10

The connection between waves and rays of light is described by wave optics. The wave theory of light was put forward by Huygens' in 1678 and later on modified by Fresnel. According to this theory, light is a form of energy which travels in the form of transverse wave. The speed of light in a medium depends upon the nature of medium. In this chapter, we will study about the various phenomena (i.e. interference of light, diffraction of light) related to the wave nature of light.

WAVE OPTICS

Concept Physics classes, for:-11,12,NEET & JEE

By:- Er. S B Choudhary . (B-tech in electrical)

Contact number :- 8076546954, 8130843551

|TOPIC 1|

Huygens' Principle

As discussed above, the speed of light in a medium depends upon the nature of the medium. Huygens supposed the existence of a hypothetical medium called "luminiferous ether" which filled the entire space.This medium was supposed to be massless with extremely high elasticity and very low density.

CHAPTER CHECKLIST

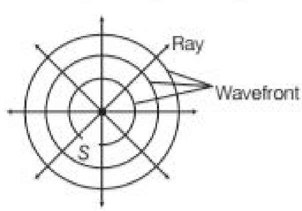
- Huygens' Principle
- Interference of Light
- Diffraction of Light

WAVEFRONT

It is the locus of points (wavelets) having the same phase (a surface of constant phase) of oscillations. A wavelet is the point of disturbance due to propagation of light. A line perpendicular to a wavefront is called a **ray**.

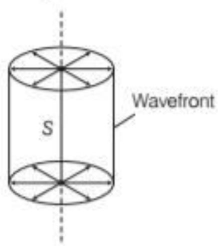
Depending on the shape of source of light, wavefronts can be of three types, which are given below

(i) **Spherical wavefront** When the source of light is a point source, the wavefront is a sphere with centre as the source.



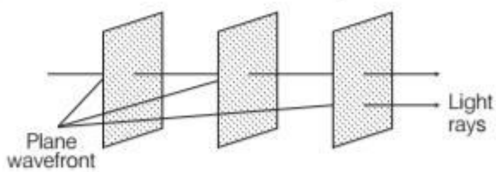
Spherical wavefront

(ii) **Cylindrical wavefront** When the source of light is linear, e.g. a straight line source, slit etc. as shown in the figure. All the points equidistant from the source lie on a cylinder. Therefore, the wavefront is cylindrical in shape.



Cylindrical wavefront

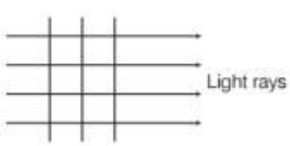
(iii) **Plane wavefront** When the point source or linear source of light is at very large distance, a small portion of spherical or cylindrical wavefront appears to be plane. Such a wavefront is called a plane wavefront.



Plane wavefront

Hence, the wavefront is a surface of constant phase.

The speed with which the wavefront moves outwards from the source is called the **speed of the wave**. The energy of the wave travels in a direction perpendicular to the wavefront.



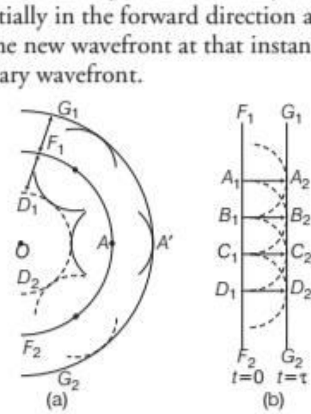
Plane wavefront

HUYGENS' PRINCIPLE

It is essentially a geometrical construction, which gives the shape of the wavefront at any time and allows us to determine the shape of the wavefront at a later time. According to Huygens' principle,

(i) Each point on the given wavefront (called primary wavefront) is the source of a secondary disturbance (called secondary wavelets) and the wavelets emanating from these points spread out in all directions with the speed of the wave.

(ii) A surface touching these secondary wavelets, tangentially in the forward direction at any instant gives the new wavefront at that instant. This is called secondary wavefront.



In Fig.(a), F_1F_2 is the section of the given spherical wavefront and G_1G_2 is the new wavefront in the forward direction. In Fig.(b), F_1F_2 is the section of the given plane wavefront and G_1G_2 is the new wavefront in the forward direction.

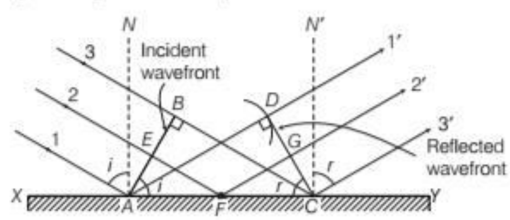
Note Huygens' argued that the amplitude of the secondary wavelets is maximum in the forward direction and zero in the backward direction. Hence, the backward secondary wavefront is absent.

Refraction and Reflection of Plane Waves Using Huygens' Principle

Huygens' principle can be used to explain the phenomena of reflection and refraction of light on the basis of wave theory of light.

Laws of Reflection at a Plane Surface

Let 1, 2, 3 be the incident rays and $1', 2', 3'$ be the corresponding reflected rays.



Laws of reflection by Huygens' principle

If c is the speed of the light, t is the time taken by light to go from B to C or A to D or E to G through F, then

$$t = \frac{EF}{c} + \frac{FG}{c} \quad \ldots\text{(i)}$$

In ΔAEF, $\sin i = \frac{EF}{AF}$

In ΔFGC, $\sin r = \frac{FG}{FC}$

or $t = \frac{AF \sin i}{c} + \frac{FC \sin r}{c}$

$$\Rightarrow \; t = \frac{AC \sin r + AF (\sin i - \sin r)}{c} \qquad [\because FC = AC - AF]$$

For rays of light from different parts on the incident wavefront, the values of AF are different. But light from different points of the incident wavefront should take the same time to reach the corresponding points on the reflected wavefront.

So, t should not depend upon AF. This is possible only, if

$$\sin i - \sin r = 0$$

i.e. $\sin i = \sin r$

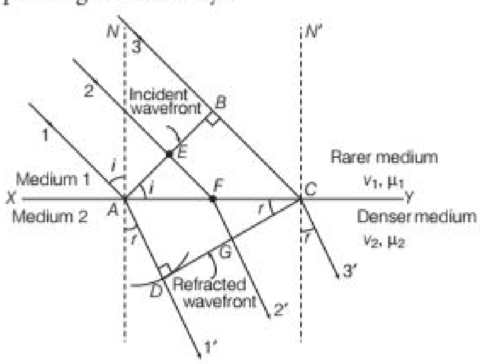
or $\angle i = \angle r$ ...(ii)

which is the **first law of reflection.**

Further, the incident wavefront AB, the reflecting surface XY and the reflected wavefront CD are all perpendicular to the plane of the paper. Therefore, incident ray, normal to the mirror XY and reflected ray all lie in the plane of the paper. This proves the **second law of reflection**.

Laws of Refraction (Snell's Law) at a Plane Surface

Let 1, 2, 3 be the incident rays and $1', 2', 3'$ be the corresponding refracted rays.



Laws of refraction by Huygens' principle

If v_1, v_2 are the speeds of light in the two media and t is the time taken by light to go from B to C or A to D or E to G through F, then $t = \frac{EF}{v_1} + \frac{FG}{v_2}$

In ΔAFE, $\sin i = \frac{EF}{AF}$

In ΔFGC, $\sin r = \frac{FG}{FC}$

$$\Rightarrow \; t = \frac{AF \sin i}{v_1} + \frac{FC \sin r}{v_2} \qquad ...(iii)$$

$$\Rightarrow \; t = \frac{AC \sin r}{v_2} + AF\left(\frac{\sin i}{v_1} - \frac{\sin r}{v_2}\right)$$

For rays of light from different parts on the incident wavefront, the values of AF are different. But light from different points of the incident wavefront should take the same time to reach the corresponding points on the refracted wavefront. So, t should not depend upon AF. This is possible only, if

$$\frac{\sin i}{v_1} - \frac{\sin r}{v_2} = 0$$

$$\Rightarrow \; \frac{\sin i}{\sin r} = \frac{v_1}{v_2} \qquad ...(iv)$$

Now, if c represents the speed of light in vacuum, then $\mu_1 = \frac{c}{v_1}$ and $\mu_2 = \frac{c}{v_2}$ are known as the **refractive indices** of medium 1 and medium 2, respectively.

In terms of refractive indices, Eq. (iv) can be written as

$$\mu_1 \sin i = \mu_2 \sin r$$

$$\Rightarrow \; \boxed{\mu = \frac{\sin i}{\sin r}}$$

This is known as **Snell's law of refraction.**

Further, if λ_1 and λ_2 denote the wavelengths of light in medium 1 and medium 2, respectively and if the distance BC is equal to λ_1, then the distance AD will be equal to λ_2, thus

$$\frac{\lambda_1}{\lambda_2} = \frac{BC}{AD} = \frac{v_1}{v_2}$$

or $\frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$

$$\Rightarrow \; \nu_1 = \nu_2 \qquad \left[\because \frac{v}{\lambda} = \nu\right]$$

Hence, the frequency does not change on refraction.

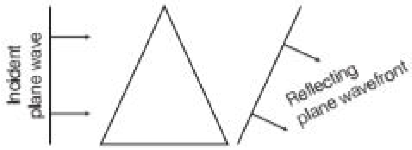
Thus, **frequency** ν being a characteristic of the source, remains the same as light travels from one medium to another.

Also, **wavelength** is directly proportional to the (phase) speed and inversely proportional to refractive index.

$$\therefore \quad \lambda' = \frac{\lambda}{\mu}, \; \mu = \frac{\lambda}{\lambda'} = \frac{c\lambda}{v\lambda} = \frac{c}{v}$$

Behaviour of Prism, Lens and Spherical Mirror Towards Plane Wavefront

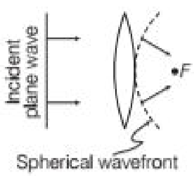
(i) **Behaviour of a prism** Since, the speed of light waves are less in glass, so the lower portion of the incoming wavefront (which travels through the greatest thickness of glass prism) will get delayed resulting in a tilt in the emerging wavefront.



Reflection of plane wave by a thin prism

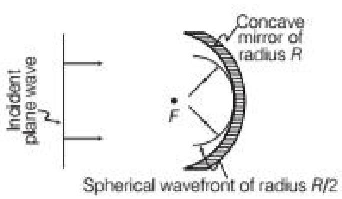
(ii) **Behaviour of a lens** The central part of the incident plane wave traverses the thickest portion of the lens and is delayed the most.

Due to this, the emerging wavefront has a depression at the centre. Therefore, the wavefront becomes spherical and converges to the point F which is known as the focus.



Reflection of plane wave by convex lens

(iii) **Behaviour of a spherical mirror** The central part of the incident wavefront travels the largest distance before reflection from the concave mirror. Hence, gets delayed, as a result of which the reflected wavefront is spherical which converges at the focal point F.



Reflection of plane wave by concave mirror

EXAMPLE |1|

(i) When monochromatic light is incident on a surface separating two media, the reflected and refracted lights both have the same frequency as the incident frequency. Explain, why?

(ii) When light travels from a rarer to a denser medium, the speed decreases. Does the reduction in speed imply a reduction in the energy carried by the light wave?

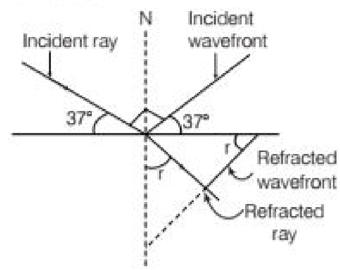
Sol. (i) Reflection and refraction arises through interaction of incident light with the atomic constituents of matter. Atoms may be viewed as oscillators, which take up the frequency of the external agency (light) causing forced oscillations.

The frequency of light emitted by a charged oscillator equals its frequency of oscillation. Thus, the frequency of scattered light equals the frequency of incident light.

(ii) No, the energy carried by a wave depends on the amplitude of the wave, not on the speed of wave propagation.

EXAMPLE |2| A plane wavefront is incident from air ($\mu = 1$) at an angle of 37° with a horizontal boundary of a refractive medium from air of refractive index $\mu = \frac{3}{2}$. Find the angle of refracted wavefront with the horizontal boundary.

Sol. It has been given that incident wavefront makes 37° with horizontal. Hence, incident ray makes 37° with normal as the ray is perpendicular to the wavefront.



Now, by Snell's law, $\dfrac{\sin 53^\circ}{\sin r} = \dfrac{3}{2}$

$$\Rightarrow \qquad \sin r = \frac{2}{3} \times \sin 53^\circ$$

$$\therefore \qquad r = \sin^{-1}(0.66 \times 0.79)$$

$$= 31.33^\circ$$

which is same as angle of refractive wavefront with horizontal.

DOPPLER'S EFFECT IN LIGHT

According to this effect, whenever there is a relative motion between a source of light and observer, the apparent frequency of light received by observer is different from the true frequency of light emitted actually from the source of light. Astronomers call the increase in wavelength due to Doppler effect as **red shift,** since a wavelength in the middle of the visible region of spectrum moves towards the red end of the spectrum.

When waves are received from a source moving towards the observer, there is an apparent decrease in wavelength, this is referred to as **blue shift**.

The fractional change in frequency is given by

$$\frac{\Delta \nu}{\nu} = -\frac{v_{\text{radial}}}{c}$$

where, v_{radial} is the component of the source velocity along the line joining the observer to the source relative to the observer. v_{radial} is considered positive, when the source moves away from the observer. The above formula is valid only when the speed of the source is small compared to that of light.

EXAMPLE |3| What speed should a galaxy move with respect to us so that the sodium light at 589 nm is observed at 589.6 nm?

Sol. Since, $\nu\lambda = c$,

$$\Rightarrow \quad \frac{\Delta\nu}{\nu} = -\frac{\Delta\lambda}{\lambda} \quad \text{[for small changes in } \nu \text{ and } \lambda]$$

Here, $\Delta\lambda = (589.6 - 589)\text{ nm} = 0.6\text{ nm}$

We know that, $\frac{\Delta\nu}{\nu} = -\frac{\Delta\lambda}{\lambda} = -\frac{v_{radial}}{c}$

$$\because \quad v_{radial} = +c\left(\frac{0.6}{589}\right)$$

$$= 3.06 \times 10^5 \text{ m/s}$$

$$= 306 \text{ km/s}$$

Therefore, the galaxy is moving 306 km/s away from us.

Applications of Doppler's Effect in Light

Some important applications of Doppler's effect are as given below

(i) Measuring the speed of stars and galaxies.
(ii) Measuring speed of rotation of the sun.
(iii) Estimation of velocity of aeroplanes, rockets and submarines, etc.

TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. Huygens' principle of secondary wavelets may be used to
(a) find the velocity of light in vacuum
(b) explain the particle's behaviour of light
(c) find the new position of a wavefront
(d) explain photoelectric effect

2. Which one of the following phenomena is not explained by Huygens' construction of wavefront?
(a) Refraction (b) Reflection
(c) Diffraction (d) Origin of spectra

3. The direction of wavefront of a wave with the wave motion is
(a) parallel (b) perpendicular
(c) opposite (d) at an angle of θ

4. Ray diverging from a point source on a wavefront are
(a) cylindrical (b) spherical
(c) plane (d) cubical

5. According to Huygens' principle, each point of the wavefront is the source of
(a) secondary disturbance
(b) primary disturbance
(c) third disturbance
(d) fourth disturbance

6. When light is refracted into a denser medium
(a) its wavelength and frequency both increases
(b) its wavelength increases but frequency remains unchanged
(c) its wavelength decreases but frequency remains the same
(d) its wavelength and frequency both decreases

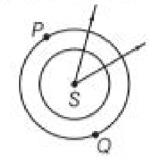
7. The Doppler effect is produced if
(a) the source is in motion
(b) the detector is in motion
(c) Both (a) and (b)
(d) None of the above

8. In the context of Doppler effect in light, the term red shift signifies
(a) decrease in frequency
(b) increase in frequency
(c) decrease in intensity
(d) increase in intensity

VERY SHORT ANSWER Type Questions

9. Define a wavefront. **Foreign 2009**

10. In the given figure, there are two points P and Q, what is the phase difference between them?



11. State Huygens' principles of secondary wavelets.

12. If a plane wavefront is incident on a prism, then draw the refracted wavefront.

13. Draw the wavefront coming out from a convex lens when a point source of light is placed at its focus. **Foreign 2009**

SHORT ANSWER Type Questions

14. (i) Differentiate between a ray and a wavefront.
    (ii) What is the phase difference between any two points on a wavefront?

15. What is the shape of the wavefront on earth for sunlight? **NCERT Exemplar**

16. Construct a diagram to show the wave characteristics of light.

17. Light of wavelength 5000 Å propagating in air gets partly reflected from the surface of water. How will the wavelengths and frequencies of the reflected and refracted light be affected? **All India 2015**

18. Consider a point at the focal point of convergent lens. Another convergent lens of short focal length is placed on the other side. What is the nature of the wavefronts emerging from the final image? **NCERT Exemplar**

19. Discuss Doppler's effect in the electromagnetic waves.

20. Define a wavefront. Using Huygens' principle, verify the laws of reflection at a plane surface. **CBSE 2018**

21. Is Huygens' principle valid for longitudinal sound waves? **NCERT Exemplar**

LONG ANSWER Type I Questions

22. Define the following terms and give its source of origin.
    (i) Spherical wavefront
    (ii) Plane wavefront
    (iii) Cylindrical wavefront

23. Choose the statement as right or wrong and justify.
    (i) Light is longitudinal wave, which gives the sensation of vision.
    (ii) A wavefront is a continuous locus of all points in which all particles vibrate in different phase.
    (iii) Rays of light are always normal to its wavefront.

24. Using Huygens' geometrical construction of wavefronts, show how a plane wave gets reflected from a surface. Hence, verify laws of reflection. **[CBSE 2019 Delhi]**

25. Use Huygens' principle to show how a plane wavefront propagates from a denser to rarer medium. Hence, verify Snell's law of refraction. **Delhi, 2015**

26. Define a wavefront. Use Huygens' geometrical construction to show the propagation of plane wavefront from a rarer medium
    (i) to a denser medium.
    (ii) undergoing refraction, hence derive Snell's law of refraction. **Foreign 2012**

27. Use Huygens' principle to verify the laws of refraction. **Delhi 2011**

28. What is the shape of the wavefront in each of the following cases?
    (i) Light diverging from point source.
    (ii) Light emerging out of a convex lens when a point source is placed at its focus.
    (iii) The portion of the wavefront of light from a distant star intercepted by the earth.
    NCERT

29. Define the term wavefront. State Huygen's principle. Consider a plane wavefront incident on a thin convex lens. Draw a proper diagram to show how the incident wavefront traverses through the lens and after refraction focusses on the focal point of the lens, giving the shape of the emergent wavefront. **All India 2016**

30. (i) Use Huygens' geometrical construction to show the behaviour of a plane wavefront,
    (a) passing through a biconvex lens
    (b) reflected by a concave mirror.
    (ii) When monochromatic light is incident on a surface separating two media, why does the refracted light have the same frequency as that of the incident light?

31. You have learnt in the text how Huygens' principle leads to the laws of reflection and refraction.Use the Huygens' principle to deduce directly that a point object placed in front of a plane mirror produces a virtual image whose distance from the mirror is equal to the distance of the object from the mirror. **NCERT**

32. Give three applications of Doppler's effect in light.

LONG ANSWER Type II Questions

33. (i) State Huygens' principle. Using this principle, draw a diagram to show how a plane wavefront incident at the interference of the two media gets refracted when it propagates from a rarer to a denser medium.
Hence, verify Snell's law of refraction.
(ii) Is the frequency of reflected and refracted light same as the frequency of incident light? **Delhi 2013**

34. (i) Use Huygens' geometrical construction to show how a plane wavefront at $t = 0$ propagates and produces a wavefront at a later time.
(ii) Verify, using Huygens' principle, Snell's law of refraction of a plane wave propagating from a denser to a rarer medium.
(iii) When monochromatic light is incident on a surface separating two media, the reflected and refracted light both have the same frequency. Explain, why ? **Delhi 2013C**

35. (i) A plane wavefront approaches a plane surface separating two media. If medium 1 is optically denser and medium 2 is optically rarer, using Huygens' principle, explain and show how a refracted wavefront is constructed?
(ii) Verify Snell's law.
(iii) When a light wave travels from a rarer to a denser medium, the speed decreases. Does it imply reduction in its energy? Explain.

NUMERICAL PROBLEMS

36. Light of wavelength 5000 Å falls on a plane reflecting surface. What are the wavelength and frequency of the reflected light? For what angle of incidence is the reflected ray normal to the incident ray? **NCERT**

37. (i) The refractive index of glass is 1.5. What is the speed of light in glass? (Speed of light in vacuum is 3×10^8 ms^{-1})
(ii) Is the speed of light in glass independent of the colour of light? If not, which of the two colours red and violet travels slower in a glass prism? **NCERT**

38. The 6563 Å H_α-line emitted by hydrogen in a star is found to be red shifted by 15 Å. Estimate the speed with which the star is receding from the earth. **NCERT**

39. The spectral line for a given element in light received from a distant star is shifted towards the longer wavelength by 0.032%. Deduce the velocity of star in the line of sight.

40. Monochromatic light of wavelength 589 nm is incident from air on a water surface. What are the wavelength, frequency and speed of
(i) reflected and
(ii) refracted light? (μ of water is 1.33) **NCERT**

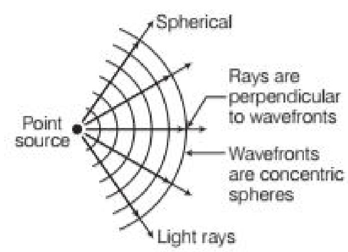
HINTS AND SOLUTIONS

1. (c) Every point on a given wavefront act as a secondary source of light and emits secondary wavelets which travels in all directions with the speed of light in the medium. A surface touching all these secondary wavelets tangentially in the forward direction, gives new wavefront at that instant of time.

2. (d) Huygens' construction does not explains quantisation of energy and it is not able to explain origin of spectrum.

3. (b) Wavefront is a surface perpendicular to a ray but a wavefront moves in the direction of the light.

4. (b) Wavefronts emitting from a point source are spherical wavefronts.



5. (a) According to Huygens' principle, each point of the wavefront is the source of a secondary disturbance and the wavelength emanating from these points spread out in all directions with the speed of the wave.

6. (c) Wavelength is dependent on refractive index medium by

$$\frac{\lambda_1}{\lambda_2}=\frac{\mu_2}{\mu_1}$$

So, in denser medium, $\mu_2>\mu_1$, so $\lambda_1>\lambda_2$ (i.e. wavelength decreases as the light travels from rarer to denser medium)

$\because \quad c = \nu\lambda$

7. (c) In the case of Doppler's effect, there is a relative motion between source and detector.

8. (a) When source moves away from the observer, frequency observed is smaller than that emitted from the source and (as if light emitted is yellow but it will be observed as red) this shift is called red shift.

9. Wavefront is the locus of points (wavelets) having the same phase of oscillations.

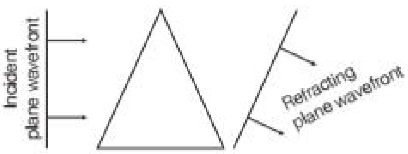
10. Since, the two points P and Q are at the same locus of the source S, so the phase difference between them equals to zero.

11. According to Huygens' principle,

(i) Each point on the given wavefront is the source of secondary disturbance and the wavelet emanating from these points spread out in all directions with the speed of wave.

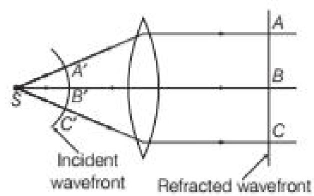
(ii) A surface touching these secondary wavelets, tangentially in the forward direction at any instant gives the new wavefront at that instant.

12.



Reflection of plane wave by a thin prism

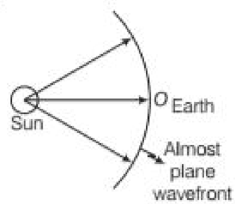
13. The wavefront in the given condition is shown below in figure.



14. Refer to the text on pages 400 and 401.

15. We know that, the sun is at very large distance from the earth. Assuming sun as spherical, it can be considered as point source situated at infinity.

Due to the large distance the radius of wavefront can be considered as large (infinity) and hence, wavefront is almost plane.



16. Refer to the text on pages 400 and 401.

17. The frequency and wavelength of reflected wave will not change. The refracted wave will have same frequency. The velocity of light in water is given by

$v = f\lambda$

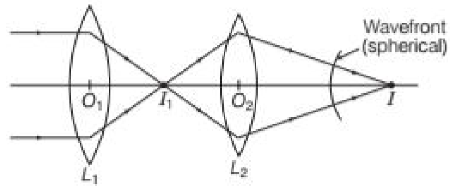
where, v = velocity of light

f = frequency of light

λ = wavelength of light

If velocity will decrease, then wavelength (λ) will also decrease.

18. Consider the ray diagram shown below



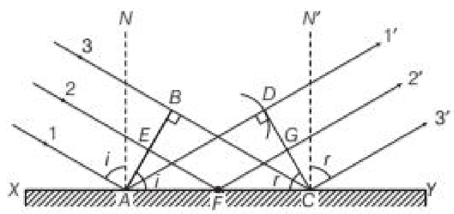
The point image I_1 due to L_1 is at the focal point. Now, due to the converging lens L_2, let the final image formed be I which is a point image, hence the wavefront for this image will be of spherical symmetry.

19. Refer to the text on pages 403 and 404.

20. Wavefront It is the locus of points (wavelets) having the same phase (a surface of constant phase) of oscillations.

Laws of reflection at a plane surface (On Huygens' principle)

Let 1, 2, 3 be the incident rays and 1′, 2′, 3′ be the corresponding reflected rays.



Laws of reflection by Huygens' principle

If c is the speed of the light, t is the time taken by light to go from B to C or A to D or E to G through F, then

$$t = \frac{EF}{c} + \frac{FG}{c} \quad \text{...(i)}$$

In ΔAEF, $\sin i = \dfrac{EF}{AF}$

In ΔFGC, $\sin r = \dfrac{FG}{FC}$

or $$t = \frac{AF \sin i}{c} + \frac{FC \sin r}{c}$$

or $$t = \frac{AC \sin r + AF (\sin i - \sin r)}{c}$$

$(\because FC = AC - AF)$

For rays of light from different parts on the incident wavefront, the values of AF are different. But light from different points of the incident wavefront should take the same time to reach the corresponding points on the reflected wavefront.

So, t should not depend upon AF. This is possible only if

$$\sin i - \sin r = 0$$

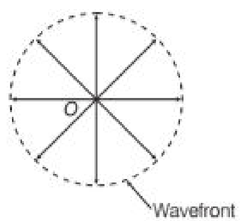
i.e. $$\sin i = \sin r$$

or $$\angle i = \angle r \quad \text{...(ii)}$$

which is the **first law of reflection.**

Further, the incident wavefront *AB*, the reflecting surface *XY* and the reflected wavefront *CD* are all perpendicular to the plane of the paper. Therefore, incident ray, normal to the mirror *XY* and reflected ray all lie in the plane of the paper. This is **second law of reflection**.

21. When we are considering a point source of sound wave. The disturbance due to the source propagates in spherical symmetry, i.e. in all directions.



The formation of wavefront is in accordance with Huygens' principle. So, Huygens' principle is valid for longitudinal sound waves also.

22. Refer to text on pages 400 and 401.

23. Refer to text on pages 400 and 401.
(i) wrong (ii) wrong
(iii) Right

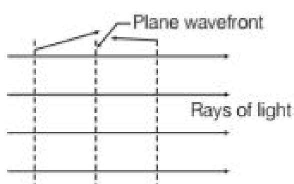
24. Refer to the text on pages 401 and 402.

25. Refer to text on page 402.

26. Refer to text on pages 400 and 402.

27. Refer to text on page 402.

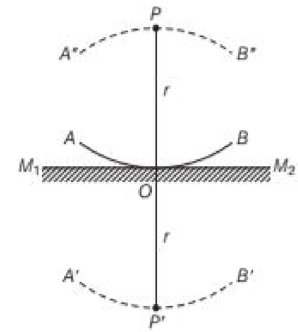
28. (i) Refer to text on page 402.
(ii) Refer to Q. 13 on page 405.
(iii) As, the star (i.e. source of light) is very far off, i.e. at infinity, the wavefront intercepted by the earth must be a plane wavefront.



29. Refer to the text on pages 400, 401 and 403.

30. (i) Refer to the text on pages 402 and 403.
(ii) Refer to the Example 1(i) on page 403.

31. In the figure, *P* is a point object placed at a distance *r* from a plane mirror M_1M_2. With *P* as centre and $PO = r$ as radius, draw a spherical arc *AB*. This is the spherical wavefront from the object, incident on M_1M_2.



32. Refer to text on page 404.

33. (i) Refer to text on pages 401 and 402.
(ii) The frequency of reflected and refracted light remains same as the frequency of incident light because frequency only depends on the source of light.

34. (i) Refer to text on page 401.
(ii) Refer to text on page 402.
(iii) Refer to Example 1(i) on page 403.

35. (i) and (ii) refer to text on page 402.
(iii) Refer to Example 1(ii) on page 403.

36. On the reflection, there is no change in wavelength and frequency. So, wavelength of reflected light will be 5000Å. Frequency of the reflected light,

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{5 \times 10^{-7}} = 6 \times 10^{14}\,\text{Hz}$$

For $i = 45°$, reflected ray becomes normal to the incident ray.

37. (i) Here, refractive index, $\mu = 1.5$

$$c = 3 \times 10^8\,\text{m s}^{-1}, v = ?$$

As, $\mu = \dfrac{c}{v}$

$$\Rightarrow \quad v = \frac{c}{\mu} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8\,\text{ms}^{-1}$$

(ii) No, the refractive index and the speed of light in a medium depend on wavelength, i.e. colour of light. We know that, $\mu_v > \mu_r$. Therefore, $v_{\text{violet}} < v_{\text{red}}$. Hence, violet component of white light travels slower than the red component.

38. Here, $\lambda = 6563\,\text{Å}$, $\Delta\lambda = +15\,\text{Å}$ and $c = 3 \times 10^8\,\text{ms}^{-1}$

Since, the star is receding away, hence its velocity *v* is negative.

$$\therefore \quad \Delta\lambda = -\frac{v\lambda}{c} \text{ or } v = -\frac{c\,\Delta\lambda}{\lambda}$$

$$= -\frac{3 \times 10^8 \times 15}{6563} = -6.86 \times 10^5\,\text{m s}^{-1}$$

Negative sign shows that star is receding away from the earth.

39. Here, $\frac{\Delta\lambda}{\lambda} = \frac{0.032}{100}, v = ?$

Since, the wavelength of light from a star is shifting towards longer wavelength side, then $\Delta\lambda$ is positive. Hence, star is moving away from the earth, i.e. v is negative.

$$\therefore \quad v = \frac{-\Delta\lambda}{\lambda}c = \frac{-0.032}{100} \times 3 \times 10^8$$

$$= -9.6 \times 10^4 \text{ m/s}$$

40. Here, wavelength, $\lambda = 589$ nm,

$c = 3 \times 10^8$ m/s, $\mu = 1.33$

(i) For reflected light,

Wavelength, $\lambda = 589 \text{ nm} = 589 \times 10^{-9}$ m

$$\Rightarrow \quad \nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{589 \times 10^{-9}}$$

$$= 5.09 \times 10^{14} \text{ Hz}$$

Speed, $v = c = 3 \times 10^8$ m/s

(ii) For refracted light,

$$\lambda' = \frac{\lambda}{\mu} = \frac{589 \times 10^{-9}}{1.33}$$

$$= 4.42 \times 10^{-7} \text{ m}$$

As, frequency remains unaffected on entering another medium, therefore

$$\nu' = \nu = 5.09 \times 10^{14} \text{ Hz}$$

$$\therefore \text{Speed, } v' = \frac{c}{\mu} = \frac{3 \times 10^8}{1.33} = 2.25 \times 10^8 \text{ m/s}$$

|TOPIC 2|

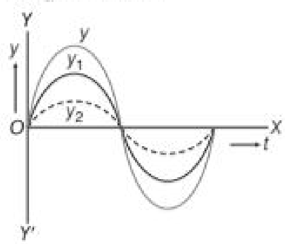
Interference of Light

SUPERPOSITION PRINCIPLE

According to this principle, at a particular point in the medium, the resultant displacement (y) produced by a number of waves is the vector sum of the displacements produced by each of the waves ($y_1, y_2, \ldots$).

i.e. $$y = y_1 + y_2 + y_3 + y_4 + \ldots$$

Clearly, each wave contributes as if the other wave is not present. The superposition principle which was stated first for mechanical waves is equally applicable to the electromagnetic (light) waves.



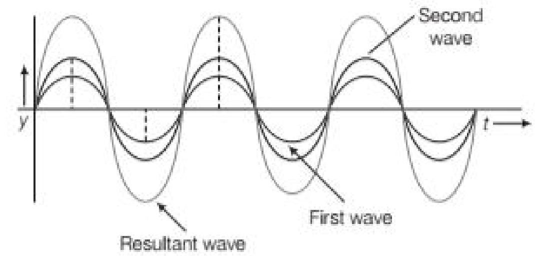
Superposition of waves

INTERFERENCE OF LIGHT WAVES

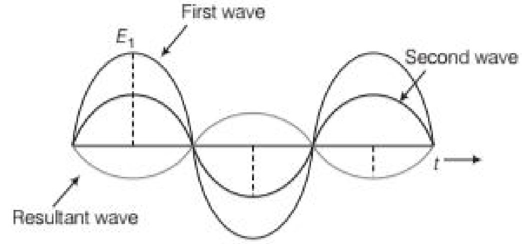
When two light waves of exactly equal frequency having constant phase difference w.r.t. time travelled on same direction and superimpose (overlap) with each other, then intensity of resultant wave does not remain uniform in space.

This phenomenon of formation of maximum intensity at some points and minimum intensity at some other points by two identical light waves travelling in same direction is called the **interference of light**.

At the points, where the resultant intensity of light is maximum, interference is said to be **constructive**. At the points, where the resultant intensity of light is minimum, the interference is said to be **destructive**.



(a) Constructive interference



(b) Destructive interference

Theory of Interference of Waves

Let the waves from two sources of light be represented as

$$y_1 = a \sin \omega t \quad \text{and} \quad y_2 = b \sin (\omega t + \phi)$$

where, a and b are the respective amplitudes of the two waves and ϕ is the constant phase angle by which second wave leads the first wave. Applying superposition principle, the magnitude of the resultant displacement of the waves is

$$y = y_1 + y_2$$

$\Rightarrow$ $$y = a \sin \omega t + b \sin (\omega t + \phi)$$

$\Rightarrow$ $$y = a \sin \omega t + b \sin \omega t \cdot \cos\phi + b \cos \omega t \cdot \sin \phi$$

$$[\because \sin (A + B) = \sin A \cos B + \cos A \sin B]$$

$\Rightarrow$ $$y = (a + b \cos \phi) \sin \omega t + b \sin \phi \cos \omega t$$

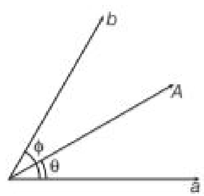
Putting $a + b \cos \phi = A \cos \theta$ and $b \sin \phi = A \sin \theta$

we get, $$y = A \cos \theta \cdot \sin \omega t + A \sin \theta \cdot \cos \omega t$$

or $$y = A \sin (\omega t + \theta)$$

where, A is the resultant amplitude and θ is the resultant phase difference.

$$\therefore \quad A = \sqrt{a^2 + b^2 + 2ab \cos \phi} \text{ and } \tan \theta = \frac{b \sin \phi}{a + b \cos \phi}$$



Resultant of amplitudes *a* and *b*

As, intensity is directly proportional to the square of the amplitude of the wave, i.e. $I \propto a^2$

So, for two different cases,

$$I_1 = ka^2, I_2 = kb^2$$

$\therefore$ $$I_R = kA^2 = k(a^2 + b^2 + 2ab \cos \phi)$$

$\therefore$ $$\boxed{I_R = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi}$$

For constructive interference

I should be maximum, for which

$$\cos \phi = \text{maximum} = +1$$

$\therefore$ Phase difference, $\phi = 0, 2\pi, 4\pi, \ldots$

i.e. $$\phi = 2n\pi, \text{ where } n = 1, 2, \ldots$$

If Δx be the path difference between the interfering waves, then

$$\Delta x = \frac{\lambda}{2\pi} \phi = \left(\frac{\lambda}{2\pi}\right)(2n\pi)$$

$\Rightarrow$ $$\Delta x = n\lambda$$

and $$\boxed{I_{\text{max}} \propto (a + b)^2}$$

Hence, condition for constructive interference at a point is that, the phase difference between the two waves reaching the point should be zero or an even integral multiple of 2π. Equivalent path difference between the two waves reaching the point should be zero or an integral multiple of full wavelength.

For destructive interference

I should be minimum, for which

$$\cos \phi = \text{minimum} = -1$$

$\therefore$ Phase difference, $\phi = \pi, 3\pi, 5\pi, \ldots$

i.e. $$\phi = (2n - 1) \pi$$

where, $$n = 1, 2, \ldots$$

The corresponding path difference between the two waves is

$$\Delta x = \left(\frac{\lambda}{2\pi}\right)\phi = \left(\frac{\lambda}{2\pi}\right)(2n - 1) \pi$$

$\Rightarrow$ $$\Delta x = (2n - 1) \frac{\lambda}{2}$$

and $$\boxed{I_{\text{min}} \propto (a - b)^2}$$

Hence, condition for destructive interference at a point is that, the phase difference between the two waves reaching the point should be an odd integral multiple of π or path difference between the two waves reaching the point should be an odd integral multiple of half wavelength.

Comparison of Intensities of Maxima and Minima

As, $$I_{\text{max}} \propto (a + b)^2 \text{ and } I_{\text{min}} \propto (a - b)^2$$

$\therefore$ $$\frac{I_{\text{max}}}{I_{\text{min}}} = \frac{(a + b)^2}{(a - b)^2} = \frac{\left(\frac{a}{b} + 1\right)^2}{\left(\frac{a}{b} - 1\right)^2}$$

$\Rightarrow$ $$\boxed{\frac{I_{\text{max}}}{I_{\text{min}}} = \frac{(r + 1)^2}{(r - 1)^2}}$$

where, $r = \frac{a}{b}$ (ratio of amplitudes)

Interference and Energy Conservation

In the interference pattern,

$$I_{\text{max}} = k(a + b)^2, \ I_{\text{min}} = k(a - b)^2$$

Average intensity of light in the interference pattern,

$$I_{\text{av}} = \frac{I_{\text{max}} + I_{\text{min}}}{2} = \frac{k(a + b)^2 + k(a - b)^2}{2}$$

$$= \frac{2k(a^2+b^2)}{2} = k(a^2+b^2)$$

Intensity of light is simply being redistributed, i.e. energy is being transferred from regions of destructive interference to the regions of constructive interference.

Thus, the principle of energy conservation is being obeyed in the process of interference of light.

EXAMPLE |1| Light waves from two coherent sources having intensities I and $2I$ cross each other at a point with a phase difference of 60°. What is the resultant intensity at that point?

Sol. Here, $I_1 = I$ and $I_2 = 2I$, $\phi = 60°$

$\therefore$ Resultant intensity,

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$
$$= I + 2I + 2\sqrt{I \times 2I}\cos 60°$$
$$= 3I + I\sqrt{2} = I(3+\sqrt{2}) = 4.414\, I$$

EXAMPLE |2| Light wave from two coherent sources of intensities in ratio 64:1 produces interference. Calculate the ratio of maxima and minima of the interference pattern.

Sol. Ratio of intensities of coherent sources,

i.e. $\frac{I_1}{I_2} = \frac{64}{1} = \frac{8^2}{1^2}$; $\frac{I_{max}}{I_{min}} = ?$

We have, $I_1 \propto a^2$, $I_{max} \propto (a+b)^2$

and $I_2 \propto b^2$, $I_{min} \propto (a-b)^2$

$\Rightarrow \frac{I_1}{I_2} = \frac{a^2}{b^2} = \frac{64}{1}$

$\Rightarrow \frac{a}{b} = \frac{8}{1}$; $a = 8b$

So, $\frac{I_{max}}{I_{min}} = \frac{(a+b)^2}{(a-b)^2} = \frac{(8b+b)^2}{(8b-b)^2} = \frac{81b^2}{49b^2}$

$= 81 : 49$

COHERENT AND INCOHERENT SOURCES

Light sources are of two types, i.e. coherent and non-coherent light sources. The sources of light which emit light waves of same wavelength, same frequency and are in same phase or having constant phase difference are known as **coherent sources.**

Two such sources of light, which do not emit light waves with constant phase difference are called **incoherent sources.**

Need of Coherent Sources for the Production of Interference Pattern

As discussed earlier, when two monochromatic waves of intensity I_1, I_2 and phase difference ϕ meet at a point, then the resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Here, the term $2\sqrt{I_1\, I_2}\cos\phi$ is called interference term.

There are two possibilities.

(i) If $\cos\phi$ remains constant with time, then the total intensity at any point will be constant. The intensity will be maximum $(\sqrt{I_1} + \sqrt{I_2})^2$ at points, where $\cos\phi$ is 1 and minimum $(\sqrt{I_1} - \sqrt{I_2})^2$ at points, where $\cos\phi$ is -1. The sources in this case are coherent.

(ii) If $\cos\phi$ varies continuously with time assuming both positive and negative value, then the average value of $\cos\phi$ will be zero over time interval of measurement. The interference term averages to zero. There will be same intensity $I = I_1 + I_2$ at every point. The two sources in this case are incoherent.

Note In practice, coherent sources are produced either by dividing the wavefront or by dividing the amplitude of the incoming waves.

Requirements for Obtaining Two Coherent Sources of Light

Following are the requirements (conditions) for obtaining two coherent sources of light

(i) Coherent sources of light should be obtained from a single source by some device.

Two coherent sources can be obtained either by
- (a) the source and its virtual image (Lloyd's mirror).
- (b) the two virtual images of the same source (Fresnel's biprism)
- (c) two real images of the same source (Young's double slit).

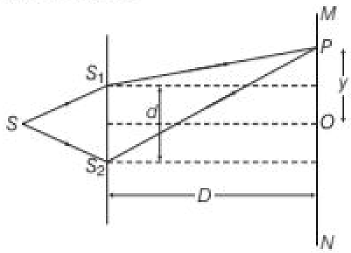
(ii) The two sources should give monochromatic light.

(iii) The path difference between light waves from two sources should be small.

(iv) Coherent sources can be produced by two methods:
- (a) By division of wavefront (Young's double slit experiment).
- (b) By division of amplitude (partial reflection or refraction in thin films).

YOUNG'S DOUBLE SLIT EXPERIMENT

Suppose S_1 and S_2 are two fine slits, a small distance d apart. They are illuminated by a strong source S of monochromatic light of wavelength λ. MN is a screen at a distance D from the slits.



Young's double slit arrangement to produce interference pattern

Consider a point P at a distance y from O, the centre of screen.

The path difference between two waves arriving at point P is equal to $S_2P - S_1P$.

Now,

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(y + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(y - \frac{d}{2}\right)^2\right]$$

$$= 2\,yd$$

Thus, $S_2P - S_1P = \dfrac{2\,yd}{S_2P + S_1P}$

But $\quad S_2P + S_1P \approx 2D$

$\therefore \quad S_2P - S_1P \approx \dfrac{dy}{D}$

For constructive interference (Bright fringes)

Path difference $= \dfrac{dy}{D} = n\lambda$, where, $n = 0, 1, 2, 3, \ldots$

$\therefore \quad y = \dfrac{nD\lambda}{d} \quad [\because n = 0, 1, 2, 3, \ldots]$

Hence, for $n = 0$, $y_0 = 0$ at O central bright fringe

for $n = 1$, $\quad y_1 = \dfrac{D\lambda}{d}$ for 1st bright fringe

for $n = 2$, $\quad y_2 = \dfrac{2D\lambda}{d}$ for 2nd bright fringe

for $n = n$, $\quad y_n = \dfrac{nD\lambda}{d}$ for nth bright fringe

The separation between two consecutive bright fringes is $\quad \beta = \dfrac{nD\lambda}{d} - \dfrac{(n-1)D\lambda}{d} = \dfrac{D\lambda}{d}$

For destructive interference (Dark fringes)

Path difference $= \dfrac{dy}{D} = (2n-1)\dfrac{\lambda}{2}$

or $\quad y = (2n-1)\dfrac{D\lambda}{2d}$, where, $n = 1, 2, 3, \ldots$

Hence, for $n = 1$, $\quad y_1' = \dfrac{D\lambda}{2d}$ for 1st dark fringe

for $n = 2$, $\quad y_2' = \dfrac{3D\lambda}{2d}$ for 2nd dark fringe

for $n = n$, $\quad y_n' = (2n-1)\dfrac{D\lambda}{2d}$ for nth dark fringe

The separation between two consecutive dark fringes is

$$\beta' = (2n-1)\frac{D\lambda}{2d} - \{2(n-1) - 1\}\frac{D\lambda}{2d} = \frac{D\lambda}{d}$$

Fringe Width

The distance between two consecutive bright or dark fringes is called **fringe width** W.

$\therefore$ Fringe width, $\boxed{W = \dfrac{D\lambda}{d}}$

The above formula is free from n that means the width of all fringes is same.

Fringe width is directly proportional to λ. Hence, the fringes of red light (longer wavelength) are broader than the fringes of blue light (shorter wavelength).

Intensity of the Fringes

For a bright fringe, $\phi = 2n\pi$

and $\quad \cos\phi = \cos 2n\pi = 1$

So, $\quad I_R = I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = 4I$

[as, $I_1 = I_2 = I$ in YDSE]

$\therefore$ Intensity of a bright fringe $= 4I$

For a dark fringe, $\phi = (2n-1)\pi$

$\Rightarrow \quad \cos\phi = -1$

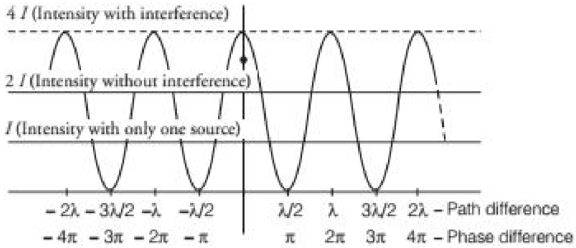
So, $\quad I_R = I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = 0$

$\therefore$ Intensity of a dark fringe $= 0$

Note If YDSE apparatus is immersed in a liquid of refractive index μ, then wavelength of light and hence fringe width decreases μ times.

Distribution of Intensity

The distribution of intensity in Young's double slit experiment is shown below



Relation among Intensity, Amplitude of the Wave and Width of the Slit

If W_1 and W_2 are widths of two slits from which intensities of light I_1 and I_2 emanate, then

$$\boxed{\frac{I_1}{I_2} = \frac{a^2}{b^2} = \frac{W_1}{W_2}}$$

where, a and b are the respective amplitudes of two waves.

EXAMPLE |3| Two slits are made one millimetre apart and the screen is placed one metre away. What is the fringe separation when blue-green light of wavelength 500 nm is used? **NCERT**

Sol. Here, $d = 1\text{mm} = 1\times10^{-3}\text{ m},\ D = 1\text{ m},$

$$\lambda = 500\text{nm} = 500\times10^{-9}\text{ m} = 5\times10^{-7}\text{ m}$$

As, fringe width, $\beta = \frac{D\lambda}{d}$

$$\therefore \quad \beta = \frac{1\times5\times10^{-7}}{1\times10^{-3}} = 5\times10^{-4}\text{ m}$$

$$= 0.5\text{ mm}$$

EXAMPLE |4| In Young's experiment, the width of the fringes obtained with light of wavelength 6000 Å is 2 mm. What will be the fringe width, if the entire apparatus is immersed in a liquid of refractive index 1.33?

Sol. As, $\beta = \frac{D\lambda}{d}$

and $\beta_1 = \frac{D\lambda_1}{d}$

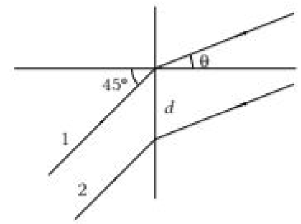
$$\therefore \quad \frac{\beta_1}{\beta} = \frac{\frac{D\lambda_1}{d}}{\frac{D\lambda}{d}} = \frac{\lambda_1}{\lambda} = \frac{1}{\mu}$$

d

or $\beta_1 = \frac{\beta}{\mu} = \frac{2}{1.33}$

$= 1.5\text{ mm}$

EXAMPLE |5| Distance between the slits, in YDSE, shown in figure is $d = 20\ \lambda$, where λ is the wavelength of light used.



Find the angle θ, where

(i) central maxima (where path difference is zero) is obtained.

(ii) third order maxima is obtained.

Sol. Ray 1 has a longer path than that of ray 2 by a distance $d\sin45°$, before reaching the slits. Afterwards ray 2 has a path longer than ray 1 by a distance $d\sin\theta$. The net path difference is therefore, $d\sin\theta - d\sin45°$.

(i) Central maximum is obtained, where net path difference is zero,

$$d\sin\theta - d\sin45° = 0 \Rightarrow \theta = 45°$$

(ii) Third order maxima is obtained, where net path difference is 3 λ, i.e.

$$d\sin\theta - d\sin45° = 3\lambda$$

$$\Rightarrow \quad \sin\theta = \sin45° + \frac{3\lambda}{d}$$

Putting $d = 20\ \lambda$, we have

$$\sin\theta = \sin45° + \frac{3\lambda}{20\lambda}$$

$$\therefore \quad \theta = 59°$$

Conditions for Sustained Interference

In order to obtain a well-defined observable interference pattern, the intensity at points of constructive and destructive interference must be maintained maximum and almost zero, respectively.

For this, following conditions must be satisfied

(i) The two sources producing interference must be coherent.

(ii) The two interfering waves must have the same plane of polarisation.

(iii) The two sources must be very close to each other and the pattern must be observed at a larger distance to have sufficient width of the fringe $\left(\frac{D\lambda}{d}\right)$.

(iv) The sources must be monochromatic, otherwise the

(iv) The sources must be monochromatic, otherwise the fringes of different colours will overlap.

(v) The two waves must be having same amplitude for better contrast between bright and dark fringes.

Fringe Shift

If refracting slab of thickness t is placed in front of one of the two slits of Young's double slit experiment, then fringe pattern gets shifted by n fringes and is given by

$$(\mu - 1)\, t = n\lambda$$

If both slits are covered by refracting surfaces of thicknesses t_1 and t_2 and refractive indices (μ_1, μ_2), then fringe pattern gets shifted by n fringe and is given by

$$(\mu_2 - \mu_1)\, t = n\lambda$$

Note The topic Young's double slit experiment has been asked frequently in the previous years 2015, 2014, 2012, 2011.

TOPIC PRACTICE 2

OBJECTIVE Type Questions

1. A thin film of oil is spread on the surface of water. The beautiful colours exhibited in the light of sun is due to

(a) dispersion of light (b) polarisation of light
(c) interference of light (d) diffraction of light

2. The phase difference between the two light waves reaching at a point P is 100π. Their path difference is equal to

(a) 10λ (b) 25λ (c) 50λ (d) 100λ

3. In the phenomenon of interference, energy is

(a) destroyed at destructive interference
(b) created at constructive interference
(c) conserved but it is redistributed
(d) same at all points

4. Two light waves superimposing at the mid-point of the screen are coming from coherent sources of light with phase difference π rad. Their amplitudes are 2 cm each. The resultant amplitude at the given point will be

(a) 8 cm (b) 2 cm (c) 4 cm (d) zero

5. The ratio of maximum and minimum intensities of two sources is 4 : 1. The ratio of their amplitudes is

(a) 1 : 81 (b) 3 : 1 (c) 1 : 9 (d) 1 : 16

6. The interference is produced by two waves of intensity ratio 16 : 9. The ratio of maximum and minimum intensities in interference pattern is

(a) 4 : 3 (b) 49 : 1
(c) 25 : 7 (d) 256 : 81

7. Two identical and independent sodium lamps act as

(a) coherent sources (b) incoherent sources
(c) Either (a) and (b) (d) None of these

8. In Young's double slit experiment, distance between slits in kept 1 mm and a screen is kept 1 m apart from slits. If wavelength of light used is 500 nm, then fringe spacing is

(a) 0.5 mm (b) 0.5 cm (c) 0.25 mm (d) 0.25 cm

9. In a Young's double-slit experiment, the source is white light. One of the holes is covered by a red filter and another by a blue filter. In this case, **NCERT Exemplar**

(a) there shall be alternate interference patterns of red and blue
(b) there shall be an interference pattern for red distinct from that for blue
(c) there shall be no interference fringes
(d) there shall be an interference pattern for red mixing with one for blue

VERY SHORT ANSWER Type Questions

10. Define the term "coherent sources" which are required to produce interference pattern in Young's double slit experiment.

11. Why we cannot obtain interference using two independent sources of light?

12. How will the fringe pattern change, if the screen is moved away from the slits?

13. How does the fringe width in Young's double slit experiment change when the distance of separation between the slits and screen is doubled? **All India 2012**

14. If the source of light used in Young's double slit experiment is changed from red to violet, what will happen to the fringe?

15. How does the angular separation of interference fringes change in Young's experiment, if the distance between the slits is increased? **Delhi 2008**

16. In Young's double slit experiment, the intensity of central maxima is I. What will be the intensity at the same place, if one slit is closed?

17. In a moving car, radio signals are interrupted sometimes. Why?

SHORT ANSWER Type Questions

18. Two light waves of amplitudes a and b interfere with each other. Calculate the ratio of intensities of a maxima to that of a minima.

19. What are the conditions for obtaining two coherent sources of light?

20. In Young's double slit experiment, the two slits are illuminated by two different lamps having same wavelength of light. Explain with reason, whether interference pattern will be observed on the screen or not. **All India 2017 C**

21. How will the interference pattern in Young's double slit experiment get affected, when

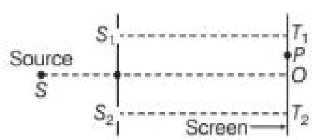
(i) distance between the slits S_1 and S_2 reduced.

(ii) the entire set up is immersed in water?

Justify your answer in each case. **Delhi 2011**

22. In Young's double slit experiment, two coherent sources S_1 and S_2 are placed at a distance D from screen, such that a bright fringe is obtained at a distance x from the centre line of screen. Give the value of x for which there is a bright fringe.

23. Consider a two slit interference arrangement (shown in figure) such that the distance of the screen from the slits is half the distance between the slits. Obtain the value of D in terms of λ such that the first minima on the screen falls at a distance D from the centre O.



NCERT Exemplar

24. Give four conditions for obtaining sustained interference.

25. If two waves of equal intensities $I_1 = I_2 = I_0$, meet at two locations P and Q with path differences Δ_1 and Δ_2, respectively. What will be the ratio of resultant intensity at points P and Q?

26. Answer the following questions:

(i) When a low flying aircraft passes overhead, we sometimes notice a slight shaking of the picture on our TV screen. Suggest a possible explanation.

(ii) As, you have learnt in the text, the principle of linear superposition of wave displacement is basic to understanding intensity distributions in diffraction and interference patterns.

What is the justification of this principle? **NCERT**

LONG ANSWER Type I Questions

27. (i) Why are coherent sources necessary to produce a sustained interference pattern?

(ii) In Young's double slit experiment using monochromatic light of wavelength λ, the intensity of light at a point on the screen, where path difference is λ, is K unit. Find out the intensity of light at a point, where path difference is $\lambda/3$. **Delhi 2012, NCERT**

28. Choose the statement as right or wrong and justify.

(i) For a coherent source, initial phase difference between two sources must have same finite value.

(ii) Resultant intensity of interference is given by $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$.

(iii) Two independent light bulbs fail to produce interference.

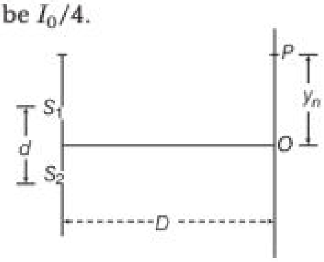
29. In Young's double slit experiment, derive the condition for

(i) constructive interference and

(ii) destructive interference at a point on the screen.

30. In Young's double slit experiment, describe briefly how bright and dark fringes are obtained on the screen kept in front of a double slit. Hence, obtain the expression for the fringe width.

31. The intensity at the central maxima (O) in a Young's double slit experiment is I_0. If the distance OP equals one-third of fringe width of the pattern, show that the intensity at point P would be $I_0/4$.



Foreign 2011

32. (a) If one of two identical slits producing interference in Young's experiment is covered with glass, so that the light intensity passing through it is reduced to 50%, find the ratio of the maximum and minimum intensity of the fringe in the interference pattern.

(b) What kind of fringes do you except to observe, if white light is used instead of monochromatic light? **CBSE 2018**

LONG ANSWER Type II Questions

33. (i) What is the effect on the interference fringes to a Young's double slit experiment when

(a) the width of the source slit is increased?

(b) the monochromatic source is replaced by a source of white light? Justify your answer in each case.

(ii) The intensity at the central maxima in Young's double slit experiment set up is I_0. Show that the intensity at a point, where the path difference is $\lambda/3$ is $I_0/4$. **Foreign 2012**

34. (i) Consider two coherent sources S_1 and S_2 producing monochromatic waves to produce interference pattern.

Let the displacement of the wave produced by S_1 be given by $Y_1 = a\cos\omega t$ and the displacement by S_2 be $Y_2 = a\cos(\omega t + \phi)$.

Find out the expression for the amplitude of the resultant displacement at a point and show that the intensity at that point will be

$$I = 4a^2\cos^2\frac{\phi}{2}$$

Hence, establish the conditions for constructive and destructive interference.

(ii) What is the effect on the interference fringes in Young's double slit experiment when (a) the width of the source slit is increased; (b) the monochromatic source is replaced by a source of white light? **Delhi 2015**

35. (i) (a) Two independent monochromatic sources of light cannot produce a sustained interference pattern. Give reason.

(b) Light waves each of amplitude a and frequency ω, emanating from two coherent light sources superpose at a point.

If the displacements due to these waves is given by $y_1 = a\cos\omega t$ and $y_2 = a\cos(\omega t + \phi)$, where, ϕ is the phase difference between the two, obtain the expression for the resultant intensity at the point.

(ii) In Young's double slit experiment, using monochromatic light of wavelength λ, the intensity of light at a point on the screen, where path difference is λ, is K units. Find out the intensity of light at a point where path difference is $\lambda/3$. **All India 2014**

36. State the importance of coherent sources in the phenomenon of interference. In Young's double slit experiment to produce interference pattern, obtain the conditions for constructive and destructive interference, hence deduce the expression for the fringe width. How does the fringe width get affected, if the entire experimental apparatus of Young's double slit experiment is immersed in water? **All India 2011**

37. (i) In a Young's double slit experiment, derive the conditions for constructive and destructive interference. Hence, write the expression for the distance between two consecutive bright or dark fringe.

(ii) What change in the interference pattern do you observe, if the two slits S_1 and S_2 are taken as point sources?

(iii) Plot a graph of intensity distribution *versus* path difference in this experiment.

NUMERICAL PROBLEMS

38. Light waves from two coherent sources having intensities I and $4I$ cross each other at a point with a phase difference of 90°. What is the resultant intensity at that point?

39. Light waves from coherent sources arrive at two points on a screen with path difference of 0 and $\lambda/2$. Find the ratio of intensities at the points. **All India 2017 C**

40. In a double slit experiment using light of wavelength 600 nm, the angular width of the fringe formed on a distant screen is 0.1°. Find the spacing between the two slits. **NCERT**

41. In Young's double slit experiment using light of wavelength 630 nm, angular width of a fringe formed on a distant screen is 0.2°. What is the spacing between the two slits?

42. In Young's double slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1.2 cm, determine the wavelength of light used in the experiment. **NCERT**

43. The amplitudes of light waves from two slits in Young's experiment are in ratio $\sqrt{2}:1$, what is the ratio of slit widths?

44. Widths of two slits in Young's experiment are in the ratio 4 : 1. What is the ratio of the amplitudes of light waves from them?

45. A beam of light, consisting of two wavelengths 560 nm and 420 nm, is used to obtain interference fringes in a Young's double slit experiment. Find the least distance from the central maxima, where the bright fringes due to both the wavelengths coincide. The distance between the two slits is 4 mm and the screen is at a distance of 1m from the slits. **Delhi 2010 C**

46. A beam of light consisting of two wavelengths 650 nm and 520 nm, is used to obtain interference fringes in a Young's double slit experiment.

(i) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 650 nm.

(ii) What is the least distance from the central maximum, where the bright fringes due to both the wavelengths coincide? **NCERT**

47. A beam of light consisting of two wavelenghts, 800 nm and 600 nm is used to obtain the interference fringes in a Young's double slit experiment on a screen placed 1.4 m away. If the two slits are separated by 0.28 mm, calculate the least distance from the central bright maximum where the bright fringes of the two wavelengths coincide. **All India 2012**

48. Laser light of wavelength 630 nm incident on a pair of slits produces an interference pattern in which the bright fringes are separated by 7.2 mm. Calculate the wavelength of another source of laser light which produce interference fringes separated by 8.1 mm using same pair of slits. **All India 2011**

49. In a Young's double slit experiment, the angular width of the fringe is found to be 0.2° on a screen placed 1 m away. The wavelength of light used is 600 nm. What will be the angular width of the fringe, if the entire experimental apparatus is immersed in water? Take, refractive index of water to be 4/3. **NCERT**

50. The ratio of the intensities at minima to the maxima in the Young's double slit experiment is 9 : 25. Find the ratio of the widths of the two slits. **All India 2014**

51. In Young's double slit experiment, the two slits 0.15 mm apart are illuminated by monochromatic light of wavelength 450 nm. The screen is 1 m away from the slits.

Find the distance of the second

(i) bright fringe.

(ii) dark fringe from the central maxima.

HINTS AND SOLUTIONS

1. (c) The light reflected from the oil film produced two coherent waves and these waves are superposed (interference) and produce beautiful colours.

2. (c) Given, $\Delta\phi = 100\pi$

We know, change in phase difference,

i.e., $\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x$

where, Δx = path difference

$\Rightarrow \quad \Delta x = \Delta\phi \times \frac{\lambda}{2\pi} = 100\pi \times \frac{\lambda}{2\pi} = 50\lambda$

3. (c) In the phenomenon of interference, energy in conserved but it is redistributed.

4. (d) Resultant amplitude

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

Here $\quad A_1 = A_2 = 2\,\text{cm} \Rightarrow \phi = \pi\,\text{rad}$

$$A = \sqrt{(2)^2 + (2)^2 + 2\times 2\times 2\times \cos\pi}$$

$$A = \sqrt{4+4-8} \text{ or } A = 0$$

5. (b) Given, $\frac{I_{max}}{I_{min}} = \frac{4}{1}$

We know, $\frac{I_{max}}{I_{min}} = \left(\frac{r+1}{r-1}\right)^2 = \frac{4}{1}$

$\Rightarrow \quad \frac{r+1}{r-1} = \frac{2}{1}$

$\Rightarrow \quad r+1 = 2r-2 \text{ or } r = 3$

$\therefore$ The ratio of amplitudes $\frac{A_1}{A_2} = r = 3$

Hence, $\frac{r_1}{r_2} = 3$ or 3 : 1

6. (b) Let the intensities of two waves is I_1 and I_2, respectively. Given, $I_1 : I_2 = 16 : 9$

$\therefore \quad \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} \Rightarrow \frac{16}{9} = \frac{a_1^2}{a_2^2}$

$$\frac{4}{3} = \frac{a_1}{a_2} \Rightarrow a_2 = \frac{3}{4} \cdot a_1$$

$$\therefore \quad \frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{\left(a_1 + \frac{3}{4}a_1\right)^2}{\left(a_1 - \frac{3}{4}a_1\right)^2} = \frac{\left(\frac{7}{4}\right)^2}{\left(\frac{1}{4}\right)^2} = \frac{49}{1}$$

i.e., $I_{max} : I_{min} = 49:1$

7. (b) Two identical and independent sodium lamps (i.e., two independent sources of light) can never be coherent. Hence, no coherence between the light emitted by different atoms.

8. (a) Fringe spacing,

$$\beta = \frac{D\lambda}{d} = \frac{1 \times 5 \times 10^{-7}}{1 \times 10^{-3}} \text{ m}$$

$$(1 \text{ nm} = 10^{-9} \text{ m})$$

$$= 5 \times 10^{-4} \text{ m} = 0.5 \text{ mm}$$

9. (c) For the interference pattern to be formed on the screen, the sources should be coherent and emits lights of same frequency and wavelength.

In a Young's double-slit experiment, when one of the holes is covered by a red filter and another by a blue filter. In this case due to filteration only red and blue lights are present. In YDSE monochromatic light is used for the formation of fringes on the screen. Hence, in this case there shall be no interference fringes.

10. Those sources of light which emit light waves of same wavelength, same frequency and are in same phase or having constant phase difference are called coherent sources.

11. This is because two independent sources of light cannot be coherent, as their relative phases are changing randomly.

12. With increase of D, fringe width also increases as,

$$\beta = \frac{D\lambda}{d}$$

or $\quad \beta \propto D$

13. As we know that, fringe width,

$$\beta = \frac{\lambda D}{d}$$

Here, $\quad D' = 2D$

So, $\quad \beta' = \frac{2\lambda D}{d}$

$\Rightarrow \quad \beta' = 2\beta$

14. If the source of light used in Young's double slit experiment is changed from red to violet, then their consecutive fringes will come closer.

15. Angular separation decreases with the increase of separation between the slits as, $\theta = \frac{\lambda}{d}$

where, d = separation between two slits.

16. When one slit is closed, amplitude becomes $\frac{1}{2}$ and hence intensity becomes $\frac{1}{4}$th and there will be no interference.

17. While moving in a car, there are periodic interruptions in a radio signal that we hear. It occurs on account of interfering of the radio waves, i.e. a common form of interference called multi path interference.

18. Refer to the text on page 410.

19. Refer to the text on page 411.

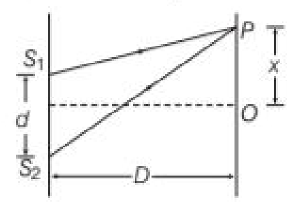
20. No, interference pattern will be observed on the screen. This is because, the source will serve as incoherent sources. For details refer to text on page 411.

21. (i) The fringe width of interference pattern increases with the decrease in separation between S_1 and S_2 as

$$\beta \propto \frac{1}{d}$$

(ii) The fringe width decreases as wavelength gets reduced, when interference set up is taken from air to water as, $\quad \beta \propto \lambda$

22.



For a bright fringe, path difference = $n\lambda$

and so $\quad n\lambda = \frac{x_n d}{D}$

$\Rightarrow \quad x_n = \frac{n\lambda D}{d}$, where $n = 1, 2, 3, \ldots$

$\therefore \quad x_n = \frac{\lambda D}{d}$, when $n = 1$

and $\quad x_n = \frac{2\lambda D}{d}$, when $n = 2$

23. From the given figure, of two slit interference arrangement, we can write

$$T_2P = T_2O + OP = D + x$$

and $\quad T_1P = T_1O - OP = D - x$

$$S_1P = \sqrt{(S_1T_1)^2 + (PT_1)^2} = \sqrt{D^2 + (D - x)^2}$$

and $\quad S_2P = \sqrt{(S_2T_2)^2 + (T_2P)^2} = \sqrt{D^2 + (D + x)^2}$

The minima will occur when $S_2P - S_1P = (2n - 1)\frac{\lambda}{2}$

i.e. $[D^2 + (D + x)^2]^{1/2} - [D^2 + (D - x)^2]^{1/2} = \frac{\lambda}{2}$

[for first minima, $n = 1$]

If $x = D$, we can write, $[D^2 + 4D^2]^{1/2} - [D^2 + 0]^{1/2} = \frac{\lambda}{2}$

$$\Rightarrow \quad [5D^2]^{1/2} - [D^2]^{1/2} = \frac{\lambda}{2}$$

$$\Rightarrow \quad \sqrt{5}D - D = \frac{\lambda}{2}$$

$$\Rightarrow \quad D(\sqrt{5}-1) = \lambda/2 \text{ or } D = \frac{\lambda}{2(\sqrt{5}-1)}$$

Putting $\sqrt{5} = 2.236$

$$\Rightarrow \quad \sqrt{5} - 1 = 2.236 - 1 = 1.236$$

$$D = \frac{\lambda}{2(1.236)} = 0.404\ \lambda$$

24. Refer to the text on page 413.

25. By formula, $\frac{I_{max}}{I_{min}} = \frac{(r+1)^2}{(r-1)^2}$ **[Ans.** 1 : 1**]**

26. (i) We notice a slight shaking of the picture on our TV screen because a low flying aircraft reflects the TV signal and there may be an interference between the direct signal and the reflected signal which results in shaking.

(ii) The superposition principle follows the linear character of the differential equation governing wave motion. If y_1 and y_2 be the solutions of wave equation, so there can be any linear combination of y_1 and y_2. When the amplitudes are large and non-linear effects are important, then the situation is more complicated.

27. (i) Refer to text on page 413.

(ii) Intensity of light at a point on the screen is given by

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2 \cos\phi}$$

For the path difference λ, phase difference is 2π.

As, sources are coherent and taken out of the same source in Young's double slit experiment,

$$I_1 = I_2 = I$$

$$\Rightarrow \quad I_R = 2I + 2I\cos 2\pi$$

$$\Rightarrow \quad I_R = 4I$$

$$\Rightarrow \quad 4I = K \text{ unit} \qquad \text{....(i)}$$

For the path difference, $\frac{\lambda}{3}$ corresponding to phase difference of $\frac{2\pi}{3}$,

$$I_R = 2I + 2I\cos\frac{2\pi}{3} = 2I - I = I \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we conclude

$$I_R = \frac{K}{4} \text{ unit}$$

28. (i) Refer to text on page 411.

(ii) Refer to text on page 410.

(iii) Two independent light bulbs are an example of incoherent sources of light and hence fail to produce interference. For details, refer to text on page 437.

29. Refer to text on page 410.

30. Refer to text on page 412.

31. Given, $OP = y_n$

The distance OP equals one-third of fringe width of the pattern.

i.e $\quad y_n = \frac{\beta}{3} = \frac{1}{3}\left(\frac{D\lambda}{d}\right) = \frac{D\lambda}{3d}$

$$\Rightarrow \quad \frac{dy_n}{D} = \frac{\lambda}{3}$$

Path difference $= S_2P - S_1P = \frac{dy_n}{D} = \frac{\lambda}{3}$

$\therefore$ Phase difference, $\phi = \frac{2\pi}{\lambda} \times$ path difference

$$= \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3}$$

If intensity at central fringe is I_0, then intensity at a point P, where phase difference ϕ, is given by

$$I = I_0 \cos^2\phi$$

$$\Rightarrow \quad I = I_0\left(\cos\frac{2\pi}{3}\right)^2 = I_0\left(-\cos\frac{\pi}{3}\right)^2$$

$$= I_0\left(-\frac{1}{2}\right)^2 = \frac{I_0}{4}$$

Hence, the intensity at point P would be $\frac{I_0}{4}$.

32. (a) Given, $I_1 = I_0$

$I_2 = 50\%$ of I_1

i.e., $\quad I_2 = \frac{I_0}{2}$

Now, ratio of maximum and minimum intensity is given as

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2$$

$$= \left(\frac{\sqrt{I_0} + \sqrt{\frac{I_0}{2}}}{\sqrt{I_0} - \sqrt{\frac{I_0}{2}}}\right)^2 = \frac{\left(\sqrt{I_0} + \sqrt{\frac{I_0}{2}}\right)^2}{\left(\sqrt{I_0} - \sqrt{\frac{I_0}{2}}\right)^2}$$

$$= \frac{\left(1 + \frac{1}{\sqrt{2}}\right)^2}{\left(1 - \frac{1}{\sqrt{2}}\right)^2} = \left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)^2 = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

$$= \left(\frac{3+2\sqrt{2}}{3-2\sqrt{2}}\right) \times \left(\frac{3+2\sqrt{2}}{3+2\sqrt{2}}\right) = \frac{(3+2\sqrt{2})^2}{(3)^2 - (2\sqrt{2})^2}$$

$$= 17 + 12\sqrt{2}$$

(b) When a white light source is used, the interference patterns due to different component colours of white light overlap incoherently. The central bright fringe for different colours is at centre.

So, central bright fringe is white.

As $\lambda_{blue} < \lambda_{red}$, fringe closest on either side of central bright fringe is blue and the farthest is red.

After few fringes, no clear pattern of fringes will be visible.

33. (i) (a) For interference fringes to be seen, $\frac{s}{S} \le \frac{\lambda}{d}$ condition should be satisfied where, s = size of the source and d = distance of the source from the plane of two slits. As, the source slit width increases, fringe pattern gets less and less sharp. When the source slit is so wide, the above condition does not get satisfied and the interference pattern disappears.

(b) The interference pattern due to the different colour components of white light overlap. The central bright fringes for different colours are at the same position. Therefore, central fringes are white. And on the either side of the central fringe (i.e. central maxima), coloured bands will appear.The fringe closed on either side of central white fringe is red and the farthest will be blue. After a few fringes, would be clear fringe pattern is seen.

(ii) Refer to Q. 31 on page 415.

34. (i) Given, the displacements of two coherent sources $y_1 = a\cos\omega t$ and $y_2 = a\cos(\omega t + \phi)$

By principle of superposition,

$y = y_1 + y_2 = a\cos\omega t + a\cos(\omega t + \phi)$

$y = a\cos\omega t + a\cos\omega t\cos\phi - a\sin\omega t\sin\phi$

$y = a(1+\cos\phi)\cos\omega t + (-a\sin\phi)\sin\omega t$

Let $a(1+\cos\phi) = A\cos\theta$ ...(i)

and $a\sin\phi = A\sin\theta$ ...(ii)

$\therefore$ $y = A\cos\theta\cos\omega t - A\sin\theta\sin\omega t$

$\Rightarrow$ $y = A\cos(\omega t + \theta)$ ...(iii)

Squaring and adding Eqs. (i) and (ii), we get

$$(A\cos\theta)^2 + (A\sin\theta)^2$$

$$= a^2(1+\cos\phi)^2 + (a\sin\phi)^2 \; A^2(\cos^2\theta + \sin^2\theta)$$

$$= a^2(1+\cos^2\phi + 2\cos\phi) + a^2\sin^2\phi$$

$$\Rightarrow A^2 \times 1 = a^2 + a^2 + 2a^2\cos\phi = 2a^2(1+\cos\phi)$$

$$\Rightarrow \quad A^2 = 2a^2\left(2\cos^2\frac{\phi}{2}\right) = 4a^2\cos^2\left(\frac{\phi}{2}\right)$$

If I is the resultant intensity, then $I = 4a^2\cos^2\frac{\phi}{2}$

Refer to text on page 434 for conditions for constructive and destructive interference.

(ii) (a) Refer to Q. 21 (i) on page 415 (b) Q. 32 (b) on page 416.

35. (i) (a) Two independent monochromatic sources of light cannot produce a sustained interference pattern because their relative phases are changing randomly. When d is negligibly small, fringe width β which is proportional to $1/d$ may become too large. Even a single fringe may occupy the screen. Hence, the pattern cannot be detected.

(b) Refer to Q. 34 (i) on page 416.

(ii) Refer to Q. 27 (ii) on page 415.

36. Refer to text on page 411.

Refer to text on pages 410 and 412.

Refer to Q. 21 (ii) on page 415.

37. Refer to text on pages 410, 412 and 413.

38. $5I$, refer to Example 1 on page 411.

39. As, $I = I_0\cos^2\phi$

$\because$ Phase difference, $\phi = \frac{2\pi x}{\lambda}$

So, $$I_1 = I_0\cos^2\left(\frac{2\pi\times 0}{\lambda}\right) = I_0$$

and $$I_2 = I_0\cos^2\left(\frac{2\pi}{\lambda}\times\lambda/2\right)$$

$$= I_0\cos^2(\pi) = I_0$$

$\therefore$ $$\frac{I_1}{I_2} = \frac{1}{1}$$

40. Here, $\lambda = 600\text{ nm} = 600\times10^{-9}\text{ m} = 6\times10^{-7}\text{ m}$

$$\Rightarrow 0.1^\circ = \frac{0.1\pi}{180}\text{ rad}$$

From angular width, $\theta = \frac{\lambda}{d}$

$$\Rightarrow \; d = \frac{\lambda}{\theta} = \frac{6\times10^{-7}}{\frac{\pi}{180}\times0.1} = 3.44\times10^{-4}\text{ m}$$

41. Angular width, $\theta = \frac{\lambda}{d}$

Here, $\theta = 0.2^\circ$ $\lambda = 630\text{ nm} = 630\times10^{-9}\text{m}$

$$\Rightarrow \quad d = \frac{630\times10^{-9}\times180}{0.2\times\pi} = 1.80\times10^{-4}\text{m}$$

42. Here, slit width, $d = 0.28\text{ mm} = 0.28\times10^{-3}$ m

Distance between slit and screen, $D = 1.4$ m

$y = 1.2\text{ cm} = 1.2\times10^{-2}\text{ m}, n = 4, \lambda = ?$

For constructive interference, $y = n\lambda\frac{D}{d}$ or $\lambda = \frac{yd}{nD}$

$$= \frac{1.2\times10^{-2}\times0.28\times10^{-3}}{4\times1.4} = 6\times10^{-7}\text{ m}$$

43. We know that, $\frac{I_1}{I_2} = \frac{a^2}{b^2} = \frac{W_1}{W_2}$

$$\Rightarrow \quad \frac{W_1}{W_2} = \frac{(\sqrt{2})^2}{(1)^2} = \frac{2}{1}$$

44. We know that $\frac{W_1}{W_2} = \frac{I_1}{I_2} = \frac{a^2}{b^2} = \frac{4}{1} \Rightarrow \frac{a}{b} = 2:1$

45. Given, distance between the screen and slit, $D = 1$ m

Slit width, $d = 4\times10^{-3}$ m

$$\lambda_1 = 560\text{ nm}, \lambda_2 = 420\text{ nm}$$

Let nth order bright fringe of λ_1 coincides with $(n+1)$th order bright fringe of λ_2.

$\therefore \quad \dfrac{Dn\lambda_1}{d} = \dfrac{D(n+1)\lambda_2}{d} \quad (\lambda_1 > \lambda_2)$

$\Rightarrow \quad n\lambda_1 = (n+1)\lambda_2$

$\Rightarrow \quad \dfrac{n+1}{n} = \dfrac{\lambda_1}{\lambda_2}$

$\therefore \quad 1 + \dfrac{1}{n} = \dfrac{560 \times 10^{-9}}{420 \times 10^{-9}} \Rightarrow 1 + \dfrac{1}{n} = \dfrac{4}{3}$

$\therefore \quad n = 3$

$\therefore$ Least distance from the central fringe where bright fringe of two wavelengths coincides

= distance of 3rd order bright fringe of λ_1

$\therefore \quad y_n = \dfrac{3D\lambda_1}{d} = \dfrac{3 \times 1 \times 560 \times 10^{-9}}{4 \times 10^{-3}}$

$= 0.42 \times 10^{-3}$ m

$\therefore \quad y_n = 0.42$ mm

3rd bright fringe of λ_1 and 4th bright fringe of λ_2 coincide at 0.42 mm from central fringe.

46. Here, $\lambda_1 = 650$ nm $= 650 \times 10^{-9}$ m,

$\lambda_2 = 520$ nm $= 520 \times 10^{-9}$ m

Suppose, d = distance between two slits

D = distance of screen from the slits

(i) For third bright fringe, $n = 3$

$\therefore \quad y = n\lambda_1 \dfrac{D}{d}$

$= 3 \times 650 \dfrac{D}{d}$ nm $= 1950 \dfrac{D}{d}$ nm

(ii) Refer to Q. 33 of topic practice 2. on page 416.

$y = 2600 \dfrac{D}{d}$ nm

47. 12 mm; refer to Q. 45 on page 417.

48. According to the question, $\dfrac{\beta_1}{\beta_2} = \dfrac{\lambda_1}{\lambda_2}$

[$\because$ D and d are same]

Here, $\beta_1 = 7.2 \times 10^{-3}$ m, $\beta_2 = 8.1 \times 10^{-3}$ m

and $\lambda_1 = 630 \times 10^{-9}$ m

Wavelength of another source of laser light,

$\lambda_2 = \dfrac{\beta_2}{\beta_1} \times \lambda_1$

$= \dfrac{8.1 \times 10^{-3}}{7.2 \times 10^{-3}} \times 630 \times 10^{-9}$ m

$\therefore \quad \lambda_2 = 708.75 \times 10^{-9}$ m

$= 708.75$ nm

49. Here, $\theta_1 = 0.2°$, $D = 1$ m, $\lambda_1 = 600$ nm, $\mu = 4/3$

Angular width of a fringe, $\theta = \dfrac{\lambda}{d}$

$\dfrac{\theta_2}{\theta_1} = \dfrac{\lambda_2}{\lambda_1} = \dfrac{1}{\mu} = \dfrac{3}{4}$

$\Rightarrow \quad \theta_2 = \dfrac{3}{4} \times \theta_1 = \dfrac{3}{4} \times 0.2° = 0.15°$

50. $\dfrac{I_{min}}{I_{max}} = \dfrac{9}{25} = \dfrac{(a-b)^2}{(a+b)^2}$

$\Rightarrow \quad \dfrac{a-b}{a+b} = \dfrac{3}{5}$

$\Rightarrow \quad a = 4b$

As, $\dfrac{W_1}{W_2} = \dfrac{a^2}{b^2} = \dfrac{(4b)^2}{b^2} = \dfrac{16}{1}$

51. (i) The distance of nth order bright fringe from central fringe is given by

$y_n = \dfrac{nD\lambda}{d}$

For second bright fringe,

$y_2 = \dfrac{2D\lambda}{d} = \dfrac{2 \times 1 \times 4.5 \times 10^{-7}}{1.5 \times 10^{-4}}$

$y_2 = 6 \times 10^{-3}$ m

The distance of the second bright fringe

$y_2 = 6$ mm

(ii) The distance of nth order dark fringe from central fringe is given by

$y'_n = (2n-1)\dfrac{D\lambda}{2d}$

For second dark fringe, $n = 2$

$y'_n = (2 \times 2 - 1)\dfrac{D\lambda}{2d} = \dfrac{3D\lambda}{2d}$

$\Rightarrow \quad y'_n = \dfrac{3}{2} \times \dfrac{1 \times 4.5 \times 10^{-7}}{1.5 \times 10^{-4}} = 4.5 \times 10^{-3}$ m

$\therefore$ The distance of the second dark fringe

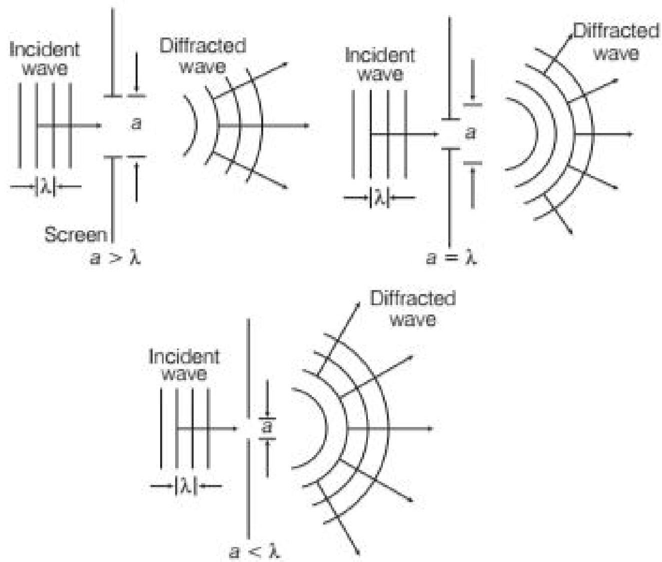
$y'_n = 4.5$ mm

|TOPIC 3|

Diffraction of Light

DIFFRACTION OF LIGHT

The phenomenon of bending of light around the sharp corners and the spreading of light within the geometrical shadow of the opaque obstacles is called diffraction of light. The light thus deviates from its linear path. The deviation becomes much more pronounced, when the dimensions of the aperture or the obstacle are comparable to the wavelength of light.



Diffraction of waves for slits of different width

Note Diffraction is a general characteristic exhibited by all types of waves. For visible light, λ is very small ($\approx 10^{-6}$ m).
Therefore, diffraction of visible light is not so common as obstacles/apertures of this size are hardly available.

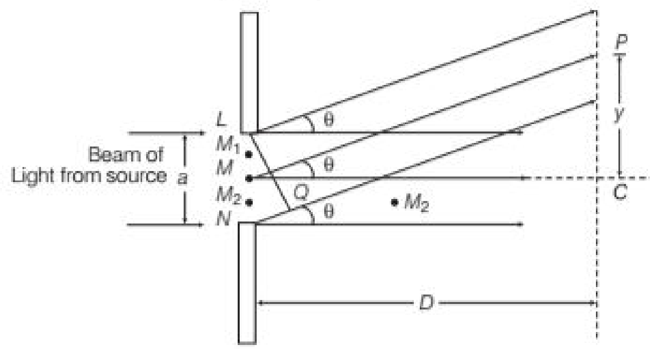
According to Fresnel, diffraction occurs on the account of mutual interference of secondary wavelets starting from portions of the wavefront which are not blocked by the obstacle or from portions of the wavefront which are allowed to pass through the aperture.

Diffraction of Light at a Single Slit

A parallel beam of light with a plane wavefront is made to fall on a single slit LN. As width of the slit $LN = a$ is of the order of wavelength of light, therefore diffraction occurs when beam of light passes through the slit.

The wavelets from the single wavefront reach the centre C on the screen in same phase. Hence, interfere constructively to give central maximum (bright fringe).

The diffraction pattern obtained on the screen consists of a central bright band, having alternate dark and weak bright bands of decreasing intensity on both sides.



Geometry of single slit diffraction

Consider a point P on the screen at which wavelets travelling in a direction, make an angle θ with MC. The wavelets from points L and N will have a path difference equal to NQ.

From the right angled ΔLNQ, we have

$$NQ = LN \sin\theta$$

or

$$NQ = a\sin\theta$$

To establish the condition for secondary minima, the slit is divided into 2, 4, 6,… equal parts such that corresponding wavelets from successive regions interfere with path difference of $\lambda/2$.

Or for **nth secondary minima**, the slit can be divided into $2n$ equal parts.

Hence, for nth secondary minima,

$$\text{Path difference} = \frac{a}{2}\sin\theta = \frac{\lambda}{2}$$

or

$$\boxed{\sin\theta_n = \frac{n\lambda}{a}, \ (n = 1, 2, 3, \dots)}$$

To establish the condition for secondary maxima, the slit is divided into 3, 5, 7, … equal parts such that corresponding wavelets from alternate regions interfere with path difference of $\lambda/2$.

Or for **nth secondary maxima**, the slit can be divided into $(2n+1)$ equal parts.

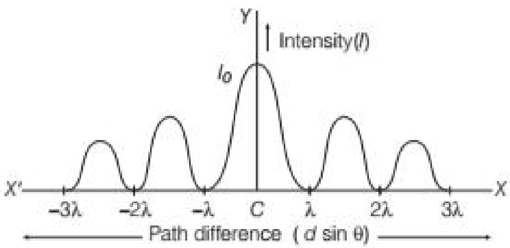
Hence, for nth secondary maxima,

$$a \sin \theta_n = (2n+1)\frac{\lambda}{2} \qquad [n = 1, 2, 3, \ldots]$$

or

$$\boxed{\sin \theta_n = (2n+1)\frac{\lambda}{2a}}$$

Hence, the diffraction pattern can be graphically shown as below



The point C corresponds to the position of central maxima. And the position $-3\lambda, -2\lambda, -\lambda, \lambda, 2\lambda, 3\lambda \ldots$ are secondary minima. The above conditions for diffraction maxima and minima are exactly reverse of mathematical conditions for interference maxima and minima.

Width of Central Maximum

It is the distance between first secondary minimum on either side of the central bright fringe C.

For first secondary minimum,

$$a \sin \theta = \lambda \quad \text{or} \quad \sin \theta = \frac{\lambda}{a} \qquad \ldots\text{(i)}$$

If θ is small, $\sin \theta \approx \theta = \dfrac{y}{D}$ $\qquad \ldots$(ii)

From Eqs. (i) and (ii), we get

$$\frac{y}{D} = \frac{\lambda}{a} \quad \text{or} \quad y = \frac{D\lambda}{a}$$

$$\boxed{\text{Width of central maximum} = 2y = \frac{2D\lambda}{a}}$$

As, the slit width a increases, width of central maximum decreases.

$$\therefore \quad \boxed{\text{Angular width of central maxima, } 2\theta = \frac{2\lambda}{a}}$$

EXAMPLE |1| A beam of light whose wavelength is 4000 Å is diffracted by a single slit of width 0.2 mm. Give the angular position of the second minima.

Sol. Given, wavelength, $\lambda = 4000$ Å

Slit width, $a = 0.2$ mm $= 0.2 \times 10^{-3}$ m

Width of second minima = ?

For nth minima, $a \sin \theta = n\lambda$

$$\therefore \quad 0.2 \times 10^{-3} \sin \theta = 2 \times 4000 \times 10^{-10}$$

For θ to be very small; $\sin \theta = \theta$

$$\therefore \quad \theta = \frac{2 \times 4000 \times 10^{-10}}{0.2 \times 10^{-3}}$$

$$= 4 \times 10^{-3} \text{ rad}$$

EXAMPLE |2| A slit 4 cm wide is irradiated with microwaves of wavelength 2 cm. Find the angular spread of central maximum, assuming incidence normal to the plane of the slit.

Sol. Here, $a = 4 \text{ cm} = 4 \times 10^{-2}$ m,

$\lambda = 2 \text{ cm} = 2 \times 10^{-2}$ m

$\therefore$ Angular spread of central maximum (2θ) is

$$2\theta = \frac{2\lambda}{a} = \frac{2 \times 2 \times 10^{-2}}{4 \times 10^{-2}} = 1 \text{ rad}$$

EXAMPLE |3| Angular width of central maximum in the Fraunhoffer diffraction pattern of a slit is measured. The slit is illuminated by light of wavelength 6000 Å. When the slit is illuminated by light of another wavelength, then the angular width decreases by 30%. Calculate the wavelength of this light. The same decrease in the angular width of central maximum is obtained when the original apparatus is immersed in a liquid. Find the refractive index of the liquid.

Sol. Angular width of central maximum $= \dfrac{2\lambda}{a}$

$$\lambda_1 = 600 \text{ nm}, \theta_1 = \frac{2\lambda_1}{a} \qquad \ldots\text{(i)}$$

$$\theta_2 = \theta_1 \times 0.7 = \frac{2\lambda_2}{a} \qquad \ldots\text{(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{1}{0.7} = \frac{\lambda_1}{\lambda_2} = \frac{600}{\lambda_2}$$

Wavelength, $\lambda_2 = 420$ nm

When immersed in liquid, $\lambda_2 = \lambda_1/\mu$

$$\theta_2 = \theta_1 \times 0.7 \qquad \ldots\text{(iii)}$$

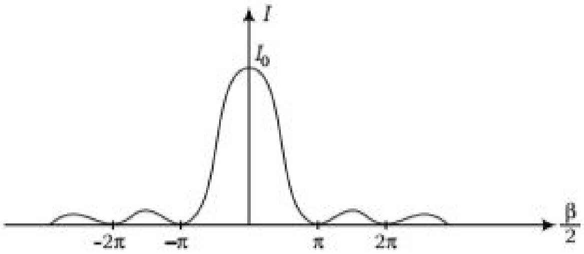
$$\Rightarrow \quad \frac{2\lambda_1/\mu}{a} = \frac{2\lambda_1}{a} \times 0.7$$

$$\frac{1}{0.7} = \mu$$

$\therefore$ Refractive index of the liquid,

$$\mu = \frac{10}{7} = 1.42$$

EXAMPLE |4| Intensity curve for a single slit diffraction pattern is shown below. Find the ratio of the intensities of the secondary maxima to the intensity of the central maximum for the single-slit Fraunhofer diffraction pattern.



Sol. To a good approximation, the secondary maxima lie midway between the zero points. From figure, we see that this corresponds to $\beta/2$ values of $3\pi/2, 5\pi/2, 7\pi/2,$

$$\therefore \quad \frac{I_1}{I_0} = \left[\frac{\sin(3\pi/2)}{(3\pi/2)}\right]^2 = \frac{1}{9\pi^2/4} = 0.045$$

and
$$\frac{I_2}{I_0} = \left[\frac{\sin(5\pi/2)}{5\pi/2}\right]^2 = \frac{1}{25\,\pi^2/4} = 0.016$$

i.e. the first secondary maxima (the ones adjacent to the central maximum) have an intensity of 4.5% that of the central maximum and the next secondary maxima have an intensity of 1.6% that of the central maximum.

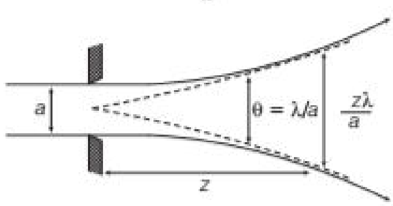
Difference in Diffraction Pattern at a Single Slit due to Monochromatic Light and White Light

For monochromatic light, the diffraction is of alternate bright and dark bands of unequal widths. The central bright fringe has maximum intensity and the intensity of successive secondary maxima decreases rapidly.

If the source is of white light, the diffraction pattern is coloured. The central maximum is white but other bands are coloured . As band width $\propto \lambda$, therefore red band width is wider than the violet band width.

Validity of Ray Optics/ Fresnel's Distance

When a slit or hole of size a is illuminated by a parallel beam, it is diffracted with an angle $\theta \approx \dfrac{\lambda}{a}$.



Diffraction of a parallel beam

In travelling a distance z, size of beam is $z\lambda / a$.

So, taking $\quad \dfrac{z\lambda}{a} \geq a$

or $\quad z \geq \dfrac{a^2}{\lambda}$

Now, distance z_F is called Fresnel's distance $[z_F = a^2/\lambda]$.

Spreading of light due to diffraction is comfortable upto distance $z_F/2$. For distance much greater than z_F, spreading due to diffraction is also prominant. So, the image formation can be explained by ray optics for distances less than z_F.

EXAMPLE |5| For what distance is ray optics a good approximation when the aperture is 3 mm wide and wavelength is 500 nm?

Sol. Here, $a = 3\,\text{mm} = 3\times10^{-3}\,\text{m}$

and $\lambda = 500\,\text{nm} = 5\times10^{-7}\,\text{m}$

According to Fresnel's distance,

$$z_F = \frac{a^2}{\lambda} = \frac{(3\times10^{-3})^2}{5\times10^{-7}} = 18\,\text{m}$$

Difference between Interference and Diffraction

(i) The interference pattern has a number of equally spaced bright and dark bands. Where as the diffraction pattern has a central bright maximum, which is twice as wide as the other maxima. The intensity falls as we go to successive maxima away from the centre on either side.

(ii) We calculate the interference pattern by superposing two waves originating from the two narrow slits. The diffraction pattern is a superposition of a continuous family of waves originating from each point on a single slit.

(iii) For a single slit of width a, the first null of the interference pattern occurs at an angle of λ/a. At the same angle of λ/a, we get a maxima (not a null) for two narrow slits separated by a distance a.

One must understand that both distances between two slits in Young's double slit experiment, d and a width of each slit have to be quite small, to be able to observe good interference and diffraction patterns respectively. e.g. d must be of the order of a millimetre or so and must be even smaller of the order of 0.1 of 0.2 mm.

TOPIC PRACTICE 3

OBJECTIVE Type Questions

1. Light seems to propagate in rectilinear path because
(a) its speed is very large
(b) its wavelength is very small
(c) reflected from the upper surface of atmosphere
(d) it is not absorbed by atmosphere

2. In diffraction from a single slit the angular width of the central maxima does not depends on
(a) λ of light used
(b) width of slit
(c) distance of slits from the screen D
(d) ratio of λ and slit width.

3. What should be the slit width to obtain 10 maxima of the double slit pattern within the central maxima of the single slit pattern of slit width 0.4 mm?
(a) 0.4 mm (b) 0.2 mm
(c) 0.6 mm (d) 0.8 mm

4. In a single diffraction pattern observed on a screen placed at D m distance from the slit of width d m, the ratio of the width of the central maxima to the width of other secondary maxima is
(a) 2 : 1 (b) 1 : 2
(c) 1 : 1 (d) 3 : 1

VERY SHORT ANSWER Type Questions

5. What is the condition for first minima in case of diffraction due to a single slit?

6. How does the angular separation between fringes in single slit diffraction experiment change when the distance of separation between the slit and screen doubled?

7. Explain how the intensity of diffraction pattern changes as the order(n) of the diffraction band varies? **All India 2017**

8. If the wavelength of light decreases, then Fresnel's distance increases or decreases?

9. What is the basic difference between diffraction and interference of light?

SHORT ANSWER Type Questions

10. For a single slit of width a, the first minimum of the interference pattern of a monochromatic light of wavelength λ occurs at an angle of $\frac{\lambda}{a}$. At the same angle of $\frac{\lambda}{a}$, we get a maximum for two narrow slits separated by a distance a. Explain. **Delhi 2014**

11. Draw the intensity pattern for single slit diffraction and double slit interference. Hence, state two differences between interference and diffraction patterns. **All India 2017**

12. State briefly two features which can distinguish the characteristic features of an interference pattern from those observed in the diffraction pattern due to a single slit. **All India 2011**

13. Why is the diffraction of sound waves more evident in daily experience than that of light wave? **NCERT Exemplar**

14. In deriving the single slit diffraction pattern, it was stated that the intensity is zero at angles $\frac{n\lambda}{d}$. Justify this by suitably dividing the slit to bring out the cancellation. **NCERT**

LONG ANSWER Type I Question

15. (a) In a single slit diffraction experiment, the width of the slit is made double the original width. How does this affect the size and intensity of the central diffraction band? Explain.
(b) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the obstacle. Explain, why? **CBSE 2018**

LONG ANSWER Type II Questions

16. (i) Using Huygens' construction of secondary wavelets explain how a diffraction pattern is obtained on a screen due to a narrow slit on which a monochromatic beam of light is incident normally.
(ii) Show that the angular width of the first diffraction fringe is half that of the central fringe.
(iii) Explain why the maxima at $\theta = \left(n + \frac{1}{2}\right)\frac{\lambda}{a}$ become weaker and weaker with increasing n? **All India 2015**

17. (i) Obtain the conditions for the bright and dark fringes in diffraction pattern due to a single narrow slit illuminated by a monochromatic source. Explain clearly, why the secondary maxima go on becoming weaker with increasing of their order?
(ii) When the width of the slit is made double, how would this affect the size and intensity of the central diffraction band? Justify your answer. **Foreign 2012**

NUMERICAL PROBLEMS

18. What should be the width of each slit to obtain 10th maxima of the double slit pattern within the central maxima of single slit pattern?

19. A parallel beam of light of wavelength 500 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1 m away. It is observed that the first minimum is at a distance of 2.5 mm from the centre of the screen. Find the width of the slit. **All India 2013, NCERT**

20. Yellow light ($\lambda = 6000$ Å) illuminates a single slit of 1×10^{-4} m. Calculate the distance between two dark lines on either side to the central maximum, when the diffraction pattern is viewed on a screen kept 1.5 m away from the slit. **All India 2011**

21. A parallel beam of light of 500 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1 m away. It is observed that the first minima is at a distance of 2.5 mm from the centre of the screen. Calculate the width of the slit. **All India 2013**

22. Two wavelength of sodium light of 590 nm and 596 nm are used in turn to study the diffraction taking place at a single slit of aperture 2×10^{-6} m. The distance between the slit and the screen is 1.5 m.Calculate the separation between the position of first maxima of the diffraction pattern obtained in the two cases. **All India 2014**

23. Two wavelengths of sodium light 590nm and 596 nm are used in turn to study the diffraction at a single slit 4mm. The distance between the slit and the screen is 2m. Calculate the separation between the positions of the first maximum of diffraction pattern in two cases.

HINTS AND SOLUTIONS

1. (b) The wavelength of visible light is very small, so it seems to propagate in rectilinear path.

2. (c) Angular width of central maxima,

$$2\theta = 2\lambda/e.$$

Thus, θ does not depend on D i.e., distance between the slit and the screen.

3. (b) As, the path difference $a\theta$ is λ,

then $\theta = \dfrac{\lambda}{a}$

$\Rightarrow \dfrac{10\lambda}{d} = \dfrac{2\lambda}{a}$

$\Rightarrow a = \dfrac{d}{5} = \dfrac{10}{5} = 0.2$ mm

So, the width of each slit is 0.2 mm.

4. (a) Width of central maxima = $2\lambda D/e$

Width of other secondary maxima = $\lambda D/e$

$\therefore$ Width of central maxima : Width of other secondary maxima = 2:1.

5. For first minima, $\sin\theta_1 = 1\times\lambda/a = \lambda/a$

6. Angular width, $2\theta = \dfrac{2\lambda}{a}$

i.e. it is independent of the distance of separation between the slit and screen.

7. In diffraction pattern, intensity of alternate dark and weak bright bands decreases as compare to central bright band.

8. As, Fresnel distance $\propto \dfrac{1}{\lambda}$, hence Fresnel's distance increases as λ decreases.

9. The interference pattern has a number of equally spaced bright and dark bands whereas the diffraction pattern has a central bright maximum, which is twice as wide as the other maxima.

10. In diffraction, angular position, $\theta = \dfrac{\Delta x}{a}$

For first minima, $\Delta x = \lambda$,

$\therefore \theta = \dfrac{\lambda}{a}$

In interference, $d = a$ (given) and angular position,

$$\theta = \frac{\Delta x}{a}$$

$\therefore$ Angular position of first maxima ($\Delta x = \lambda$)

11. **For intensity pattern curve** Refer to text on pages 413 and 423.

For difference Refer to text on page 424.

12. Refer to text on page 424.

13. As, we know that the frequencies of sound waves lie between 20 Hz to 20 kHz, so their wavelength ranges between 15 m to 15 mm. The diffraction occurs, if the wavelength of waves is nearly equal to slit width.

As, the wavelength of light waves is 7000×10^{-10} m to 4000×10^{-10} m, the slit width is very near to the wavelength of sound waves as compared to light waves. Thus, the diffraction of sound waves is more evident in daily life than that of light waves.

14. Let the slit be divided into n smaller slits each of width,

$$d' = \frac{d}{n}.$$

The angle, $\theta = \frac{n\lambda}{d} = \frac{n\lambda}{d'n} = \frac{\lambda}{d'}$

Therefore, each of the smaller slit would send zero intensity in the direction of θ. Hence, for the entire single slit, intensity at angle $\frac{n\lambda}{d}$ would be zero.

15. (a) We know that, width of central maximum is given as

$$2y = \frac{2D\lambda}{a}$$

where, a = width of slit.

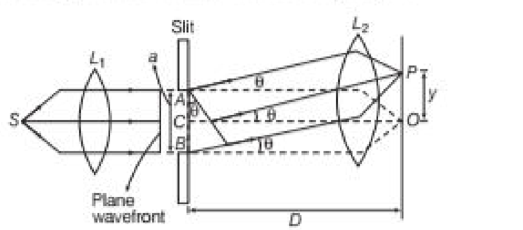
when $a = 2a$

$\therefore$ Width of central maximum $= \frac{2D\lambda}{2a} = \frac{\lambda D}{a}$

Thus, the width of central maximum became half. But in case of diffraction, intensity of central maxima does not changes with slit width. Thus, the intensity remains same in both cases.

(b) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the obstacle because the waves diffracted from the edge of circular obstacle interfere constructively at the centre of the shadow resulting in the formation of a bright spot.

16. (i)



Consider a parallel beam of light from a lens falling on a slit AB. As, diffraction occurs, the pattern is focused on the screen with the help of lens L_2. We will obtain a diffraction pattern that is central maximum at the centre O, flanked by a number of dark and bright fringes called secondary maxima and minima.

Each point on the plane wavefront AB sends out the secondary wavelets in all directions. The waves from points equidistant from the centre C, lying on the upper and lower half, reach point O with zero path difference and hence, reinforce each other producing maximum intensity at O.

(ii) Let λ and a be the wavelength and slit width of diffracting system respectively. Let O be the position of central maximum.

Condition for the first minimum is given by

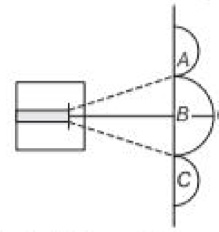
$$a \sin\theta = m\lambda \quad \text{... (i)}$$

Let θ be the angle of diffraction.

As, diffraction angle is small

$\therefore \quad \sin\theta = \theta$

For first diffraction minimum, $\theta = \theta_1$ (let)



For the first minimum, take $m = 1$

$$a\theta_1 = \lambda \Rightarrow \theta_1 = \frac{\lambda}{a}$$

Now, angular width, $AB = \theta_1$

Angular width, $BC = \theta_1$

Angular width, $AC = 2\theta_1$

(iii) On increasing the value of n, the part of slit contributing to the maximum decreases. Hence, the maximum becomes weaker.

17. (i) Refer to text on page 412.

On increasing the order, the part of slit contributing to the maximum decreases. Hence, the secondary maxima becomes weaker.

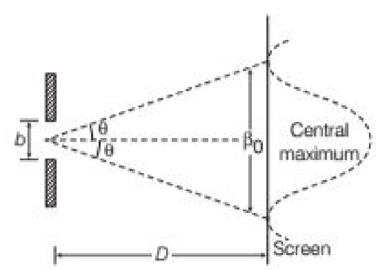
(ii) As, the number of point sources increases, their contribution towards intensity also increases. intensity varies as square of the slit width. Thus, when the width of the slit is made double the original width, intensity will get four times of its original value.

Width of central maximum is given by $\beta = \frac{2D\lambda}{b}$

where, D = distance between screen and slit,

λ = wavelength of the light

and b = size of slit.



So, with the increase in size of slit, the width of central maxima decreases. Hence, double the size of the slit would result as half the width of the central maxima.

18. As, the path difference is λ

So, $a\theta = \lambda$

$\Rightarrow \theta = \lambda / a$

$\Rightarrow \frac{10\lambda}{d} = \frac{2\lambda}{a}$

$\Rightarrow a = \frac{d}{5} = \frac{10}{5} = 2\text{m}$

19. Given, $D = 1$ m, $n = 1$, $y = 2.5$ mm

$= 2.5 \times 10^{-3}$ m

$\lambda = 500$ nm $= 5 \times 10^{-7}$ m, $d = ?$

The condition for minima is $d\frac{y}{D} = n\lambda$

$$\Rightarrow d = \frac{n\lambda D}{y} = \frac{1 \times 5 \times 10^{-7} \times 1}{2.5 \times 10^{-3}}$$

$= 2 \times 10^{-4}$ m $= 0.2$ mm

20. Given, $\lambda = 6000$ Å $= 6 \times 10^{-7}$ m and $d = 1 \times 10^{-4}$ m

Separation between slit and screen, $D = 1.5$ m

$\therefore$ The separation between two dark lines on either side of the central maximum

= fringe width of central maximum

$$= \frac{2D\lambda}{d} = \frac{2 \times 1.5 \times 6 \times 10^{-7}}{1 \times 10^{-4}}$$

$= 18 \times 10^{-3}$ m $= 18$ mm

21. The distance of the nth minima from the centre of the screen is given by $x_n = \frac{nD\lambda}{a}$ ...(i)

where, D = distance of slit from screen = 1 m

λ = wavelength of the light = 500 nm

$= 500 \times 10^{-9}$ m,

$n = 1$,

$x_n = 2.5$ mm $= 2.5 \times 10^{-3}$ m,

and a = width of the slit for first minima = ?

Putting these values in Eq. (i), we get

$$2.5 \times 10^{-3} = \frac{(500 \times 10^{-9})}{a}$$

$\Rightarrow a = 2 \times 10^{-4}$ m $= 0.2$ m

22. For $\lambda_1 = 590$ nm

Location of 1st maxima, $y_1 = (2n+1)\frac{D\lambda_1}{2a}$

If $n = 1 \Rightarrow y_1 = \frac{3D\lambda_1}{2a}$

For $\lambda_2 = 596$ nm

Location of 2nd maxima, $y_2 = (2n+1)\frac{D\lambda_2}{2a}$

If $n = 1 \Rightarrow y_2 = \frac{3D\lambda_2}{2a}$

$\therefore$ Path difference $= y_2 - y_1 = \frac{3D}{2a}(\lambda_2 - \lambda_1)$

$$= \frac{3 \times 1.5}{2 \times 2 \times 10^{-6}}(596 - 590) \times 10^{-9} = 6.75 \times 10^{-3} \text{ m}$$

23. Given, $\lambda_1 = 590$ nm, $\lambda_2 = 596$ nm,

$D = 2$ m, $d = 4$ mm

Refer to Sol. 50 (page 459) location of 1st maxima,

$$y_1 = (2n+1)\frac{D\lambda_1}{2a}$$

if $1 = 1, \Rightarrow y_1 = \frac{3D\lambda_1}{2a}$

For $\lambda_2 = 596$ nm, location of 2nd maxima,

$$y_2 = (2n+1)\frac{D\lambda_2}{2a}$$

if $n = 1, \Rightarrow y_2 = \frac{3D\lambda_2}{2a}$

$$y_2 - y_1 = \frac{3D}{2a}(\lambda_2 - \lambda_1)$$

$$= \frac{3 \times 2}{2 \times 4 \times 10^{-3}}(596 - 590) \times 15^{-9}$$

$= 4.5 \times 10^{-6}$ m

SUMMARY

- **Wave optics** describes the connection between the waves and rays of light.
- **Wavefront** is the locus of points (wavelets) having the same phases of oscillations.
- Wavefronts can be of three types
  (i) Spherical wavefront
  (ii) Cylindrical wavefront
  (iii) Plane wavefront
- **Huygens' Principle** is essentially a geometrical construction, which gives the shape of a wavefront at any time allows us to determine the shape of the wavefront at a later time.
- **Doppler's Effect in Light** According to this effect, whenever there is a relative motion between a source of light and observer, then the apparent frequency of light emitted from the light source.
- **Superposition Principle of Waves** states that at a particular point in the medium, the resultant displacement produced by the number of the displacements produced by each of the waves.
- **Interference of Light Waves** is the phenomenon of redistribution of light energy in a medium on the account of superposition of light waves from two coherent sources.
- **Relation Between Intensity, Amplitude of the Wave and Width of Slit**
  It is given by, $\frac{I_1}{I_2} = \frac{a^2}{b^2} = \frac{W_1}{W_2}$
- **Conditions for Interference** The intensity at the points of constructive and destructive interference must be maintained maximum and zero, respectively.
- **Diffraction of Light** Bending of light around the sharp corners an spreading of light with in the geometrical shadow of opaque obstacles is called diffraction of light.
- In diffraction pattern, angular width of the central maxima is
$$2\theta = \frac{2\lambda}{a}$$
- **Difference between Interference and Diffraction** The interference pattern has number of equally spaced dark and bright bands while the diffraction pattern has central bright maximum.
- **Interference and Energy Conservation** Intensity of light is simply redistributed, i.e., energy is being transferred from the regions of destructive interference. So, the principle of energy conservation is obeyed in interference process.
- **Coherent Sources** of light emit the light waves with constants phase difference.
- **Incoherent Sources** of light emit the light do not waves with a constants phase difference.
- **Young's Double Slit Experiment**
  (i) For constructive interference (bright fringes),
  Path difference $= \frac{dy}{D} = n\lambda$
  $$\Rightarrow \quad y = \frac{nD\lambda}{d}$$
  (ii) For destructive interference (dark fringes), path difference
  $$\frac{dy}{D} = (2n-1)\frac{\lambda}{2}$$
  $$\Rightarrow \quad y = (2n-1)\frac{D\lambda}{2d}$$
- **Intensity of Fringes**
  (i) For bright fringe, $I_R - I_1 + I_2 + 2\sqrt{I_1 I_2}$
  (ii) For dark fringe, $I_R = I_1 - I_2 - 2\sqrt{I_1 I_2}$
- **Fringe Width** is given as, $W = \frac{D\lambda}{d}$
  (i) Resolving power of a microscope is
  $$\text{RP} = \frac{1}{\Delta d} = \frac{2\mu \sin\beta}{1.22\lambda}$$
  (ii) Resolving power of a telescope is
  $$\text{RP} = \frac{1}{d\theta} = \frac{D}{1.22\lambda}$$

CHAPTER PRACTICE

OBJECTIVE Type Questions

1. If a source is at infinity, then wavefronts reaching to observer are
(a) cylindrical (b) spherical
(c) plane (d) conical

2. In Huygens' wave theory, the locus of all points in the same state of vibration is called
(a) a half period zone (b) oscillator
(c) a wavefronts (d) a ray

3. A monochromatic light refracts by the medium of refractive index 1.5 in vacuum. The wavelength of refracted wave will be
(a) equal
(b) increase
(c) decrease
(d) depend upon the intensity of refracted light

4. The source of light is moving towards observer with relative velocity of 3 kms^{-1}. The fractional change in frequency of light observed is
(a) 3×10^{-3} (b) 3×10^{-5}
(c) 10^{-5} (d) None of these

5. The reason of interference is
(a) phase difference
(b) change of amplitude
(c) change of velocity
(d) intensity

6. Two distinct light bulbs as sources
(a) can produce an interference pattern
(b) cannot produce a sustained interference pattern
(c) can produce an interference pattern, if they produce light of same frequency
(d) can produce an interference pattern only when the light produced by them is monochromatic in nature

7. From a single slit, the first diffraction minima is obtained at 30° for a light of 6500 Å wavelength. The width of the slit is
(a) 3250 Å (b) 1.3 μ
(c) 5.4×10^{-4} km (d) 1.2×10^{-2} cm

8. In Young's double slit experiment two disturbance arriving at a point P have phase difference of $\pi/2$. The intensity of this point expressed as a fraction of maximum intensity I_0 is
(a) $\frac{3}{2}I_0$ (b) $\frac{1}{2}I_0$ (c) $\frac{4}{3}I_0$ (d) $\frac{3}{4}I_0$

9. If the width of slit is decreased in a single slit diffraction, then the width of central maxima will
(a) increase
(b) decrease
(c) remain unchanged
(d) not depend on the width of slit.

ASSERTION AND REASON

Directions (Q. Nos. 10-16) *In the following questions, two statements are given- one labeled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

10. **Assertion** In the field of geometrical optics, light can in assumed to approximately travel in straight line.

Reason The wavelength of visible light is very small in comparison to the dimensions of typical mirrors and lenses, then light can be assumed to approximately travel in straight line.

11. **Assertion** When monochromatic light is incident on a surface separating two media, the reflected and refracted light both have the same frequency as the incident frequency.

Reason Speed of light and wavelength of light both changes in refraction and hence, the ratio $v=c/\lambda$ is a constant.

12. **Assertion** The emergent plane wavefront is tilted on refraction of a plane wave by a thin prism.

Reason The speed of light waves is more in glass and the base of the prism is thicker than the top.

13. **Assertion** If we have a point source emitting waves uniformly in all directions, the locus of point which have the same amplitude and vibrate in the same phase are spheres.

Reason Each point of the wavefront is the source of a secondary disturbance and the wavelets emanating from these points spread out in all directions with the speed of the wave.

14. **Assertion** Increase in the wavelength of light due to Doppler's effect is red shift.

Reason When the wavelength increases, then wavelength in the middle of the visible region of the spectrum moves towards the red end to the spectrum.

15. **Assertion** No interference pattern is detected when two coherent sources are infinitely close to each other.

Reason The fringe width is inversely proportional to the distance between the two slits.

16. **Assertion** If the initial phase difference between the light waves emerging from the slits of Young's double slit experiment is π-radian, the central fringe will be dark.

Reason Phase difference is equal to $\frac{2\pi}{\lambda}$ times the path difference.

CASE BASED QUESTIONS

Directions (Q.No. 17) *These questions are case study based questions. Attempt any 4 sub-parts from each question. Each question carries 1 mark.*

17. **The Wavefront**

In 1678, a Dutch scientist, Christian Huygens' propounded the wave theory of light. According to him, wave theory introduced the concepts of wavefront. Light travels in the form of waves.

A wavefront is the locus of points (wavelets) having the same phase (a surface of constant phase) of oscillations. A wavelet is the point of disturbance due to propagation of light. Wavefront may also be defined as the hypothetical surface on which the light waves are in the same phase.

(i) Huygens' original theory of light assumed that, light propagates in the form of

(a) minute elastic particles
(b) transverse electromagnetic wave
(c) transverse mechanical wave
(d) longitudinal mechanical wave

(ii) A wave normal

(a) is parallel to a surface at the point of incidence of a wavefront
(b) is the line joining the source of light and an observer
(c) gives the direction of propagation of a wavefront at a given point
(d) is the envelope that is tangential to the secondary wavelets

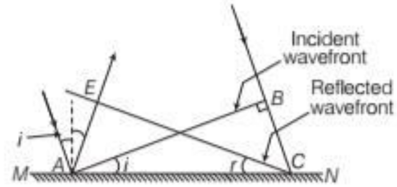
(iii) Ray diverging from a point source form a wavefront that is

(a) cylindrical
(b) spherical
(c) plane
(d) cubical

(iv) According to Huygens' principle, a wavefront propagates through a medium by

(a) pushing medium particles
(b) propagating through medium with speed of light
(c) carrying particles of same phase along with it
(d) creating secondary wavelets which forms a new wavefront

(v) In case of reflection of a wavefront from a reflecting surface,



I. points A and E are in same phase.
II. points A and C are in same phase.
III. points A and B are in same phase.
IV. points C and E are in same phase.

Which of the following is correct?

(a) Both I and II
(b) Both II and III
(c) Both III and IV
(d) Both I and IV

VERY SHORT ANSWER Type Questions

18. State the conditions that must be satisfied for two light sources to be coherent.

19. In Young's double slit experiment, if the distance between the slits is halved, what changes in the fringe width will take place?

20. Suppose while performing double slit experiment, the space between the slits and the screen is filled with water. How does the interference pattern change ?

21. 5000 Å monochromatic light passes through a slit having 0.05 mm width. How much does it spread?

SHORT ANSWER Type Questions

22. Derive fringe separation formula for YDSE.

23. Obtain the expression for the maximum slit width for diffraction.

LONG ANSWER Type II Questions

24. What do you understand by coherent sources? Obtain expression for fringe width of a bright fringe. Write expression for the angular width of fringe.

25. Describe Young's experiment for interference of light. Obtain the formula for fringe width. What is the shape of the fringes?

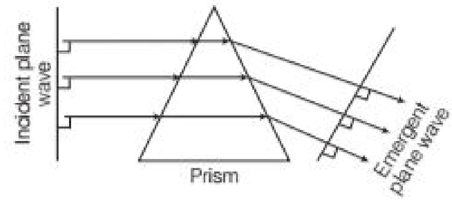
ANSWERS

1. (*c*)	**2.** (*c*)	**3.** (*c*)	**4.** (*c*)	**5.** (*a*)
6. (*b*)	**7.** (*b*)	**8.** (*b*)	**9.** (*a*)	

10. (*a*) The branch of optics in which one completely neglects the finiteness of the wavelength is called geometrical optics. The wavelength of light is very small as compared to the dimensions of objects (such as mirror, lenses etc.) and hence, it can be neglected and assumed to travel in a straight line.

11. (*b*) Reflection and refraction arise through interaction of incident light with constituents of matter. Atoms may be viewed as oscillators, which take up the frequency of the external agency (light) causing forced oscillations. The frequency of light emitted by a charged oscillator equals its frequency of oscillation. Thus, the frequency of scattered light equals the frequency of incident light.

12. (*c*) Since, the speed of light waves is less in glass, the lower portion of the incoming wavefront (which travels through the greatest thickness of glass) will get delayed resulting in a tilt in the emerging wavefront.



13. (*a*) According to Huygens' principle each point of the wavefront is the source of a secondary disturbance and the wavelets emanating from these point spread out in all directions with the space of wave.

These wavelets emanating from the wavefront are usually referred to as secondary wavelets and if we draw a common tangent to all these spheres, we obtain the new position of the wavefront at a later time.

14. (*a*) Increase in wavelength of light when the source move away from the observer due to Doppler's effect is called red shift. The visible regions shifts towards red end of electromagnetic spectrum and hence called red shift.

15. (*a*) As, we know, fringe width β *i.e.*, $= \frac{\lambda D}{d}$

So, smaller the distance between the slits (d), then larger will be fringe width (β).

Hence, single fringe will cover whole screen and pattern will not be visible.

16. (*b*) Given, initial phase difference $= \phi_{S_1} - \phi_{S_2} = \pi$

At central maximum, $\Delta x = 0$ (path difference $= \Delta x$)

$\Rightarrow$ Total phase difference

$$= \phi_{S_1} - \phi_{S_2} + \frac{2\pi}{\lambda}\Delta x$$

At central maximum,

$$\Delta\phi = \pi + \frac{2\pi}{\lambda} \times \Delta x = \pi + 0$$

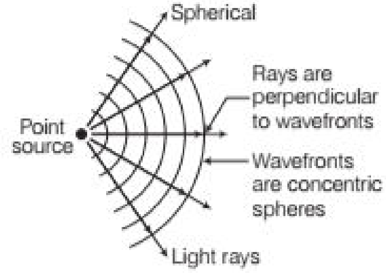
or $\Delta\phi = \pi =$ odd multiple of π.

Hence, at central maximum dark band is obtained.

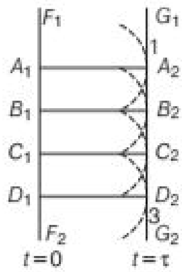
17. (i) (*d*) Huygens' original wave theory of light assumes that light propagates in the form of longitudinal mechanical wave.

(ii) (*c*) A wave normal is a line perpendicular to the wavefront. It gives the direction of a moving wave.

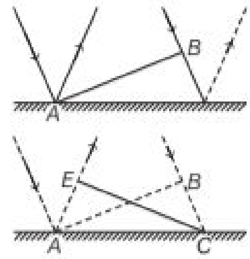
(iii) (*b*) Wavefronts emanating from a point source are spherical wavefronts.



(iv) (*d*) According to Huygens' principle, each point of the wavefront is the source of a secondary disturbance and the wavelets emanating from these points spread out in all directions with the speed of the wave. These wavelets emanating from the wavefront ($F_1 F_2$) are usually referred to as secondary wavelets and if we draw a common tangent to all these spheres as shown below, we obtain the new position of the wavefront ($G_1 G_2$) at a later time (τ).



(v) (*c*) Figure shows *AB* as incident wavefront, so *A* and *B* are in same phase.



By the time *B* reaches *C*, secondary wavelet from *A* reaches *E*.

So, points *C* and *E* are same time intervals apart as they are in same phase.

18. Refer to text on page 411.

19. Fringe width, $W \propto \frac{1}{d}$, hence W is doubled.

20. The pattern will remain same, but slightly shifted.

21. Refer to Example 2 on page 423. [**Ans.** 0.2 rad]

22. Refer to text on page 412.

23. Refer to text on page 423.

24. Refer to text on page 411 and 412.

25. Refer to text on page 412 and 413.