +3.5e, +5.5e$
(d) $+5e, -8e, +7e$

3. An object has charge of 1 C and gains 5.0×10^{18} electrons. The net charge on the object becomes **CBSE SQP (Term-I)**

(a) -0.80 C (b) $+0.80$ C
(c) $+1.80$ C (d) $+0.20$ C

4. The number of electrons that must be removed from an electrically neutral silver dollar to give it a charge of $+2.4$ C is

(a) 2.5×10^{19} (b) 1.5×10^{19}
(c) 1.5×10^{-19} (d) 2.5×10^{-19}

5. Two identical metallic spheres having charges $+4q$ and $-2q$ are placed with their centres r distance apart. Force of attraction between the spheres is F. If the two spheres are brought in contact and then placed at the same distance r

spheres is F. If the two spheres are brought in contact and then placed at the same distance r
apart, the force between them

(a) F (b) $F/2$
(c) $F/4$ (d) $F/8$

6. In the following configuration of charges, force on charge q_2 by q_1 is given by

(a) $\mathbf{F}_{21}=\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r^2}\cdot\hat{\mathbf{r}}_{21}$

(b) $\mathbf{F}_{21}=\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r^2}(-\hat{\mathbf{r}}_{21})$

(c) $\mathbf{F}_{21}=\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r^3}\cdot\hat{\mathbf{r}}_{21}$

(d) $\mathbf{F}_{21}=\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r^3}\cdot(-\hat{\mathbf{r}}_{21})$

(here, $r=r_{21}=|\mathbf{r}_2-\mathbf{r}_1|$)

7. If charges q, q and $-q$ are placed at vertices of an equilateral triangle of side l. If $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are the forces on the charges respectively, then

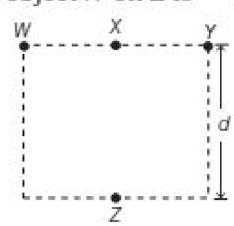
(a) $|\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3|=\sqrt{3}\dfrac{kq^2}{l^2}$

(b) $|\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3|=0$

(c) $|\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3|=3\sqrt{2}\dfrac{kq^2}{l^2}$

(d) $|\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3|=\sqrt{2}\dfrac{kq^2}{l^2}$

8. Four objects W, X, Y and Z each with charge $+q$ are held fixed at four points of a square of side d as shown in the figure. Objects X and Z are on the mid-points of the sides of the square. The electrostatic force exerted by object W on object X is F, then the magnitude of the force exerted by object W on Z is **CBSE SQP (Term-I)**

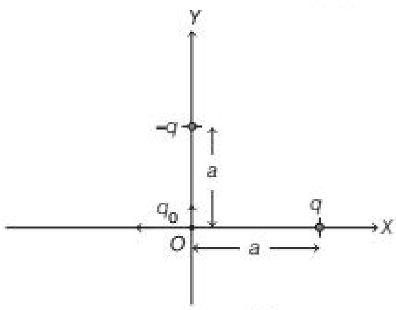


(a) $\dfrac{F}{7}$ (b) $\dfrac{F}{5}$

(c) $\dfrac{F}{3}$ (d) $\dfrac{F}{2}$

9. Three charges $q, -q$ and q_0 are placed as shown in figure. The magnitude of the net force on the charge q_0 at point O is $\left(\text{Take, } K = \frac{1}{4\pi\varepsilon_0}\right)$

CBSE 2021 (Term-I)



(a) 0
(b) $\frac{2Kqq_0}{a^2}$
(c) $\frac{\sqrt{2}Kqq_0}{a^2}$
(d) $\frac{1}{\sqrt{2}} \frac{Kqq_0}{a^2}$

10. Two point charges placed in a medium of dielectric constant 5 are at a distance r between them, experience an electrostatic force F. The electrostatic force between them in vacuum at the same distance r will be

(a) $5F$ (b) F (c) $F/2$ (d) $F/5$

11. The magnitude of electric field due to a point charge $2q$, at distance r is E. Then, the magnitude of electric field due to a uniformly charged thin sphere shell of radius R with total charge q at a distance $\frac{r}{2}$ $(r >> R)$ will be

CBSE SQP (Term-I)

(a) $\frac{E}{4}$
(b) 0
(c) $2E$
(d) $4E$

12. Two point charges $+8q$ and $-2q$ are located at $x = 0$ and $x = L$, respectively.
The point on X-axis at which net electric field is zero due to these charges, is

(a) $8L$ (b) $4L$ (c) $2L$ (d) L

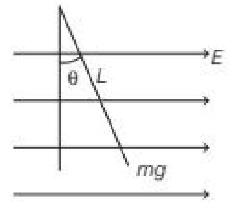
13. Two parallel large thin metal sheets have equal surface densities 26.4×10^{-12} C/m^2 of opposite signs. The electric field between these sheets is

CBSE SQP (Term-I)

(a) 1.5 N/C
(b) 1.5×10^{-16} N/C
(c) 3×10^{-10} N/C
(d) 3 N/C

14. A small object with charge q and weight mg is attached to one end of a string of length L attached to a stationary support. The system is placed in a uniform horizontal electric field E, as shown in the given figure. In the presence of the field, the string makes a constant angle θ with the vertical. The sign and magnitude of q

CBSE SQP (Term-I)



(a) positive with magnitude mg/E
(b) positive with magnitude $mg/E \tan\theta$
(c) negative with magnitude $mg/E \tan\theta$
(d) positive with magnitude $E \tan\theta / mg$

15. Unit of electric field is

Or Unit of electric field intensity is

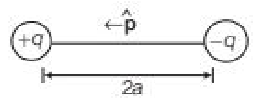
Or The unit of intensity of electric field is

(a) N/m (b) C/N (c) N/C (d) J/N

16. Electric field of a system of charges does not depend on

(a) position of charges forming the system
(b) distance of point (at which field is being observed) from the charges forming system
(c) value of test charge used to find out the field
(d) separation of charges forming the system

17. For the dipole shown,



Dipole moment is given by

(a) $\mathbf{p} = q \times 2a\hat{\mathbf{p}}$
(b) $\mathbf{p} = \frac{1}{2} q \times 2a\hat{\mathbf{p}}$
(c) $\mathbf{p} = -q \times 2a\hat{\mathbf{p}}$
(d) $\mathbf{p} = 4q \times 2a\hat{\mathbf{p}}$

18. An electric dipole of moment p is placed parallel to the uniform electric field. The amount of work done in rotating the dipole by 90° is

CBSE SQP (Term-I)

(a) $2pE$
(b) pE
(c) $pE/2$
(d) zero

19. Which statement is true for Gauss' law?

(a) All the charges whether inside or outside the gaussian surface contribute to the electric flux.
(b) Electric flux depends upon the geometry of the gaussian surface.
(c) Gauss' theorem can be applied to non-uniform electric field.
(d) The electric field over the gaussian surface remains continuous and uniform at every point.

20. A square sheet of side a is lying parallel to XY-plane at $z = a$. The electric field in the region is $\mathbf{E} = cz^2\hat{\mathbf{k}}$. The electric flux through the sheet is

CBSE SQP (Term-I)

(a) a^4c (b) $\frac{1}{3}a^3c$ (c) $\frac{1}{3}a^4c$ (d) 0

21. Gauss' law is true only if force due to charges varies as

(a) r^{-1} (b) r^{-2} (c) r^{-3} (d) r^{-4}

22. For a given surface, the $\oint \mathbf{E} \cdot d\mathbf{S} = 0$. From this, we can conclude that

(a) E is necessarily zero on the surface
(b) E is perpendicular to the surface at every point
(c) the total flux through the surface is zero
(d) the flux is only going out of the surface

23. A charge on a sphere of radius 2 cm is 2 μC while charge on sphere of radius 5 cm is 5μC. Find the ratio of an electric field on distance of 10 cm from centre of the sphere.

(a) 1 : 1 (b) 2 : 5 (c) 5 : 2 (d) 4 : 25

ASSERTION AND REASON

Directions (Q. Nos. 24-31) *In the following questions, two statements are given- one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

24. **Assertion** If a point charge be revolved in a circle around another charge as the centre of circle, then work done by electric field will be zero.

Reason Work done is equal to dot product of force and displacement.

25. **Assertion** A positive point charge initially at rest in a uniform electric field starts moving along electric lines of force. (Neglect all other forces except electric forces).
Reason A point charge released from rest in an electric field always moves along the line of force.

26. **Assertion** Electric force between two charges always acts along the line joining the two charges

Reason Electric force is a conservative force.

27. **Assertion** When a neutral body acquires positive charge, its mass decreases.
Reason A body acquires positive charge when it loses electrons.

28. **Assertion** In a non-uniform electric field, a dipole will have translatory as well as rotatory motion.

Reason In a non-uniform electric field, a dipole experiences a force as well as torque.

29. **Assertion** **E** outside vicinity of a conductor depends only on the local charge density σ and it is independent of the other charges present anywhere on the conductor.
Reason **E** in outside vicinity of a conductor is given by $\frac{\sigma}{\varepsilon_0}$.

30. **Assertion** Upon displacement of charges within a closed surface, **E** at any point on the surface does not change.

Reason The flux crossing through a closed surface is independent of the location of charge within the surface.

31. **Assertion** If Gaussian surface does not enclose any charge, then **E** at any point on the Gaussian surface must be zero.
Reason No net charge is enclosed by Gaussian surface, so net flux passing through the surface is zero.

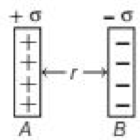
CASE BASED QUESTIONS

Directions (Q.Nos. 32-34) *These questions are case study based questions. Attempt any 4 sub-parts from each question. Each question carries 1 mark.*

32. **Parallel Sheet of Charge**
Surface charge density is defined as the charge per unit surface area of surface charge distribution.

i.e. $\sigma = \frac{dq}{dS}$.

Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs having magnitude of $17.0 \times 10^{-22}\ \text{Cm}^{-2}$ as shown below.



The intensity of electric field at a point is $E = \frac{\sigma}{\varepsilon_0}$

where, ε_0 = permittivity of free space.

(i) E in the outer region of the first plate is

(a) 17×10^{-22} N/C (b) 1.5×10^{-15} N/C

(c) 1.9×10^{-10} N/C (d) zero

(ii) E in the outer region of the second plate is

(a) 17×10^{-22} N/C (b) 1.5×10^{-15} N/C

(c) 1.9×10^{-10} N/C (d) zero

(iii) E between the plates is

(a) 17×10^{-22} N/C (b) 1.5×10^{-15} N/C

(c) 1.9×10^{-10} N/C (d) zero

(iv) The ratio of **E** from right side of **B** at distances
2 cm and 4 cm, respectively is

(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) $1 : \sqrt{2}$

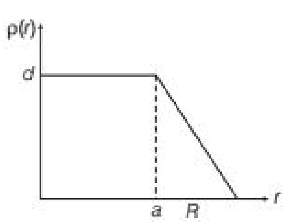
(v) In order to estimate the electric field due to a thin finite plane metal plate, the gaussian surface considered is

(a) spherical (b) cylindrical

(c) straight line (d) None of these

33. Electric Field due to Nuclear Charge

The nuclear charge (Ze) is non-uniformly distributed within a nucleus of radius R. The charge density $\rho(r)$ [charge per unit volume is dependent only on the radial distance r from the centre of the nucleus as shown in figure. The electric field is only along the radial direction.



(i) The electric field at $r = R$ is

(a) independent of a

(b) directly proportional to a

(c) directly proportional to a^2

(d) inversely proportional to a

(ii) Net charge on given system is

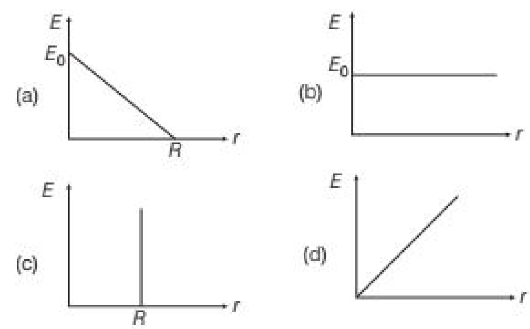
(a) $Q = \int \rho_r\ (4\pi r^2)dr$ (b) $Q = \int \rho_r\ (\pi r^2)dr$

(c) $Q = \int \rho_r\ \frac{r^2}{2}\,dr$ (d) $Q = \int \rho_r\ (4\pi r^2)$

(iii) For $a = 0$, the value d (maximum value of ρ as shown in the figure) is

(a) $\frac{3Ze^2}{4\pi R^3}$ (b) $\frac{3Ze}{\pi R^3}$ (c) $\frac{4Ze}{3\pi R^3}$ (d) $\frac{Ze}{3\pi R^3}$

(iv) The correct graph representing the variation of E with r is



(v) The electric field within the nucleus is generally observed to be linearly dependent on r. This implies

(a) $a = 0$ (b) $a = \frac{R}{2}$

(c) $a = R$ (d) $a = \frac{2R}{3}$

34. Faraday cage

A Faraday cage or Faraday shield is an enclosure made of a conducting material. The fields within a conductor cancel out with any external fields, so the electric field within the enclosure is zero. These Faraday cages act as big hollow conductors. You can put things to shield them from electrical fields. Any electrical shocks the cage receives, pass harmlessly around the outside of the cage.

(i) Which of the following material can be used to make a Faraday cage?

(a) Plastic (b) Glass

(c) Copper (d) Wood

(ii) Example of a real-world Faraday cage is

(a) car (b) plastic box

(c) lightning rod (d) metal rod

(iii) What is the electrical force inside a Faraday cage, when it is struck by lightning?

(a) The same as the lightning

(b) Half that of the lightning

(c) Zero

(d) A quarter of the lightning

(iv) An isolated point charge $+q$ is placed inside the Faraday cage. Its surface must have charge equal to

(a) zero (b) $+q$

(c) $-q$ (d) $+2q$

(v) A point charge of 2C is placed at centre of Faraday cage in the shape of cube with surface of 9 cm edge. The number of electric field lines passing through the cube normally will be

(a) 1.9×10^5 N-m^2/C, entering the surface

(b) 1.9×10^5 N-m^2/C, leaving the surface

(c) 2.01×10^{11} N-m^2/C, leaving the surface

(d) 2.01×10^5 N-m^2/C, entering the surface

VERY SHORT ANSWER Type Questions

35. Electric charge is additive in nature. Explain.

36. Give one difference between the conductors and insulators.

37. "Electrostatic forces are much stronger than the gravitational forces". Give one example to justify this statement.

38. A point charge is placed at the centre of a hollow conducting sphere of internal radius r and outer radius $2r$. The ratio of the surface charge density of the inner surface to that of the outer surface will be .... **CBSE 2020**

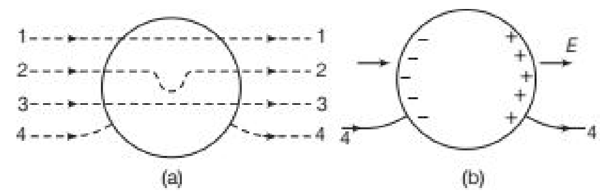
39. The work done in moving a charge particle between two points in a uniform electric field, does not depend on the path followed by the particle. Why? **CBSE 2020**

40. Draw the pattern of electric field lines, when a point charge $-Q$ is kept near an uncharged conducting plate. **CBSE 2019**

41. Draw the pattern of electric field lines due to an electric dipole. **CBSE 2019**

42. Draw a pattern of electric field lines due to two positive charges placed a distance d apart. **CBSE 2019**

43. A metallic solid sphere is placed in a uniform electric field as shown below.



Which path is followed by electric field lines?

44. A point charge $+Q$ is placed in the vicinity of a conducting surface. Draw the electric field lines between the surface and the charge. **All India 2017 C**

45. The electric field induced in a dielectric when placed in an external field is 1/10 times the electric field. Calculate relative permittivity of the dielectric.

SHORT ANSWER Type Questions

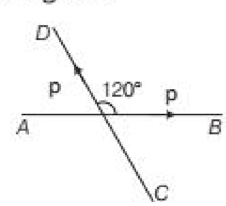
46. Two identical metallic spherical shells A and B having charges $+4Q$ and $-10Q$ are kept a certain distance apart.

A third identical uncharged sphere C is first placed in contact with sphere A and then with sphere B, then spheres A and B are brought in contact and then separated.

Find the charge on the spheres A and B.

47. Deduce the expression for the electric field **E** due to a system of two charges q_1 and q_2 with position vectors r_1 and r_2 at a point r with respect to a common origin.

48. Two small identical dipoles AB and CD, each of dipole moment **p** are kept at an angle of 120° as shown in the figure.



What is the resultant dipole moment of this combination? If this system is subjected to electric field (**E**) directed along positive x-direction, what will be the magnitude and direction of the torque acting on this?

49. Derive the expression for the torque acting on an electric dipole, when it is held in a uniform electric field. Identify the orientation of the dipole in the electric field, in which it attains a stable equilibrium. **CBSE 2020**

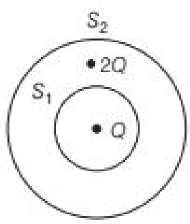
50. What is the use of Gaussian surface? Also, mention the importance of Gauss' theorem.

51. A point charge causes an electric flux of -3.1×10^4 N-m^2/C to pass through a spherical Gaussian surface.

(i) Find the value of the point charge.

(ii) If the radius of the Gaussian surface is doubled, how much flux would pass through the surface?

52. S_1 and S_2 are two parallel concentric spheres enclosing charges Q and $2Q$ as shown in the figure.



(i) What is the ratio of the electric flux through S_1 and S_2?

(ii) How will the electric flux through the sphere S_1 change, if a medium of dielectric constant $5\varepsilon_0$ is introduced in the space inside S_1 in place of air?

LONG ANSWER Type I Questions

53. A spherical conducting shell of inner radius R_1 and outer radius R_2 has a charge Q. A charge q is placed at the centre of the shell.

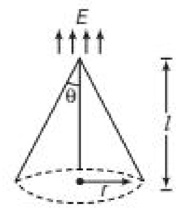
(i) What is the surface charge density on the
(a) inner surface?
(b) outer surface of the shell?

(ii) Write the expression for the electric field at a point $X > R_2$ from the centre of the shell.

54. A large plane sheet of charge having surface charge density 5×10^{-6} C/m^2 lies in XY-plane. Find the electric flux through a circular area of radius 0.1 m, if the normal to the circular area makes an angle of 60° with Z-axis.

[Take, $\varepsilon_0 = 8.85\times10^{-12}$ C^2/N-m^2]

55. Consider a region bounded by conical surface as shown in the figure.



In this region, **E** is in vertical upward direction, then find the electric field when electric flux is passing through curved surface.

56. A hollow conducting sphere of inner radius r_1 and outer radius r_2 has a charge Q on its surface. A point charge $-q$ is also placed at the centre of the sphere.

(a) What is the surface charge density on the (i) inner and (ii) outer surface of the sphere?

(b) Use Gauss' law of electrostatics to obtain the expression for the electric field at a point lying outside the sphere.

Or

(a) An infinitely long thin straight wire has a uniform linear charge density λ. Obtain the expression for the electric field (E) at a point lying at a distance x from the wire, using Gauss' law.

(b) Show graphically the variation of this electric field E as a function of distance x from the wire.

57. Two large charged plane sheets of charge densities σ and -2σ C/m^2 are arranged vertically with a separation of d between them. Deduce expressions for the electric field at points (i) to the left of the first sheet, (ii) to the right of the second sheet and (iii) between the two sheets. **CBSE 2019**

Or

A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q.

(a) A charge q is placed at the centre of the shell. Find out the surface charge density on the inner and outer surfaces of the shell.

(b) Is the electric field inside a cavity (with no charge) zero; independent of the fact whether the shell is spherical or not? Explain.

58. Total charge $-Q$ is uniformly spread along length of a ring of radius R. A small test charge $+q$ of mass m is kept at the centre of the ring and is given a gentle push along the axis of the ring.

(i) Show that the particle executes a simple harmonic oscillation.

(ii) Obtain its time period.

LONG ANSWER Type II Questions

59. (i) Derive the expression for the electric intensity at any point P, at distance r from the centre of an electric dipole, making angle α, with its axis.

(ii) Two point charges $+4\,\mu C$ and $+1\,\mu C$ are separated by a distance of 2 m in air. Find the point on the line joining charges at which the net electric field of the system is zero.

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60. (a) Derive an expression for the electric field at any point on the equatorial line of an electric dipole.

(b) Two identical point charges, q each are kept 2 m apart in air. A third point charge Q of unknown magnitude and sign is placed on the line joining the charges such that the system remains in equilibrium. Find the position and nature of Q.

61. (a) Define an ideal electric dipole. Give an example.

(b) Derive an expression for the torque experienced by an electric dipole in a uniform electric field. What is net force acting on this dipole?

(c) An electric dipole of length 2 cm is placed with its axis making an angle of 60° with respect to uniform electric field of 10^5 N/C.

If it experiences a torque of $8\sqrt{3}$ N-m, calculate the

(i) magnitude of charge on the dipole and (ii) its potential energy.

62. (i) State Gauss' law. Using this law, obtain the expression for the electric field due to an infinitely long straight conductor of linear charge density λ.

(ii) A wire AB of length L has linear charge density $\lambda = kx$, where x is measured from the end A of the wire. This wire is enclosed by a Gaussian hollow surface. Find an expression for the electric flux through this surface. **All India 2017C**

ANSWERS

1. (*c*)	**2.** (*b*)	**3.** (*d*)	**4.** (*b*)	**5.** (*d*)
6. (*a*)	**7.** (*b*)	**8.** (*b*)	**9.** (*c*)	**10.** (*a*)
11. (*c*)	**12.** (*c*)	**13.** (*d*)	**14.** (*b*)	**15.** (*c*)
16. (*c*)	**17.** (*a*)	**18.** (*b*)	**19.** (*d*)	**20.** (*c*)
21. (*b*)	**22.** (*c*)	**23.** (*b*)		

24. (*a*) Force by electric field will be perpendicular to the displacement.

25 (*c*) If the field lines are curved, then the charged particle follows the straight line path along the direction of tangent drawn to electric field lines at its starting point.

26. (*b*)

27. (*a*)

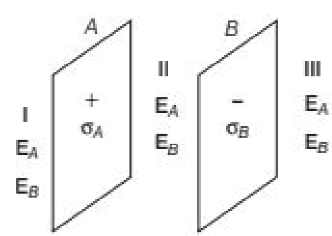
28. (*a*)

29. (*d*) **E** in outside vicinity of conductor's surface depends on all the charges present in the space, but expression $\mathbf{E} = \frac{\sigma}{\varepsilon_0}$.

30. (*d*) Due to displacement of charge within closed surface E at any point may change. But net flux crossing the surface will not change.

31. (*d*) **E** at any point on Gaussian surface may be due to outside charges also.

32. (i) (*d*) There are two plates A and B having surface charge densities $\sigma_A = 17.0 \times 10^{-22}$ C/m^2 on A and $\sigma_B = -17.0 \times 10^{-22}$ C/m^2 on B, respectively.



According to Gauss' theorem, if the plates have same surface charge density but having opposite signs, then the electric field in region I is zero.

$$\mathbf{E}_{\text{I}} = \mathbf{E}_A + \mathbf{E}_B = \frac{\sigma}{2\varepsilon_0} + \left(-\frac{\sigma}{2\varepsilon_0}\right) = 0$$

(ii) (*d*) The electric field in region III is also zero.

$$\mathbf{E}_{\text{III}} = \mathbf{E}_A + \mathbf{E}_B$$

$$= \frac{\sigma}{2\varepsilon_0} + \left(-\frac{\sigma}{2\varepsilon_0}\right) = 0$$

(iii) (*c*) In region II, the electric field

$$\mathbf{E}_{\text{II}} = \mathbf{E}_A - \mathbf{E}_B = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$

$$= \frac{\sigma}{\varepsilon_0} = \frac{\sigma_A \text{ or } \sigma_B}{\varepsilon_0}$$

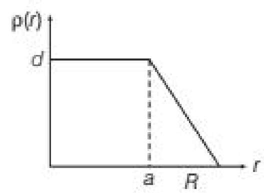
$$= \frac{17.0 \times 10^{-22}}{8.85 \times 10^{-12}}$$

$$E = 1.92 \times 10^{-10} \text{ NC}^{-1}$$

(iv) (*c*) Since, electric field due to an infinite.-plane sheet of charge does not depend on the distance of observation point from the plane sheet of charge. So, for the given distances, the ratio of E will be 1 : 1.

(v) (*b*) In order to estimate the electric field due to a thin finite plane metal plate, we take a cylindrical cross-sectional area A and length $2r$ as the gaussian surface.

33. (i) (*a*) Electric field at $r = R$



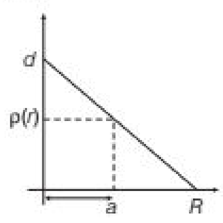
$$E = \frac{KQ}{R^2}; Q = \text{Total charge within the nucleus} = Ze$$

So, $E = \dfrac{KZe}{R^2}$

So, electric field is independent of a.

(ii) (*a*) Net charge, $Q = \int \rho_r 4\pi r^2 \, dr$

(iii) (*b*) As, $\dfrac{d}{R} = \dfrac{\rho_r}{R-r}$



$$Q = \int_0^R \frac{d}{R}(R-r)\, 4\pi r^2 dr$$

$$= \frac{4\pi d}{R}\left(R\int_0^R r^2 dr - \int_0^R r^3 dr\right)$$

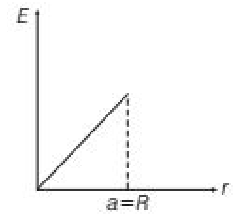
$$= \frac{4\pi d}{R}\left(\frac{R^4}{3} - \frac{R^4}{4}\right) = \frac{\pi d R^3}{3}$$

$$Q = Ze = \frac{\pi d R^3}{3}$$

$$\Rightarrow d = \frac{3Ze}{\pi R^3}$$

(iv) (*d*) From the formula of uniformly (volume) charged sphere $E = \dfrac{\rho r}{3\varepsilon_0}$

$\Rightarrow E \propto r$, so correct graph is as shown,



(v) (*c*) For $E \propto r, \rho$ should be constant throughout that of nucleus. This will be possible only when $a = R$.

34. (i) (c), (ii) (a), (iii) (c), (iv) (b)

(v) (*c*) According to Gauss' law, electric flux,

$$\phi = \frac{q}{\varepsilon_0} = \frac{2}{8.86 \times 10^{-12}} = 2.01 \times 10^{11} \text{ N-m}^2/\text{C}$$

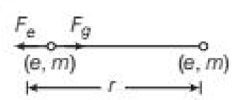
(leaving the surface)

35. Refer to text on page 2.

36. Conductors are those substances which can be used to carry or conduct electric charge/electron from one point to other. Insulators are those substances which cannot conduct electricity.

37. This statement can be proved by taking comparison of electric and magnetic forces between two protons.

Let the distance between two protons having charge $+e$ and mass m is placed at a distance r from each other as shown in figure.



$$\therefore \quad F_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \qquad \text{[Coulomb's law]}$$

$$= \frac{1}{4\pi\,\varepsilon_0}\frac{e \times e}{r^2}$$

$$\Rightarrow \quad F_g = G\frac{m_1 m_2}{r^2} \text{ (gravitation law)} = G\frac{m \times m}{r^2}$$

where, F_e and F_g are electric and gravitational force respectively, on putting the values, we get

$$\frac{F_e}{F_g} = \frac{\dfrac{1}{4\pi\varepsilon_0}\dfrac{e}{r^2}}{\dfrac{Gm^2}{r^2}} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{m^2}\cdot\frac{1}{G}$$

$$= \frac{(9\times10^9)(1.6\times10^{-19})^2}{(1.67\times10^{-27})^2}\times\frac{1}{(6.67\times10^{-11})}$$

$$= 1.2\times10^{36} \doteq 10^{36}$$

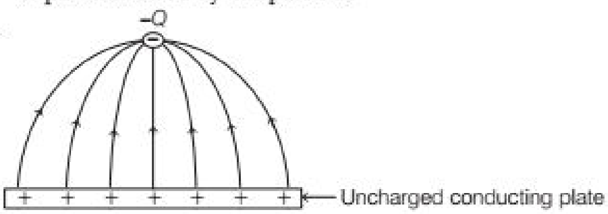
$\Rightarrow \quad F_e >> F_g$

Hence, the electric force is much stronger than gravitational force.

38. 4 : 1, for detail refer to sol 14 given on page 19 and 20.

39. It is because force acting on charge particle in an electric field is conservative which does not depend on path. Hence, work done by electric force is independent of path followed by the particle.

40.

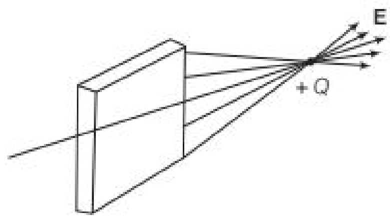


41. Refer to figure on pages 14 and 15.

42. Refer to diagram on pages 14 and 15.

43. Path 4 is followed by electric field lines. Since, there are no electric field lines within the metallic sphere and field lines are normal at each point on the surface.

44.



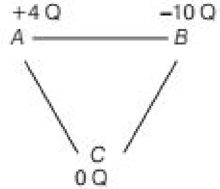
45. We know that, $\quad E_r = K = \dfrac{\varepsilon_0}{\varepsilon}$

Here, $\quad \varepsilon = \dfrac{1}{10}\varepsilon_0$

$\therefore \quad \varepsilon_r = \dfrac{\varepsilon_0}{\dfrac{1}{10}\varepsilon_0} = 10$

Hence, relative permittivity of the dielectric becomes 10 times of the original value.

46. When C is in contact with A, the charge developed on C is $-4Q$. When this is brought in contact with B, the charges are distributed and becomes $(-10-4)-14Q$ on B and C each.



At last when A and B are brought in contact again charges are re-distributed and become $(-14+4) = 10\,Q$ on each.

47. Refer to text on page 13.

48. Refer to Q. 18 on page 30.

49. Refer to text on page 28.

50. Refer to text on pages 36 and 37.

51. Refer to Example 5 on page 39.

52. Refer to Q. 31 on page 41.

53. Refer to text on pages 38 and 39.

54. According to Gauss' law, the electric field due to a plane sheet of charge is

$$E = \frac{\sigma}{2\varepsilon_0}$$

Given that, $\sigma = 5\times10^{-6}$ C/m^2

So, $\quad E = \dfrac{5\times10^{-6}}{2\times8.85\times10^{-12}} = \dfrac{5\times10^{6}}{17.7}$

$= 282.5\times10^3$ V/m

$\therefore \quad d\phi = \mathbf{E}\cdot d\mathbf{A}$

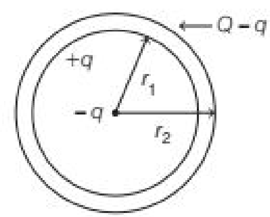
$\Rightarrow \quad \phi = EA\cos\theta$

$= 282.5\times10^3\times3.14\times10^{-2}\times\cos 60°$

$= 4.44$ kVm $= 4.44\times10^3$ N-m^2/C

55. Use Gauss' theorem, $\int \mathbf{E}\cdot d\mathbf{S} = \dfrac{q}{\varepsilon_0}$

56. (a) When a charge $-q$ is placed at the centre of the hollow conducting sphere, then charge induced on the inner surface is $+q$ and on outer surface is $-q$. But charge Q is already present on its outer surface. So, net charge on outer surface is $(Q-q)$ as shown.



Therefore, surface charge density on

(i) inner surface is $\dfrac{q}{4\pi r_1^2}$

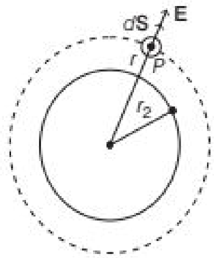
(ii) and on outer surface is $\dfrac{Q-q}{4\pi r_2^2}$.

(b) At a point lying outside the sphere by Gauss' law, the electric flux is given by,

$$\phi_E = \oint_s \mathbf{E}\cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

where, q is the net charge enclosed.

As **E** and $d\mathbf{S}$ are in same direction as shown.



$$\therefore \quad E(4\pi r^2) = \frac{Q-q}{\varepsilon_0}$$

$\Rightarrow$ Electric field, $E = \dfrac{Q-q}{4\pi\varepsilon_0 r^2}$

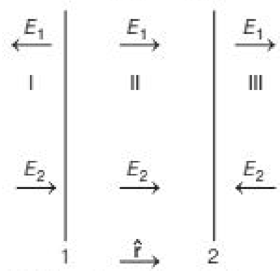
Or

(a) Refer to text on pages 37 and 38 (Replacing r by x).

(b) Refer to text and graph given on page 38 (Replacing r by x).

57. (i) Electric field to the left of plate 1 (region I)

$$E_I = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}} - \frac{2\sigma}{2\varepsilon_0}\hat{\mathbf{r}}$$



(ii) Electric field to the right of plate 2 (region III)

$$E_{III} = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}} - \frac{2\sigma}{2\varepsilon_0}\hat{\mathbf{r}}$$

(iii) Electric field between two plates (region II)

$$E_{II} = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}} + \frac{2\sigma}{2\varepsilon_0}\hat{\mathbf{r}}\text{-}$$

Or

(a) Refer to Sol. 14 on pages 19 and 20.

(b) Yes, the electric field inside a cavity is zero irrespective of shape because the cavity has zero net charge.

58. Refer to Q. 14 on page 30.

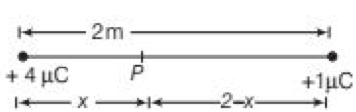
(i) Use relation, $E = \dfrac{qr}{4\pi\varepsilon_0 (r^2 + a^2)^{3/2}}$ to find

$F = qE = -kz$

(ii) Use relation, $T = 2\pi\sqrt{\dfrac{m}{k}}$

59. (i) Refer to text on page 27.

(ii)



Let the net electric field be zero at point P at a distance x from charge $+4\mu C$, then

$$\frac{1}{4\pi\varepsilon_0}\frac{4\times10^{-6}}{x^2} - \frac{1\times10^{-6}\times1}{4\pi\varepsilon_0\times(2-x)^2} = 0$$

$$\Rightarrow \quad \frac{4}{x^2} = \frac{1}{(2-x)^2},$$

$$\Rightarrow \quad \frac{2}{x} = \frac{1}{2-x}$$

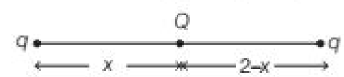
$$\Rightarrow \quad x = 4 - 2x$$

$$\Rightarrow \quad x = \frac{4}{3}\text{ m}$$

60. (a) Refer to text on page 26.

(b) Let P be the point at which the system of charges is in equilibrium, then

$$F(x) = F(2-x)$$



$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{x^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{(2-x)^2}$$

$$\Rightarrow \quad \frac{1}{x^2} = \frac{1}{(2-x)^2}$$

$$\Rightarrow \quad x = (2-x) \Rightarrow x = 1$$

Thus, the charge Q should be placed at the centre of line joining two given charges. Also the two given charges are identical, i.e. having same nature, so the third charge could be of any nature (positive or negative). As the forces on it at the centre are equal and opposite.

61. (a) Refer to text on page 25.

(b) Refer to text on page 28.

(c) Given, length of electric dipole, $2l = 2\text{ cm} = 2\times10^{-2}$ m

$\theta = 60°$

$E = 10^5$ N/C

(i) $\because \quad \tau = pE\sin\theta$

$$p = \frac{\tau}{E\sin\theta}$$

$$\Rightarrow \quad 2lq = \frac{\tau}{E\sin\theta}$$

$$\Rightarrow \quad q = \frac{\tau}{(2l)E\sin\theta}$$

$$= \frac{8\sqrt{3}}{2\times10^{-2}\times10^5\times\sin60°} = 8\times10^{-3}\text{ C}$$

(ii) Potential energy $= -pE\cos\theta = -(2l\times q)E\cos60°$

$$= -2\times10^{-2}\times8\times10^{-3}\times10^5\times\frac{1}{2}$$

$$= -8 \text{ joule}$$

62. (i) Refer to text on pages 36, 37 and 38.

(ii) Use $dQ = \lambda\cdot dl$ and Gauss' theorem $\phi = \dfrac{Q}{\varepsilon_0}$