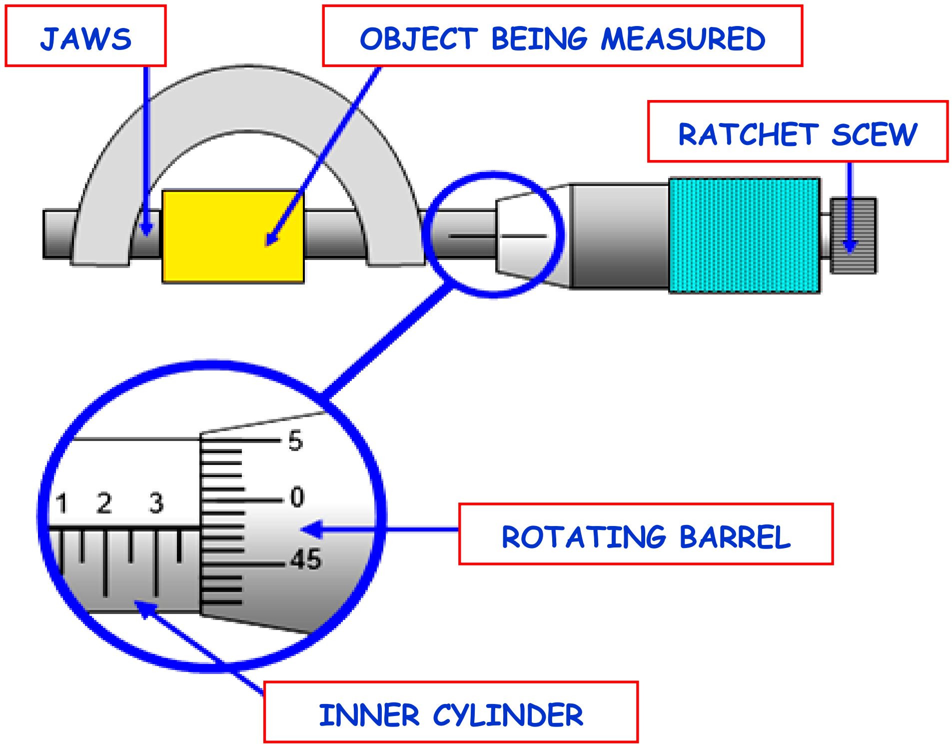
PHYSICS

for JEE, NEET & BOARD

11th

UNIT DIMENSIONS & MEASUREMENTS



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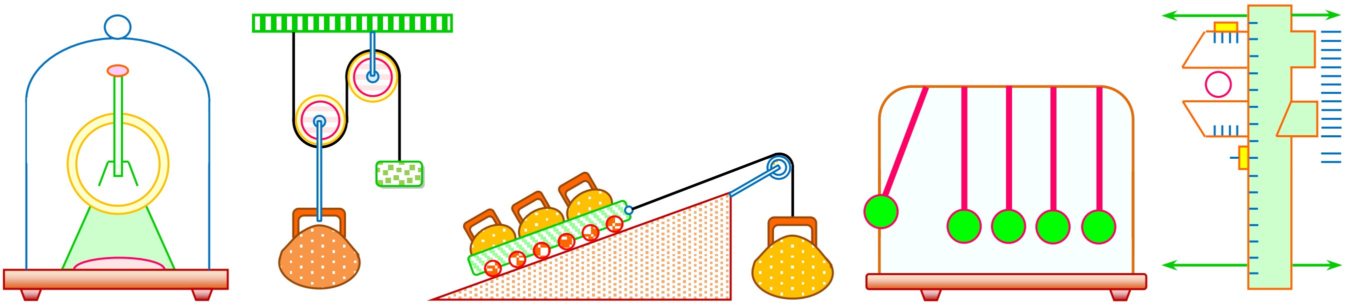

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UNIT DIMENSIONS AND MEASUREMENTS

1 INTRODUCTION: PHYSICS

"Physics is a fundamental science concerned with understanding the natural phenomena that occurs in our universe".

Natural phenomena such as flow of water, heating of objects, sound of waterfall, rainbow, thunderbolt, energy coming from nucleus … etc., which are very exciting to us and a number of events taking place around us which are very useful in our life. All events in nature take place according to some basic laws and *revealing these laws of nature from the observed events is physics.*

"The scientist does not study physics because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful. If nature were not beautiful, it would not be worth knowing, and if nature were not worth knowing, life would not be worth living"

BY: Henri Poincare

The main objective of physics is to use the limited number of fundamental laws that governs natural phenomena to develop theories that can predict the results of future experiments.

The fundamental laws used in developing theories are expressed in the language of mathematics, the tool that provides a bridge between theory and experiments.

Between 1600 and 1900, three broad areas were developed, which is together called classical physics.

(i) Classical Mechanics deals with the study of the motion of particles and fluids.

(ii) Thermodynamics deals with the study of temperature, heat transfer and properties of aggregations of many particles.

(iii) Electromagnetism deals with electricity, magnetism, electromagnetic wave, and optics.

These three areas explain all the physical phenomena with which we are familiar.

But by 1905 it became apparent that classical ideas failed to explain several phenomena. Then some new theories were developed in what is called Modern Physics.

Three important theories in modern physics are

(i) Special Relativity: A theory of the behavior of particles moving at high speeds. It led to a radical revision of our ideas of space, time and energy.

(ii) Quantum Mechanics: A theory dealing with the behavior of particles at the submicroscopic level as well as the macroscopic world.

(iii) General Relativity: A theory that relates the force of gravity to the geometrical properties of space.

It is also useful in the study of other subjects like biotechnology, geophysics, geology etc.

SCOPES AND EXCITEMENT

The scope of physics is very wide. We can understand the scope of physics by looking at its various sub-discipline. It covers a very wide variety of natural phenomena. It deals with the phenomena from microscopic level to macroscopic level. The microscopic domain includes atomic, molecular and nuclear phenomena. The macroscopic domain includes phenomena at laboratory, terrestrial and astronomical scales.

For example forces we encounter in nature are nuclear forces, chemical forces and forces exerted by ropes, springs, fluids, electric charges, magnets, the earth and the Sun. Their ranges and relative strength can be summarized as shown below in the table.

Forces	Relative Strength	Range
Strong	1	10^{-15} m
Electromagnetic	10^{-2}	Infinite
Weak	10^{-6}	10^{-17} m
Gravitational	10^{-38}	Infinite

Similarly the range of distance we study in physics vary from 10^{-14} m (size of nucleus) to 10^{+25} m (size of universe)

The range of masses includes in study of physics varies from 10^{-30} kg (mass of electron) to 10^{55} kg (mass of universe)

The range of time interval varies from 10^{-22} sec (time taken by a light to cross a nuclear distance) to 10^{18} sec. (life time of sun).

So we can say scope of physics is really wide.

The study of physics is very exciting in many ways.

Excitement of Physics can be seen in every field.

- ❖ Advancement of technology has upgraded the entire scenario of entertainment, starting from a radio to the most advanced cyber park.
- ❖ Communication system has been also deepened its root by bridging distant areas closer.
- ❖ Advances in health science, which has enabled operations without surgery.
- ❖ Telescopes & satellite have broken the limits of knowledge of the undiscovered universe.
- ❖ Exploring the sources of energy from the unexplored sources.
- ❖ Made possible the reach of man beyond the earth, towards the cosmos.
- ❖ Use of Robots in a hazardous places is highly beneficial.

2 PHYSICAL QUANTITIES

All the quantities by means of which we describe the laws of nature and which can be measured are called physical quantities. Examples are Mass, length, time, force, velocity, acceleration etc.

'Beauty and intelligence' are not physical quantities, because they can't be measured.

All the physical quantities have been classified into two parts

Physical Quantities

Fundamental — *Derived*

2.1 FUNDAMENTAL PHYSICAL QUANTITY

It is an elementary physical quantity, which doesn't require any other physical quantity to express it. It means it cannot be resolved further in terms of any other physical quantity. It is also known as basic physical quantity.

In mechanics length, mass and time are only three basic physical quantities. Other basic physical quantities are electric current, temperature, luminous intensity and amount of substance.

2.2. DERIVED PHYSICAL QUANTITY

All those physical quantities, which can be derived from the combination of two or more fundamental quantities or can be expressed in terms of basic physical quantities, are called derived physical quantities.

Examples: Velocity, density, force, energy etc.

Velocity can be expressed in terms of distance and time

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

Density can be expressed in terms of mass and length

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{\text{Mass}}{(\text{Length})^3}$$

3 UNITS

We know that the results of physics are based on experimental observation and quantitative measurement, so it is also known as **quantitative science**.

Now to express the magnitude of any physical quantity we need a unit.

Suppose we say that the distance between two stations *A* and *B* is 500. Without a unit, we are unable to understand how far *B* is from *A*.

But when we say that distance is 500 m or 500 km then we get a picture of how far *B* is from *A,* since we know the standard length of meter. Similarly, when we say that the speed of an automobile is 20, unless we also specify the units meter per second or kilometer per hour it is meaningless. When we want to know the amount of fuel needed for a long trip it is difficult to arrive at an answer without knowing the unit of speed.

So we can say that all physical quantity needs a unit.

The need of unit felt long-long ago in *A.D.* 1120. The king of England decided the unit of length, which was yard. The yard was the distance from the tip of his nose to the end of outstretched arm.

Similarly, the original standard for the foot adopted by the French was the length of royal foot of king Louis XIV. This standard prevailed until 1799, until the legal standard in France became the meter.

In this way we see that different standards were decided in different periods of time and in different parts of the world. Due to this reason again a major problem was faced which can be understood from this example. Suppose a visitor from another place is talking about a length of 8 "glitches" and we don't know the meaning of the unit glitch. So this talk is meaningless for us. Then the need of a standard unit was felt which should be universal.

In 1960, an international committee was established to set a standard of unit for physical quantities. This system is called **International System (SI) of units**.

3.1 UNITS

To measure a physical quantity we require a standard of that physical quantity. This standard is called **unit of that physical quantity**. It can be defined in this way.

"Measurement of any physical quantity involves comparison with a certain basic internationally accepted reference standard called unit.

Measure of physical quantity = Numerical value × Unit

Length of a rod = 8 m

where 8 is numerical value and m (metre) is unit of length.

3.2 FUNDAMENTAL AND DERIVED UNITS

There are a large number of physical quantities and every quantity needs a unit.

However, all the quantities are not independent. For example, if a unit of length is defined, a unit of area or volume is automatically obtained. Thus, we can define a set of fundamental quantities and all other physical quantities, which can be expressed in terms of fundamental quantities, are derived quantities.

Fundamental Units: The units of fundamental physical quantities are called fundamental units.

In mechanics, unit of length, mass and time are m (meter), kg (kilogram) and s (second), respectively, which are called fundamental units.

Derived Units: The units of all other physical quantities, which can be obtained from fundamental units, are called derived units.

For example units of velocity, density and force are m/s, kg/m^3, kg m/s^2 respectively.

Who decides the units In 1960, an international committee established a set of standards for these fundamental quantities. This system is called the International System (SI) of units. In this system units of length, mass and time are the meter, kilogram, and second, respectively. Other standard basic SI units established by the committee are for temperature (the kelvin), electric current (the ampere), Luminous intensity (the candela) and for the amount of substance (the mole)

These seven units are the basic/fundamental SI units as shown below in tables with symbol.

S.No.	Fundamental Physical quantity	Units	Symbol
1.	Length	metre	m
2.	Mass	kilogram	kg
3.	Time	second	s
4.	Electric current	ampere	A
5.	Luminous intensity	candela	cd
6.	Temperature	kelvin	K
7.	Amount of Substance	mole	mol

However definition of units is under constant review and is changed from time to time. A body named "General conference on Height and Measures' holds the meeting to decide any changes in units.

Other International Systems are

(1) The F.P.S. System is the British Engineering system of units. In this system unit of length is foot, of mass is pound and of time is second.

(2) The C.G.S. system is the Gaussian system. In this system units of length, mass and time are centimetre, gram and second respectively.

(3) The M.K.S. System in this system the units of length, mass and time are metre, kilogram and second respectively.

3.3 DEFINITIONS OF FUNDAMENTAL UNIT

Length: To measure the distance between any two points in the space we use the term length and to specify the magnitude we use a unit of length.

Most common unit of length is metre.

In 1799 legal standard of length became metre, which was defined as one ten-millionth the distance from the Equator to the North Pole.

In 1960, the length of metre was defined as the distance between two lines on a specific platinum iridium bar stored under controlled condition.

Recently the metre was defined as 1650,763.73 wavelength of orange red light emitted from a krypton-86. However in October 1983, the metre was redefined as the distance traveled by light in vaccum during a time of 1/299792458 second.

Mass: The mass is a basic property of matter. The S.I. unit of mass is kilogram, which is defined as the mass of a specific platinum-iridium alloy cylinder kept at the international Bureau of Heights and Measures at Severs, France. This mass standard was established in 1887, and there has been no change since that time because platinum-iridium is an unusually stable alloy. A duplicate is kept at the National Institute of Standards and Technology in Gaithersburg.

Time: The concept of time is very old. Before 1960, the standard of time was defined in terms of mean solar day. The mean solar second was the basic unit of time, which was defined as $\left(\frac{1}{60}\right)\left(\frac{1}{60}\right)\left(\frac{1}{24}\right)$ of a mean solar day.

In 1967, the definition of second was modified using the characteristic frequency of a particular kind of cesium atom as the "reference clock".

The SI unit of time, the second, is defined as 91926631770 periods of the radiation from cesium – 133 atoms.

SI Base Quantities and Units

Base Quantity	SI Unit		
	Name	Symbol	Definition
Length	metre	m	The metre is the length of the path traveled by light in vacuum during a time interval of 1/299,792,458 of a second. (1983)
Mass	kilogram	kg	The kilogram is equal to the mass of the international prototype of the kilogram (a platinum-iridium alloy cylinder) kept at International Bureau of Weights and Measures, at Sevres, near Paris, France. (1989)
Time	second	s	The second is the duration of 9,192,631, 770 periods of the radiation corresponding to the transition between the two-hyperfine levels of the ground state of the cesium-133 atom. (1967)
Electric current	ampere	A	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in *vacuum*, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length. (1948)
Thermo dynamic temperature	kelvin	K	The kelvin, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (1967)
Amount of substance	mole	mol	The mole is the amount of substance of a system, which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12. (1971)
Luminous intensity	candela	cd	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian. (1979)

Some SI Derived Units expressed in SI Base Units

Physical quantity	SI Unit	
	Name	Symbol
Area	square metre	m^2
Volume	cubic metre	m^3
Speed, velocity	metre per second	m/s or ms^{-1}
Angular velocity	radian per second	rad/s or rad s^{-1}
Acceleration	metre per second square	m/s^2 or ms^{-2}
Angular acceleration	radian per second square	rad/s^2 or rad s^{-2}
Wave number	per metre	m^{-1}
Density, mass density	kilogram per cubic metre	kg/m^3 or kg m^{-3}
Current density	ampere per square metre	A/m^2 or Am^{-2}
Magnetic field strength, magnetic intensity, magnetic moment density	ampere per metre	A/m or Am^{-1}
Concentration (of amount of substance)	mole per cubic metre	mol/m^3 or mol m^{-3}
Specific volume	cubic metre per kilogram	m^3/kg or m^3 kg^{-1}
Luminance, intensity of illumination	candela per square metre	cd/m^2 or cd m^{-2}
Kinematic viscosity	square metre per second	m^2/s or m^2 s^{-1}
Momentum	kilogram metre per second	kg m s^{-1}
Moment of inertia	kilogram metre square	kg m^2
Radius of gyration	metre	m
Linear/superficial/volume expansivities	per kelvin	K^{-1}
Flow rate	cubic metre per second	m^3 s^{-1}

SI Derived Units with Special names

Physical quantity	SI Unit			
	Name	Symbol	Expression in terms of other units	Expression in terms of SI base Units
Frequency	hertz	Hz	—	s^{-1}
Force	newton	N	—	kg m s^{-2}
Pressure, stress	pascal	Pa	N/m^2 or Nm^{-2}	$kgm^{-1}s^{-2}$
Energy, work, quantity of heat	joule	J	Nm	kg m^2 s^{-2}
Power, radiant flux	watt	W	J/s or Js^{-1}	kg m^2 s^{-3}
Quantity of electricity, Electric charge	coulomb	C	—	A-s
Electric potential, Potential difference, Electromotive force	volt	V	W/A or WA^{-1}	Kg m^2s^{-3} A^{-1}
Capacitance	farad	F	C/V	A^2s^4 kg^{-1} m^{-2}
Electric resistance	ohm	Ω	V/A	kg m^2 s^{-3} A^{-2}
Conductance	mho	S	A/V	m^{-2} kg^{-1} s^3 A^2
magnetic flux	weber	Wb	Vs or J/A	kg m^2 s^{-2} A^{-1}
Magnetic field, magnetic flux density, magnetic induction	tesla	T	Wb/m^2	kg s^{-2} A^{-1}
Inductance	henry	H	Wb/A	kg m^2 s^{-2} A^{-2}
Luminous flux, luminous Power	lumen	lm	—	cd /sr
Illuminance	lux	lx	lm/m^2	m^{-2} cd sr^{-1}
Activity of a radio nuclide/radioactive source	becquerel	Bq		s^{-1}

3.4 CONVERSION OF UNITS

In making calculations, it is essential that all the physical quantities used should be consistent in units. If we want to know the total amount of fuel consumed by an automobile during a long trip, we cannot simply add one purchase made in litres to another made in gallons. So it is necessary to convert different units of a physical quantity to a desired unit.

Suppose we wish to convert 5 miles per hour (mi/h) into meter per second (m/s).

Given that 1 mile = 1.6 km

$$5.0\,\frac{\text{mile}}{\text{hour}} = \frac{5\times1.6\ \text{km}}{1\times60\ \text{minutes}}$$

$$= \frac{8}{60}\frac{\text{km}}{\text{minutes}} = \frac{8\times1000}{60\times60}\frac{\text{meter}}{\text{second}}$$

$$= \frac{20}{9}\frac{\text{meter}}{\text{second}} = 2.22\ \text{ms}^{-1}$$

3.5 PREFIXES AND MULTIPLICATION FACTORS

The magnitude of physical quantities vary over a wide range as discussed in scope of physics, so standard prefixes for certain power of 10 was decided by CGPM, which are shown below in table.

The most commonly used prefixes are given below in tabular form.

Power of 10	Prefix	Symbol
18	exa	E
15	peta	P
12	tera	T
9	giga	G
6	mega	M
3	kilo	k
2	hecto	h
1	deka	da
–1	deci	d
–2	centi	c
– 3	mili	m
– 6	micro	μ
– 9	nano	n
–12	pico	p
–15	femto	f
–18	atto	a

Conversion Table

Length

1 in. = 2.54 cm
1 m = 39.37 in. = 3.281 ft
1 ft = 0.3048 m = 30.48 cm
12 in. = 1 ft
3 ft = 1 yd
1 yd = 0.9144 m
1 km = 0.621 mi

Force

1 N = 10^5 dyne = 0.2248 lbf
1lbf = 4.448 N
1 dyne = 10^{-5} N = 2.248 × 10^{-6} lbf

Velocity

1 mi/h = 1.47 ft/s = 0.447 m/s = 1.61 km/h
1 m/s = 100 cm/s = 3.28 ft/s
1 mi/min = 60 mi/h = 88 ft/s

1 mi = 1.609 km
1 mi = 5280 ft
1 Å = 10^{-10} m
1 μm = 10^{-6} m I= 10^{4} Å
1 light-year (ly) = 9.461 × 10^{15} m

Area

1 m^2 = 10^4 cm^2 = 10.76 ft^2
1 ft^2 = 0.0929 m^2 = 144 $in.^2$
1 $in.^2$ = 6.452 cm^2
1 m^2 = 10.76 ft^2
1 mi^2 = 460 acres = 2.590 km^2

Volume

1 m^3 = 10^6 cm^3 = 6.102 × 10^4 $in.^3$
1 ft^3 = 1728 $in.^3$ = 2.83 × 10^{-2} m^3
1 liter = 1000 cm^3 = 1.0576 qt = 0.0353 ft^3
1 ft^3 = 7.481 gal = 28.32 liters = 2.832 × 10^{-2} m^3
1 gal = 3.786 liters = 281 $in.^3$
1 in^3 = 16.39 cm^3

Mass

1 kg = 1000 g
1 kg = 2 lb
1 u = 1.66 × 10^{-27} kg

Magnetic Field

1 G = 10^{-4} T
1 T = 1 Wb m^{-2} = 10^4 G
1 u = 1.66 × 10^{-27} kg

Acceleration

1 m/s^2 = 3.28 ft/s^2 = 100 cm/s^2
1 ft/s^2 = 0.3048 m/s^2 = 30.48 cm/s^2

Pressure

1 bar = 10^5 N/m^2 = 14.50 $lb/in.^2$
1 atm = 760 mm Hg = 76.0 cm Hg
1 atm = 14.7 $lb/in.^2$ = 1.013 × 10^5 N/m^2
1 Pa = 1 N/m^2 = 1.45 × 10^{-4} $lb/in.^2$

Time

1 acre = 43,560 ft^2
1 day = 24 h = 1.44 × 10^3 min = 8.64 × 10^4 s

Energy

1 J = 0.738 ft. lb = 10^7 ergs
1 cal = 4.186 J
1 Btu = 252 cal = 1.054 × 10^3 J
1 eV = 1.6 × 10^{-19} J

931.5 MeV is equivalent to 1 u
1 kWh = 3.60 × 10^6 J

Power

1000 kg = 1 tonne
1 W = 1J/s = 0.738 ft.lb/s
1 Btu/ h = 0.293 W

Angle and Angular speed

π rad = 180°
1 rad = 57.30°
1° = 1.745 × 10^{-2} rad
1 rev min^{-1} = 0.1047 rad s^{-1}
1 rad s^{-1} = 9.549 rev min^{-1}

Illustration 1

Question: **The SI unit of force is newton such that 1N = 1kg ms^{-2}. In C.G.S. system, force is expressed in dyne. How many dyne of force is equivalent to a force of 5 N?**

Solution: Let 1 N = n dynes

$$\Rightarrow \left(\frac{1\,kg\,m}{s^2}\right) = n\left(\frac{g-cm}{s^2}\right)$$

or $$\frac{1000g \times (100)cm}{s^2} = n\left(\frac{g-cm}{s^2}\right)$$

$$\Rightarrow n = 10^5$$

∴ 5 N = 5 × 10^5 dyne.

Illustration 2

Question: **Young's modulus of a metal is 21×10^{10} N/m^2. Express it in dyne/cm^2.**

Solution:
$$Y = 21 \times 10^{10}\, Nm^{-2}$$
$$= 21 \times 10^{10}\, kgm^{-1} s^{-2}$$
$$= 21 \times 10^{10} (10^3\, g)(10^2\, cm)^{-1} s^{-2}$$
$$= 21 \times 10^{11} (gcms^{-2}) \times cm^{-2}$$
$$\therefore \quad Y = \mathbf{21 \times 10^{11}\, dyne\, cm^{-2}}$$

Illustration 3

Question: **the sun's angular diameter is measured to be 1920". The distance *D* of the Sun from the Earth is 1.496×10^{11} m. What is the diameter of the Sun ?**

Solution: Sun's angular diameter α

= 1920"

= $1920 \times 4.85 \times 10^{-6}$ rad

= 9.31×10^{-3} rad

Sun's diameter

$$d = \alpha D$$
$$= (9.31 \times 10^{-3}) \times (1.496 \times 10^{11}) m$$

= 1.39×10^9 m

4 DIMENSIONAL ANALYSIS

The word **dimension** has a special meaning in Physics. It usually denotes the physical nature of a quantity. Whether a distance is measured in units of feet or meters or furlongs, it is a distance. We say its dimension is **length.**

The symbols we use to express the dimensions are only for fundamental physical quantities so we use seven symbols for seven fundamental quantities as given below. Remaining all derived physical quantities are expressed in terms of dimensions of fundamental quantities.

"The dimensions of a physical quantities are the powers to which the base quantities are raised to represent that quantity"

Dimension of fundamental quantities are

Length	L
Mass	M
Time	T
Electric current	A
Temperature	K
Luminous Intensity	cd
Amount of substance	mol

Volume occupied by an object = Length × breadth × height

Dimensions of volume are = L^3

As the volume is independent of mass and time. It is said to be zero dimension in mass M°, zero dimension in time T° and three dimensions in length. All dimensions are written together in this way $M° L^3 T°$.

$$\text{Force} = \text{Mass} \times \text{acceleration} = \text{Mass} \times \frac{\text{Total path displacement}}{(\text{Time})^2}$$

$$\text{So the dimensions of force are} = \frac{M\,L}{T^2} = MLT^{-2}$$

This equation of dimensions is also known as dimensional equation.

Dimensions of commonly used Physical Quantities

S.No.	Physical Quantity (Mechanics)	SI Units	Dimensional formula
1.	Velocity = displacement/time	m/s	M^0LT^{-1}
2.	Acceleration = velocity/time	m/s^2	M^0LT^{-2}
3.	Force = mass × acceleration	$kg\text{-}m/s^2$ = Newton or N	MLT^{-2}
4. 5. 6.	Work = force × displacement Energy Torque = force × perpendicular distance	$kg\text{-}m^2/s^2$ = N-m = Joule or J N-m	ML^2T^{-2}
7.	Power = work/time	J/s or watt	ML^2T^{-3}
8.	Momentum = mass × velocity	Kg-m/s	MLT^{-1}
9.	Impulse = force × time	Kg-m/s or N-s	MLT^{-1}
10. 11.	Angle = arc/radius Strain = $\frac{\Delta L}{L}$ or $\frac{\Delta V}{V}$	radian or rad no units	$M^0L^0T^0$
12. 13. 14.	Stress = force/area Pressure = force/area Modulus of elasticity = stress/strain	N/m^2 N/m^2 N/m^2	$ML^{-1}T^{-2}$ $ML^{-1}T^{-2}$ $ML^{-1}T^{-2}$
15. 16.	Frequency = 1/ time period Angular velocity = angle/time	per sec or hertz (Hz) rad/s	$M^0L^0T^{-1}$ $M^0L^0T^{-1}$
17.	Moment of inertia = (mass) (distance)^2	$kg\text{-}m^2$	ML^2T^0
18.	Surface tension = force/length	N/m	ML^0T^{-2}
19.	Gravitational constant $= \frac{\text{Force} \times (\text{distance})^2}{(\text{mass})^2}$	$N\text{-}m^2/kg^2$	$M^{-1}L^3T^{-2}$

S.No.	Physical Quantity	SI Units	Dimensional formula
1.	Thermodynamic temperature	kelvin (K)	$M^0L^0T^0K$
2.	Heat	joule	ML^2T^{-2}
3.	Specific heat	$Jkg^{-1}K^{-1}$	$M^0L^2T^{-2}K^{-1}$
4.	Latent heat	$J\ kg^{-1}$	$M^0L^2T^{-2}$
5.	Universal gas constant	$J\ mol^{-1}\ K^{-1}$	$ML^2T^{-2}K^{-1}mol^{-1}$
6.	Boltzmann's constant	JK^{-1}	$ML^2T^{-2}K^{-1}$

7.	Stefan's constant	$Js^{-1}m^{-2}K^{-4}$	$MT^{-3}K^{-4}$
8.	Planck's constant	Js	ML^2T^{-1}
9.	Solar constant	$J\,m^{-2}s^{-1}$	ML^0T^{-3}
10.	Thermal conductivity	$Js^{-1}m^{-1}K^{-1}$	$MLT^{-3}K^{-1}$
11.	Thermal resistance	$Kscal^{-1}$	$M^{-1}L^{-2}T^3K$
12.	Enthalpy	cal	ML^2T^{-2}
13.	Entropy	cal K^{-1}	$ML^2T^{-2}K^{-1}$

Quantities having same Dimensions

Dimension	Quantity
$[M^0L^0T^{-1}]$	Frequency, angular frequency, angular velocity, velocity gradient and decay constant
$[M^1L^2T^{-2}]$	Work, internal energy, potential energy, kinetic energy, torque, moment of force
$[M^1L^{-1}T^{-2}]$	Pressure, stress, Young's modulus, bulk modulus, modulus of rigidity, energy density
$[M^1L^1T^{-1}]$	Momentum, impulse
$[M^0L^1T^{-2}]$	Acceleration due to gravity, gravitational field intensity
$[M^1L^1T^{-2}]$	Thrust, force, weight, energy gradient
$[M^1L^2T^{-1}]$	Angular momentum and Planck's constant
$[M^1L^0T^{-2}]$	Surface tension, Surface energy (energy per unit area)
$[M^0L^0T^0]$	Strain, refractive index, relative density, angle, solid angle, distance gradient, relative permittivity (dielectric constant), relative permeability etc.
$[M^0L^2T^{-2}]$	Latent heat and gravitational potential
$[ML^2T^{-2}K^{-1}]$	Thermal capacity, gas constant, Boltzmann constant and entropy
$[M^0L^0T^1]$	$\sqrt{\frac{l}{g}}, \sqrt{\frac{m}{k}}, \sqrt{\frac{R}{g}}$, where l = length g = acceleration due to gravity, m = mass, k = spring constant, R = Radius of earth
$[M^0L^0T^1]$	$\frac{L}{R}, \sqrt{LC}, RC$ where L = inductance, R = resistance, C = capacitance
$[ML^2T^{-2}]$	$I^2Rt, \frac{V^2}{R}t, VIt, qV, LI^2, \frac{q^2}{C}, CV^2$ where I = current, t = time, q = charge, L = inductance, C = capacitance, R = resistance

Note : A physical constant may or may not have dimensions.

Illustration 4

Question: **Find the dimensional formula of angular velocity.**

Solution: [angular velocity] $= \dfrac{[\theta]}{[t]} = \dfrac{M^{\circ}L^{\circ}T^{\circ}}{T} = \mathbf{M^{\circ}L^{\circ}T^{-1}}$

Illustration 5

Question: **Find the dimensional formula for Planck's constant (*h*).**

Solution: $E = \dfrac{hc}{\lambda}$

$$[h] = \frac{[E\lambda]}{[c]} = \frac{ML^2T^{-2}L}{LT^{-1}} = \mathbf{ML^2T^{-1}}$$

4.1 APPLICATION OF DIMENSIONAL ANALYSIS

(i) In conversion of units from one system to other

This is based on the fact that the product of the numerical value and its corresponding unit is constant.

Numerical value × Unit = constant

So when the unit will change, numerical value will also change.

Gravitational constant $G = 6.67 \times 10^{-11}$ Nm^2/kg^2 in SI units. We can convert it in C.G.S. system in this way.

The dimensional formula of G is $M^{-1}L^3T^{-2}$

Let n_1 represents magnitude in S.I. unit and n_2 in C.G.S. unit

$$n_1 (M_1^{-1} L_1^3 T_1^{-2}) = n_2 \left(M_2^{-1} L_2^3 T_2^{-2}\right)$$

$$\Rightarrow n_2 = n_1 \left(\frac{M_1}{M_2}\right)^{-1} \left(\frac{L_1}{L_2}\right)^{3} \left(\frac{T_1}{T_2}\right)^{-2}$$

$$= 6.67 \times 10^{-11} \left(\frac{1kg}{1gm}\right)^{-1} \left(\frac{1m}{1cm}\right)^{3} \left(\frac{1s}{1\,s}\right)^{-2}$$

$$= 6.67 \times 10^{-11} \left(\frac{1000\,gm}{1gm}\right)^{-1} \left(\frac{100\ cm}{1\ cm}\right)^{3} \left(\frac{1s}{1\ s}\right)^{-2}$$

$= 6.67 \times 10^{-11} \times 10^{-3} \times 10^{6} = 6.67 \times 10^{-8}$ dyne cm^2/g^2

(ii) To check the dimensional correctness of a given physical relation

This is based on the principle of homogeneity. According to this principle the dimensions of each term on both sides of an equation must be the same.

If the dimensions of each term on both sides of equation are same then it is called dimensionally correct equation. But it may or may not be physically correct. If an equation is physically correct then it will be also dimensionally correct.

Time period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

Dimension of left hand side = T

Dimension of right hand side $= \sqrt{\dfrac{L}{LT^{-2}}} = \sqrt{T^2} = T$

In the above equation, the dimensions of both sides are same. So the given formula is dimensionally correct.

(iii) To establish the relation among various physical quantities

If we know the various factors on which a physical quantity depends, then we can find a relation among different factors.

It is used as a research tool to derive new relations.

Stoke's law has been developed using this principle.

This law explains the motion of a spherical ball in a viscous liquid, where viscous force due to liquid opposes the motion.

It is found experimentally that viscous force depends on radius (r) of the sphere, velocity of the sphere (v) and the viscosity η of the liquid

$$F \propto (\eta^x r^y v^z)$$

$$F = k\,\eta^x r^y v^z$$

Where k is a dimensionless constant. To be a correct equation, it should be dimensionally correct

$$MLT^{-2} = (ML^{-1}T^{-1})^x (L)^y (LT^{-1})^z$$

$$MLT^{-2} = M^x L^{-x+y+z} T^{-x-z}$$

equating the power of similar quantities

$$x = 1,\ -x + y + z = 1 \text{ and } -x - z = -2$$

$$\Rightarrow \quad x = y = z = 1$$

$$F = k\eta r v$$

From experiment $k = 6\pi$

$$F = 6\pi\eta r v$$

(iv) To find dimensions of physical constants or co-efficients. Suppose there is a physical equation with a constant, we can find out the dimensions of that constant by substuting the dimensions of all other quantities.

Example:

$$F = \frac{Gm_1m_2}{r^2}$$

Where G is universal gravitational constant

$$G = \frac{Fr^2}{m_1m_2}$$

$$= \frac{(MLT^{-2})(L^2)}{(M^2)}$$

$$= M^{-1}L^3T^{-2}$$

4.2 LIMITATIONS OF DIMENSIONAL ANALYSIS

The method of dimensions has the following limitations:

(i) By this method the value of dimensionless constant cannot be calculated.

(ii) By this method the equation containing trigonometric, exponential and logarithmic terms cannot be analyzed.

(iii) If a physical quantity in mechanics depends on more than three factors, then relation among them cannot be established because we can have only three equations by equalizing the powers of M, L and T.

(iv) It doesn't tell whether the quantity is vector or scalar.

Illustration 6

Question: **The velocity of sound in a gas depends on its pressure and density. Obtain a relation between velocity, pressure and density.**

Solution: Let the relationship is expressed as $v = kP^a \rho^c$, where *P* is pressure, ρ is density and *v* is velocity of sound.

Now, $[v] = [LT^{-1}], [P] = [ML^{-1}T^{-2}], [\rho] = [ML^{-3}]$

$\therefore \quad [M^0 LT^{-1}] = [ML^{-1}T^{-2}]^a [ML^{-3}]^c$

or $\quad [M^0 LT^{-1}] = [M^{a+c} L^{-a-3c} T^{-2a}]$

Equating the powers of *M*, *L* and *T*, we have

$a + c = 0, -a - 3c = 1, -2a = -1$

$\Rightarrow \quad a = \frac{1}{2}, c = \frac{-1}{2}$

$\therefore$ Expression for velocity of sound is of the form $v = k\sqrt{\frac{P}{\rho}}$.

Actual formula is $v = \sqrt{\frac{\gamma P}{\rho}}$ where $\gamma = \frac{C_P}{C_V}$ (adiabatic exponent).

Illustration 7

Question: **In a new system of units, unit of mass is taken as 50 kg, unit of length is taken as 100 m and unit of time is 1 minute. What will be the weight of a body in this system, if in SI system, its weight is 10 N.**

Solution: Let the weight of the body in new system is *X* units

$\Rightarrow \quad 10 \text{ N} = X$ units

Let M_1, L_1, T_1 and M_2, L_2, T_2 be the symbols for mass, length and time in the two system respectively, then

$10[M_1 L_1 T_1^{-2}] = X[M_2 L_2 T_2^{-2}]$

$M_1 = 1 \text{ kg}, M_2 = 50 \text{ kg}, L_1 = 1 \text{ m}, L_2 = 100 \text{ m}, T_1 = 1 \text{ s}, T_2 = 60 \text{ s}$

$\Rightarrow \quad X = \frac{10}{50} \times \frac{1}{100} \times \left(\frac{1}{60}\right)^{-2} = \frac{36000}{5000} = 7.2 \text{ units}$

$\therefore \quad 10 \text{ N} = 7.2$ units.

Illustration 8

Question: **The period *P* of a simple pendulum is the time for one complete swing. How does *P* depend on the mass *m* of the bob, the length ℓ of the string, and the acceleration due to gravity *g*?**

Solution: We begin by expressing the period *P* in terms of the other quantities as follows:

$P = km^x \ell^y g^z$

Where *k* is a constant and *x*, *y* and *z* are to be determined. Next we insert the dimensions of each quantity

$P = M^x L^y L^z T^{-2z}$

$= M^x L^{y+z} T^{-2z}$

On equating the powers of each dimension on either side of the equation.

T: $1 = -2z$

M: $0 = x$

L: $0 = y + z$

These equations are easily solved and yield $x = 0$,

$z = -\frac{1}{2}$, and $y = +\frac{1}{2}$. Thus,

$$P = k\sqrt{\frac{\ell}{g}}$$

This analysis will not yield the value of *k*, but we have found, perhaps surprisingly, that the period does not depend on the mass. A derivation in terms of the forces acting on the bob shows that $k = 2\pi$.

Illustration 9

Question: **The equation of state of some gases can be expressed as $\left(P + \frac{a}{V^2}\right)(V - b) = RT$. Here, *P* is the pressure, *V* the volume, *T* the absolute temperature, and *a*, *b*, *R* are constants. Find the dimensions of *a*.**

Solution: Dimensions of $\frac{a}{v^2}$ will be same as dimensions of pressure

$$\frac{[a]}{[L^3]^2} = ML^{-1}T^{-2}$$

$$[a] = \mathbf{ML^5T^{-2}}$$

Illustration 10

Question: **If energy (E), momentum (P) and force (F) are chosen as fundamental units. Then find the dimensions of mass in new system.**

Solution: The dimensions of E, P and F in terms of M, L and T are

$[E] = ML^2T^{-2}$

$[P] = MLT^{-1}$

$[F] = MLT^{-2}$

Let $[M] = E^a P^b F^C$

or $[M] = (ML^2T^{-2})^a (MLT^{-1})^b (MLT^{-2})^C$

Equating the powers of M, L and T, we have $a = -1$, $b = 2$ and $c = 0$

Hence $[M] = \mathbf{E^{-1} P^2}$

Illustration 11

Question: **Consider the equation $F = \frac{\alpha^2}{\beta} e^{-\frac{\beta x}{E}}$, where *F* is force *x* is distance *E* is energy and α, β are constants. What are the dimensions of α and β ?**

Solution: As argument of exponential functions is dimensionless, so

Dimensions of $\left[\frac{\beta x}{E}\right] = [M^0 L^0 T^0]$

$$\frac{[\beta][L]}{[ML^2T^{-2}]} = [M^0L^0T^0]$$

$$[\beta] = [MLT^{-2}]$$

Now $\frac{\alpha^2}{\beta}$ will have dimensions of force

$$\Rightarrow \left[\frac{\alpha^2}{\beta}\right] = [F]$$

$$\frac{[\alpha^2]}{[MLT^{-2}]} = [MLT^{-2}]$$

$$[\alpha] = M^1L^1T^{-2}$$

5 MEASUREMENT

MEASUREMENT OF LENGTH

Vernier calliper and screw gauge are the instruments used for measuring length with high accuracy. From a normal scale, accuracy of 1 mm is obtained, while with these instruments, accuracies upto 0.01 mm can be achieved.

LEAST COUNT

The smallest measurement that can be taken with the help of a measuring instrument is called its least count. Like, in a simple wristwatch, the second's hand moves through a smallest time interval of 1 s. So, we can measure a minimum time of 1 s. Thus, the least count of the wristwatch is 1 s similarly the least count of a normal metre scale is 1 mm.

Following two points are worth remembering regarding the least count.

(i) Least count represents the permissible error in the measurement.

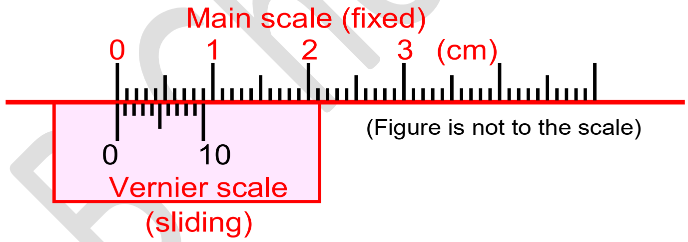
(ii) It gives the limit of resolution of the instrument.

The following illustrations will make the above statements clear.

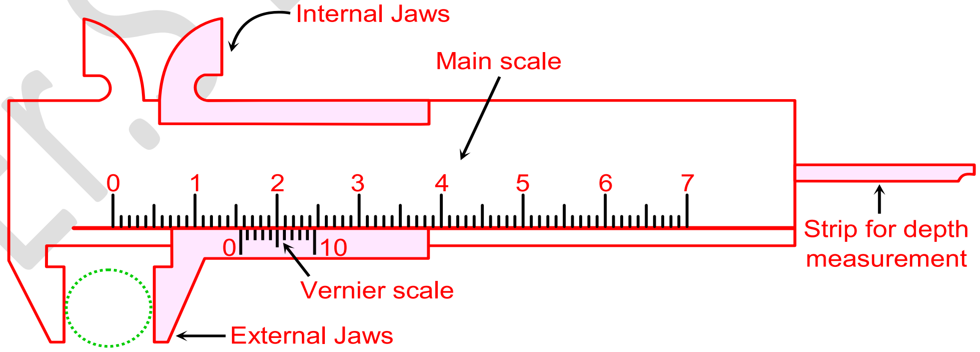
For example a simple geometry box scale has smallest division of 0.1 cm marked on it. This is the least count. When a measurement is taken from this scale, the result can be 5.1 cm or 7.9 cm, but it cannot be 5.15 cm or 7.93 cm. Reporting the answer as 7.93 cm gives a wrong information, it shows that the least count is 0.01 cm i.e., instrument has greater resolution.

5.1 VERNIER CALLIPER

It consists of a fixed main scale marked on a metallic strip. Another metallic strip slides on it with a vernier scale on it. Scales are marked in such a way that when zero's of both scales are aligned, n^{th} division of vernier scale coincides with $(n-1)^{th}$ division of main scale. In the figure shown, 10^{th} division of vernier scale coincides with 9^{th} division of main scale.



Construction:



External and internal jaws are used for external and internal measurements respectively. The strip at the end is used for depth measurement.

Working:

n Vernier Scale Divisions (VSD) = $(n-1)$ Main Scale Divisions (MSD)

$$\therefore \quad 1\,\text{VSD} = \left(\frac{n-1}{n}\right)\text{MSD}$$

Difference between 1 MSD and 1 VSD is, 1 MSD – 1 VSD = $1-\frac{n-1}{n}=\frac{1}{n}\text{MSD}$

The figure $\frac{1}{n}\text{MSD}$ is called Vernier Constant (V.C.) or Least Count (L.C.) of the vernier. It is the smallest distance, which can be accurately measured by the vernier.

For example if main scale division be of 1 mm and 50 divisions of vernier scale coincide with 49 divisions of main scale.

MSD = 1 mm

VSD = $\frac{49\text{ mm}}{50}$, $\text{L.C.}=\frac{1}{50}\text{mm}=0.02\text{ mm}$

For example if main scale division be of 1 mm and 10 divisions of vernier scale coincide with 9 divisions of main scale.

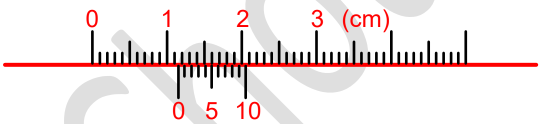
MSD = 1 mm

VSD = $\frac{9\text{ mm}}{10}=0.9\text{ m}$. So least count = 1 MSD = 1 VSD = 1 mm – 0.9 mm = 0.1 mm

READING A VERNIER

Let zero of vernier scale be between two main scale divisions. The division on main scale just before zero of vernier scale is the main scale reading.

Now, count the number of the division of vernier scale that coincides with a division on main scale. Multiply this number to L.C. of the vernier. This is called vernier scale reading. It is to be added to main scale reading.



The least count of the scale shown is 0.1 mm

The main scale reading = 11 mm

The vernier scale reading = 5 × 0.1 = 0.5 mm

Hence measurement = 11.5 mm.

ZERO ERROR

When jaws of vernier touch each other, the vernier should read 0.0 i.e., Zeros of main scale and vernier scale should coincide. If it is not so, the instrument has a zero error.

If a vernier shows a non-zero reading when jaws are touching each other, it is the zero error, which is to be algebraically subtracted from the reading taken.

ZERO CORRECTION

Zero correction is the correction to be applied to the reading taken to get correct measurement. It is of same magnitude as that of the zero error but is of opposite sign. Zero correction is to be algebraically added to the reading taken.

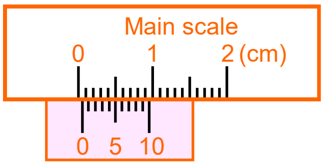
POSITIVE AND NEGATIVE ZERO ERROR

When the jaws of vernier touch each other.

If zero of vernier scale is to the right of the zero of main scale, then zero error is positive.

If zero of vernier scale is to the left of zero of main scale, then zero error is negative.

For example the figure shows a situation when the jaws of vernier are touching each other.



Each main scale division is of 1 mm and 5th division of vernier scale coincides with a main scale division.

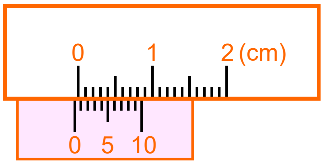
$$L.C. = \frac{1}{10} = 0.1\,mm$$

∴ Zero error = +5 × 0.1 = 0.5 mm

This error is to be subtracted from the reading taken for measurement.

Also, zero correction = – 0.5 mm.

For example figure shows reading when jaws of vernier are touching each other.



Each main scale division is 1 mm.

$$L.C. = \frac{1}{10} = 0.1\,mm$$

5th division of vernier coincides with a main scale division.

∴ Zero error = – 5 × 0.1 = – 0.5 mm

This error is to be algebraically subtracted from the reading taken.

Zero correction = – (– 0.5) mm

= + 0.5 mm

This is to be added to the reading taken.

Precautions (to be taken)

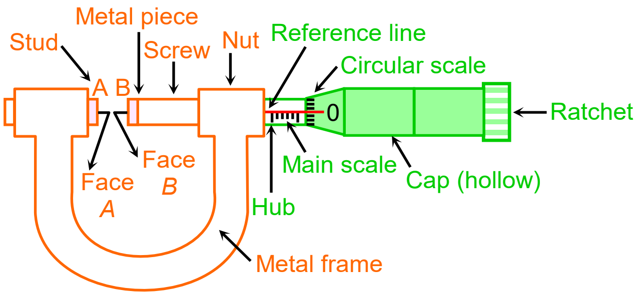
1. Motion of vernier scale on main scale should be made smooth (by oiling if necessary).
2. Vernier constant and zero error should be carefully found and properly recorded.
3. The body should be gripped between the jaws firmly but gently (without undue pressure on it from the jaws).
4. Observations should be taken at right angles at one place and taken at least at three different places.

Sources of Error

1. The vernier scale may be loose on main scale.
2. The jaws may not be at right angles to the main scale.
3. The graduations on scale may not be correct and clear.
4. Parallax may be there in taking observations.

5.2 SCREW GAUGE (OR MICROMETER)

It works on the principle of advancement of a screw when it is rotated inside mating threads.



When the screw is rotated inside the nut by rotating the cap (connected internally to the screw), the screw advances or retracts depending on direction of rotation.

A reference line is marked on the Hub and main scale divisions are marked on it (see figure). Circular scale markings (usually 50 or 100 divisions) are made on the cap.

Least count:

$$L.C. = \frac{\text{Main scale division value}}{\text{Number of divisions on circular scale}} = \frac{\text{Linear distance advanced in one rotation}}{\text{Number of divisions on circular scale}}$$

If main scale division value is 1 mm and there are 100 divisions in circular scale, then

$$L.C. = \frac{1}{100} = 0.01\,\text{mm}$$

Reading: Main scale reading is taken on the main scale. To find circular scale reading, the number of the division of circular scale that coincides with reference line is counted. If n^{th} division coincides, then

Final reading = Main scale reading + n × (L.C.)

Illustration 12

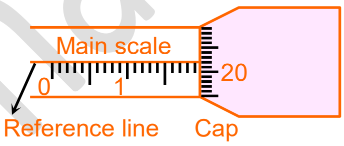
Question: **A screw gauge has main scale divisions of 1 mm each and its circular scale has 50 divisions. While measuring the diameter of a ball, cap end is after 19^{th} marking of main scale and 21^{st} division of circular scale coincides with the reference line. What is the measured diameter of the ball?**

Solution: $L.C. = \frac{1\,\text{mm}}{50} = 0.02\text{ mm}$

Main scale reading = 19 mm

Circular scale reading = 21 × 0.02 = 0.42 mm

∴ Diameter = 19 + 0.44 = 19.44 mm



5.3 ZERO ERROR AND ITS CORRECTION

If the zero mark of circular scale coincides with the reference line when there is no gap between the faces *A* and *B*, then there is no zero error. If the zero mark does not coincide, there is a zero error.

If the zero of circular scale is left behind the reference line when the faces meet, zero error is positive. The magnitude of error is to be deducted from the reading taken.

If zero of circular scale advances beyond the reference line when the faces meet, the zero error is negative. The magnitude of error is to be added to the reading taken.

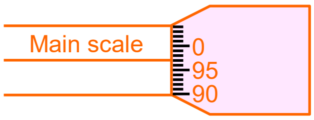
For example least count of a screw gauge is 0.01 mm. When the faces touch each other, main scale zero marking is not visible and 97^{th} division of circular scale coincides with the reference line.

As zero of main scale is not visible, the circular cap has advanced by 100 – 97 = 3 divisions.

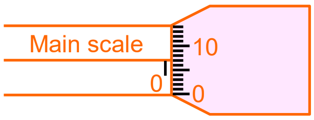
Hence zero error = – 3 × 0.01

= – 0.03 mm

Zero correction = + 0.03 mm



For example a micrometer has L.C. = 0.02 mm. When the faces are touching each other, only zero marking of main scale is visible and 7^{th} division of circular scale is coinciding with the reference line. Find zero error of the instrument. The circular scale is left behind by 6 divisions



∴ Zero error = + 7 × 0.02 = + 0.14 mm

Zero correction = – 0.14 mm

Precaution (to be taken)

1. While taking an observation, the screw must always be turned only in one direction so as to avoid the backlash error.
2. At each place, take readings in pairs i.e. in two directions at right angles to each other.
3. The wire must be straight and free from kinks.
4. Always rotate the screw by the ratchet and stop as soon as it gives one tick sound only.
5. While taking a reading, rotate the screw in only one direction so as to avoid the backlash error.

Sources of Error

1. The screw may have friction.
2. The screw gauge may have backlash error.
3. Circular scale divisions may not be of equal size.
4. The wire may not be uniform.

6 SIGNIFICANT FIGURES

When a certain measurement of length is taken by a scale of least count 0.01 cm, the result is say, 7.62 cm. In this measurement, there exists a maximum possible error of 0.01 cm. Thus, we can say that, in the above measured value, '7' and '6' are certain digits and last digit '2' is uncertain digit. The measured value, in this case, is said to have three significant figures. Following are a few important points regarding significant figures.

(i) Significant figures represent the precision of measurement, which depends on the least count of the measuring instruments.

(ii) A choice of different units does not change the number of significant digits in a measurement.

(iii) Significant figures in a result after calculations are, the number of certain digits plus one uncertain digit.

6.1 THE RULES FOR DETERMINING THE NUMBER OF SIGNIFICANT FIGURES ARE AS FOLLOWS

(i) All the non-zero digits are significant.

Example: 156,78 contains five significant figure.

(ii) All the zeros between two non-zero digits are significant no matter where the decimal point is: Example: 108.006 contains six significant figures.

(iii) If the number is less than 1, the zeros on the right of decimal point but to the left of 1st non-zero digit are not significant.

Example: In 0.002308 the under lined zeros are not significant.

(iv) All the zeros to the right of the last non-zero digit (trailing zeros) in a number without a decimal point are not significant, unless they come from experiment.

Thus 123 m = 12300 cm = 123000 mm has three significant figures. The trailing zeros are not significant.

But if these are obtained from a measurement, they are significant.

(v) The trailing zeros in a number with a decimal point are significant.

The number 3.500 or 0.06900 have four significant figure.

Now note that choice of change of different units does not change the number of significant digits or figures in measurement.

☞ *the length* 2.308 cm *has four significant figures, but in different units, the same value can be written as 0.02308* m *or 23.08* mm. *All these number have the same number of significant figures. It shows that location of decimal point does not matter in determining the number of significant figures.*

When there are zeros at the right end of the number, then there may be some confusion.

☞ *If length is 500* mm *and we don't know least count of the measuring instrument, then we can't be sure that last digits (zeros) are significant or not.*

Scientific Notation

If the scale had marking only at each meter, then the digit 5 can be obtained by eye approximation. So only 5 is significant figure, but if the markings are at each centimeter, then only 5,0 of the reading will be significant. If the scale used have marking in millimeters, all three digits 5,00 are significant. To remove such ambiguities in determining the number of significant figures, the best way is to report every measurement in scientific notation, every number should express as $a \times 10^b$ where a is between 1 and 10 and b is any +ve or –ve power of 10, and decimal is placed after the first digit.

Now the confusion mention above can be removed.

4.700 m = 4.700×10^2 cm = 4.700×10^3 mm

Here power of 10 is irrelevant to the determination of significant figures.

6.2 SIGNIFICANT FIGURE IN ALGEBRAIC OPERATION

To know the number of significant figures after an algebric operation (Addition, subtraction, multiplication and division) certain rules can be followed which are as follows.

(i) In multiplication or division, the number of significant digits in the final result should be equal to the number of significant digits in the quantity, which has the minimum number of significant digits.

(ii) In addition or subtraction the final result should retain as many decimal places as are there in the number with the least decimal place.

6.3 ROUNDING OFF THE UNCERTAIN DIGIT: (LEAST SIGNIFICANT DIGIT):

The least significant digit is rounded according the rules given below.

(i) If the digit next to the least significant (Uncertain) digit is more than 5, the digit to be rounded is increased by 1.

(ii) If the digit next to the one rounded is less than 5, the digit to be rounded is left unchanged.

(iii) If the digit next to the one rounded is 5, then the digit to be rounded is increased by 1 if it is odd and left unchanged if it is even.

☞ ***The insignificant digits are dropped from the result if they appear after the decimal point. Zero replaces them if they appear to the left of the decimal point.***

Example: Suppose we have to round off to three significant digits to 15462.

In 15462, third significant digit is 4. This digit is to be rounded. The digit next to it is 6 which is greater than 5. The third digit should, therefore be increased by 1. The digits to be dropped should be replaced by zeros, because they appear to the left of decimal point thus 15462 becomes 15500 on rounding to three significant figure.

Illustration 13

Question: **If mass of an object measured is 4.237 gm (four significant figure) and its volume is measured to be 2.51 (three significant figure) cm³, then find its density in proper significant figures.**

Solution: Density = $\frac{\text{mass}}{\text{volume}} = \frac{4.237}{2.57}$

= 1.6486 gm/cm³.

But it should be up to three significant digits. Density = **1.65 gm/cm³**

Illustration 14

Question: **Find out the sum of the numbers 436.32 gm, 227.2 gm and 0.301 gm by arithmetic addition.**

Solution:

```
 436.32
 227.2
   0.301
―――――――――
 663.821
```

But the least precise measurement (227.2) gm is correct to only one decimal place. So final should be rounded off to one decimal place.

So sum will be **663.8 gm**

Illustration 15

Question: **Round off the following numbers to three significant digits**

(a) **14.745,**

(b) **14.750,**

(c) **14.650×10^{12}.**

Solution: **(a)** The third significant digit in 14.745 is 7. The number next to it is less than 5. So 14.745 becomes 14.7 on rounding to three significant digits.

(b) 14.750 will become 14.8 because the digit to be rounded is odd and the digit next to it is 5.

(c) 14.650×10^{12} will become 14.6×10^{12} because the digit to be rounded is even and the digit next to it is 5.

7 ERROR ANALYSIS IN EXPERIMENTS

Measurement is an important aspect of physics. Whenever we want to know about a physical quantity, we take its measurement.

Instruments used in measurement are called measuring instruments.

7.1 ACCURACY AND PRECISION OF INSTRUMENTS

Accuracy of an instrument represents the closeness of the measured value of actual value. Precision of an instrument represents the resolution of the instrument. It depends on least count.

For example a physical quantity is measured from two instruments *A* and *B*. The reading of *A* is 2.54 cm (say) and that of *B* is 2.516 cm. The actual value is 2.53 cm. The first reading is closed to actual value, it has more accuracy. The second reading is less accurate, but the instrument B has greater resolution as it can measure upto 3 decimal places.

Sensitivity: the ratio of output signal or response of the instrument to a change of input or measured variable.

Resolution: the smallest change in measured value to which the instrument will respond.

7.2 ERRORS

The measured value of a physical quantity is usually different from its true value. The result of any experimentally measured value contains some uncertainty. This uncertainty is called error.

In general, the errors in measurements can be broadly classified into two categories as

(a) Systematic errors and

(b) Random errors

(c) Accidental errors

Systematic errors: These are the errors occurring due to the instruments used, imperfection in experimental technique or carelessness of the person performing the experiment. These errors tends to move in one direction only.

Random errors: The random errors occur irregularly and therefore random in magnitude and sign. These errors tends to move in both directions. They may arise due to unpredictable fluctuation while performing the experiment. These errors can be minimized by taking a large number of observations for the same physical quantity.

Accidental errors: Some time the measured value is to much small or to much larger then their average value. This is called accidental error. These values are dropped before calculating the average value.

Instrumental errors: These errors depend on the instruments used for measurement. Mainly following two types of errors occur due to instruments.

(i) **Zero error:** It is a type of systematic error. It occurs due to improper positioning of zero mark of the instrument.

(ii) **Least count error:** This is the error associated with the resolution of the instrument. Least count has been discussed in details in the earlier section.

For example to measure the length of an object its one end is placed at the zero mark of the scale and other end is found to come at 10^{th} millimeter mark from the start. This implies that the length of the object is 10 mm. But, if one end of the object is placed at 1^{st} mm mark instead of zero mark, the measurement will come out be 11 mm. So a zero error of 1 mm is introduced in the measurement.

7.3 ABSOLUTE ERROR, RELATIVE ERROR AND PERCENTAGE ERROR

Mean of observations: Suppose the observations of a physical quantity are X_1, X_2, X_3 …….. X_n.

Arithmetic mean of these observations X_{mean} is defined as

$X_{mean} = (X_1 + X_2 + X_3 + \ldots\ldots + X_n)/n$

or $X_{mean} = \frac{1}{n}\sum_{i=1}^{n} x_i$

The arithmetic mean of these observations is taken as best possible value of the quantity under the given conditions of measurement.

Absolute Error: The magnitude of the difference between the true value of the quantity and the individual measurement value is called the absolute error of the measurement.

(Arithmetic mean is taken as true value of number of observations.)

$\Delta X_1 = X_{mean} - X_1$,

$\Delta X_2 = X_{mean} - X_2$,

…. ….. ….

…. ….. ….

$\Delta X_n = X_{mean} - X_n$

The absolute error may be positive or negative.

Absolute Mean Error: The average of the mod of errors is called the average or mean error.

$\Delta X_{mean} = (|\Delta X_1| + |\Delta X_2| + |\Delta X_3| \ldots\ldots |\Delta X_n|)/n$

Relative Error: The relative error is the ratio of the mean absolute error ΔX_{mean} to the mean value X_{mean} of the quantity measured.

Relative error = $\Delta X_{mean} / X_{mean}$

Percentage Error: When the relative error is expressed in percent, it called the percentage error

Thus, percentage error = $(\Delta X_{mean} / X_{mean}) \times 100\%$

7.4 COMBINATION OF ERRORS

When we perform some experiment, different observations are used to get a result using algebraic operations like addition, subtraction, multiplication, division etc.

Now we need to calculate the errors in combination of various mathematical operations.

(A) Errors in sum or difference

Let $X = A \pm B$

Further, let ΔA is the absolute error in the measurement of A, ΔB the absolute error in the measurement of B and ΔX is the absolute error in the measurement of x.

Then,
$$X + \Delta X = (A \pm \Delta A) \pm (B \pm \Delta B)$$
$$= (A \pm B) \pm (\Delta A \pm \Delta B)$$
$$= X \pm (\Delta A \pm \Delta B)$$

or $\Delta X = \pm \Delta A \pm \Delta B$

The four possible values of ΔX $(\Delta A + \Delta B)$, $(\Delta A - \Delta B)$, $(-\Delta A - \Delta B)$ and $(-\Delta A + \Delta B)$

Therefore, the maximum absolute error in X is, $\Delta X = \pm(\Delta A + \Delta B)$

i.e., the maximum absolute error in sum and difference of two quantities is equal to sum of the absolute errors in individual quantities.

(B) Errors in product

Let, $X = AB$

Then $(X \pm \Delta X) = (A \pm \Delta A)(B \pm \Delta B)$

or $X\left(1 \pm \frac{\Delta X}{X}\right) = AB\left(1 \pm \frac{\Delta A}{A}\right)\left(1 \pm \frac{\Delta B}{B}\right)$

or $1 \pm \frac{\Delta X}{X} = 1 \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \pm \frac{\Delta A}{A} \cdot \frac{\Delta B}{B}$ (as $X = AB$)

or $\pm \frac{\Delta X}{X} = \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \cdot \frac{\Delta B}{B}$

Here, $\left(\frac{\Delta A}{A} \cdot \frac{\Delta B}{B}\right)$ is a small quantity, so can be neglected.

Hence $\pm \frac{\Delta X}{X} = \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B}$

Possible values of $\frac{\Delta X}{X}$ are $\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right), \left(\frac{\Delta A}{A} - \frac{\Delta B}{B}\right), \left(-\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$ and $\left(-\frac{\Delta A}{A} - \frac{\Delta B}{B}\right)$.

Hence, maximum possible value of $\frac{\Delta X}{X} = \pm\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$

Therefore maximum fractional error in product of two (or more) quantities is equal to sum of fractional errors in the individual quantities.

(C) Errors in division

Let, $X = \frac{A}{B}$

Then, $X \pm \Delta X = \frac{A \pm \Delta A}{B \pm \Delta B}$

or $X\left(1 \pm \frac{\Delta X}{X}\right) = \frac{A\left(1 \pm \frac{\Delta A}{A}\right)}{B\left(1 \pm \frac{\Delta B}{B}\right)}$

or $\left(1 \pm \frac{\Delta X}{X}\right) = \left(1 \pm \frac{\Delta A}{A}\right)\left(1 \pm \frac{\Delta B}{B}\right)^{-1}$ $\left(\text{as} \quad X = \frac{A}{B}\right)$

As $\frac{\Delta B}{B} << 1$, so expanding binomially, we get $\left(1 \pm \frac{\Delta X}{X}\right) = \left(1 \pm \frac{\Delta A}{A}\right)\left(1 \pm \frac{\Delta B}{B}\right)$

or $1 \pm \frac{\Delta X}{X} = 1 \pm \frac{\Delta A}{A} \mp \frac{\Delta B}{B} \pm \frac{\Delta A}{B} \cdot \frac{\Delta B}{B}$

Here, $\frac{\Delta A}{A} \cdot \frac{\Delta B}{B}$ is small quantity, so can be neglected.

Therefore, $\pm \frac{\Delta X}{X} = \pm \frac{\Delta A}{A} \mp \frac{\Delta B}{B}$

Possible values of $\frac{\Delta X}{X}$ are $\left(\frac{\Delta A}{A} - \frac{\Delta B}{B}\right), \left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right), \left(-\frac{\Delta A}{A} - \frac{\Delta B}{B}\right)$ and $\left(-\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$

Therefore, the maximum value of $\frac{\Delta X}{X} = \pm\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$

or, the maximum value of fractional error in division of two quantities is equal to the sum of fractional errors in the individual quantities.

(D) Errors in quantity raised to some power

Let $X = \frac{A^n}{B^m}$

Then $\ln(X) = n\ln(A) - m\ln(\mathbf{B})$

Differentiating both sides, we get $\frac{dX}{X} = n.\frac{dA}{A} - m\frac{dB}{B}$

In terms of fractional error we may write $\pm\frac{\Delta X}{X} = \pm n\frac{\Delta A}{A} \mp m\frac{\Delta B}{B}$

Therefore maximum value of $\frac{\Delta X}{X} = \pm\left(n\frac{\Delta A}{A} + m\frac{\Delta B}{B}\right)$

Illustration 16

Question: **The original length of a wire is (153.7 ± 0.6) cm. It is stretched to (155.3 ± 0.2) cm. Calculate the elongation in the wire with error limits.**

Solution: Elongation (l) = 155.3 – 153.7 = 1.6 cm

$\Delta l = \pm(\Delta l_1 + \Delta l_2)$

$= \pm(0.6 + 0.2) = \pm 0.8$ cm.

∴ Elongation = **(1.6 ± 0.8) cm**

Illustration 17

Question: **The measures of the lengths of a rectangle are l = (3.00 ± 0.01) cm and breadth b = (2.00 ± 0.02) cm. What is the area of the rectangle?**

Solution: Area = $lb = 3 \times 2 = 6$ cm^2

Error $= \pm 6\left(\frac{0.01}{3} + \frac{0.02}{2}\right) = \pm 0.08$ cm^2

Area **= (6.00 ± 0.08)cm^2**

Illustration 18

Question: **The change in the velocity of a body is (12.5 ± 0.2) m/s in a time (5.0 ± 0.3) s. Find the average acceleration of the body within error limits.**

Solution: Here V = (12.5 ± 0.2) m/s; t = (5.0 ± 0.3) s

∴ $a = \frac{12.5}{5.0} = 2.5$ ms^{-2}

Also $\frac{\Delta a}{a} = \pm\left(\frac{\Delta V}{V} + \frac{\Delta t}{t}\right) = \pm\left(\frac{0.2}{12.5} + \frac{0.3}{5.0}\right) = \pm 0.08$

$\Delta a = \pm(0.08 \times 2.5) = \pm 0.2$

a = **(2. 5 ± 0.2) ms^{-2}**

Illustration 19

Question: **The heat dissipated in a resistance can be obtained by the measurement of resistance, the current and time. If the maximum error in the measurement of these quantities is 1%, 2% and 1% respectively, what is the maximum error in determination of the dissipated heat?**

Solution: Heat produced H is given by

$H = I^2Rt$

$$\therefore \quad \frac{\Delta H}{H} = 2\frac{\Delta I}{I} + \frac{\Delta R}{R} + \frac{\Delta t}{t}$$

For maximum percentage error,

$$\frac{\Delta H}{H}\times 100 = 2\frac{\Delta I}{I}\times 100 + \frac{\Delta R}{R}\times 100 + \frac{\Delta t}{t}\times 100$$

$$= 2\times 2\% + 1\% + 1\% = \mathbf{6\%}$$

Illustration 20

Question: **Calculate focal length of a spherical mirror from the following observations: object distance $u = (10 \pm 0.1)$ cm and image distance $v = (10 \pm 0.1)$ cm.**

Solution: As $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{f} = \frac{1}{10} + \frac{1}{10}$, $f = 5$ cm

$$\frac{\Delta f}{f^2} = \frac{\Delta u}{u^2} + \frac{\Delta v}{v^2},\ \Delta f = \left[\frac{\Delta u}{u^2} + \frac{\Delta v}{v^2}\right]f^2 = \left[\frac{0.1}{100} + \frac{0.1}{100}\right](25) = \frac{5}{100}\ \text{cm}$$

$\Delta f = 0.05$ cm

$\therefore \quad f = \mathbf{(5 \pm 0.05)\ cm}$

Illustration 21

Question: **The diameter of a wire as measured by a Screw gauge was found to be 1.328, 1.330, 1.325, 1.326, 1.334 and 1.336 cm. Calculate (i) mean value of diameter (ii) absolute error in each measurement (iii) mean absolute error (iv) fractional error, (v) percentage error. Also, express the result in terms of absolute error and percentage error.**

Solution: Observed values of diameter of wire are given.

(i) $D_m = \frac{1.328 + 1.330 + 1.325 + 1.326 + 1.334 + 1.336}{6}$

$= 1.330$

(ii) $\Delta D_1 = 1.330 - 1.328 = +0.002\ cm$

$\Delta D_2 = 1.330 - 1.330 = 0.000\ cm$

$\Delta D_3 = 1.330 - 1.325 = 0.005\ cm$

$\Delta D_4 = 1.330 - 1.326 = 0.004\ cm$

$\Delta D_5 = 1.330 - 1.334 = -0.004\ cm$

$\Delta D_6 = 1.330 - 1.336 = -0.006\ cm$

(iii) Mean absolute error, ΔD_{mean}

$= |\Delta D_1| + |\Delta D_2| + |\Delta D_3| + |\Delta D_4| + |\Delta D_5| + |\Delta D_6|$

$= \frac{0.021}{6} = 0.0035 = 0.004$

(iv) Fractional error = $\frac{\Delta D_{mean}}{D} = \pm\frac{0.004}{1.330} = \pm 0.003$

(v) percentage error

$= \pm 0.003 \times 100\% = \pm 0.3\%$

(vi) Diameter of wire

$= (1.330 \pm 0.003)\, cm$

or $D = 1.330\ \text{cm} \pm 0.3\%$

Illustration 22

Question: **The time period of oscillation of simple pendulum is given by $t = 2\pi\sqrt{l/g}$. What is the accuracy in the determination of g if 10 cm length is known to 1 mm accuracy and 0.5 s time period is measured from time of 100 oscillations with a watch of 1 second resolution.**

Solution: Here $\frac{\Delta l}{l} = \frac{0.1}{10}$

Δt = 1 second, and

time of 100 oscillations. $t = 100 \times 0.5 = 50$ s

$\therefore \quad \frac{\Delta t}{t} = \frac{1}{50}$

From $\quad t = 2\pi\sqrt{\frac{l}{g}}$

$t^2 = 4\pi^2 \frac{l}{g}$

$g = 4\pi^2 \frac{l}{t^2}$

$\therefore \quad \frac{\Delta g}{g} = \pm\left(\frac{\Delta l}{l} + 2\frac{\Delta t}{t}\right)$

$\therefore \quad$ error, $\frac{\Delta g}{g} \times 100 = \pm\left(\frac{0.1}{10} + \frac{2\times 1}{50}\right) \times 100 = \pm$ **5%**

PROFICIENCY TEST

The following questions deal with the basic concepts of this section. Answer the following briefly. Go to the next section only if your score is at least 80%. Do not consult the Study Material while attempting these questions.

1. The value of a force on a body is 20 N in SI units. What is the value of this force in cgs units, that is, dynes?

2. Convert an energy of 4.2 joule into ergs.

3. The density of air is 1.293 kg m^{-3}. Express this value in c.g.s. units.

4. The SI and CGS units of energy are joule and erg respectively. How many ergs are equal to one joule ?

5. The average wavelength of light from a sodium lamp is 5893 Å. Express it in metre and in nm.

6. Find the dimensions of the following quantities

(i) Acceleration (ii) Angle
(iii) Density (iv) Kinetic energy
(v) Gravitational constant (vi) Permeability

7. Name the two physical quantity represented by dimensions $ML^{-1}T^{-2}$.

8. Which one has dimensions different from the remaining three of the following quantities; Force, work, energy and torque?

9. If velocity, time and force were chosen as basic quantities, find the dimensions of mass.

10. Calculate α in the given equation : $(\text{velocity})^{\alpha} = (\text{pressure difference})^{3/2} \times (\text{density})^{-3/2}$

11. Assuming that the critical velocity of flow of a liquid through a narrow tube depends on the radius of the tube, density of the liquid and viscosity of the liquid, find the expression for critical velocity.

12. Check by the method of dimensional analysis whether the following relations are correct or not.

(i) $v = \sqrt{\frac{P}{D}}$ where v = velocity of sound and P = pressure, D = density of medium.

(ii) $n = \frac{1}{2l}\sqrt{\frac{F}{m}}$, where n = frequency of vibration, l = length of the string.

F = stretching force, m = mass per unit length of the string.

13. The velocity v of a particle depends on the time t according to the equation $v = a + bt + \frac{c}{d+t}$.

Write the dimensions of a, b, c and d.

14. In the equation $y = A \sin(\omega t - kx)$, obtain the dimensional formula of ω and k. Given, x is distance and t is time.

15. The centripetal force F acting on a particle moving uniformly in a circle may depend upon mass (m), velocity (v) and radius (r) of the circle. Derive the formula for F using the method of dimensions.

16. State the number of significant figures.
(a) $0.007\ m^2$ (b) $2.64 \times 10^{24} kg$ (c) $0.2370\ g\ cm^{-3}$

17. Round off to three significant digits (i) 0.03927 kg (ii) 4.085×10^5

18. Find the sum of 2.0347 and 15.7 taking care of significant figure.

19. Compute the addition of 3.8×10^{-6} with 4.2×10^{-5} with due regard to significant figure.

20. Subtract 2.5×10^4 from 3.9×10^5 with due regard to significant figures. (rounded to one place of decimal)

21. Calculate area enclosed by a circle of diameter 1.06 m to correct number of significant figures.

22. The lengths of two rods are measured as $l_1 = (6.62 \pm 0.01)$cm and $l_2 = (4.84 \pm 0.03)$cm Calculate the difference in lengths using error limits.

23. A body travels uniformly a distance of (13.8 ± 0.2)m in a time (4.0 ± 0.3)s. What is the velocity of the body within error limits?

24. The diameter and length of a cylinder measured with the help of vernier callipers of least count 0.01 cm, are 1.24 cm and 5.25 cm. Calculate the volume of the cylinder and maximum permissible error in its volume. How can the measurement of volume be more accurate?

25. What is the percentage error in volume of a sphere, when error in measuring its radius is 3 %?

ANSWERS TO PROFICIENCY TEST

1. 2×10^6 dynes

2. 4.2×10^7 erg

3. 0.001293 g cm^{-3}

4. 1 joule = 10^7 erg.

5. 5893×10^{-10} m ; 589.3 nm

6. (i) LT^{-2}, (ii) dimensionless. (iii) ML^{-3}, (iv) ML^2T^{-2} , (v) $M^{-1}L^3T^{-2}$, (vi) $MLT^{-2}A^{-2}$

7. Pressure and stress

8. Force

9. [mass] = $[FTV^{-1}]$

10. 3

11. $\frac{k\eta}{rp}$

13. LT^{-1}, LT^{-2}, L, T

14. $[M^{\circ}L^{\circ}T^{-1}]$, $[M^0L^{-1}T^0]$

15. $F = K\frac{mv^2}{r}$

16. (a) 1 (b) 3 (c) 4

17. **(i) 0.0393 kg**
(ii) 4.08×10^5 s

18. 17.7

19. 4.6×10^{-5}

20. 3.6×10^5

21. 0.882 m^2

22. $1.78\,cm \pm 0.04$ cm

23. $(3.45 \pm 0.3)ms^{-1}$

24. 1.8 %

25. $\pm 9\%$

SOLVED OBJECTIVE EXAMPLES

***Example* 1:**

The dimensional formula for Planck's constant (*h*) is

(a) $ML^{-2}T^{-3}$ (b) ML^2T^{-2} (c) ML^2T^{-1} (d) $ML^{-2}T^{-2}$

Solution:

$$E = \frac{hc}{\lambda}$$

$$[h] = \frac{[E\lambda]}{[c]} = \frac{ML^2T^{-2}L}{LT^{-1}} = \mathbf{ML^2T^{-1}}$$

∴ **(c)**

***Example* 2:**

Of the following quantities, which one has dimensions different from the remaining three?

(a) energy per unit volume

(b) force per unit area

(c) product of voltage and charge per unit volume

(d) angular momentum per unit mass

Solution:

$$[\text{energy per unit volume}] = \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$$

$$[\text{force per unit area}] = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

[product of voltage and charge per unit volume] = $ML^{-1}T^{-2}$

[angular momentum per unit mass]= $ML^2T^{-1}L.M^{-1} = L^2T^{-1}$

∴ **(d)**

***Example* 3:**

E, m, J and G denote energy, mass, angular momentum and gravitational constant respectively. Then the dimensions of EJ^2/m^5G^2 are

(a) angle (b) length (c) mass (d) time

Solution:

$$\frac{[EJ^2]}{[m^5G^2]} = \frac{(ML^2T^{-2})(ML^2T^{-1})^2}{(M)^5(M^{-1}L^3T^{-2})^2} = \mathbf{M^0L^0T^0}$$

∴ **(a)**

***Example* 4:**

When 2.0347 is added to 15.7, the sum is

(a) 17.7347 (b) 17.734 (c) 17.73 (d) 17.7

Solution:

Since the least number of places after decimal is 1, the final result will have same. On rounding off 17.7347 to one place after decimal it becomes 17.7

∴ **(d)**

Example **5:**

A body travels uniformly a distance of (13.8 ± 0.2)m in a time (4.0 ± 0.3)s. The velocity of the body within error limits is

(a) $(3.45 \pm 0.2)ms^{-1}$ **(b)** $(3.45 \pm 0.3)ms^{-1}$ **(c)** $(3.45 \pm 0.4)ms^{-1}$ **(d)** $(3.45 \pm 0.5)ms^{-1}$

Solution:

Here $S = (13.8 \pm 0.2)cm$; $t = (4.0 \pm 0.3)s$

$\therefore$ $V = \frac{13.8}{4.0} = 3.45\ ms^{-1}$

Also $\frac{\Delta V}{V} = \pm\left(\frac{\Delta S}{S} + \frac{\Delta t}{t}\right) = \pm\left(\frac{0.2}{13.8} + \frac{0.3}{4.0}\right) = \pm 0.0895$

$\Delta V = \pm 0.3$ (rounding off to one place of decimal)

$V = (3.45 \pm 0.3)ms^{-1}$

$\therefore$ **(b)**

Example **6:**

The length and breadth of a metal sheet are 3.124 m and 3.002 m respectively. The area of this sheet up to four correct significant figures is

(a) 9.376 m² **(b) 9.378 m²** **(c) 9.379 m²** **(d) 9.388 m²**

Solution:

Given length (l) = 3.124 m and breadth (b) = 3.002 m.

We know that area of the sheet $(A) = l \times b = 3.124 \times 3.002 = 9.378248\,m^2$.

Since both length and breadth have four significant figures, therefore area of the sheet after rounding off to four significant figures is 9.378 m^2

$\therefore$ **(b)**

Example **7:**

The value of universal gravitation constant $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}. The value of G in units of g^{-1} cm^3 s^{-2} is

(a) 6.67×10^{-8} **(b)** 6.67×10^{-7} **(c)** 6.67×10^{-9} **(d)** 6.67×10^{-10}

Solution:

$G = 6.67 \times 10^{-11}\ Nm^2\ kg^{-2}$

$= 6.67 \times 10^{-11} \times (kg\ ms^{-2}) \times (m^2) \times (kg)^{-2}$

$= 6.67 \times 10^{-11} \times [\ (1000\ g\ ) \times (100\ cm) \times s^{-2}] \times (100\ cm)^2 \times (1000\ g)^{-2}$

$= 6.67 \times 10^{-11} \times 10^5 \times 10^4 \times 10^{-6}\ g^{-1}\ cm^3\ s^{-2}$

$= \mathbf{6.67 \times 10^{-8}\ g^{-1}\ cm^3\ s^{-2}}$

$\therefore$ **(a)**

Example **8:**

If energy (E), momentum (P) and force (F) are chosen as fundamental units. The dimensions of mass in new system is

(a) $E^{-1} P^3$ **(b)** $E^{-1} P^2$ **(c)** $E^{-2} P^2$ **(d) none of these**

Solution:

The dimensions of E, P and F in terms of M, L and T are

$[E] = ML^2T^{-2}$

$[P] = MLT^{-1}$

$[F] = MLT^{-2}$

Let $[M] = E^a P^b F^C$

or $[M] = (ML^2T^{-2})^a (MLT^{-1})^b (MLT^{-2})^C$

Equating the powers of M, L and T, we have $a = -1$, $b = 2$ and $c = 0$

Hence $[M] = \mathbf{E^{-1} P^2}$

$\therefore$ **(b)**

***Example* 9:**

Two full turns of the circular scale of a screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on circular scale is 50. Further, it is found that screw gauge has a zero error of – 0.03 mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in line with the main scale as 35. The diameter of the wire is

(a) 3.67 mm (b) 3.38 mm (c) 3.32 mm (d) 3.73 mm

Solution:

Least count $= \frac{0.5\,\text{mm}}{50} = 0.01\,\text{mm}$

Zero error = – 0.03 mm (Given)

Measured diameter = 3 mm + 35 × 0.01 mm = 3.35 mm

Correct diameter = 3.35 – (–0.03 mm) = 3.38 mm

$\therefore$ **(b)**

***Example* 10:**

A body of mass *m* = 3.513 kg is moving along the *x*-axis with a speed of 5.00 ms^{-1}. The magnitude of momentum is recorded as

(a) 17.56 kg ms^{-1} (b) 17.57 kg ms^{-1} (c) 17.6 kg ms^{-1} (d) 17.565 kg ms^{-1}

Solution:

$p = mv = 3.513 \times 5.00 = 17.565$ kg ms^{-1}

Since the result cannot have more than three significant figures

$p = 17.6$ kg ms^{-1}

$\therefore$ **(c)**

SOLVED SUBJECTIVE EXAMPLES

Example 1:

Find the dimensions of the following quantities

(i) Acceleration **(ii) Angle** **(iii) Density**
(iv) Kinetic energy **(v) Gravitational constant**

Solution:

(i) Acceleration = $\frac{\text{Velocity}}{\text{Time}}$ $\therefore$ [Acceleration] = $\frac{[\text{Velocity}]}{[\text{Time}]} = \frac{LT^{-1}}{T} = LT^{-2}$

(ii) Angle = $\frac{\text{Distance}}{\text{Distance}}$ $\therefore$ angle is dimensionless.

(iii) Density = $\frac{\text{Mass}}{\text{Volume}}$ $\therefore$ [Density] = $\frac{[\text{Mass}]}{[\text{Volume}]} = \frac{M}{L^3} = ML^{-3}$

(iv) Kinetic energy = $\frac{1}{2}$ Mass × Velocity^2

$\therefore$ [Kinetic energy] = [Mass] × [Velocity]^2 = ML^2T^{-2}

(v) Constant of gravitation occurs in Newton's law of gravitation

$F = G\frac{m_1m_2}{d^2}$

$\therefore$ $[G] = \frac{[F]\,[d^2]}{[m_1][m_2]} = \frac{MLT^{-2}L^2}{MM} = M^{-1}L^3T^{-2}$

Example 2:

Check by the method of dimensional analysis whether the following relations are correct.

(i) $v = \sqrt{\frac{P}{D}}$ **where v = velocity of sound and**

P = pressure, D = density of medium.

(ii) $n = \frac{1}{2l}\sqrt{\frac{F}{m}}$ **, where n = frequency of vibration, l = length of the string.**

F = stretching force, m = mass per unit length of the string.

Solution:

(i) [R.H.S.] = $\sqrt{\frac{[P]}{[D]}} = \sqrt{\frac{ML^{-1}T^{-2}}{ML^{-3}}} = LT^{-1}$

[L.H.S] = [v] = LT^{-1} **Hence the relation is correct.**

(ii) [R.H.S.] = $\frac{1}{[l]}\sqrt{\frac{[F]}{[m]}} = \frac{1}{L}\sqrt{\frac{MLT^{-2}}{ML^{-1}}} = \frac{1}{L}LT^{-1} = T^{-1}$

[L.H.S.] = $\left[\frac{A\ \text{Number}}{\text{Time}}\right] = \frac{1}{T} = \mathbf{T^{-1}}$. **Hence the result is correct.**

Example 3:

Round off to three significant digits

(i) 0.03927 kg **(ii) 4.085 × 10^5**

Solution:

(i) 0.0393 kg ($\because$ the last digit 7 > 5)

(ii) 4.08 × 10^5 s ($\because$ the digit to be rounded off is 5 and preceding digit is even)

***Example* 4:**

The velocity *v* of a particle depends on the time *t* according to the equation $v = a + bt + \frac{c}{d+t}$. Write the dimensions of *a*, *b*, *c* and *d*.

Solution:

From principle of homogeneity

$[a] = [v]$

or $[a] = LT^{-1}$

$[bt] = [v]$

or $[b] = \frac{[v]}{[t]} = \frac{LT^{-1}}{T}$

or $[b] = LT^{-2}$

Similarly, $[d] = [t] = T$

Further, $\frac{[c]}{[d+t]} = [v]$

or $[c] = [v]\,[d+t]$

or $[c] = LT^{-1}.T$

or $[c] = L$

***Example* 5:**

The moment of inertia of a body rotating about a given axis is 6.0 kg m^2 in the SI system. What is the value of the moment of inertia in a system of units in which the unit of length is 5 cm and the unit of mass is 10 g?

Solution:

The dimensions of moment of inertia are (ML^2). We have

$n_1(u_1) = n_2(u_2)$

or $n_1(M_1L_1^2) = n_2(M_2L_2^2)$

$\therefore \quad n_2 = \frac{n_1(M_1L_1^2)}{(M_2L_2^2)} = n_1\left(\frac{M_1}{M_2}\right)\left(\frac{L_1}{L_2}\right)^2$

Given $n_1 = 6.0$, $M_1 = 1$ kg, $L_1 = 1$ m, $M_2 = 10$ g and $L_2 = 5$ cm.

Therefore, $n_2 = 6.0 \times \left(\frac{1\text{kg}}{10\text{g}}\right) \times \left(\frac{1\text{m}}{5\text{cm}}\right)^2$

$= 6.0 \times \left(\frac{1000\text{g}}{10\text{g}}\right) \times \left(\frac{100\text{cm}}{5\text{cm}}\right)^2$

$= 6.0 \times 100 \times (20)^2 =$ **2.4×10^5 units**

***Example* 6:**

The time period *T* of the oscillations of a large star, oscillating under its own gravitational attraction, may depend on its mean radius *R*, its mean density ρ and the gravitation constant *G*. Using dimensional analysis, show that *T* is independent of *R* and is given by $T = \frac{k}{\sqrt{\rho G}}$ where *k* is a dimensionless constant.

Solution:

Let $T = kR^a\rho^bG^c$

Substituting the dimensions, we have

$[T] = (L)^a(ML^{-3})^b(M^{-1}L^3T^{-2})^c$

Equating powers of M, L and T, we have

$0 = b - c$

$0 = a - 3b + 3c$

and 1 = -2c

which give a = 0, b = -1/2 and c = -1/2

Hence $T = kR^0 \rho^{-\frac{1}{2}} G^{-\frac{1}{2}}$

or $T = \dfrac{k}{\sqrt{\rho G}}$

Thus *T* is independent of *R*

***Example* 7:**

A rectangular plate has a length of (21.3 ± 0.2) cm and a width of (9.80 ± 0.10) cm. Find the area of the plate.

Solution:

Area (A) = *lb* = 21.3 × 9.8 = 209 cm^2

$$\frac{\Delta A}{A} = \pm\left(\frac{0.2}{21.3} + \frac{0.1}{9.8}\right) = \pm 0.019$$

$\Delta A = \pm(0.019 \times 209) = \pm 3.97\text{cm}^2 = \pm 4\text{ cm}^2$

A = $\mathbf{(209 \pm 4)cm^2}$

***Example* 8:**

The heat dissipated in a resistance can be obtained by the measurement of resistance, the current and time. If the maximum error in the measurement of these quantities is 1%, 2% and 1% respectively, what is the maximum error in determination of the dissipated heat?

Solution:

Heat produced *H* is given by

$$H = \frac{I^2 Rt}{J}$$

$$\therefore \quad \frac{\Delta H}{H} = 2\frac{\Delta I}{I} + \frac{\Delta R}{R} + \frac{\Delta t}{t} - \frac{\Delta J}{J}$$

For maximum percentage error,

$$\frac{\Delta H}{H} \times 100 = 2\frac{\Delta I}{I} \times 100 + \frac{\Delta R}{R} \times 100 + \frac{\Delta t}{t} \times 100 + \frac{\Delta J}{J} \times 100$$

= 2 ×2% + 1% + 1% + 0% = **6%**

***Example* 9:**

In an experiment, values of two resistances are measured to be r_1 = (5.0 ± 0.2) ohm and r_2 = (10.0 ± 0.1) ohm. Find the values of total resistance in parallel with limits of percentage error.

Solution:

Combined resistance in parallel (R_p) is given by

$$\frac{1}{R_p} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{r_2 + r_1}{r_1 r_2}$$

$R_p = \dfrac{r_1 r_2}{r_1 + r_2} = \dfrac{50}{15} = 3.33\ \Omega$ As $\dfrac{1}{R_p} = \dfrac{1}{r_1} + \dfrac{1}{r_2}$

Differentiating we get

$$\frac{-dR_p}{R_{p^2}} = \frac{-dr_1}{r_1^2} - \frac{dr_2}{r_2^2}$$

or $\dfrac{dR_p}{R_{p^2}} = \dfrac{dr_1}{r_1^2} + \dfrac{dr_1}{r_2^2}$

$$\frac{dR_p}{(3.33)^2} = \frac{0.2}{(5.0)^2} + \frac{0.1}{(10.0)^2} \Rightarrow dR_p = 0.1\Omega$$

So $R_p = (3.3 \pm 0.1)\Omega$

***Example* 10:**

A helicopter can hover when the power of its engine is *P*. A second helicopter is an exact copy of the first one but its linear dimensions are double those of the original. What power output is needed to enable this second helicopter to hover?

Solution

The required power for the hovering helicopter should depend on acceleration due to gravity g, linear size of helicopter L, average density of its material ρ and density of air ρ_0

So, let $P \propto g^a \times L^b \times \rho^c \times \rho_0^d$

Or $P = k g^a L^b \rho^c \rho_0^d$ where k is a dimensionless constant

$$[M^1 L^2 T^{-3}] = k[LT^{-2}]^a [L]^b [ML^{-3}]^c [ML^{-3}]^d$$

$\Rightarrow$ $c + d = 1$

$a + b - 3c - 3d = 2$

$-2a = -3 \Rightarrow a = \frac{3}{2}$

Or $b - 3(c+d) = \frac{1}{2}$

$b - 3 = \frac{1}{2}$ $[\because c + d = 1]$

$b = \frac{7}{2}$

So it is clear that $P \propto L^{\frac{7}{2}}$

$$P' = 2^{\frac{7}{2}} P = 8\sqrt{2}P \text{ so } P' = 11.31P$$

So power of the second helicopter should be 11.31 times the power of the first helicopter.

TIPS & TRICKS

- *The standard of Weight and Measures Act was passed in India in 1976. It recommended the use of SI in all fields of science, technology, trade and industry.*
- *The dimensions of many physical quantities, especially those in heat, thermodynamics, electricity and magnetism in terms of mass, length and time alone become irrational. Therefore, SI is adopted which uses 7 basic units.*
- *The dimensions of a physical quantity are the powers to which basic units should be raised to represent the derived unit of that physical quantity.*
- *The dimensional formula is very helpful in writing the unit of a physical quantity in terms of the basic units.*
- *The dimensions of a physical quantity do not depend on the system of units.*
- *A physical quantity that does not have any unit must be dimensionless.*
- *The pure numbers are dimensionless.*
- *Generally, the symbols of those basic units, whose dimension (power) in the dimensional formula is zero, are omitted from the dimensional formula.*
- *It is wrong to say that the dimensions of force are* MLT^{-2}. *On the other hand we should say that the dimensional formula for force is* MLT^{-2} *and that the dimensions of force are 1 in mass, 1 in length and – 2 in time.*
- *The ratio of two similar quantities is dimensionless.*
- *The physical relation involving logarithm, exponential, trigonometric ratios, numerical factors etc. cannot be derived by the method of dimensional analysis.*
- *Physical relations involving addition or subtraction sign cannot be derived by the method of dimensional analysis.*
- *If units or dimensions of two physical quantities are same, these need not represent the same physical characteristics. For example torque and work have the same units and dimensions but their physical characteristics are different.*
- *The standard units must not change with space and time. That is why atomic standard of length and time have been defined. Attempts are being made to define the atomic standard for mass as well.*
- *The unit of time, the second, was initially defined in terms of the rotation of the earth around the sun as well as that about its own axis. This time standard is subjected to variation with time. Therefore, the atomic standard of time has been defined.*
- *Any repetitive phenomenon, such as an oscillating pendulum, spinning of earth about its axis, etc. can be used to measure time.*
- *The product of numerical value of the physical quantity (n) and its unit (U) remains constant.*
- *That is: nU = constant or* $n_1U_1 = n_2U_2$.

- *The product of numerical value (n) and unit (U) of a physical quantity is called magnitude of the physical quantity. Thus: Magnitude = nU.*
- *The order of magnitude of a quantity means its value (in suitable power of 10) nearest to the actual value of the quantity.*
- *Angle is exceptional physical quantity, which though is a ratio of two similar physical quantities (angle = arc/radius) but still requires a unit (degrees or radians) to specify it along with its numerical value.*
- *Solid angle subtended at a point inside the closed surface is 4π steradian.*
- *A measurement of a physical quantity is said to be accurate if the systematic error in its measurement is relatively very low. On the other hand, the measurement of a physical quantity is said to be precise if the random error is small.*
- *A measurement is most accurate if its observed value is very close to the true value.*
- *Errors are always additive in nature.*
- *For greater accuracy, the quantity with higher power should have least error.*
- *The absolute error in each measurement is equal to the least count of the measuring instrument.*
- *Percentage error = relative error × 100.*
- *The unit and dimensions of the absolute error are same as that of quantity itself.*
- *Absolute error is not dimensionless quantity.*
- *Relative error is dimensionless quantity.*
- *Least Count* $= \dfrac{\text{value of 1 part on main scale (s)}}{\text{Number of parts on vernier scale (n)}}$
- *Least count of vernier calipers* $= \begin{Bmatrix}\text{value of 1 part of} \\ \text{main scale (s)}\end{Bmatrix} - \begin{Bmatrix}\text{value of 1 part of} \\ \text{vernier scale (v)}\end{Bmatrix}$
  - *Least count of vernier calliper = 1 MSD – 1 VSD*
  - *Where MSD = Main Scale Division*
  - *VSD = Vernier Scale Division*
- *Least count of screw gauge* $= \dfrac{\text{Pitch (p)}}{\text{Number of parts on circular scale (n)}}$
- *Smaller the least count, higher is the accuracy of measurement.*
- *Larger the number of significant figures after the decimal in a measurement, higher is the accuracy of measurement.*
- *Significant figures do not change if we measure a physical quantity in different units.*
- *Significant figures retained after mathematical operation (like addition, subtraction, multiplication and division) should be equal to the minimum significant figures involved in any physical quantity in the given operation.*
- *Significant figures are the number of digits upto, which we are sure about their accuracy.*
- *If a number is without a decimal and ends in one or more zeros, then all the zeros at the end of the*

number may not be significant. To make the number of significant figures clear, it is suggested that the number may be written in exponential form. For example 20300 may be expressed as 203.00 × 10^2*, to suggest that all the zeros at the end of 20300 are significant.*

- *1 inch = 2.54 cm*
- *1 foot = 12 inches = 30.48 cm = 0.3048 m*
- *1 mile = 5280 ft = 1.609 km*
- *1 yard = 0.9144 m*
- *1 slug = 14.59 kg*
- *1 barn =* $10^{-28}\ \text{m}^2$
- *1 liter =* $10^3\ \text{cm}^3 = 10^{-3}\ \text{m}^3$
- *1 km/h =* $\frac{5}{18}\ \text{m/s}$
- *1 m/s = 3.6 km/h*
- *1* $\text{g/cm}^3 = 1000\ \text{kg/m}^3$
- *1 atm = 76 cm of Hg = 1.013 ×* $10^5\ \text{N/m}^2$
- *1* $\text{N/m}^2 = Pa$ (Pascal)
- *When we add or subtract two measured quantities, the absolute error in the final result is equal to the sum of the absolute errors in the measured quantities.*
- *When we multiply or divide two measured quantities, the relative error in the final result is equal to the sum of the relative errors in the measured quantities.*

EXERCISE - I

NEET SINGLE CHOICE CORRECT

1. If velocity (V), acceleration (A) and force (F) are taken as fundamental quantities instead of mass (M), length (L) and time (T), the dimensions of Young's modulus of elasticity would be (with M, L and T as fundamental, [Young's modulus] = $ML^{-1}T^{-2}$

(a) FA^2V^{-2} (b) FA^2V^{-3} (c) FA^2V^{-4} (d) FA^2V^{-5}

2. The number of particles crossing per unit area perpendicular to *X*-axis in unit time is

$N = -D\,\dfrac{n_2 - n_1}{x_2 - x_1}$

Where n_1 and n_2 are number of particles per unit volume for the value of x_1 and x_2 respectively. The dimensions of diffusion constant *D* are

(a) $M^\circ LT^2$ (b) $M^\circ L^2T^{-4}$ (c) $M^\circ LT^{-3}$ (d) $M^\circ L^2T^{-1}$

3. The dimensional formula for latent heat is

(a) $M^\circ L^2T^{-2}$ (b) MLT^{-2} (c) ML^2T^{-2} (d) ML^2T^{-1}

4. The result after adding 3.8×10^{-6} with 4.2×10^{-5} with due regard to significant figure is

(a) 4.58×10^{-5} (b) 0.458×10^{-4} (c) 4.6×10^{-5} (d) 45.8×10^{-6}

5. A dimensionally consistent relation for the volume *V* of a liquid of coefficient of viscosity η flowing per second through a tube of radius *r* and length *l* and having a pressure difference *p* across its end is $([\eta] = ML^{-1}T^{-1})$

(a) $V = \pi pr^4/8\eta l$ (b) $V = \pi\eta l/8\,pr^4$ (c) $V = 8\,p\eta l/\pi r^4$ (d) $V = \pi p\eta\,/8lr^4$

6. The volume of a sphere is 1.76 cm^3. The volume of 25 such spheres taking into account the significant figure is

(a) $0.44 \times 102\ cm^3$ (b) $44.0\ cm^3$ (c) $44\ cm^3$ (d) $44.00\ cm^3$

7. Which of the following has not been expressed in proper units? [stress = force /Area, surface tension = force/length]

(a) stress/strain = N/m^2 (b) surface tension = N/m
(c) energy = kg × m/s (d) pressure = N/m^2

8. Suppose the acceleration due to gravity at a place is 10 m/s^2. Its value in $cm/(minute)^2$ is

(a) 36×10^5 (b) 6×10^4 (c) 36×10^4 (d) none of these

9. If $x = a^n$, then fractional error $\dfrac{\Delta x}{x}$ is equal to

(a) $\pm\left(\dfrac{\Delta a}{a}\right)^n$ (b) $\pm n\left(\dfrac{\Delta a}{a}\right)$ (c) $\pm n\log_e\dfrac{\Delta a}{a}$ (d) $\pm n\log\dfrac{\Delta a}{a}$

10. What is the percentage error in the measurement of time period of a pendulum if maximum errors in the measurement of '*l*' and *g*' are 2% and 4% respectively?

(a) 6% (b) 4% (c) 3% (d) 5%

11. A physical quantity is represented by $X = M^a L^b T^{-c}$. If percentage error in the measurement of *M*, *L* and *T* are α%, β% and γ% respectively, then the maximum percentage error in calculating X is

(a) $(\alpha a - \beta b + \gamma c)\%$ (b) $(\alpha a + \beta b + \gamma c)\%$ (c) $(\alpha a - \beta b - \gamma c)\%$ (d) none of the above

12. The density of a cube is measured by measuring its mass and the length of its sides. If the maximum errors in the measurement of mass and length are 3% and 2% respectively, then the maximum error in the measurement of density is

(a) 9% (b) 7% (c) 5% (d) 1%

13. What is a vernier constant?

(a) It is the value of one main scale division divided by the total number of division on the main scale

(b) It is the value of one vernier scale division divided by the total number of divisions on the vernier scale

(c) It is the difference between value of one main scale division and one vernier scale division.

(d) It is not the least count of the vernier scale.

14. Choose the wrong statement for zero error and zero correction.

(a) If the zero of the vernier scale does not coincide with the zero of the main scale then the instrument is said to be having a zero error.

(b) Zero correction has a magnitude equal to zero error but sign is opposite to that of the zero error.

(c) Zero error is positive when the zero of vernier scale lies to the left of the zero of the main scale.

(d) None of these is wrong.

15. In a vernier callipers *N* divisions of vernier scale coincides with *N* – 1 divisions of main scale (in which length of one divisions is 1 mm). The least count of the instrument is (in mm)

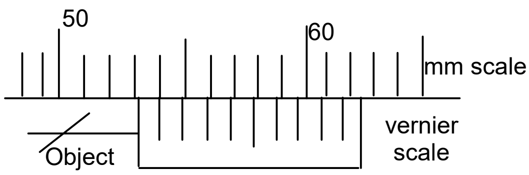
(a) N (b) $N - 1$ (c) $\frac{1}{N}$ (d) $\frac{1}{N-1}$

EXERCISE - II

IIT-JEE SINGLE CHOICE CORRECT

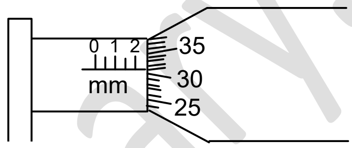
1. What is the reading of vernier callipers shown in figure

(a) 54.6 mm (b) 53.1 mm
(c) 52.7 mm (d) 54.7 mm



2. What is the reading of micrometer screw gauge shown in figure.

(a) 2.31 mm (b) 2.29 mm
(c) 2.36 mm (d) 2.41 mm



3. Precision in measurement depends on

(a) zero error (b) parallax
(c) least count of instrument (d) calibration of instrument

4. The number of significant figures in the number 0.020740×10^{-3} are

(a) 6 (b) 5 (c) 7 (d) 4

5. 0.205 – 0.2014 can be expressed as

(a) 0.0036 (b) 3.6×10^{-3} (c) 4.00×10^{-3} (d) 4×10^{-3}

6. The density of wood is 0.5 in CGS system of units. The corresponding value in MKS unit is

(a) 500 (b) 0.5 (c) 5×10^{-2} (d) 5000

7. If n is the numerical value of a physical quantity in the system in which its unit is u, then which of the following relation is correct?

(a) $\frac{n}{u} = \text{constant}$ (b) $\frac{u}{n} = \text{constant}$ (c) $nu = \text{constant}$ (d) $n^2u = \text{constant}$

8. In an experiment to measure the diameter of a wire using a screw gauge of resolution 0.001 cm, which of the following may correctly represents the measurement?

(a) 01.235 cm (b) 01.2351 cm (c) 01.24 cm (d) 01.23500 cm

9. The breadth of a thin rectangular sheet is measured as 10.1 cm. The uncertainty in the measurement is

(a) ± 1% (b) ± 0.5 % (c) ± 0.1 % (d) ± 5%

10. The least count of a stop watch is $\frac{1}{5}$ s. The time of 20 oscillations of a pendulum is measured as 25 s. The percentage error in the measurement of time will be

(a) 0.1% (b) 0.8% (c) 1.8% (d) 8%

11. A measured value 0.003749 expressed in two significant digits is
(a) 0.0 (b) 0.0038 (c) 0.0037 (d) 2.8×10^{-3}

12. The density of a cube is found by measuring its mass and the length of its side. If the maximum errors in the measurement of mass and length are 0.3% and 0.2% respectively, the maximum error in the measurement of density is
(a) 0.3% (b) 0.5% (c) 0.9% (d) 1.1%

13. The kinetic energy of a particle depends on the square of speed of the particle. If error in measurement of speed is 40%, the error in the measurement of kinetic energy will be
(a) 40% (b) 80% (c) 96% (d) 20%

14. The length of a rod is measured by four different instruments and the measurements are reported as
(A) 500.0 mm (B) 50.0 cm (C) 0.500 m (D) 5.0×10^{-4} km
We can conclude that
(a) (A) is most accurate measurement
(b) (C) is most accurate measurement
(c) (A), (C) and (D) are equally accurate measurements
(d) the accuracy of all the measurements is the same

15. The linear momentum *p* of a particle is given as a function of time *t* as $p = At^2 + Bt + C$. The dimensions of constant *B* are
(a) $ML^{-1}T^{-1}$ (b) $ML^{-1}T^{-2}$ (c) MLT^{-2} (d) MLT^{-1}

ONE OR MORE THAN ONE CHOICE CORRECT

1. A length of 5.0×10^1 cm when converted into meter can be written as
(a) 0.5 m (b) 0.50 m (c) 5.0×10^{-1} m (d) 5.00×10^{-1} m

2. A dimensionless quantity *y* is represented by the formula $y = \frac{a - bc}{d + e}$. Which of following is/are correct?
(a) Dimensions of *d* are *e* are same
(b) *abc* and *de* have some dimensions
(c) $\frac{bc}{ae + d}$ is dimensionless
(d) *de* + *bc* is not meaningful

3. Let $y = A \sin(\omega t - kx)$ represents the variation of distance *y* of a particle with time *t*. Which of the following is/are not meaningful?
(a) $\frac{y}{A} + \omega$ (b) $\frac{y}{\omega} + \frac{At}{kx}$ (c) $A - kx$ (d) $A + \frac{\omega}{k}$

4. In a screw gauge, there are 100 divisions in circular scale and each main scale division is of 1 mm. With no gap between the jaws, 98th divisions coincide with the main scale and zero of main scale is not visible. While measuring the diameter of a ball, the circular scale is between 3 mm mark and 4 mm mark. 76th division of circular scale coincides with the reference line. Select the correct alternative
(a) the least count of micrometer is 0.01 mm3
(b) zero error is – 0.02 mm
(c) zero error is 0.02 mm
(d) diameter of the ball is 3.76 mm

5. Which of the following functions of A and B cannot be performed if A and B possess different dimensions?

(a) $A + B$ (b) $A - B$ (c) $\frac{A}{B}$ (d) None of these

6. The radius of a spherical ball is (10.4 ± 0.4) cm. Select the correct alternative

(a) the percentage error in radius is 4%
(b) the percentage error in radius is 0.4%
(c) the percentage error in volume is 12%
(d) the percentage error in volume is 1.2 cm^3

7. A dimensionless quantity

(a) may have a unit
(b) must have a unit
(c) may not have a unit
(d) must not have a unit

8. The equation of a stationary wave is $y = 2A\sin\left(\frac{2\pi ct}{\lambda}\right)\cos\left(\frac{2\pi x}{\lambda}\right)$. Which of the following is/are correct?

(a) unit of ct is same as that of λ
(b) unit of x is same as that of λ
(c) unit of $\frac{2\pi c}{\lambda}$ is same as that of $\frac{2\pi x}{\lambda ct}$
(d) unit of $\frac{c}{\lambda}$ is same as that of $\frac{x}{\lambda}$

9. A voltmeter has a least count of 0.1 V and an ammeter has a least count of 0.1 A. The potential drop V across a resistance is measured as 10.0 V and current through it is measured as 1.0 A. Select the correct alternative

(a) the value of R is $(1.0 \pm 0.1) \times 10^1\ \Omega$
(b) the relative error in measurement of current is 0.1
(c) the accuracy in measurement of potential drop is 0.01
(d) the value of R is $(10 \pm 0.2)\Omega$

10. The dimensions of the quantities in one (or more) of the following pairs are the same. Identify the pair (s)

(a) torque and work
(b) angular momentum and work
(c) energy and Young's modulus
(d) light year and wavelength

EXERCISE - III

MATCH THE FOLLOWING

1. Some physical quantities are given in **Column I** and some possible SI units in which these quantities may be expressed are given in **Column II**. Match the physical quantities in **Column I** with the units in **Column II.**

Column -I	Column -II
I. GM_eM_s G – universal gravitational constant, M_e – mass of the earth, M_s – mass of the Sun	**A.** (volt) (coulomb) (metre)
II. $\frac{3RT}{M}$ R – universal gas constant, T – absolute temperature, M – molar mass	**B.** (kilogram) (metre)^3 (second)$^{-2}$
III. $\frac{F^2}{q^2B^2}$ F – force, q – charge, B – magnetic field	**C.** (metre)^2 (second)$^{-2}$
IV. $\frac{GM_e}{R_e}$ G – universal gravitational constant, M_e – mass of the earth, R_e – radius of the earth	**D.** (farad) (volt)^2 (kg)$^{-1}$ **E.** (kilogram)^0 (meter)^2 (second)$^{-2}$

REASONING TYPE

Directions: Read the following questions and choose

(A) If both the statements are true and statement-2 is the correct explanation of **statement-1**.
(B) If both the statements are true but statement-2 is not the correct explanation of **statement-1**.
(C) If statement-1 is True and **statement-2** is False.
(D) If statement-1 is False **and statement-2** is True.

1. **Statement-1**: Number of significant figures in 0.005 is one and that in 0.500 is three.
Statement-2: This is because zeros are not significant.
(a) A (b) B (c) C (d) D

2. **Statement-1**: Surface tension and surface energy have the same dimensions.
Statement-2: Both have different S.I. unit.
(a) A (b) B (c) C (d) D

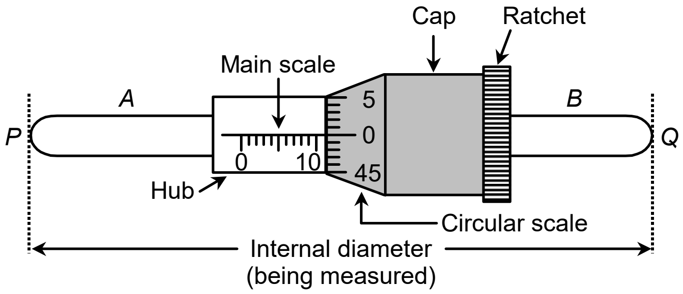
3. **Statement-1**: When we change the unit of measurement of a quantity, its numerical value changes.
Statement-2: Smaller the unit of measurement smaller is its numerical value.
(a) A (b) B (c) C (d) D

4. **Statement-1**: Dimensionless constants are the quantities having numerical value.
Statement-2: All constants are dimensionless.
(a) A (b) B (c) C (d) D

5. **Statement-1**: The time period of a pendulum is given by the formula, $T = 2\pi\sqrt{\frac{l}{g}}$.
Statement-2: According to the principle of homogeneity of dimensions, only that formula may be correct in which the dimensions of L.H.S. is equal to dimensions of R.H.S.
(a) A (b) B (c) C (d) D

LINKED COMPREHENSION TYPE

Internal micrometer is a measuring instrument used to measure internal diameter (ID) of a large cylinder bore with high accuracy.

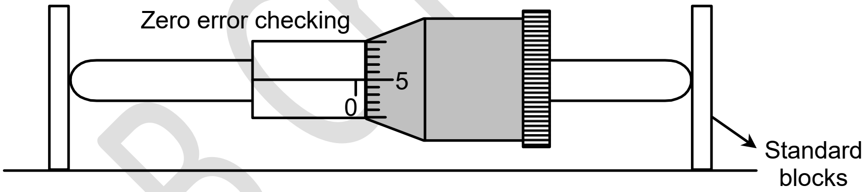


Construction is shown in figure. There is one fixed rod *B* (to the right in figure) and one moving rod *A* (to the left in figure). It is based on the principle of advancement of a screw when it is rotated in a nut with internal threads. Main scale reading can be directly seen on the hub, which is fixed with respect to rod *B*. When the cap is rotated, rod *A* moves in or out depending on direction of rotation. The circular scale reading is seen by checking which division of circular scale coincides with the reference line. This is to be multiplied by L.C. to get circular scale reading.

Least count = value of 1 circular scale division

$$= \frac{\text{pitch}}{\text{number of divisions on circular scale}}$$

Length of rod *A* is chosen to match the ID (*PQ*) to be measured. Zero error is checked by taking reading between standard blocks fixed at nominal value of ID to be measured.



Zero error is positive if cap end is on the right side of the main scale and negative it is on the left side.

1. In an internal micrometer, main scale division is of 0.5 mm and there are 50 divisions in circular scale. The least count of the instrument is

(a) 0.005 mm (b) 0.001 mm (c) 0.05 mm (d) 0.01 mm

2. In the above instrument, while measuring an internal diameter. ID is set of 321 mm with no zero error. If cap end is after 7^{th} division and 17^{th} division of main scale coincides with the reference line, the ID is

(a) 321.717 mm (b) 321.87 mm (c) 328.17 mm (d) 324.67 mm

3. During zero setting of the above instrument, the end of the cap is on left side of the zero of main scale (i.e., zero of main scale is not visible) and 41^{st} division of circular scale coincides with the reference time, the zero error is

(a) – 0.09 mm (b) + 0.41 mm (c) – 0.41 mm (d) + 0.09 mm

EXERCISE - IV

SUBJECTIVE PROBLEMS

1. Determine the number of significant figures in the following numbers
(a) 23 cm, (b) 3.589 s, (c) 4.67×10^3 m/s, (d) 0.0032 m.

2. Carry out the following arithmetic operations:
(a) the sum of the numbers 756, 37.2, 0.83, and 2.5;
(b) the product 3.2×3.563;
(c) the product 5.6×3.14.

3. The time period (*t*) of vibration of a liquid drop depends on surface tension (*S*), radius (*R*) of the drop and density (ρ) of the liquid. Find *t*.

4. The height *h* to which a liquid of density ρ and surface tension *S* rises in a capillary tube of radius *r* may depend upon ρ, σ, *r* and *g*, where *g* is the acceleration due to gravity. Experiments show that *h* is inversely proportional to *r*. With this experimental information, use the dimensional method to show that $h = \frac{kS}{r\rho g}$. (k is the dimensionless constant)

5. The Young's modulus of steel in the CGS system is 2.0×10^{12} dyne cm^{-2} . Express it in the SI system.

6. A cube has a side of length 1.2×10^{-2}m. Calculate its volume.

7. We measure the period of oscillation of a simple pendulum. In successive measurements, the readings turn out to be 2.63 s, 2.56s, 2.42s, 2.71s and 2.80 s. Calculate the (a) absolute errors, (b) relative error (c) percentage error.

8. A physical quantity ρ is related to four variables α, β, γ and η as follows $\rho = \frac{\alpha^3\beta^2}{\sqrt{\gamma\eta}}$. The percentage errors of measurements in α, β, γ and η are 1%, 3%, 4% and 2 % respectively. Find the percentage errors in ρ.

9. A wire is of mass (0.3 ± 0.003)g. The radius is (0.5 ± 0.005)mm and length is (6.0 ± 0.06)cm then find the % error in density.

10. In 2 complete revolutions of the circular scale, distance moved on the pitch scale is 1 mm. There are 50 divisions on the circular scale. There is no zero error. The diameter of a wire is measured with the help of this gauge. It reads 6 divisions on the main scale and 32 divisions on the cap. If each main scale division is 0.5 mm, what is the diameter of the wire?

11. A vernier calipers has 50 divisions on its vernier scale. The smallest division on main scale is 1 mm. When the jaws are touching each other, zero of vernier scale is between zero and 1st division of main scale and 8th division of vernier scale coincides with one of the divisions of main scale. While taking a measurement, zero of vernier scale is between 15th and 16th division of main scale and 39th division of vernier scale coincides with one of the divisions of main scale. What is the correct value of measurement?

12. The kinetic energy of a particle moving along x-axis varies with the distance x of the particle from origin as $K = \frac{A + x^3}{B(x^{1/4}) + C}$. Write the dimensional formula for A^2B.

13. If $(n + 1)^{th}$ divisions of vernier side coincides with n^{th} division of main scale then find the least count of the vernier. It is given that one main scale division is equal to a units.

14. The pitch of a screw quage is 0.5 mm and there are 50 divisions on the circular scale. In measuring the thickness of a metal plate, there are five divisions on the main scale and 34th division concides with the reference line. Find the thickness of the metal plate

15. The length of cylinder is measured with the help of a vernier callipers whose smallest division on the main scale is 0.5 mm and nine divisions of the main scale are equal to ten divisions of the vernier scale. It is observed that 78th divisions of the main scale coincides with the sixth division of the vernier scale. Find the length of the cylinder.

ANSWERS

EXERCISE – I

NEET SINGLE CHOICE CORRECT

1. (c)	2. (d)	3. (a)	4. (c)	5. (a)
6. (b)	7. (c)	8. (c)	9. (b)	10. (c)
11. (b)	12. (a)	13. (c)	14. (c)	15. (c)

EXERCISE – II

IIT-JEE SINGLE CHOICE CORRECT

1. (d)	2. (c)	3. (c)	4. (a)	5. (c)
6. (c)	7. (c)	8. (c)	9. (a)	10. (b)
11. (b)	12. (a)	13. (a)	14. (a)	15. (c)

ONE OR MORE THAN ONE CHOICE CORRECT

1. (b, c)	2. (a, b, d)	3. (a, c, d,)	4. (a, b)	5. (a, b)
6. (a, c)	7. (a), (c)	8. (a, b)	9. (a, b, c)	10. (a, d)

EXERCISE – III

MATRIX MATCH TYPE

1. I – (A, B), II – (C, D, E), III – (C, D, E), (IV) – (C, D, E)

REASONING TYPE

1. (c)	2. (d)	3. (c)	4. (c)	5. (b)

LINKED COMPREHENSION TYPE

1. (d)	2. (d)	3. (a)

EXERCISE – IV

SUBJECTIVE PRBOLEMS

1. (a) 2 (b) 4 (c) 3 (d) 2

2. (a) 796 (b) 11 (c) 18

3. $t = k\sqrt{\dfrac{R^3\rho}{S}}$

5. 2.0×10^{11} N m^{-2}

6. 1.7×10^{-6} m^3

7. (a) 0.01s , 0.06 s, 0.20 s, 0.09 s, 0.18 s
(b) .042
(c) 4.2%

8. 13%

9. 4%

10. 3.32 mm

11. 15.60 mm

12. $[M^1 L^{\frac{19}{4}} T^{-2}]$

13. $\dfrac{1}{n+1}$ units

14. 2.84 mm

15. 36.8 mm

Q.1 Which pair have not equal dimensions-

(1) Energy and torque

(2) Force and impulse

(3) Angular momentum and Plank's constant

(4) Elastic modulus and pressure

Q.2 The dimension of Plank's constant equals to that of-

(1) Energy (2) Momentum

(3) Angular momentum (4) Power

Q.3 The dimensions of universal gravitational constant are-

(1) ML^2T^{-1} (2) $M^{-2}L^3T^{-2}$

(3) $M^{-2}L^2T^{-1}$ (4) $M^{-1}L^3T^{-2}$

Q.4 The ratio of the dimension of Plank's constant and that of the moment of inertia is the dimension of-

(1) Velocity

(2) Angular momentum

(3) Time

(4) Frequency

Q.5 The velocity v of a particle at time t is given by $v = at + \frac{b}{t+c}$, where a, b and c are constants. The dimensions of a, b and c are respectively.

(1) LT^{-2}, L and T (2) L^2, T and LT^2

(3) LT^2, LT and L (4) L, LT and T^2

Q.6 'Parsec' is the unit of-

(1) time

(2) distance

(3) frequency

(4) angular acceleration

Q.7 Dimension of electrical resistance is-

(1) $ML^2T^{-3}A^{-1}$ (2) $ML^2T^{-3}A^{-2}$

(3) $ML^3T^{-3}A^{-2}$ (4) $ML^{-1}L^3T^3A^2$

Q.8 Which two of the following five physical parameters have the same dimensions ?

(a) energy density

(b) refractive index

(c) dielectric constant

(d) Young's modulus

(e) magnetic field

(1) (a) and (d) (2) (a) and (e)

(3) (b) and (d) (4) (c) and (e)

Q.9 If the dimensions of a physical quantity are given by $M^aL^bT^c$, then the physical quantity will be-

(1) Force if a = 0, b = –1, c = –2

(2) Pressure if a = 1, b = –1, c = –2

(3) Velocity if a = 1, b = 0, c = –1

(4) Acceleration if a = 1, b = 1, c = –2

Q.10 The dimension of $(\mu_0 \in_0)^{-1/2}$ are :

(1) $[L^{-1/2}T^{1/2}]$ (2) $[L^{1/2}T^{-1/2}]$

(3) $[L^{-1}T]$ (4) $[LT^{-1}]$

Q.11 The density of a material in CGS system of units is 4 g/cm^3. In a system of units in which unit of length is 10 cm and unit of mass is 100g, the value of density of material will be –

(1) 0.04 (2) 0.4

(3) 40 (4) 400

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM - 2

Q.1 Two physical quantities of which one is a vector and the other is a scalar having the same dimensional formula are-

(1) Work and energy

(2) Troque and work

(3) Impulse and momentum

(4) Power and pressure

Q.2 The fundamental unit which has same power in the dimensional formula of surface tension and viscosity is-

(1) Mass (2) Length

(3) Time (4) None

Q.3 The ratio of one micron to one nanometre is-

(1)10^3 (2) 10^{-3}

(3) 10^{-6} (4) 10^{-1}

Q.4 The equation of a wave is given by Y = Asin $\omega\left(\frac{x}{v}-k\right)$ where ω is the angular velocity and v is the linear velocity. The dimension of k is-

(1) LT (2) T

(3) T^{-1} (4) T^2

Q.5 Temperature can be expressed as a derived quantity in terms of which of the following-

(1) Length and mass

(2) Mass and time

(3) Length, mass and time

(4) In terms of none of these

Q.6 The time dependence of a physical quantity P is given by P = P_0 exp($-\alpha t^2$), where α is a constant and t is time. The constant α

(1) dimensionless

(2) has dimensions T^{-2}

(3) has dimensions of P

(4) has dimensions T^2

Q.7 Density of wood is 0.5 gm/cc in the CGS system of units. The corresponding value in MKS units is-

(1) 500 (2) 5

(3) 0.5 (4) 5000

Q.8 Joule × s is the unit of-

(1) Energy

(2) Momentum

(3) Angular momentum

(4) Power

Q.9 In a particular system the units of length mass and time are chosen to be 10 cm, 10 g and 0.1 s respectively. The unit of force in this system will be equal to-

(1) 0.1 N (2) 1 N

(3) 10 N (4) 100 N

Q.10 Match list I with list II and select the correct answer by using the codes given below the lists

List I **(Item)**	**List-II** **(Units of length)**
A. Distance between earth & stars	1. Micron
B. Inter atomic distance in a solid	2. Angstrom
C. Size of nucleus	3. Light year
D. Wavelength of infrared laser	4. Fermi
	5. Kilometre

Codes	A	B	C	D
(1)	5	4	2	1
(2)	3	2	4	1
(3)	5	2	4	3
(4)	3	4	1	2

Q.11 Which one of the following quantities has not been expressed in proper units ?

(1) Stress/Strain = N/m^2

(2) Surface tension = N/m

(3) Energy = kg - m/s

(4) Pressure = N/m^2

Q.12 Which of the following is not the unit of time ?

(1) Micro second (2) Leap year

(3) Lunar months (4) Parallactic second

Q.13 Which of the following is smallest unit ?

(1) Milimetre (2) Angstrom

(3) Fermi (4) Metre

Q.14 Which the following functions of A and B may be performed if A and B possess different dimensions?

(1) A/B (2) A + B

(3) A – B (4) None

Q.15 Which relation is wrong ?

(1) 1 Calorie = 4.18 Joules

(2) 1Å = 10^{-10} m

(3) 1 MeV = 1.6 × 10^{-13} Joules

(4) 1 Newton = 10^{-5} Dynes

Q.16 The dimensional formula of angular velocity is-

(1) $M^0L^0T^{-1}$ (2) MLT^{-1}

(3) $M^0L^0T^{-1}$ (4) ML^0T^{-2}

Q.17 Which of the following is not the unit of length ?

(1) micron (2) light year

(3) angstrom (4) radian

Q.18 Parsec is the unit of-

(1) Speed (2) Time

(3) Distance (4) None of the above

Q.19 From the following pairs, choose the pair that does not have identical dimensions-

(1) Impulse and momentum

(2) Work and torque

(3) Moment of inertia and moment of force

(4) Angular momentum and Planck's constant

Q.20 A force F is given by F = at + bt^2, where t is time. The dimension of 'a' and 'b' are

(1) $[M\,L\,T^{-3}]$ and $[M\,L\,T^{-4}]$

(2) $[M\,L\,T^{-4}]$ and $[M\,L\,T^{-3}]$

(3) $[M\,L\,T^{-1}]$ and $[M\,L\,T^{-2}]$

(4) $[M\,L\,T^{-2}]$ and $[M\,L\,T^{0}]$

Q.21 The mechanical equivalent of heat J is-

(1) constant

(2) a physical quantity

(3) a conversion factor

(4) none of the above

Q.22 If the energy $E = G^p h^q c^r$ where G is the universal gravitational constant, h is the Planck's constant and c is the velocity of light, then the values of p, q and r are, respectively-

(1) –1/2, 1/2 and 5/2

(2) 1/2, –1/2 and –5/2

(3) –1/2, 1/2 and 3/2

(4) 1/2, 1/2 and –3/2

Q.23 Match list I with II and select the correct answer:

(A) spring constant	(1) $M^1L^2T^{-2}$
(B) pascal	(2) $M^0L^0T^{-1}$
(C) hertz	(3) $M^1L^0T^{-2}$
(D) joule	(4) $M^1L^{-1}T^{-2}$

	A	B	C	D
(1)	3	4	2	1
(2)	4	3	1	2
(3)	4	3	2	1
(4)	3	4	1	2

Q.24 Match the following -

(a) Angular momentum	(1) $M^{-1}L^2T^{-2}$
(b) Torque	(2) MT^{-2}
(c) Gravitational constant	(3) ML^2T^{-2}
(d) Tension	(4) ML^2T^{-1}

(1) (c) $\rightarrow$ 2, (d) $\rightarrow$ 1

(2) (a) $\rightarrow$ 4, (b) $\rightarrow$ 3

(3) (a) $\rightarrow$ 3, (c) $\rightarrow$ 1

(4) (b) $\rightarrow$ 2, (a) $\rightarrow$ 1

Q.25 A kilowatt hour is equal to-

(1) 3.6×10^6 joule (2) 3.6×10^4 joule

(3) 3.6×10^3 joule (4) 6×10^{-4} joule

Q.26 The value of Planck's constant is-

(1) 6.63×10^{-34} J/s (2) 6.63×10^{-34} kg-m^2/s

(3) 6.63×10^{-34} kg-m^2 (4) 6.63×10^{-34} J-s^{-2}

Q.27 Units of Stefan constant is-

(1) watt-m^2-K^4 (2) watt-m^2/K^4

(3) watt/m^2-K (4) watt/m^{2}K^4

Q.28 Dimension of relative density is-

(1) kg m^{-3} (2) ML^{-3}

(3) dimensionless (4) M^{2}L^{-6}

Q.29 Planck's constant has dimensions of-

(1) Energy (2) Momentum

(3) Frequency (4) Angular momentum

Q.30 The equation of state of some gases can be expressed as $\left(P+\frac{a}{V^2}\right)(V-b)$ =RT, where P is the pressure, V is the volume, T is the absolute temperature and a, b and R are constants. The dimension of 'a' are-

(1) [ML^{5}T^{-2}] (2) [ML^{-1}T^{-2}]

(3) [L^3] (4) [L^6]

Q.31 Which of the following does not have the same unit as others **?**

(1) watt-sec (2) kilowatt-hour

(3) eV (4) J-sec

Q.32 Which of the following pairs does not have similar dimensions ?

(1) Planck's constant & angular momentum

(2) Tension and surface tension

(3) Angle and strain

(4) Stress and pressure

Q.33 If dimensions of A and B are different, then which of the following operation is valid ?

(1) $\frac{A}{B}$ (2) $e^{-A/B}$

(3) A – B (4) A + B

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Q.1 The unit of power is-

(1) kilowatt (2) kilowatt-hour

(3) dyne (4) joule

Q.2 The unit of energy is-

(1) J/s (2) watt-day

(3) kilowatt (4) g-cm/s^2

Q.3 In the S.I. system, the unit of temperature is-

(1) degree centigrade

(2) Kelvin

(3) degree Celsius

(4) degree Fahrenheit

Q.4 In the S.I. system the unit of energy is-

(1) erg (2) calorie

(3) joule (4) electron volt

Q.5 Unit of pressure in S.I. system is-

(1) atmosphere

(2) dynes per square cm

(3) pascal

(4) bar

Q.6 Which of the following is not a unit for energy ?

(1) Kilo watt hour (2) Newton- meter

(3) (weber) (ampere) (4) None of these

Q.7 In SI unit the angular acceleration has unit of-

(1) $Nmkg^{-1}$ (2) ms^{-2}

(3) rad.s^{-2} (4) Nkg^{-1}

Q.8 Surface tension has unit of-

(1) Joule.m^2 (2) Joule.m^{-2}

(3) Joule.m (4) Joule.m^3

Q.9 The M.K.S. units of coefficient of viscosity is-

(1) kg $m^{-1}s^{-1}$ (2) kg m s^{-2}

(3) kg m^2 s^{-1} (4) kg^{-1} m^{-1} s^2

Q.10 A dimensionless quantity-

(1) never has a unit (2) always has a unit

(3) may have a unit (4) does not exist

Q.11 [M L T^{-1}] are the dimensions of-

(1) power (2) momentum

(3) force (4) couple

Q.12 The dimensions of impulse are equal to that of-

(1) force

(2) angular momentum

(3) pressure

(4) linear momentum

Q.13 Which of the following pairs have same dimensions –

(a) Torque and work

(b) Angular momentum and work

(c) Energy and moment of inertia

(d) Light year and wavelengths

(1) a and b (2) a and d

(3) b and c (4) a , b, and d

Q.14 Which of the following does not have dimensions of length ?

(1) Fermi (2) Micro

(3) Angstrom (4) Radian

Q.15 The dimensional formula for angular momentum is –

(1) ML^2T^{-2} (2) ML^2T^{-1}

(3) MLT^{-1} (4) $M^0L^2T^{-2}$

Q.16 Which of the following statement is wrong ?

(1) Unit of K.E. is Newton-metre

(2) Unit of viscosity is poise

(3) Work and energy have same dimensions

(4) Unit of surface tension is Newton metre

Q.17 Which of the following is different from other with a point of view of dimension ?

(1) Planck's constant

(2) Coefficient of viscosity

(3) Force constant

(4) Poisson's ratio

Q.18 Dimensions of magnetic flux density is -

(1) $M^1 L^0 T^{-1} A^{-1}$ (2) $M^1 L^0 T^{-2} A^{-1}$

(3) $M^1 L^1 T^{-2} A^{-1}$ (4) $M^1 L^0 T^{-1} A^{-2}$

Q.19 The dimensions of the quantity $\frac{L}{RCV}$ are -

(1) $M^0 L^0 T^1 A^1$ (2) $M^0 L^0 T^{-1} A^{-1}$

(3) $M^0 L^0 T^0 A^1$ (4) $M^0 L^0 T^0 A^{-1}$

Q.20 A and B are two physical quantities having different dimensions. Then which of the following operation is dimensionally correct ?

(1) A + B (2) $\log \frac{A}{B}$

(3) $\frac{A}{B}$ (4) $e^{A/B}$

Q.21 Vander waal's gas equation is

$\left(P + \frac{a}{V^2}\right)$ (V–b) = RT. The dimensions of constant **a** as given above are –

(1) $M L^4 T^{-2}$ (2) $ML^5 T^{-2}$

(3) $M L^3 T^{-2}$ (4) $M L^2 T^{-2}$

Q.22 For $10^{(at+3)}$, the dimension of a is-

(1) $M^0 L^0 T^0$ (2) $M^0 L^0 T^1$

(3) $M^0 L^0 T^{-1}$ (4) None of these

Q.23 The velocity of a moving particle depends upon time t as $v = at + \frac{b}{t+c}$. Then dimensional formula for b is –

(1) $[M^0 L^0 T^0]$ (2) $[M^0 L^1 T^0]$

(3) $[M^0 L^1 T^{-1}]$ (4) $[M^0 L^1 T^{-2}]$

Q. 24 The SI unit of length is the meter. Suppose we adopt a new unit of length which equals to x meters. The area $1m^2$ expressed in terms of the new unit has a magnitude-

(1) x (2) x^2

(3) $\frac{1}{x}$ (4) $\frac{1}{x^2}$

Q.25 The units nanometre, fermi, angstrom and attometre, arranged in decreasing order will read as-

(1) angstrom, nanometre, fermi, attometre

(2) fermi, attometre, angstrom, nanometre

(3) nanometre, angstrom, fermi, attometre

(4) attometre, angstrom, fermi, nanometre

Q.26 Which of the following pairs of physical quantities has different dimensions ?

(1) stress, pressure

(2) Young's modulus, energy density

(3) density, relative density

(4) energy, torque

Q.27 If the unit of length is micrometre and the unit of time is microsecond, the unit of velocity will be-

(1) 100 m/s (2) 10 m/s

(3) micrometre/s (4) m/s

Q.28 A wave is represented by- y = a sin (At – Bx + C)

where A, B, C are constants. The Dimensions of A, B, C are

(1) T^{-1}, L, $M^0L^0T^0$ (2) T^{-1}, L^{-1}, $M^0L^0T^0$

(3) T, L, M (4) T^{-1}, L^{-1}, M^{-1}

Q.29 Which of the following is a dimensional constant ?

(1) Refractive index

(2) Dielectric constant

(3) Relative density

(4) Gravitational constant

Q.30 Two quantities whose dimensions are not same, cannot be-

(1) multiplied with each other

(2) divided

(3) added or subtracted in the same expression

(4) added together

Q.31 If force, length and time would have been the fundamental units, what would have been the dimensional formula for mass ?

(1) $F L^{-1} T^{2}$ (2) $F L T^{-2}$

(3) $F L T^{-1}$ (4) F

Q.32 If $x = at + bt^2$, where x is in metre and t in hour (hr), then unit of b will be-

(1) m^2/hr (2) m

(3) m/hr (4) m/hr^2

Q.33 The equation of the stationary wave is

$$y = 2A \sin\left(\frac{2\pi ct}{\lambda}\right)\cos\left(\frac{2\pi x}{\lambda}\right)$$

Which of the following statements is wrong ?

(1) the unit of ct is same as that of λ

(2) the unit of x is same as that of λ

(3) the unit of $2\pi c/\lambda$ is same as that of $2\pi x/\lambda t$

(4) the unit of c/λ is same as that of x/λ

Q.34 The dimension of which quantity is different from the remaining three quantities-

(1) Elastic constants

(2) Pressure

(3) Stress

(4) Angular momentum per unit mass

Q.35 Temperature can be represented as derived unit from which of the combination of units given below –

(1) mass and length

(2) mass and time

(3) mass, length and time

(4) none of these

Q.36 The unit of temperature in SI system is-

(1) degree Celsius

(2) degree Fahrenheit

(3) degree Kelvin

(4) degree Reaumur

Q.37 If the units of length and force are increased four times, then the unit of energy will-

(1) becomes 8 times

(2) becomes 16 times

(3) decrease 16 times

(4) increase 4 times

Q.38 If Force = (x/density) + C is dimensionally correct, the dimension of x are –

(1) MLT^{-2} (2) MLT^{-3}

(3) ML^2T^{-3} (4) $M^2L^{-2}T^{-2}$

Q.39 If the units of length , velocity and force are half, then the units of Power will be –

(1) doubled

(2) halved

(3) quadrupled

(4) remain unaffected

Q.40 The distance covered by a particle in time t is given by $x = a + bt + ct^2 + dt^3$. The dimensions of a and d are –

(1) L, T^{-3} (2) L, LT^{-3}

(3) L^0, T^3 (4) none of these

Q.41 Choose the wrong statement-

(1)all quantities can be expressed dimensionally in terms of the fundamental quantities

(2) a fundamental quantity cannot be represented dimensionally in terms of the rest of fundamental quantities

(3) the dimension of a fundamental quantity, in other fundamental quantities is always zero

(4) the dimension of a derived quantity is never zero in any fundamental quantity

Q.42 The period of a body under S.H.M. is represented by : $T \propto P^aD^bS^c$, where P is pressure, D is density and S is surface tension, then the values of a, b, and c are-

(1) –3/2, 1/2, 1 (2) –1, –2, 3

(3) 1/2, –3/2, –1/2 (4) 1, 2, 1/3

Q.43 When a wave transverses in a medium, the displacement of a particle located at distance x at time t is given by y = a sin(bt – cx) where a, b and c are constants of the wave. The dimension of b/c are same as that of-

(1) wave velocity (2) wavelength

(3) wave amplitude (4) wave frequency

Q.44 Which of the following system of units is not based on units of mass, length and time alone ?

(1) SI (2) MKS

(3) FPS (4) CGS

Q.45 Which of the following quantity is unitless ?

(1) Velocity gradient

(2) Pressure gradient

(3) Displacement gradient

(4) Force gradient

Q.46 The method of dimensional analysis can be used to derive which of the following relations ?

(1) $N_0e^{-\lambda t}$ (2) A sin(ωt + kx)

(3) $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ (4) None of the above

Q.47 Which of the following does not have the dimensions of force ?

(1) Potential gradient

(2) Energy gradient

(3) Weight

(4) Rate of change of momentum

Q.48 Which of the following is incorrect statement?

(1) A dimensionally correct equation may be correct

(2) A dimensionally correct equation may be incorrect

(3) A dimensionally incorrect equation may be correct

(4) A dimensionally incorrect equation is incorrect

Q.49 A dimensionless quantity -

(1) Never has a unit

(2) Always has a unit

(3) May have a unit

(4) Does not exist

Q.50 A unitless quantity-

(1) Does not exist

(2) Always has a nonzero dimension

(3) Never has a nonzero dimension

(4) May have a nonzero dimension

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 4

Q.1 A physical parameter a can be determined by measuring the parameters b, c, d and e using the relation $a = b^{\alpha} c^{\beta} / d^{\gamma} e^{\delta}$. If the maximum errors in the measurement of b, c, d and e are b_1%, c_1%, d_1% and e_1%, then the maximum error in the value of a, determined by the experiment is-

(1) $(b_1 + c_1 + d_1 + e_1)$%

(2) $(b_1 + c_1 - d_1 - e_1)$%

(3) $(\alpha b_1 + \beta c_1 - \gamma d_1 - \delta e_1)$%

(4) $(\alpha b_1 + \beta c_1 + \gamma d_1 + \delta e_1)$%

Q.2 The heat generated in a circuit is dependent upon the resistance, current and time for which the current is flown. If the error in measuring the above are as 1%, 2% and 1% the maximum error in measuring heat will be-

(1) 2% (2) 3% (3) 6% (4) 1%

Q.3 The percentage errors in the measurement of mass and speed are 2% and 3% respectively. How much will be the maximum error in the estimate of kinetic energy obtained by measuring mass and speed ?

(1) 11% (2) 8% (3) 5% (4) 1%

Q.4 One centimetre on the main scale of vernier callipers is divided into ten equal parts. If 10 divisions of vernier scale coincide with 8 small divisions of the main scale, the least count of the callipers is-

(1) 0.01 cm (2) 0.02 cm

(3) 0.05 cm (4) 0.005 cm

Q.5 While measuring acceleration due to gravity by a simple pendulum a student makes a positive error of 1% in the length of the pendulum and a negative error of 3% in the value of the time period. His percentage error in the measurement of the value of g will be-

(1) 2 % (2) 4 %

(3) 7 % (4) 10 %

Q.6 The density of a cube is measured by measuring its mass and the length of its side. If the maximum errors in the measurement of mass and length are 4% and 3% respectively, the maximum error in the measurement of the density is-

(1) 9 % (2) 13 %

(3) 12 % (4) 7 %

Q.7 A student measured the diameter of a wire using a screw gauge with least count 0.001 cm and listed the measurements. The correct measurement is-

(1) 5.3 cm (2) 5.32 cm

(3) 5.320 cm (4) 5.3200 cm

Q.8 The error in the measurement of radius of a sphere is 0.1%. The error in the measurement of volume is-

(1) 0.1 % (2) 0.3 %

(3) 0.5 % (4) 0.8 %

Q.9 The pressure on a square plate is measured by measuring the force on the plate and the length of the sides of the plate. If the maximum error in the measurement of force and length are respectively 4% and 2%, the maximum error in the measurement of pressure is-

(1) 1 % (2) 2 %

(3) 6 % (4) 8 %

Q.10 When a copper sphere is heated, maximum percentage change will be observed in-

(1) radius

(2) area

(3) volume

(4) none of these

Q.11 A wire has a mass (0.3 ± 0.003) g, radius (0.5 ± 0.005) mm and length (6 ± 0.06) cm. The maximum percentage error in the measurement of its density is-

(1) 1 (2) 2

(3) 3 (4) 4

Q.12 If the error in the measurement of radius of a sphere is 2% then the error in the determination of volume of the sphere will be-

(1) 8 % (2) 2 %

(3) 4 % (4) 6 %

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM - 5

Q.1 A quantity is represented by $X = M^aL^bT^c$. The percentage error in measurement of M, L and T are α%, β% and γ% respectively. The percentage error in X would be-

(1) $(\alpha a + \beta b + \gamma c)$ %

(2) $(\alpha a - \beta b + \gamma c)$ %

(3) $(\alpha a - \beta b - \gamma c)$ %

(4) None of these

Q.2 An experiment measure quantities a, b and c, and X is calculated from $X = ab^2/c^3$. If the percentage error in a, b and c are ±1%, ±3% and ±2% respectively, the percentage error in X will be-

(1) ± 13 % (2) ± 7 %

(3) ± 4 % (4) ± 1 %

Q.3 Zero error of an instrument introduces-

(1) Systematic errors (2) Random errors

(3) Both (4) None

Q.4 What is the fractional error is g calculated from $T = 2\pi\sqrt{\ell/g}$? Given that fractional errors in T and ℓ are ±x and ± y respectively.

(1) x + y (2) x – y

(3) 2x + y (4) 2x – y

Q.5 If error in measuring diameter of a circle is 4%, the error in the radius of the circle would be-

(1) 2% (2) 8%

(3) 4% (4) 1%

Q.6 A thin copper wire of length ℓ metre increase in length by 2% when heated through 10ºC. What is the percentage increase in area when a square copper sheet of length ℓ metre is heated through 10ºC ?

(1) 4%

(2) 8%

(3) 16%

(4) None of the above

Q.7 The period of oscillation of a simple pendulum in the experiment is recorded as 2.63 s, 2.56s, 2.42s, 2.71s and 2.80s respectively. The average absolute error is-

(1) 0.1 s (2) 0.11 s

(3) 0.01 s (4) 1.0 s

Q.8 The resistance is R = $\frac{V}{I}$ where V = 100 ± 5 volts and I = 10 ± 0.2 amperes. What is the total error in R ?

(1) 5 % (2) 7 %

(3) 5.2 % (4) $\left(\frac{5}{2}\right)$%

Q.9 The length, breadth and thickness of a strip are (10.0 ± 0.1) cm, (1.00 ± 0.01) cm and (0.100 ± 0.001) cm respectively. The most probable error in its volume will be-

(1) ± 0.03 cm^3 (2) ± 0.111 cm^3

(3) ± 0.012 cm^3 (4) None of these

Q.10 If error in measuring diameter of a circle is 4%, the error in circumference of the circle would be-

(1) 2 % (2) 8 %

(3) 4 % (4) 1 %

Q.11 The external and internal radius of a hollow cylinder are measured to be (4.23 ± 0.01) cm and (3.89 ± 0.01) cm. The thickness of the wall of the cylinder is-

(1) (0.34 ± 0.02) cm (2) (0.17 ± 0.02) cm

(3) (0.17 ± 0.01) cm (4) (0.34 ± 0.01) cm

Q.12 Percentage error in measuring the radius and mass of a solid sphere are 2% and 1% respectively. Then error in measurement of moment of inertia with respect to its diameter is-

(1) 3 % (2) 6 %

(3) 5 % (4) 4 %

Q.13 In a vernier calliper, N divisions of vernier scale coincide with (N – 1) divisions of main scale (in which 1 division represents 1mm). The least count of the instrument in cm should be-

(1) N (2) N – 1

(3) $\frac{1}{10\,N}$ (4) $\frac{1}{N-1}$

Q.14 The length of a cylinder is measured with a metre rod having least count 0.1 cm. Its diameter is measured with vernier callipers having least count 0.01 cm. Given the length is 5.0 cm and radius is 2.00 cm. The percentage error in the calculated value of volume will be-

(1) 2 % (2) 1 %

(3) 3 % (4) 4 %

Q.15 A vernier callipers has 20 divisions on the vernier scale which coincide with 19 divisions on the main scale. The least count of the instrument is 0.1 mm. The main scale divisions are of-

(1) 0.5 mm (2) 1 mm

(3) 2 mm (4) 1/4 mm

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 1 (ANSWERS)

Q.No.	1	2	3	4	5	6	7	8	9	10	11
Ans.	2	2	3	4	4	1	2	2	1	4	3

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 2 (ANSWERS)

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	2	1	1	2	4	2	1	3	1	2	3	4	3	1	4	1	4	3	3	1
Q.No.	21	22	23	24	25	26	27	28	29	30	31	32	33							
Ans.	3	1	1	2	1	2	4	3	4	1	4	2	1							

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 3 (ANSWERS)

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	1	2	2	3	3	4	3	2	1	3	2	4	1	4	2	4	4	2	4	3
Q.No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	2	3	2	4	3	3	4	2	4	3	2	4	4	4	4	3	2	4	3	2
Q.No.	41	42	43	44	45	46	47	48	49	50										
Ans.	4	1	1	1	3	4	1	3	3	3										

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 4 (ANSWERS)

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12
Ans.	4	3	2	2	3	2	3	2	4	3	4	4

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 5 (ANSWERS)

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	1	1	1	3	3	1	2	2	1	3	1	3	3	3	3