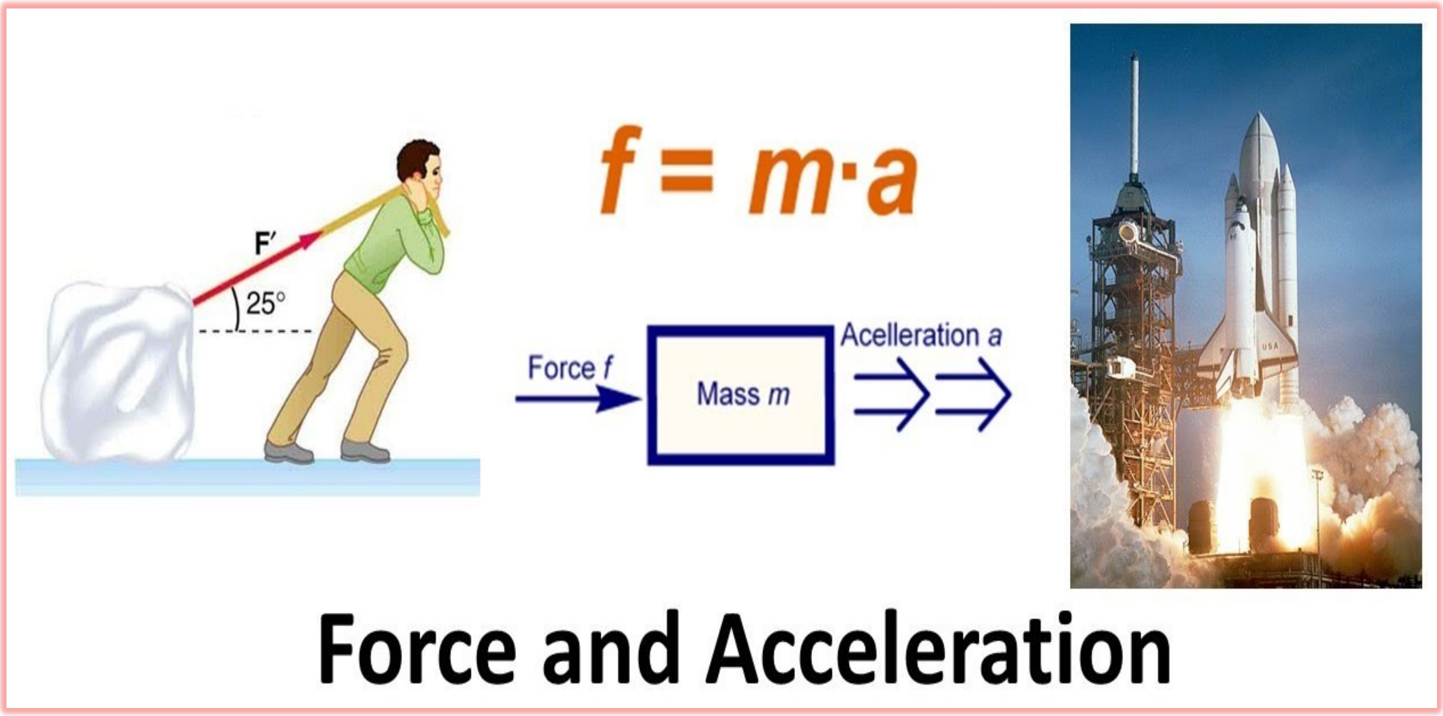
PHYSICS

for JEE, NEET & BOARD

11th

KINEMATICS



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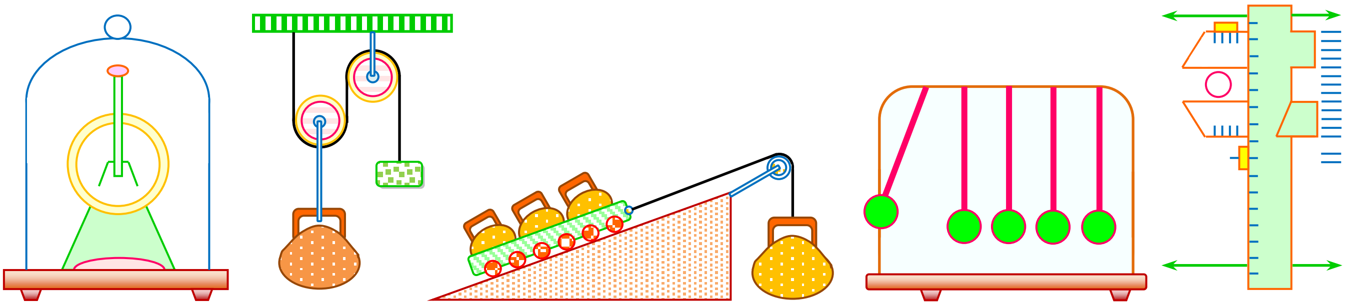

PHYSICS BOOKLET FOR JEE NEET & BOARDS

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KINEMATICS

1 KINEMATICS

When we observe the nature, we see many objects in motion. From experience we recognize that motion represents the continuous change in position of an object with time.

When we study **mechanics** the first step is to know the details of motion, which is **dynamics** part of the mechanics. The other part is **statics**, which deals with body at rest. We study dynamics in two parts: **Kinematics & Kinetics**. In kinematics we study the motion in terms of space, and time without knowing the cause of motion and in kinetics we include the cause of motion.

The present lesson is the study about motion irrespective of its cause i.e., **Kinematics**. In physics we study three types of motion: transnational, rotational and vibrational. A car moving on a highway is undergoing transnational motion, the earth's daily spin on its axis is an example of rotational motion and back-and-forth motion of a pendulum is an example of vibrational motion.

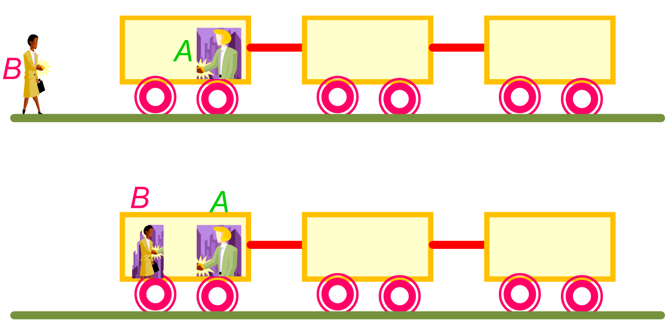
1.1 MOTION AND REST

Whether a body or particle is in motion or at rest, it depends upon the frame of reference of the observer. If an object changes its position in space with time relative to an observer, then it is said to be in motion, otherwise at rest. Now we should know "What is frame of reference?"

Frame of Reference

Suppose a person '*B*' who is on the ground observes a person '*A*' who is inside of a moving train. *B* observes that position of *A* is changing, so *A* is in motion

When *B* is also inside the train in the same compartment and train is still in motion, *B* observes that position of *A* is not changing. So *A* is at rest.



We can conclude that whether an object is at rest or motion, it depends from where the object is observed which is called frame of reference.

Here in the first case ground is frame of reference and in the second case, train is frame of reference.

There is no rule or restriction on the choice of a reference frame. We can choose it according to our convenience.

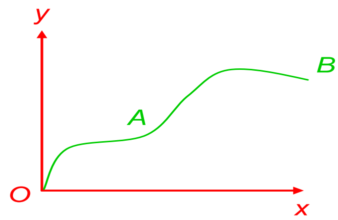
1.2 Distance (PATH LENGTH) AND Displacement

Distance: It is defined as the actual length of path covered by an object.

It is a +ve scalar quantity.

Its SI unit is m (metre).

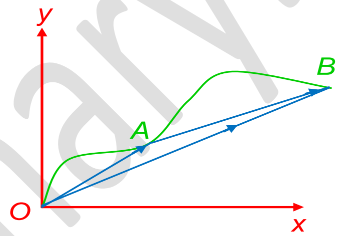
Suppose a particle moves from A to B as shown in figure. The length of curved path from A to B will be distance travelled by that particle.



Displacement: Displacement is the change in position vector i.e., vector joining initial and final position, or we can say it is a minimum possible distance between two positions.

It is a vector quantity.

If a particle moves from A to B in a given reference frame as shown in figure, then



Position vector of A $= \vec{OA} = \vec{r}_1$

Position vector B $= \vec{OB} = \vec{r}_2$

Displacement from position A to $B = \vec{AB}$

$\vec{AB} = \vec{OB} - \vec{OA} = \vec{r}_2 - \vec{r}_1$

1.3 DIFFERENCE BETWEEN DISTANCE AND DISPLACEMENT

Distance	Displacement
1. *Distance is length of actual path travelled by the particle*	1. *Displacement is minimum possible distance between two points.*
2. *It is a scalar quantity*	2. *It is a vector quantity*
3. *It can have more than two values between any two points*	3. *It has a unique value between any two points*
4. *It can never decrease with time*	4. *It can decrease with time*
5. *For a moving particle distance > 0*	5. *For a moving particle displacement >, <, = or 0.*

1.4 SPEED

(a) Speed: It is defined as the rate of change of distance, with respect to time.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{S}{T} \qquad \dots (1)$$

It is a scalar quantity.

Its S.I. unit is m/s and dimensions is $[LT^{-1}]$

(a) Instantaneous Speed

Instantaneous speed of a particle is speed at a particular instant of time. If ΔS is the distance travelled by a particle in a time interval Δt, then instantaneous speed is the limiting value of $\frac{\Delta S}{\Delta t}$ as Δt approaches zero.

$$\text{Instantaneous speed } (v) = \underset{\Delta t \to 0}{Lt} \frac{\Delta S}{\Delta t}$$

In calculus notation, this limit is called the derivative of s with respect to time, written as ds/dt

$$\text{Instantaneous speed } (v) = \frac{ds}{dt} \qquad \dots (2)$$

(b) Average Speed

The average speed of a particle is defined as ratio of total distance travelled to the total time taken

$$\text{Average Speed} = \frac{\text{Total Distance travelled}}{\text{Total time taken}}$$

If a particle travels distances S_1, S_2, S_3 … etc in time t_1, t_2, t_3 …. etc., respectively.

$$\text{Average speed} = \frac{S_1 + S_2 + S_3 \ldots S_n}{t_1 + t_2 + t_3 \ldots t_n}$$

$$V_{avg} = \frac{\sum_{i=1}^{n} S_i}{\sum_{i=1}^{n} t_i} \quad \ldots (3)$$

☞ *If a particle travels equal interval of distance at speeds v_1, v_2, v_3……… v_n respectively, then*

$$\frac{1}{v_{avg}} = \frac{1}{n}\left[\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3} \ldots\ldots + \frac{1}{v_n}\right]$$

Average speed is harmonic mean of individual speeds.

☞ *If a particle travels for equal intervals of time at speed v_1, v_2 …. v_n respectively, then*

$$V_{avg} = \frac{v_1 + v_2 + v_3 \ldots + v_n}{n}$$

In this case average speed is arithmetic mean of individual speed.

1.5 VELOCITY

It is defined as the rate of change in displacement with respect to time.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}} = \frac{\vec{S}}{T} \quad \ldots (4)$$

It is a vector quantity.

Its S.I. unit is m/s and dimensions is $[LT^{-1}]$

(a) Instantaneous Velocity

Instantaneous velocity of a particle is velocity at a particular instant of time. It is defined as rate of change of particle's position with time

If the position of a particle changes by $\Delta\vec{r}$ in a small time interval Δt, the limiting value of $\frac{\Delta\vec{r}}{\Delta t}$ as Δt approaches zero, gives the instantaneous velocity.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}$$

In calculus notation, this limit is called the derivative of r with respect to time, written as $d\vec{r}/dt$

$$\therefore \quad v = \frac{d\vec{r}}{dt} \quad \ldots (5)$$

(b) Average Velocity

The average velocity of a particle for a given interval of time is defined as the ratio of its displacement to the time taken.

$$\text{Average velocity} = \vec{v}_{av} = \frac{\text{Displacement}}{\text{Time}} = \frac{\Delta\vec{S}}{\Delta t} \quad \ldots (6)$$

Average velocity is independent of path taken between any two points. It depends only on the initial and final position of the particle since it depends on displacement.

If a particle starts from some point and returns to the same point, via any path, average velocity in this trip will be zero.

1.6 ACCELERATION

It is defined as rate of change of velocity with respect to time.

$$\text{Acceleration } (\vec{a}) = \frac{\text{change in velocity}}{\text{Time}} \qquad \dots (7)$$

It is a vector quantity

Its S.I. unit is m/s^2 and dimension is $[LT^{-2}]$

(a) Instantaneous acceleration

Instantaneous acceleration is the limiting value of average acceleration as Δt approaches to zero

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \qquad \dots (8)$$

Instantaneous acceleration is equal to the derivative of the velocity with respect to time.

Slope of the tangent on v–t graph gives the instantaneous acceleration.

Instantaneous acceleration also known as acceleration

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \qquad \dots (9)$$

When the acceleration is constant, the average acceleration is equal to the instantaneous acceleration

(b) Average acceleration

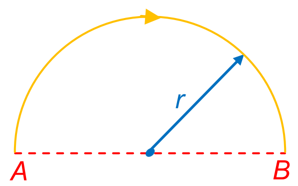
It is defined as ratio of change in velocity to the time interval in which change takes place.

Suppose a particle moving along x-axis has velocity v_1 at time t_1 and velocity v_2 at time t_2 average acceleration a_{av} is given by

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} \qquad \dots (10)$$

Illustration 1

Question: **A person moves on a semi-circular path of radius 20 m as shown in figure. If he starts at one end of the path *A* and reaches other end *B*. Find the distance covered and displacement during this motion.**



Solution: Distance covered by the person

= the length of the path

= πr

= $\pi \times 20$ = **62.8 m**

The displacement = $2r$

= 2×20 = **40 m**

Illustration 2

Question: **If a particle moves a distance at speed v_1 and comes back to initial position with speed v_2, what will be average speed?**

Solution: When the particle is going with speed v_1 and coming back with speed v_2, distance travelled by the particle in going and coming back is same, so average speed will be harmonic mean

So, $$\frac{1}{v_{avg}} = \frac{1}{2}\left(\frac{1}{v_1} + \frac{1}{v_2}\right)$$

$$v_{avg} = \frac{2v_1v_2}{v_1 + v_2}$$

Illustration 3

Question: **The distance travelled by particle in time t is given by $S = 3t^2 + 2$.**

(a) Find the instantaneous speed at $t = 3$ s.

(b) The average speed of the particle in 0 to 3 s

Solution:

(a) instantaneous speed $= \frac{ds}{dt}$

Here, $S = 3t^2 + 2$

$$\frac{ds}{dt} = 6t$$

$= 6 \times 3 =$ **18 m/s**

(b) The distance travelled during time 0 to 3 s is

$S = 3(3^2) + 2 - 2 = 27$ m

$\therefore$ Average speed $= \frac{\text{Total distance travelled}}{\text{Total time of motion}} = \frac{27}{3}$ **m/s**

Illustration 4

Question: **The position of an object moving along x-axis is given by $x = 2 + 3t^2$, where t is in seconds (a) what is its instantaneous velocity at t = 2s?(b) what is its average velocity at t = 2s and t = 4s?**

Solution: (a) Instantaneous velocity $(v) = \frac{dx}{dt} = 0 + 6t$

$v_{t=2} = 6 \times 2 =$ **12 m/s**

(b) Average velocity $= \frac{x(4) - x(2)}{4 - 2} = \frac{(2 + 3(4)^2) - (2 + 3(2)^2)}{2} =$ **18 m/s**

2 MOTION

Motion is the change in position with respect to time in a given reference frame. There are different types of motion such as motion in a straight line (Translatory motion), projectile motion, circular motion, simple harmonic motion etc. On the basis of motion with respect to the co-ordinate axes has been categorized into three types.

(1) One dimensional motion

(2) Two dimensional motion

(3) Three dimensional motion

One dimensional motion is that type of motion in which motion can be described along any one axis of co-ordinate system.

i.e., motion in a straight line

Two dimensional motion when a particle moves in a plane and it requires two axes of co-ordinate system to describe it.

i.e., circular motion, projectile motion.

Three-dimensional motion when a particle moves in space and it requires three axes of coordinate system to describe it.

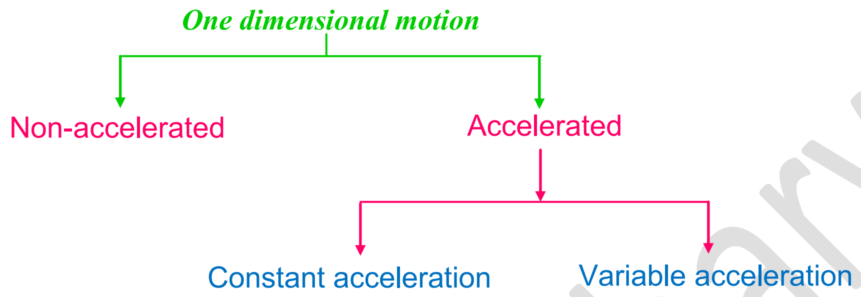
i.e., motion of an aeroplane, a balloon or a kite.

3 ONE DIMENSIONAL MOTION

One-dimensional motion can be classified in two parts

(i) Non-accelerated motion

(ii) Accelerated motion



3.1 NON-ACCELERATED MOTION

Such type of motion in which velocity is uniform, it means acceleration will be zero. In this type of motion .

$$\left.\begin{aligned} \text{Distance} &= \text{Velocity} \times \text{time} \\ \text{Velocity} &= \frac{\text{Distance}}{\text{Time}} \end{aligned}\right\} \text{(acceleration should be zero)} \qquad \dots \textbf{(11)}$$

3.2 ACCELERATED MOTION

If velocity of a particle varies with time then the motion is called accelerated motion. It can be classified in two parts:

(i) Uniformly accelerated motion

(ii) Non-uniformly accelerated motion

3.3 UNIFORMLY ACCELERATED MOTION

Such type of motion in which acceleration is constant or uniform.

In this type of motion the average acceleration equals the instantaneous acceleration.

If initial velocity v_i at time t_i changes to final velocity v_f at time t_f , then

$$\text{acceleration (a)} = \frac{v_f - v_i}{t_f - t_i}$$

Let when $t_i = 0$, $v_i = v_0$ and after time t, $v_f = v$

$$a = \frac{v - v_0}{t}$$

$$v - v_0 = at$$

$$v = v_0 + at \quad \text{(for constant } a\text{)} \qquad \dots \textbf{(12)}$$

This expression enables us to determine the velocity at any time t if the initial velocity, the acceleration, and the elapsed time are given.

From the above equation of motion we can say that velocity varies linearly with respect to time. Therefore the average velocity in any time interval can be expressed as the arithmetic mean of the initial velocity v_0, and final velocity v,

$$v_{avg} = \frac{v_0 + v}{2}$$

Now to find out the displacement as function of time, we can use this formula

$$\Delta s = v_{avg}\,\Delta t$$

Let us choose $t_1 = 0$ at which the initial position is $s_1 = 0$ and in any time t, displacement is s.

$$\Delta s = v_{avg}\,\Delta t$$

$$s - 0 = \frac{1}{2}(v + v_0)t$$

on putting $v = v_0 + at$

$$s - 0 = \frac{1}{2}(v_0 + at + v_0)t$$

$$s = \frac{1}{2}(2v_0 + at)\,t$$

$$s = v_0 t + \frac{1}{2}at^2 \qquad \ldots \textbf{(13)}$$

This expression enables us to determine the distances travelled in any time t if the initial velocity, acceleration and the time elapsed are known.

Now to obtain an expression that doesn't contain time, we can substitute t from equation (12) in above equation

$$s = \frac{1}{2}(v + v_0)t$$

$$= \frac{1}{2}(v + v_0)\left(\frac{v - v_0}{a}\right)$$

$$= \frac{1}{2}\left(\frac{v^2 - v_0^2}{a}\right)$$

$$v^2 - v_0^2 = 2as$$

$$v^2 = v_0^2 + 2as \qquad \ldots \textbf{(14)}$$

Now we want to find out the distance travelled in n^{th} second.

Let distance travels in n second by a particle is S_n and in $(n - 1)$ second is s_{n-1}, where initial velocity at $t = 0$ is v_0 and acceleration is a

$$s_n = v_0 n + \frac{1}{2}an^2$$

$$s_{n-1} = [v_0(n-1) - \frac{1}{2}a(n-1)^2]$$

$$s_n - s_{n-1} = (v_0 n + \frac{1}{2}an^2) - [v_0(n-1) - \frac{1}{2}a(n-1)^2]$$

$$S_{nth} = v_0 + \frac{a}{2}(2n - 1) \qquad \ldots \textbf{(15)}$$

3.4 REACTION TIME AND STOPPING DISTANCE

When a situation demands our immediate action, it takes some time before we really respond. Reaction time is the time a person takes to observe, think and act. For example, if a person is driving and suddenly a boy appears on the road, then the time elapse before he slams the breaks of the car is the reaction time. Reaction time depends on complexity of the situation and on an individual.

When brakes are applied to a moving vehicle, the distance it travels before stopping is called stopping distance.

Illustration 5

Question: **A car moving along a straight highway with speed of 126 kmh^{-1} is brought to rest within a distance of 200 m. What is the retardation of car (assumed uniform), and how long does it take for the car to stop?**

Solution: Given Initial speed of car (v_0) = 126 km/h = 35 m/s

Distance travelled by the car (s) = 200 m

Final velocity of car (v) = 0

$$v^2 = v_0^2 + 2as$$

$$(0)^2 = (35)^2 + 2 \times a\,(200)$$

$$400\,a = -(35 \times 35)$$

$$a = -\left(\frac{35 \times 35}{400}\right) = \frac{-49}{16} = -3.06 \text{ m/s}^2$$

Here, negative sign shows that the acceleration is opposite to the direction of motion and is called retardation.

So retardation = 3.06 m/s^2

Now retardation calculated above can be used to find the time,

$$v = v_0 + at$$

$$0 = 35 - 3.06\,t$$

$$t = \frac{35}{3.06} = 11.437 \text{ s}$$

So car will take **11.44 s**, to come to rest.

Illustration 6

Question: **A uniformly accelerating train passes successive kilometer stone with speed 18 km/hr and 27 km/hr, respectively. What will its speed be at the next kilometer stone and how long did it take to cover these 2 kilometres?**

Solution: Applying the equation

$v^2 = v_0^2 + 2ax$, we get

$$(7.5)^2 = (5)^2 + 2a \times 1000$$

$$\therefore \quad a = \frac{(7.5)^2 - (5)^2}{2000} = 0.0156 \text{ ms}^{-2}$$

The speed at next kilometer stone is given by

$$v_1^2 = v^2 + 2ax$$

$$= (7.5)^2 + 2 \times 0.0156 \times 1000 = 87.45 \Rightarrow \quad v_1 = 9.35 \text{ m/s}$$

Time to cover the distance of 2 kilometre is obtained by

$$t = \frac{v_1 - v_0}{a}$$

$$\Rightarrow \quad t = \frac{9.35 - 5}{0.0156} = 278.8 \text{ s}$$

Illustration 7

Question: **A motorist while driving at a speed of 72 km/hr sees a boy standing on the road at a distance of 52 m. He applies the brake and stops his car at a distance of 2 m from the boy. Find the acceleration caused due to the application of brake and time taken to stop the car**

Solution: $v^2 = v_0^2 + 2ax$,

we get $0^2 = 20^2 + 2a \times 50$ or $a = \mathbf{-4\ ms^{-2}}$

Applying equation,

$v = v_0 + at$, we obtain

$0 = 20 + (-4)t$

or $\boldsymbol{t = 5s}$

Illustration 8

Question: **The distance covered by a body during the 4th second is twice the distance covered by the body during the 2nd second of this journey. Find the initial velocity of the body. If its acceleration is 3 ms^{-2}.**

Solution: The distance covered in 2nd second is given by

$$S_2 = v_0 + \frac{a}{2}(2 \times 2 - 1)$$

The distance covered in the 4th second is given by

$$S_4 = v_0 + \frac{a}{2}(2 \times 4 - 1)$$

$$v_0 = \frac{a}{2} = \frac{3}{2} = \mathbf{1.5\ ms^{-1}}$$

3.5 MOTION UNDER GRAVITY

(i) Motion under gravity means an object is in motion in space under the force of gravity alone.

(ii) Motion under gravity is a uniformly accelerated motion. So equations of motion for uniformly accelerated motion can be used which are

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$\vec{s} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

$$v^2 = v_0^2 + 2\vec{a}.\vec{s}$$

(iii) Here acceleration will be acceleration due to gravity.

In SI-unit $g = 9.8$ m/s^2

In c.g.s. unit $g = 980$ cm/s^2

(iv) When an object is thrown upward or downward, in both cases same acceleration 'g' will be experienced by the object, which acts in downward direction.

(v) Here air resistance is neglected. In a real experiment air resistance cannot be neglected. It is an ideal case. Such motion is referred to as free fall.

Case 1:

When an object is thrown in upward direction (taking positive)space with initial velocity v_0.

Acceleration = $-g$ (in downward direction)

So, equation of motion will be

$v = v_0 - gt$

$h = v_0t - \frac{1}{2}gt^2$

$v^2 = v_0^2 - 2gh$

Case 2:

When an object is thrown in downward direction (taking positive) in space with initial velocity v_0.

Acceleration $= +g$ (in downward direction)

So, equation of motion will be

$v = v_0 + gt$

$s = v_0t + \frac{1}{2}gt^2$

$v^2 = v_0^2 + 2gh$

Case 3:

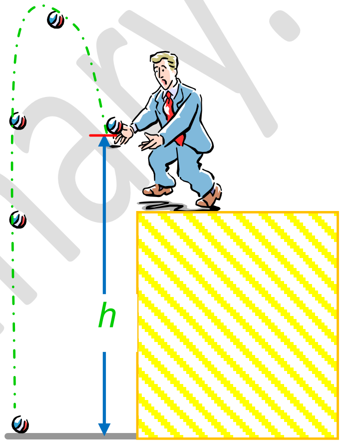
When an object is thrown in space in such a way that at first it goes up and then comes down.

To solve such types of problem, the following sign convention is used:-

Sign Convention

Motional quantities in upward-direction are taken as positive.

Motional quantities, which are in downward-direction, are taken as negative.



Let initial velocity is v_0 which is in vertically upward direction and finally comes down to the ground as shown in figure.

Initial velocity $= +v_0$ (in upward directions)

Displacement $= -h$ (in downward direction)

Acceleration $= -g$ (in downward direction)

So using equation of motion, $s = v_0t + \frac{1}{2}at^2$, we have

$-h = v_0t - \frac{1}{2}gt^2$

Illustration 9

Question: **A player throws a ball upwards with an initial speed of 29.4 m/s^{-1}**

(a) What is the direction of acceleration during the upward motion of the ball?

(b) What are the velocity and acceleration of the ball at the highest point of its motion?

(c) To what height does the ball reaches and after how long does the ball return to the player's hands (Take $g = 9.8$ m/s^2 and neglect air resistance).

Solution:

(a) Direction of acceleration is downward.

(b) At highest point velocity of the ball will be zero and acceleration will be $g =$ **9.8 m/s^2**

(c) The ball will go up till velocity will become zero

$v = 0$

$v_0 = 29.4$ m/s

$a = g = 9.8$ m/s in downward direction (opposite to the motion)

$v^2 = v_0^2 - 2gh$

$o^2 = (29.4)^2 - 2 \times 9.8$ h

$h = \frac{29.4 \times 29.4}{2 \times 9.8}$

= **39.6 m**

Let 't' be the time taken by the ball to return to the player's hands. We have,

$v_0 = 29.4$ m/s

$a = 9.8$ m/s^2

$s = 0$

$\therefore\ 0 = 29.4 \times t - \frac{1}{2} \times 9.8 \times t^2$

$\Rightarrow t = 29.4 \times 2 =$ **6 s**

Illustration 10

Question: **A rocket is fired vertically and ascends with a constant vertical acceleration of 10 ms^{-2} for 60 s. its fuel is then all used and it continues as a free particle. (a) What is the maximum altitude reached? (b) After how much time from then will the maximum height reached? (c) What is the total time elapsed from take off until the rocket strikes the earth? (g = 10 m/s^2)**

Solution: (a) The distance traversed by the rocket till its fuel ends is given by

$h_1 = v_0 t + \frac{1}{2} a t^2 = 18000$ m.

(i) The velocity of the rocket when its fuel ends

$v = v_0 + a t_1 = 600\,\text{ms}^{-1}$

(ii) The height the rocket ascends after the fuel ends.

$v_f^2 - v^2 = -2gh \Rightarrow h = 18000$ m $\quad\therefore\quad$ Maximum height = **36000 m**

(b) The time to reach the maximum height after the burning of the fuel is given by the equation.

$v_f = v - gt^2$

$\therefore\quad 0 = 600 - 10t_2$ or $t_2 = 60$s.

(c) The total time elapsed from the moment the rocket takes off to its touching the ground is given by

$y = \frac{1}{2} g t_3^2$

$\therefore\quad t = t_1 + t_2 + t_3 = 204.9$ s

4 CONCEPT OF GRTAPH

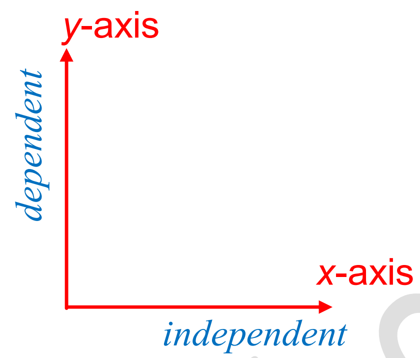
There are two types of variables

(i) *Independent variable.*

(ii) *Dependent variable.*

Independent variables are such type of variables which don't depend on other variable like, time and dependent variables are those variables which depend on some other variables i.e., velocity, distance covered, acceleration which depend on time.

When we plot the graph independent variable is plotte on x-axis and dependent variable is plotted on y-axis.

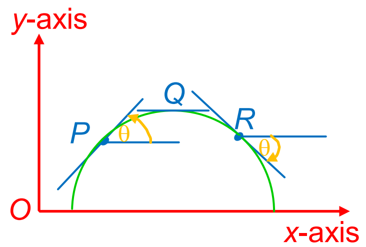


4.1 SLOPE

If a tangent at a particular point on the curve makes an angle θ with positive x–axis then tanθ gives the slope of that tangent.

If angle is measured in anticlockwise direction from positive x-axis, then slope is taken as positive and if angle is measured in clockwise direction, slope is taken as negative.

Here a curve is shown and at points P, Q and R tangents are drawn.



At P, the tangent makes an angle θ in anticlockwise with positive x-axis. So slope at P is positive.

At Q, the tangent makes zero angle with the positive x-axis. So slope at Q is zero.

At R, the tangent makes an angle θ in clockwise with positive x-axis. So slope at R is negative.

In this chapter we study mainly three types of graph.

(i) Distance–time graphs

(ii) Velocity–time graph

(iii) Acceleration–time graph

4.2 DISPLACEMENT–TIME GRAPH

(i) Slope of position vs time graph gives velocity

(ii) Slope at a particular point of the graph gives instantaneous velocity

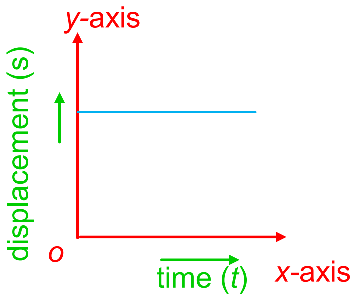
(iii) Slope of a line joining initial position to final position gives average velocity between two points.

(iv) The maximum slope at any point on the graph gives the maximum velocity.

(v) When graph will be straight line parallel to x-axis, it means slope is zero so velocity will be zero.

$$v = \tan\theta = 0 = \tan 0°$$

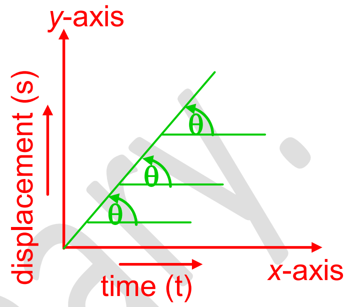
$$\Rightarrow \quad \theta = 0°$$



(vi) Slope of a straight line is constant which means velocity is constant.

(a) As shown in graph slope is positive and constant since it makes an angle θ with positive x-axis in anticlockwise direction.

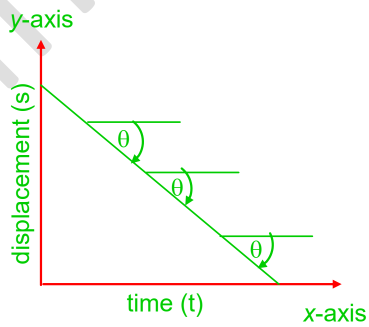
So velocity is positive and constant



(a)

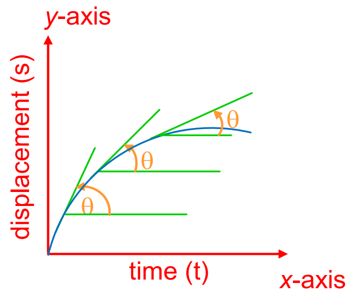
(b) In this graph, slope is negative and constant since θ is in clockwise direction.

So velocity is negative and constant



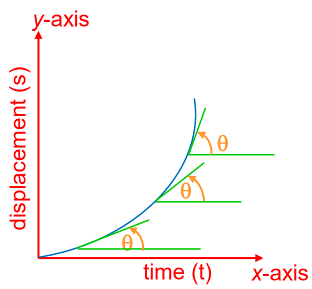
(vii) (a) Slope of a curve as shown in figure, is positive but decreasing.

So velocity is positive and decreases.



(b) In this graph slope is positive and increasing.

So velocity is positive and increases



☞ *Area under curve of a position time graph has no physical significance.*

4.3 VELOCITY–TIME GRAPH

(i) Slope of velocity vs. time graph gives acceleration.

(ii) Area under curve gives displacement and area on negative side gives negative displacement.

As shown in the graph of velocity vs. time

(i) Slope from 0 to A is positive and constant hence, acceleration is positive and constant.

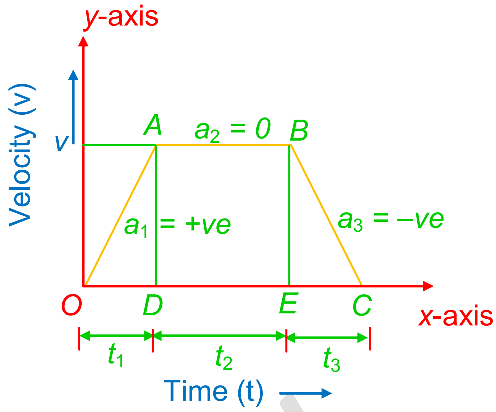
(ii) Slope from A to B is zero, hence acceleration will be zero

(iii) Slope from B to C is negative and constant, hence acceleration is negative and constant.

Distance covered in motion from O to A, A to B and B to C will be area under curve with x-axis.

Here, area of curve = area of trapazium

$= \frac{1}{2} \times v \times [t_1 + t_2 + t_3 + t_2]$



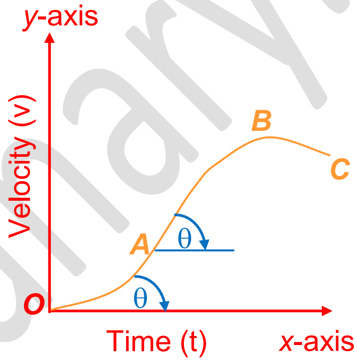
When graph of velocity vs time is in curve form as shown in figure.

(i) Slope from O to A is positive and increasing, hence acceleration is positive and increasing

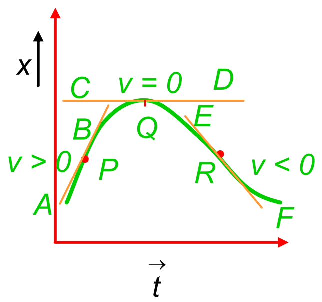
(ii) A to B slope is positive and decreasing, so acceleration is positive and decreasing.

(iii) At B slope is zero, so acceleration is zero

(iv) B to C slope is negative and increasing, so acceleration is also negative and increasing.



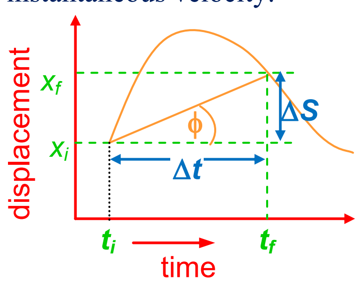
☞ The instantaneous velocity can be positive, negative or zero as shown in the position-time graph.
When the slope of position-time graph will be +ve instantaneous velocity will be +ve, such as at point P, slope of tangent AB is positive as shown in figure. So instantaneous velocity at P is positive.



When the slope of this graph will be 0 instantaneous velocity will be 0, such as at Q slope of tangent CD is 0, so instantaneous velocity at Q is 0.
When the slope of this graph is –ve, instantaneous velocity is – ve. Such as at point R, slope of tangent EF is –ve so instantaneous velocity at R is –ve.
When the velocity is constant, the average velocity is equal to the instantaneous velocity.

☞ Average velocity of the particle is equal to the slope of the straight line joining the initial and final position on the displacement–time graph as shown in figure.

$$v_{avg} = \frac{\Delta S}{\Delta t} = \tan\phi = \text{slope of chord}$$



4.4 GRAPHICAL METHOD TO PROVE $S = ut + \frac{1}{2}at^2$

Let a particle starts with velocity u and it moves under constant acceleration from A to B in t second. Its velocity versus time graph is as shown in the figure. We know area under velocity-time graph gives the distance S covered by the particle in time t.

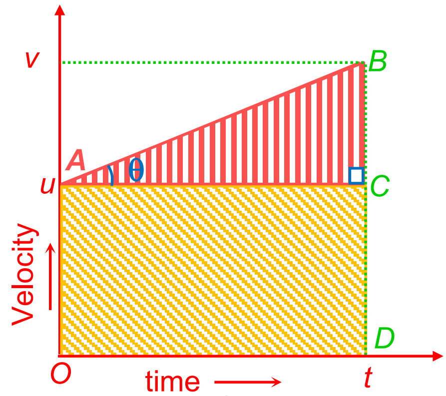
S = Area under velocity – time graph

= area of rectangle OACD + Area of triangle ACB

$= ut + \frac{1}{2} AC \times CB$

$= ut + \frac{1}{2} AC^2 \times \frac{CB}{AC}$

Also $\frac{CB}{AC} = \tan\theta$ = slope of velocity –time graph = acceleration a



$\therefore \quad S = ut + \frac{1}{2} at^2$

4.5 ACCELERATION–TIME GRAPH

(1) The slope of acceleration vs. time graphs gives rate of change of acceleration with respect to time, which has no physical significance.

(2) Area under curve of this graph gives change in velocity.

Illustration 11

Question: **The speed-time graph of a particle moving along a fixed direction is shown in figure.**

(a) Calculate the distanced traveled by the particle from $t = 0$ to $t = 10$ s.

(b) Calculate the average speed in the time interval from 0 to 10s.

(c) Calculate the distance traveled in the time interval from 2s to 6s.

(d) Calculate the average speed in interval from $t = 2$s to $t = 6$s.

Solution:

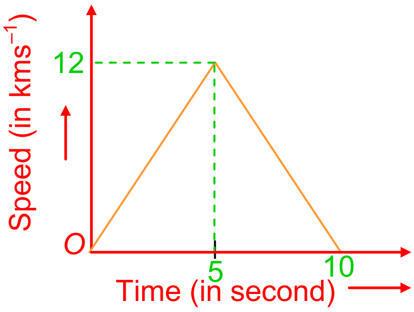
(a) Distance travelled from $t = 0$ to $t = 10$ s

= area under speed –time graph

$= \frac{1}{2} \times 10\text{s} \times 12\,\text{ms}^{-1} = \mathbf{60\ m}$

(b) Average speed over the interval from $t = 0$ to $t = 10$ s

$= \frac{60\,\text{m}}{10\,\text{s}} = \mathbf{6\ ms^{-1}}$



(c) In order to calculate distance from $t = 2$s to $t = 5$s, let us first determine separately the following and then add them up.

(i) distance covered from $t = 2$s to $t = 5$ s

(ii) distance covered from $t = 5$s to $t = 6$ s

(i) Acceleration $= \frac{12\,\text{ms}^{-1}}{5\text{s}} = 2.4\ \text{ms}^{-2}$

Velocity at the end of $2s = 2.4 \times 2\,\text{ms}^{-1} = 4.8\ \text{ms}^{-1}$

This will be regarded as initial velocity for motion over the time interval from $t = 2$ second to $t = 5$ second.

If x is the distance travelled in this time interval, then

$x\left(4.8 \times 3 + \frac{1}{2} \times 2.4 \times 9\right)\text{metre} = (14.4 + 10.8)\text{m} = \mathbf{25.2\ m}$

(ii) Deceleration = –2.4 ms^{-2}

For motion from $t = 5$s to $t = 6$s, the initial velocity will be 12 ms^{-1}. If x the distance covered in this interval of time, then

$$x' = 12\times1-\frac{1}{2}\times2.4\times1\times1 = 12-1.2 = 10.8\,m$$

Total distance = (25.2 + 10.8) metre = 36 cn

(d) Average speed $\frac{36\,m}{4s} = \mathbf{9\ ms^{-1}}$

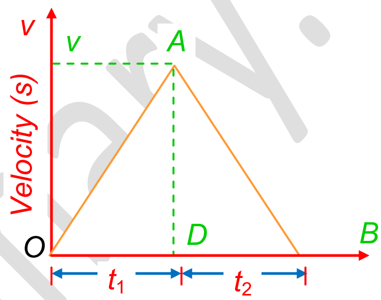
Illustration 12

Question: **A car accelerates from rest at constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time elapsed is *t* seconds, calculate**

(a) the maximum velocity reached

(b) the total distance travelled.

Solution: The situation as shown in figure



Let the car accelerate for time t_1 and decelerate for time t_2, so that total time $t_1 + t_2 = t$ Let v be the maximum velocity reached

(a) For accelerated motion,

$v = 0 + \alpha\, t_1$

or, $v = \alpha t_1$

For decelerated motion, $0 = v + (-\beta)t_2$

or $v = \beta t_2$

$\therefore \quad \alpha t_1 = \beta t_2$

or $\frac{t_2}{t_1} = \frac{\alpha}{\beta}$

Adding 1 to both sides

$$\frac{t_2 + t_1}{t_1} = \frac{\alpha+\beta}{\beta}$$

or $\frac{t}{t_1} = \frac{\alpha+\beta}{\beta}$

or $t_1 = \frac{\beta t}{\alpha+\beta}$

Hence, $v = \alpha t_1 = \alpha\left(\frac{\beta t}{\alpha+\beta}\right) = \left(\boldsymbol{\frac{\alpha\beta t}{\alpha+\beta}}\right)$

(b) Total distance traveled = Area of ΔOAB

or, $S = \frac{AD\times OB}{2}$

$= \frac{v\times t}{2}$

$= \frac{(\alpha\beta)t}{\alpha+\beta}\cdot\frac{t}{2}$

$= \boldsymbol{\frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)t^2}$

PROFICIENCY TEST–I

The following questions deal with the basic concepts of this section. Answer the following briefly. Go to the next section only if your score is at least 80%. Do not consult the Study Material while attempting these questions.

1. If velocity v of particle moving in straight line is related with distance travelled s as $v = 2\sqrt{(1+s)}$, where v is in meter/sec and s is in meter. Find the acceleration of the particle.

2. A car moves on a straight road for first half time with the constant speed of 20 m/s, and the next half time with a constant speed of 60 m/s, then calculate average speed.

3. If displacement x of a particle moving in straight line is given by $x = t^3 - 12t$ where t is time in sec and x is in meter. Find the acceleration of the particle, when velocity of the particle is zero.

4. A clock has its second hand 2.0 cm long. Find the average velocity of the tip of the second hand in 15 seconds.

5. A car travels a distance A to B at a speed of 40 km/h and returns to A at a speed of 30 km/h.
(i) What is the average speed for the whole journey ?
(II) What is the average velocity ?

6. An automobile moving with a velocity of 54 km/hr is brought to rest in a distance of 9 m by the application of brakes. Find the retardation assuming it to be uniform.

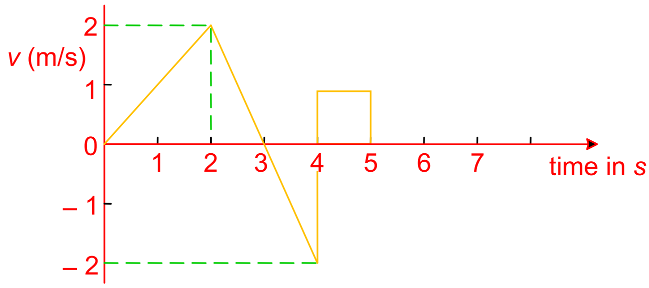
7. A stone is allowed to fall from the top of a tower and cover half the height of the tower in the last second of its journey. Find the time taken by the stone to reach the foot of the tower.

8. A particle is projected vertically upward with speed 20 m/s. Find the time after particle is at a height 15 m above the ground (g = 10 m/s^2)

9. A ball is dropped from the top of a tower. In the last second of its fall, the ball covers a distance equal to 9/25 of the height of the tower. Find the height of the tower.(g = 10 m/s^2)

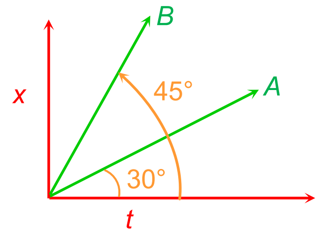
10. A balloon is ascending at the rate of 20 m/sec at a height of 105 metre above the ground when a package is dropped. How long does it take the package to reach the ground? (g = 10 m/s^2)

11. The velocity-time graph of a body moving along a straight line is as follows:

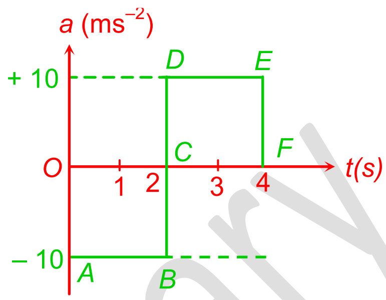


Find the displacement and distance covered by the body in 5 s.

12. The *x-t* graph for the two particles *A* and *B* are straight lines inclined at angles of 30° and 45° with the time axis. What is the ratio of $v_A : v_B$?



13 A particle starts from rest at time $t = 0$ and moves on a straight line with acceleration a (ms^{-2}) as plotted in figure. Find the time at which the speed of the particle is maximum. Also calculate the displacement of the particle from starting point after 4 is.



14. A train starting from rest accelerates uniformly for 100 s, runs at a constant speed for 5 minutes and then comes to a stop with uniform retardation in the next 150 seconds. During this motion it covers a distance of 4.25 km. Find its constant speed.

15. A stone is dropped from a height of 100 m from a balloon which rises up from the ground. Find the velocity of the balloon at that moment if the stone reaches the ground 5 seconds after it was dropped.

ANSWERS TO PROFICIENCY TEST–I

1. 2 m/s^2

2. 40 m/s

3. 12 m/s^2

4. 18.85×10^{-4} m/s

5. (i) 34.3 km/h (ii) zero

6. 12.5 m/s^2

7. $(2+\sqrt{2})$ sec

8. 1 sec and 3 sec

9. 125 m

10. 7 s

11. 3 m, 5 m

12. $1:\sqrt{3}$

13. $t = 2$ s, 40 m.

14. 36 km/hr

15. Velocity of the balloon = 5 m/s directed upward

5 MOTION IN A PLANE

In this chapter we deal with the kinematics of a particle moving in a plane, which is two dimensional motion. Some common examples of motion in a plane are the motion of projectiles and satellites and the motion of particles in circular path. As in the case of one dimensional motion, we derive the kinematic equations for two-dimensional motion from the fundamental definitions of displacement, velocity, and acceleration. To define various kinematics quantities like position, displacement, velocity and acceleration for objects moving along a plane, we need to use the language of vectors that already we have learned.

The concept of motion when a particle is moving along a straight line can be used for motion in a plane or three dimensions. When motion of a particle is in a plane, we consider plane of motion as *x*-*y*-plane. We choose the origin at the place from where the motion starts and then we consider motion along any two convenient mutually perpendicular direction as one dimensional motion. Motion in two perpendicular directions are chosen as the *x* and *y*-axes.

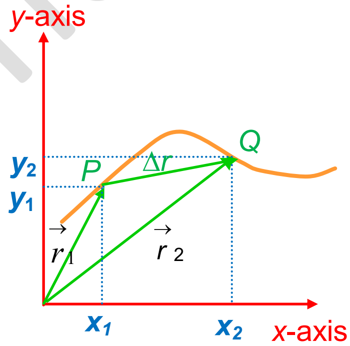
5.1 POSITION VECTOR AND DISPLACEMENT

The displacement of a particle is the difference between its final position, and initial position. Therefore, the **displacement vector** for the particle as shown in figure equals the difference between its final position vector and its initial position vector.

Suppose a particle moves in a plane along the curve as shown in figure. At time t_1, the particle is at point P, and at some later time t_2, the particle is at Q, where the indices 1 and 2 refer to initial and final values. As the particle moves from P to Q in the time interval $\Delta t = t_2 - t_1$ the position vector changes from r_1 to r_2.

$\Delta r = r_2 - r_1$ (from triangle law of vector addition). … **(16)**

The direction of Δr is indicated in figure



Let the co-ordinate of the particle at points P and Q are (x_1, y_1) and (x_2, y_2) respectively.

Then, $\vec{r}_1 = x_1\hat{i} + y_1\hat{j}$

$\vec{r}_2 = x_2\hat{i} + y_2\hat{j}$

$\vec{r}_2 - \vec{r}_1 = (x_2\hat{i} + y_2\hat{j}) - (x_1\hat{i} + y_1\hat{j})$

$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$

$\Delta\vec{r} = \Delta x\,\hat{i} + \Delta y\,\hat{j}$ where $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$

Where, $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$; $\Delta x = x_2 - x_1$; $\Delta y = y_2 - y_1$

As we see from figure the magnitude of the displacement vector is less than the distance travelled along the curved path.

5.2 VELOCITY

We define the average velocity of the particle during the time interval Δt as the ratio of the displacement to that time interval:

$$\bar{v} = \frac{\Delta\vec{r}}{\Delta t} \quad \ldots (17)$$

$\Delta\vec{r} = \Delta x\,\hat{i} + \Delta y\hat{j}$

$$\frac{\Delta \vec{r}}{\Delta t} = \frac{\Delta x \hat{i}}{\Delta t} + \frac{\Delta y}{\Delta t}\hat{j} \Rightarrow \vec{v} = v_x\hat{i} + v_y\hat{j}$$

Since displacement is a vector quantity and the time interval is a scalar quantity, we conclude that the average velocity is a vector quantity directed along $\Delta \vec{r}$.

☞ *The average velocity between points P and Q is independent of the path between the two points. This is because the average velocity is proportional to the displacement, which in turn depends only on the initial and final position vectors and not on the path taken between those two points.*

☞ *As we did with one-dimensional motion, we conclude that if a particle starts its motion at some point and returns to this point via any path, its average velocity is zero for this trip since its displacement is zero.*

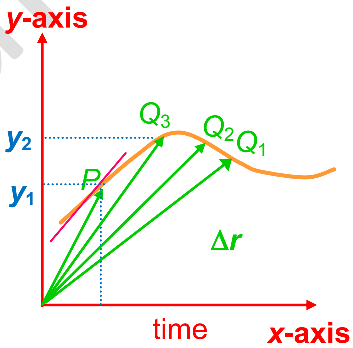
Instantaneous velocity

The instantaneous velocity $\vec{v}$, is defined as the limit of the average velocity, $\Delta \vec{r} \ / \ \Delta t$, as Δt approaches zero:

i.e. $\vec{v} = \lim\limits_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$ … **(18)**

That is, the instantaneous velocity equals the derivative of the position vector with respect to time.

Consider again the motion of a particle between two points in the *xy* plane, as shown in figure. As the time intervals over which we observe the motion become smaller and smaller, the direction of the displacement approaches that of the line tangent to the path at the point *P.*



$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}\hat{i} + \Delta \vec{y}\hat{j}}{\Delta t} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

The direction of the instantaneous velocity vector at any point in a particle's path is along a line that is tangent to the path at that point and in the direction of motion; this is illustrated in figure.

☞ *The magnitude of the instantaneous velocity vector is called the speed.*

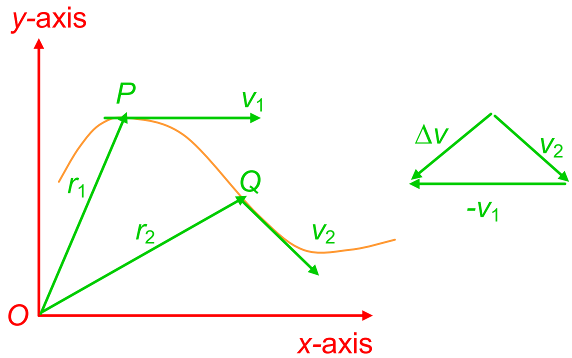
5.3 ACCELERATION

(a) Average acceleration

The average acceleration of a particle as it moves from P to Q is defined as the ratio of the change in the instantaneous velocity vector, Δv, to the elapsed time, Δt

i.e., $\vec{\bar{a}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$ … **(19)**

Since the average acceleration is the ratio of a vector quantity, $\Delta \vec{v}$, and a scalar quantity, Δt, we conclude that $\bar{a}$ is a vector quantity directed along $\Delta \vec{v}$. As is indicated in figure, the direction of $\Delta \vec{v}$ is found by adding the vector $-\vec{v}_1$ (the negative of $\vec{v}_1$) to the vector $\vec{v}_2$, since by definition



$$\Delta \vec{v} = \vec{v}_2 - \vec{v}_1$$

$$\vec{\bar{a}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta\left(\vec{v}_x \hat{i} + \vec{v}_y \hat{j}\right)}{t_2 - t_1} = a_x \hat{i} + a_y \hat{j}$$

(b) Instantaneous acceleration

The instantaneous acceleration $\vec{a}$, is defined as the limiting value of the ratio $\Delta \vec{v} / \Delta t$ as Δt approaches zero:

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{d\vec{v}}{dt} \qquad \ldots (20)$$

6 TWO DIMENSIONAL MOTION WITH CONSTANT ACCELERATION

Let us consider two-dimensional motion during which the acceleration remains constant. That is, we assume that the magnitude and direction of the acceleration remain unchanged during the motion.

As we learned, in cause of the motion a particle can be determined by its position vector $\vec{r}$. The position vector for a particle moving in the *xy* plane can be written

$$\vec{r} = x\hat{i} + y\hat{j} \qquad \ldots (21)$$

Where x, y, and $\vec{r}$ change with time as the particle moves. If the position vector is known, the velocity of the particle can be obtained on differentiating equation, which give

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$\vec{v} = v_x \hat{i} + v_y \hat{j} \qquad \ldots (22)$$

on differentiating equation (25) with respect to time, we get

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

$$= a_x \hat{i} + a_y \hat{j} \qquad \ldots (23)$$

Because $\vec{a}$ is assumed constant, its components $\vec{a}_x$ and $\vec{a}_y$ are also constants. Therefore , we can apply the equations of kinematics to the x and y components of the velocity vector and displacement vector as shown below.

$$\vec{v} = \vec{v}_0 + \vec{a}t \begin{cases} v_x = v_{x_0} + a_x t \\ v_y = v_{y_0} + a_y t \end{cases} \qquad \ldots (24)$$

$$\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \begin{cases} x = v_{x_0} t + \frac{1}{2} a_x t^2 \\ y = v_{y_0} t + \frac{1}{2} a_y t^2 \end{cases} \qquad \ldots (25)$$

$$v^2 = v_0^2 + 2as \begin{cases} v_x^2 = v_{x_0}^2 + 2a_x s_x \\ v_y^2 = v_{y_0}^2 + 2a_y s_y \end{cases} \qquad \ldots (26)$$

We can say in other words, two-dimensional motion having constant acceleration is equivalent to two independent motions in the x and y directions having constant accelerations a_x and a_y.

Examples of motion in plane are projectile motion and circular motion

7 PROJECTILE MOTION

If an object is thrown in air with some initial velocity at some angle with the horizontal, and then allowed to move under the gravity, the object moves in a curved path, such type of motion is called projectile motion and that object is known as projectile. Such a projectile may be football, a cricket ball or any object.

It is an example of two dimensional motion.

☞ *The path of projectile, which is called trajectory, is always a parabola*

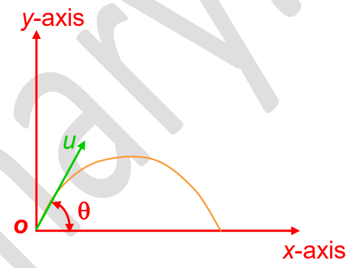
Assumptions of projectile motion

(1) The acceleration due to gravity ('*g*') is constant over the motion of projectile.

(2) The effect of air resistances is negligible

(3) The effect due to rotation of earth and curvature of the earth is negligible.

When an object is thrown at some angle to the horizontal, the motion of projectile will be in horizontal direction as well as in vertical direction. Suppose motion of projectile is in *xy*-plane of co-ordinate system as shown in figure, then motion of the object will be along the two mutually perpendicular direction together, say *x* and *y*-axes of the co-ordinate system. Consider motion along horizontal direction as *x*-axis and motion along vertical direction as *y*-axis.



We can study the two-dimensional motion as one-dimensional motion along *x*-axes and *y*-axis since its superposition gives the resultant motion.

Let us assume that at $t = 0$, projectile leaves the origin ($x = 0$, $y = 0$) with velocity u, which makes an angle θ with the horizontal.

Motion along horizontal direction (x-axis): In this direction initial velocity will be horizontal component of initial velocity with which the object is projected i.e.,

$$u_x = u \cos \theta$$

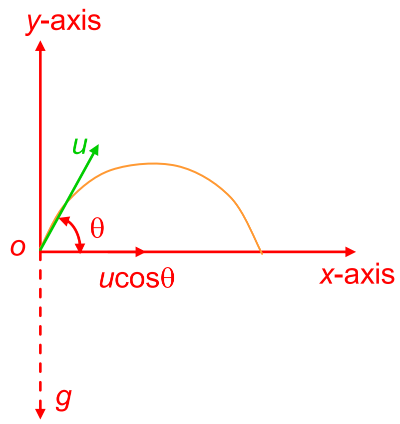
acceleration will be perpendicular component of g (acceleration due to gravity)

$$a_x = 0$$

Now we can say motion in *x*-direction is with constant velocity since acceleration is zero.

$$v_x = v_{0x} + a_x t = u_x = u \cos\theta$$

and $$x = v_0 t + \frac{1}{2} a_x t^2 = u_x t = (u\cos\theta)t$$



Motion in vertical direction (y-axis)

In this direction initial velocity will be vertical component of initial velocity with which the object is projected.

$$u_y = u \sin\theta$$

and acceleration will be '*g*' which is acting in negative *y*-axis.

$$a_y = -g \quad \text{(which is constant)}$$

It means motion along *y*-axis is with uniform acceleration, so equation of kinematics for one-dimensional motion can be used here.

$$v_y = u_y - gt = u\sin\theta - gt$$

and $\quad y = u_y t - \frac{1}{2} g t^2$

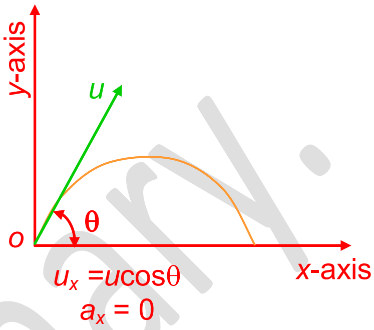
$$= (u\sin\theta)\, t - \frac{1}{2} g t^2$$

Also we have,

$$v_y^2 = u_y^2 - 2gy$$

$$= (u \sin \theta)^2 - 2gy$$

7.1 EQUATION OF TRAJECTORY



Let projected body be in motion for t second.

Consider motion in x-direction

$$u_x = u \cos \theta$$

$$a_x = 0$$

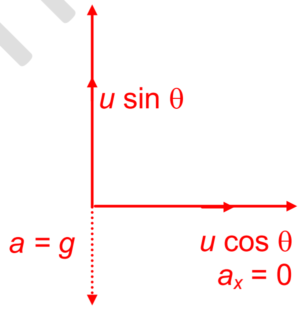
$$x = (u \cos \theta)\, t$$

$$t = \frac{x}{u\cos\theta} \qquad \text{… (i)}$$

Now consider motion in y-direction

$u_y = u\sin\theta$ and $a_y = -g$

$$y = (u\sin\theta)t - \frac{1}{2} g t^2$$



on putting value of t from equation (i)

$$y = (u\sin\theta)\frac{x}{u\cos\theta} - \frac{1}{2} g \frac{x^2}{u^2\cos^2\theta}$$

$$y = (\tan\theta)x - \frac{1}{2} g \frac{x^2}{u^2\cos^2\theta} \qquad \text{… (27)}$$

Which is valid for the angles in the range $0 < \theta < \frac{\pi}{2}$

The above equation represents a parabola. So we can say that equation of trajectory is parabola.

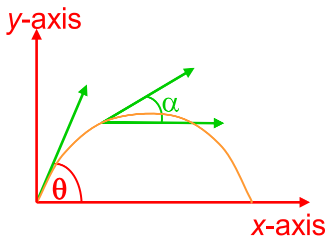
7.2 VELOCITY OF PROJECTILE IN SPACE

Now to obtain the speed v of the projectile as a function of time, first we calculate speed along x-axis and y-axis v_x and v_y respectively as a function of time

$$v = \sqrt{v_x^2 + v_y^2}$$

In x-direction velocity will remains constant. So at any time t

$$v_x = (u\cos\theta)$$



In y-direction velocity will be $v_y = u\sin\theta - gt$

$$|v| = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$$

$$v = \sqrt{u^2 - 2ugt\sin\theta + g^2t^2} \qquad \text{… (28)}$$

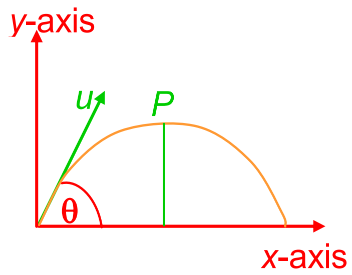
and velocity vector will be tangent to the path at any instant as shown in figure

$$\tan\alpha = \left(\frac{v_y}{v_x}\right).$$

7.3 TIME OF ASCENT

Time of ascent is the time when the projectile will attain the maximum height. At this position the vertical component of velocity vector will become zero.

As shown in figure P is point of maximum height. Now to find the time taken from initial point of projection upto maximum height, we consider motion in y-direction



$v_y = u_y - gt$

$0 = u\sin\theta - gt$

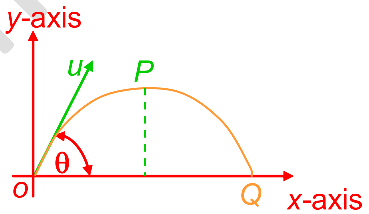
$$t = \frac{u\sin\theta}{g} \quad \ldots (29)$$

7.4 TIME OF FLIGHT

When a particle is projected in air after some time it hits the ground again, time taken in this process is called time of flight or we can say time of flight is time of motion of projectile when it is in air. It is denoted by T.

Let a particle be projected from O and again it hits the ground at point Q. In this motion distance travelled in y-direction is zero.

On applying equation of motion in y-direction.



$y = ut - \frac{1}{2}gt^2$

$0 = u\sin\theta\, T - \frac{1}{2}gT^2$

$0 = (u\sin\theta - \frac{1}{2}gT)T$

$T = 0$ or $T = \frac{2u\sin\theta}{g}$

$$T = \frac{2u\sin\theta}{g} \quad \ldots (30)$$

Time of flight can be found out in second way as given below.

The total time of flight is consists of two parts

(i) Time taken by the particle to go from O to highest point P, it is called time of ascent.

(ii) Time taken by the particle to go from the highest point P to Q where it hits the ground again. It is called **time of descent**.

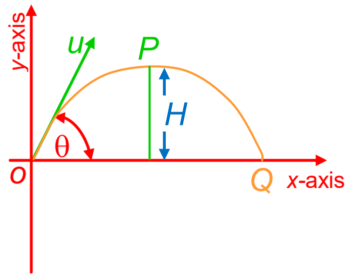
As motion from O to P and P to Q are symmetrical.

$\therefore$ time of ascent = time of descent $= \frac{u\sin\theta}{g}$

Total time of fight = time of ascent + time of descent.

$$T = \frac{2u\sin\theta}{g}$$

7.5 MAXIMUM HEIGHT OF PROJECTILE



It is the maximum vertical height attained by the particle above the point of projection during its flight. it is denoted by H.

When the projectile is at maximum height, its vertical component of velocity will be zero.

Initial velocity $v_y = u \sin\theta$

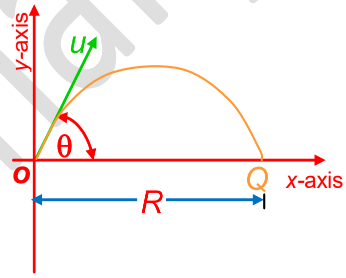
On using equation of motion along y axis.

$$v_y^2 = u_y^2 - 2gs$$

$$0 = u^2\sin^2\theta - 2gH$$

$$H = \frac{u^2 \sin^2\theta}{2g} \quad \ldots (31)$$

7.6 HORIZONTAL RANGE



Range of projectile is horizontal distance travelled by the particle during the time of flight

Motion along x-axis is with constant velocity

So distance travelled in this direction will be

$$x = (u\cos\theta)\, t$$

$$R = (u\cos\theta)T$$

Where T is time of flight

$$T = \frac{2u\sin\theta}{g}$$

on putting the value of T

$$R = (u\cos\theta)\left(\frac{2u\sin\theta}{g}\right)$$

$$R = \frac{u^2\sin 2\theta}{g} \quad \ldots (32)$$

For range to be maximum

$$R = \frac{u^2\sin 2\theta}{g}$$

$$\sin 2\theta = 1 = \sin 90°$$

$$2\theta = 90°$$

$$\theta = 45°$$

$$R_{max} = \frac{u^2}{g} \quad \ldots (33)$$

☞ *A projectile will have maximum range when it is projected at angle of 45° to the horizontal and maximum range will be* $= \frac{u^2}{g}$

☞ *When range is maximum, the maximum height H reached by the projectile* $H = \frac{u^2\sin^2 45°}{2g}$

$= \frac{u^2}{2g} = \frac{R_{max}}{4}$

☞ *For same range two angle of projectile are possible, such that two angles are complementary to each other.*

$\theta_1 + \theta_2 = 90°$,

where θ_1 and θ_2 are two different angles of projection which gives same horizontal range.

If $\theta_1 = 45° + \alpha$,

then $\theta_2 = 45° - \alpha$

Illustration 13

Question: **A ball is thrown from a field with a speed of 12.0 m/s at an angle of 45^0 with the horizontal. At what distance will it hit the field again? Take g =10.0 m/s^2**

Solution: The horizontal range $= \frac{u^2 \sin 2\theta}{g}$

$$= \frac{(12\,m/s)^2 x \sin(2x45^0)}{10\,m/s^2}$$

$$= \frac{144\,m^2/s^2}{10\,m/s^2} = 14.4\,m.$$

Thus, the ball hits the field at 14.4 m from the point of projection.

Illustration 14

Question: **Find the maximum horizontal range when the velocity of projection is 30 m/s. Find the two directions of projection to give a range of 45 m. Take $g = 10$ m/s^2.**

Solution: (i) Maximum range $R_m = \frac{u^2}{g} = \frac{30^2}{10}$ = **90 m**

(ii) Now $\frac{u^2 \sin 2\alpha}{g} = 45$

or $\sin 2\alpha = \frac{45 \times 10}{30 \times 30} = \frac{1}{2}$

or $2\alpha = 30°$ or $150°$ [$\because$ $(180 - \theta) = \sin\theta$]

or $\alpha =$ **15°** or **75°**

Therefore for a given velocity of projection and for a given range, two directions of projection are possible.

Illustration 15

Question: **What is the least velocity with which a cricket ball can be thrown through a distance of 100 m?**

Solution: Since the range is given, the least velocity of projection is that value when the angle of projection is 45°. For velocity u to be least

$\frac{u^2 \sin 2\alpha}{g} = 100$ where $\alpha = 45°$ or $\frac{u^2}{g} = 100$

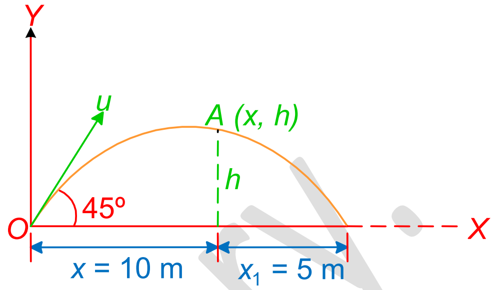
$u^2 = 100 \times 9.8 = 980$

$u = \sqrt{980}$ = **31.3 m/s**

Illustration 16

Question: **From a point on the ground at a distance 10 m from the foot of a vertical wall, a ball is thrown at an angle of 45° which just clears the top of the wall and afterwards strikes the ground at a distance 5 m on the other side. Find the height of the wall.**

Solution: Let u be the velocity of projection at an angle $\alpha = 45°$ with the horizontal.



Horizontal range $= \dfrac{u^2 \sin 2\alpha}{g}$

$= 15$

$\dfrac{u^2}{g} = 15$

Equation of the trajectory

$$y = x \tan\alpha - \frac{1}{2}\frac{gx^2}{u^2\cos^2\alpha}$$

or $$y = x \cdot 1 - \frac{1}{2}\frac{gx^2}{u^2 \times \frac{1}{2}} = x - \frac{gx^2}{u^2} = x - \frac{x^2}{15}$$

When $x = 10, y = h,$

$$\therefore \quad h = 10 - \frac{10^2}{15} = \frac{50}{15} = \mathbf{3\frac{1}{3}m}$$

7.7 RANGE OF PROJECTILE ON AN INCLINED PLANE THROUGH THE POINT OF PROJECTION

A particle is projected from a point A on an inclined plane, which is inclined at an angle β to the horizon with a velocity u at an elevation α. The direction of projection lies in the vertical plane through AB, the line of the greatest slope of the plane.

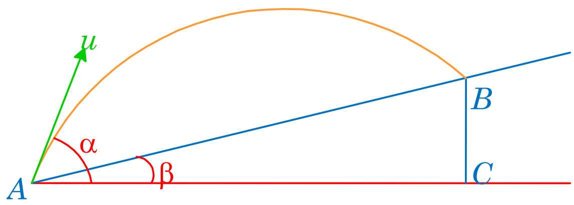


Fig. 3.

Let the particle strike the plane at B so that AB is the range on the inclined plane.

The initial velocity of projection u can be resolved into a component $u \cos(\alpha - \beta)$ along the plane and a component $u \sin(\alpha - \beta)$ perpendicular to the plane. The acceleration due to gravity g which acts vertically down can be resolved into components $g \sin\beta$ up the plane and $g \cos\beta$ perpendicular to the plane. By the principle of physical independence of forces the motion along the plane may be considered independent of the motion perpendicular to the plane. Let T be the time, which the particle takes to go from A to B. Then in this time the distance traversed by the projectile perpendicular to the plane is zero.

$$\therefore \quad 0 = u\sin(\alpha - \beta)T - \frac{1}{2}g\cos\beta\, T^2$$

$$\therefore \quad T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$$

During this time the horizontal velocity of the projectile ($u \cos\alpha$) remains constant. Hence the horizontal distance described is given by

$$AC = u\cos\alpha T = \frac{2u^2\sin(\alpha-\beta)\cos\alpha}{g\cos\beta}$$

$\therefore \quad AB = \dfrac{AC}{\cos\beta} = \dfrac{2u^2 \sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta}$

$\therefore$ Range on the inclined plane $= \dfrac{2u^2 \sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta}$

Maximum range on the inclined plane

$$R = \frac{2u^2 \sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta}$$

$$= \frac{u^2}{g\cos^2\beta}[\sin(2\alpha-\beta) - \sin\beta]$$

For given values of u and β, R is maximum when
$\sin(2\alpha - \beta) = 1$
i.e., $(2\alpha - \beta) = 90^\circ$

$$\alpha = (45^\circ + \beta/2)$$

If R_m represents the maximum range on the inclined plane,

$$R_m = \frac{u^2}{g\cos^2\beta}(1 - \sin\beta)$$

$$R_m = \frac{u^2}{g(1+\sin\beta)}$$

For a given velocity of projection, it can be shown that there are two directions of projection which are equally inclined to the direction of maximum range.

Now $\quad R = \dfrac{u^2}{g\cos^2\beta}[\sin(2\alpha-\beta) - \sin\beta]$

For given values of u, β and R, $\sin(2\alpha - \beta)$ is constant. There are two values of $(2\alpha - \beta)$ each less than 180° that can satisfy the above equation.

Let $(2\theta_1 - \beta)$ and $(2\theta_2 - \beta)$ be the two values. Then

$$2\theta_1 - \beta = 180^\circ - (2\theta_2 - \beta)$$

$$\theta_1 - \beta/2 = 90^\circ - (\theta_2 - \beta/2)$$

$$\theta_1 - (45^\circ + \beta/2) = (45^\circ + \beta/2) - \theta_2$$

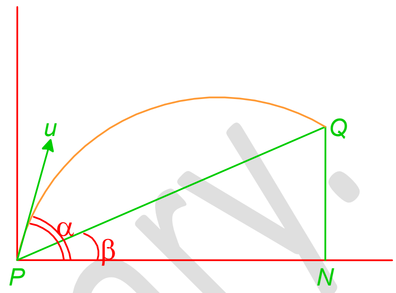
Since $(45^\circ + \beta/2)$ is the angle of projection giving the maximum range, it follows that the direction giving maximum range bisects the angle between the two angles of projection that can give a particular range

Illustration 17

Question: **A particle is projected at an angle $\alpha = 60^0$ with horizontal from the foot of a plane whose inclination to horizontal is $\beta = 15^0$. Find the value of cot β.**

Solution: Let u be the velocity of projection so that $u \cos(\alpha - \beta)$ and $u \sin(\alpha - \beta)$ are the initial velocities respectively parallel and perpendicular to the inclined plane. The acceleration in these two directions are $(-g \sin\beta)$ and $(-g \cos\beta)$.

The initial component of velocity perpendicular to PQ is $u \sin(\alpha - \beta)$ and the acceleration in this direction is $(-g \cos\beta)$. If T is the time the particle takes to go from P to Q then in time T the space described in a direction perpendicular to PQ is zero.



$$0 = u \sin(\alpha - \beta) \cdot T - \frac{1}{2} g \cos\beta \cdot T^2$$

$$T = \frac{2u\sin(\alpha - \beta)}{g\cos\beta}$$

If the direction of motion at the instant when the particle hits the plane be perpendicular to the plane, then the velocity at that instant parallel to the plane must be zero.

$\therefore \quad u \cos(\alpha - \beta) - g \sin\beta T = 0$

$$\frac{u\cos(\alpha - \beta)}{g\sin\beta} = T = \frac{2u\sin(\alpha - \beta)}{g\cos\beta}$$

$\therefore \quad \cot\beta = 2\tan(\alpha - \beta) = \mathbf{2}$

8 RELATIVE VELOCITY

The terms 'rest' and 'motion' are only relative. For example, when we say that a train is moving with velocity 30 m.p.h, we mean is that it is the velocity with which the train moves with respect to an observer on the earth who is regarded as fixed. This is not true strictly since a person on the earth unconsciously partakes the rotatory motion of the earth round its axis and the motion of earth round the sun. In addition, he shares the motion of entire solar system through space with respect to certain fixed stars. Thus there is no absolutely fixed point on the earth about which we can measure motion. Hence a person on the earth can never realise absolute motion or absolute rest.

Let us consider two motor cars A and B moving in the same direction on a road with equal speed. To a person seated in A, if he were unconscious of his motion, the car B would appear to be at rest. The line joining the two cars will always remain constant in magnitude and direction. The velocity of B relative to A or the velocity of A relative to B is zero.

On the other hand if A is moving with 30 m.p.h and B with 40 m.p.h in the same direction, a person in A would observe the car B to be drawing away from him at the rate of 10 m.p.h. This represents the velocity of B relative to A. If, however, B is moving opposite to the direction of A with velocity 40 m.p.h., for a person in A, B appears to draw away from him at the rate of 70 m.p.h. This, therefore, represents the velocity of B relative to A.

Definition of Relative velocity

When the distance between two moving points A and B is altering, either in magnitude or in direction or both, each point is said to possess a velocity relative to the other. The velocity of one of the moving points, say, A, relative to the other point B is obtained by compounding with the velocity of A, the reversed velocity of *B*. The velocity of *A* relative to *B* is the velocity with which *A* will appear to move to *B*, if *B* is reduced to rest.

If velocity *A* is $\vec{v}_A$ and that of *B* is $\vec{v}_B$ with respect to a stationary frame, then from the definition, relative velocity of *A* with respect to *B*, $\vec{v}_{AB}$ is given by

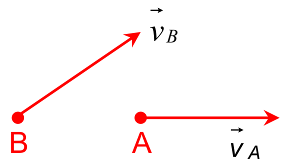


Fig. 8.

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B \qquad \ldots(34)$$

If angle between $\vec{v}_A$ and $\vec{v}_B$ is θ then

$$|\vec{v}_{AB}| = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta} \qquad \ldots(35)$$

Also angle α made by relative velocity with v_A is given by

$$\tan\alpha = \frac{v_B \sin\theta}{v_A - v_B \cos\theta} \qquad \ldots(36)$$

From the above definition of relative velocity it follows that if we impress on both the moving points *A* and *B*, a velocity equal and opposite to that of *B*, then *B* would be reduced to rest and A will have two velocities (i) its own velocity and (ii) the reversed velocity of *B*. These two can be compounded into a single velocity by the parallelogram law, which will give the velocity of *A* relative to *B*.

8.1 RELATIVE VELOCITY IN ONE-DIMENSION

Case I

If θ = 0°,

$$v_{AB} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B}$$

$$= \sqrt{(v_A - v_B)^2}$$

$$= v_A - v_B$$

If two bodies have velocity in same direction, then their difference gives the relative velocity.

Case II

If θ = π or 180°

$$v_{AB} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos 180^\circ}$$

$$v_{AB} = \sqrt{v_A^2 + v_B^2 + 2v_{AB}}$$

$$= v_A + v_B$$

When two bodies have velocity in opposite direction, then their sum gives the relative velocity.

8.2 RELATIVE VELOCITY IN TWO DIMENSIONS

The concept of relative velocity has been discussed in one-dimensional motion. Same concept is used when motion is in a plane. In this case we can consider relative velocity in a plane as superposition of relative velocity in two mutually perpendicular direction considered as motion along *x*-axis and *y*-axis. Suppose two objects *A* and *B* are moving with constant velocity $\vec{v}_A$ and $\vec{v}_B$ each with respect to same common frame of reference, say ground, then velocity of *A* relative to *B* is

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

When we will take relative motion along *x*-axis

$v_{xAB} = = v_{xA} - v_{xB}$

When we will take relative motion along *y*-axis

$v_{yAB} = v_{yA} - v_{yB}$

River-Boat Problems

River-boat problem is based on concept of relative velocity and resultant velocity. In this problem we come-across three terms

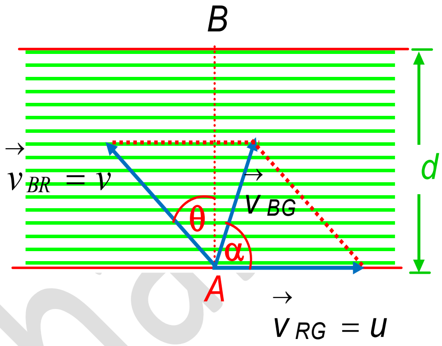
$\vec{v}_{RG}$ = velocity of river/stream with respect to ground. It is denoted by *u*.

$\vec{v}_{BR}$ = velocity of boatman with respect to river or velocity of boatman in still water. It is denoted by *v*.

$\vec{v}_{BG}$ = velocity of boatman with respect to ground.

Let the river is flowing along *x*-axis with velocity $\vec{V}_{RG}$ and width of river is *d*.

Let boat-man start from one bank of river from the point *A* with velocity $\vec{v}_{BR}$ along the direction as shown in figure.



$$\vec{v}_{BR} = \vec{v}_{BG} - \vec{v}_{RG}$$

$$\vec{v}_{BG} = \vec{v}_{BR} + \vec{v}_{RG} \qquad \ldots \text{(i)}$$

Taking *x*-component in equation (i)

$$\vec{v}_{BG} \cos\alpha = u - v\sin\theta \qquad \ldots \text{(ii)}$$

Taking *y*-component in equation (i)

$$\vec{v}_{BG} \sin\alpha = 0 + v\cos\theta \qquad \ldots \text{(iii)}$$

On squaring and adding equation (ii) and (iii)

$$v_{BG}^2 = u^2 + v^2 - 2uv\sin\theta$$

$$v_{BG} = \sqrt{u^2 + v^2 - 2uv\sin\theta}$$

on dividing equation (ii) by (i)

$$\tan\alpha = \frac{-v\cos\theta}{u - v\sin\theta}$$

When boatman cross the river, the displacement in y-direction is *d*

Time taken to cross the river is *t*

$$t = \frac{d}{v\cos\theta}$$

In this time displacement in *x*-direction is *x*

$$x = (u - v\sin\theta)\, t$$

$$x = \frac{(u - v\sin\theta)d}{v\cos\theta}$$

(a) When a boat tends to cross a river in shortest time

$$t = \frac{d}{v\cos\theta}$$

for shortest time , *v* cosθ has to be maximum

∴ cosθ has to be maximum = 1

θ = 0°

So the boats should go along *AB* (vertically opposite direction) to cross the river in shortest time.

When boat-man is crossing the river in shortest time, the horizontal distance traveled along the direction of river = *x*

$$x = u\,t = \frac{ud}{v}$$

(b) When a boat-man tends to cross the river along a shortest path (wants to reach the point just opposite from where he started)

When boat-man wants to cross the river in shortest path, horizontal distance covered should be zero.

$$x = (u - v\sin\theta)$$

$$x = \frac{(u - v\sin\theta)d}{v\cos\theta} = 0$$

$$u - v\sin\theta = 0, \quad \sin\theta = \frac{u}{v}, \qquad \theta = \sin^{-1}\left(\frac{u}{v}\right)$$

Hence, to reach just opposite point the boatman should row at an angle $\theta = \sin^{-1}\frac{u}{v}$ upstream from AB.

$\sin\theta < 1$

$\frac{u}{v} < 1$

$u < v$

so to cross the river directly velocity of river or stream should be less than velocity of boat.

If river velocity is greater than the velocity of boat, then it is impossible to reach the point B.

Illustration 18

Question: **Ram is moving due east with a velocity of 1 m/s and Shyam is moving due west with a velocity of 2 m/s. What is the velocity of Ram with respect to shyam?**

Solution: It is a dimensional motion. So, let us choose the east direction as positive and the west as negative. Now given that

v_S = velocity of Ram = 1 m/s

and v_G = velocity of Shyam = - 2m/s

Thus, v_{SG} = velocity of Ram with respect to Shyam

$= v_S - v_G = 1 - (-2) = 3$ m/s

Hence, velocity of Ram with respect to Shyam is 3 m/s due east.

Illustration 19

Question: **A police van moving on a highway with a speed of 30 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km h^{-1}. If the muzzle speed of the bullet is 150 ms^{-1}, with what speed does the bullet hit the thief's car?**

Solution: Speed of police van, $v_p = 30\,\text{kmh}^{-1} = \frac{30 \times 1000}{3600}\text{ms}^{-1} = \frac{25}{3}\text{ms}^{-1}$

Speed of thief's car, $v_t = 192\,\text{kmh}^{-1} = \frac{192 \times 1000}{3600}\text{ms}^{-1} = \frac{160}{3}\text{ms}^{-1}$

Speed of bullet, v_b = speed of police van + speed with which bullet is actually fired.

$$\therefore \qquad v_b = \left(\frac{25}{3} + 150\right)\text{ms}^{-1} = \frac{475}{3}\text{ms}^{-1}$$

Relative velocity of bullet w.r.t. thief's car,

$$v_{bt} = v_b - v_b = \left(\frac{475}{3} - \frac{160}{3}\right)\text{ms}^{-1} = 105\ \text{ms}^{-1}$$

Illustration 20

Question: **A jet air plane traveling at the speed of 500 km h^{-1} ejects its products of combustion at the speed of 1500 kmh^{-1} relative to the jet plane. What is the speed of the combustion products w.r.t. an observer on the ground?**

Solution: Speed of combustion products w.r.t. observer on the ground = ?

Velocity of jet air plane w.r.t. observer on ground = 500 km h^{-1}

If $\vec{v}_j$ and $\vec{v}_a$ represent the velocities of jet and observer respectively, then

$$v_j - v_a = 500\,\text{kmh}^{-1}$$

Similarly, if $\vec{v}_c$ represents the velocity of the combustion products w.r.t. jet plane, then

$$v_c - v_f = -1500\text{ kmh}^{-1}$$

The negative sign indicates that the combustion products move in a direction opposite to that of jet.

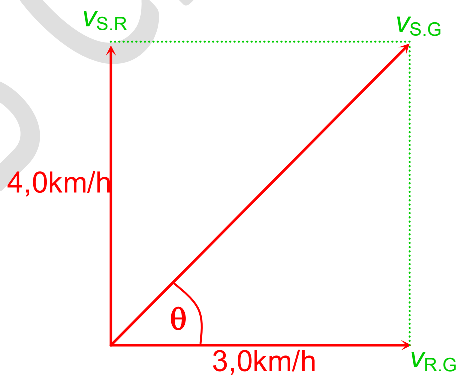
Speed of combustion products w.r.t. observer

$$= v_c - v_a = (v_c - v_j) + (v_j - v_a) = (-1500 + 500)\text{kmh}^{-1} = -1000\text{ km h}^{-1}$$

Illustration 21

Question: **A swimmer can swim in still water at a rate 4.0 km/h. If he swims in a river flowing at 3.0 km/h and keeps his direction (with respect to water) perpendicular to the current, find his velocity with respect to the ground.**

Solution: The velocity of the swimmer with respect to water is $\vec{v}_{S,R} = 4.0\,\text{km/h}$ in the direction perpendicular to the river. The velocity of river with respect to the ground is $\vec{v}_{R,G} = 3.0\,\text{km/h}$ along the length of the river. The velocity of the swimmer with respect to the ground is $\vec{v}_{S,G}$ where



$$\vec{v}_{S,G} = \vec{v}_{S,R} + \vec{v}_{S,G}$$

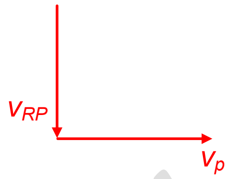
$$v_{S,G} = \sqrt{(4.0\,\text{km/h})^2 + (3.0\,\text{km/h})^2} = 5.0\text{ km/h.}$$

The angle θ made with the direction of flow is

$$\tan\theta = \frac{4.0\,\text{km/h}}{3.0\,\text{km/h}} = \frac{4}{3}$$

Illustration 22

Question: **To a standing person, the rains appears to fall vertically downward at 10 km/hr. If the person moves eastward at 5 km/hr, determine the direction of umbrella used to protect the person against rain. Also find the apparent velocity of the rain with respect to person**

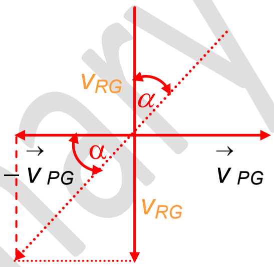


Solution: We have to find out $\vec{V}_{RP}$

$\vec{V}_{RP} = \vec{V}_{RG} - \vec{V}_{PG}$

or, $\vec{V}_{RP} = \vec{V}_{RG} + (-\vec{V}_{PG})$

So to find it, direction of velocity of person should reversed



$\vec{V}_{RP} = \vec{V}_{RG} + (-\vec{V}_{PG})$

$\therefore \vec{V}_{RP} = \sqrt{V_{RG}^2 + V_{PG}^2}$

$= \sqrt{(10)^2 + (5)^2}$

$= 5\sqrt{5}$ km/hr

The direction of resultant can be found out by

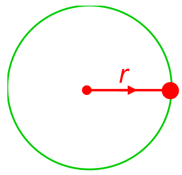
$\tan\alpha = \dfrac{V_{RG}}{V_{PG}} = \dfrac{10}{5} = 2$

$\alpha = \tan^{-1}2$

9 CIRCULAR MOTION

Circular motion is a type of motion in which a particle moves around a fixed point such that, its distance from a fixed point is constant.

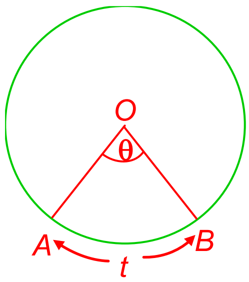
Suppose a particle moves on a circular path which is shown in figure. Here *r* is distance from the fixed point, which is centre of this circle to the particle. It will remain constant.



9.1 IMPORTANT PARAMETERS

Angular displacement: When a particle moves on a circular path, angle made at centre is called angular displacement.

Suppose a particle moving on a circular path start from *A* and after some time *t*, it is at *B* the angle made at centre during this interval of time is $\angle AOB = \theta$.



Angular displacement = θ

Its Unit is radian

Angular Velocity (ω)

Angular velocity is rate of change of angular displacement

As shown above the particle moves from *A* to *B* in *t*

Angular velocity (ω) $= \dfrac{\Delta\theta}{\Delta t} = \dfrac{\theta}{t}$ … **(37)**

Its unit is radian per second,

If angular velocity is constant, $\theta = \omega t$

Instantaneous Angular velocity

Instantaneous angular velocity is angular velocity of a particle at any moment of time.
It is given by limiting value of angular velocity when time approaches to zero

Instantaneous angular velocity = $\underset{\Delta t \to o}{Lt} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$ … (38)

i.e., rate of change of angular displacement with respect to time.

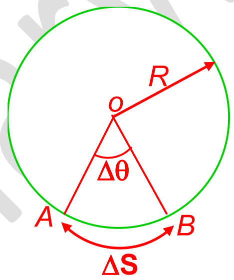
Angular Acceleration (α)

Angular acceleration is defined as rate of change of angular velocity

Average angular acceleration $(\alpha) = \frac{\Delta\omega}{\Delta t}$ … (39)

Instantaneous angular acceleration, $\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$ … (40)

Relation between angular velocity and linear velocity



A particle moves θ on a circular path of radius R. It moves from a point A to B. Angular displacement = $\Delta\theta$

Time taken = Δt

Distance travelled = Δs

angular velocity $(\omega) = \frac{\Delta\theta}{\Delta t}$

$$\Delta\theta = \frac{\Delta s}{R}$$

$$\omega = \frac{\Delta\theta}{\Delta t} \qquad \left(\because \Delta\theta = \frac{\Delta S}{R}\right)$$

$$\omega = \frac{1}{R}\frac{\Delta s}{\Delta t}$$

$$\omega = \frac{1}{R}v \qquad \left(\because v = \frac{\Delta s}{\Delta t}\right)$$

$v = \omega R$ … (41)

tangential velocity = Radius × angular velocity

Tangential acceleration $(a) = \frac{dv}{dt}$

$$= \frac{d}{dt}(\omega R) \quad (\because v = \omega R)$$

$$= R\frac{d\omega}{dt} = R\alpha$$

$a = \alpha R$ … (42)

Tangential Acceleration = (Radius) (Angular acceleration)

9.2 TYPES OF CIRCULAR MOTION

(a) Uniform circular motion

(b) Non-uniform circular motion

(a) Uniform circular motion: When a particle moves on a circular path in such a way that its angular speed is constant, such type of motion is called uniform circular motion.

Since ω is constant, angular acceleration α is zero.

☞ In this motion speed is constant but velocity is not constant since direction changes continuously so acceleration is also non-uniform.

- In uniform circular motion tangential acceleration will be zero.
- Time period is time taken in one complete revolution. It is denoted by T.

$$T = \frac{2\pi}{\omega}$$

- Frequency (n) is number of revolutions per second and $n = \frac{1}{T}$

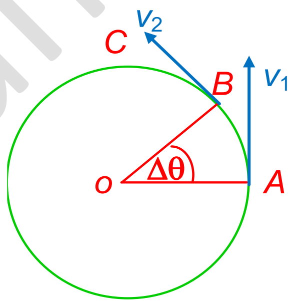
(b) Non-Uniform circular Motion

Such type of motion in which angular speed is not constant.

- In this type of motion velocity changes with magnitude as well as direction both.
- Angular acceleration (α) = $\frac{d\omega}{dt} \neq 0$
- Tangential acceleration (a) = $\frac{dv}{dt} \neq 0$

9.3 CENTRIPETAL ACCELERATION

"Centripetal" comes from a Greek word, which means centre-seeking. When an object moves on a circular path with constant angular speed, an acceleration acts at each point of the path **directed towards the centre of** the circle is called **centripetal acceleration.**



Let the angle made at the centre is $\Delta\theta$ when an object moves from A to B. The velocities at A and B are v_1 and v_2 respectively which are in tangential direction at A and B. Since motion is uniform circular

$$|\vec{v}_1| = |\vec{v}_2| = v$$

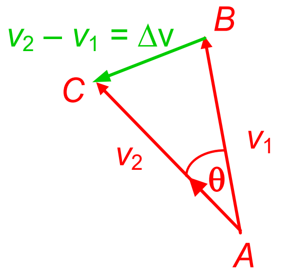
From triangle ABC

$$\vec{AB} + \vec{BC} = \vec{AC}$$

$$\vec{BC} = \vec{AC} - \vec{BC}$$

$$= \vec{v}_2 - \vec{v}_1 = \Delta\vec{v}$$

$$BC = \Delta v = v\Delta\theta$$



$$\frac{\Delta v}{\Delta t} = v\frac{\Delta\theta}{\Delta t}$$

$$\Rightarrow a = v\omega$$

$$\Rightarrow a = v\frac{v}{R} \quad (\because \omega = \frac{v}{R})$$

centripetal acceleration $= \frac{v^2}{R}$ ... **(43)**

Direction of Acceleration

Since velocity vector $|AB| = |AC| = v$

So angle $\angle ACB = \angle ABC = \alpha$

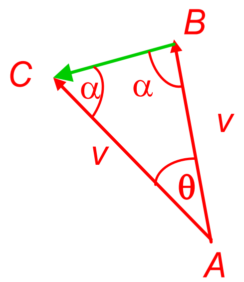
It $\Delta t \to 0$, then $\Delta\theta \to 0$

$2\alpha + \Delta\theta = 180°$

$\Delta\theta \to 0$

$2\alpha = 180°$

$\alpha = 90°$



It means Δv is perpendicular to $\vec{v}$

As $\vec{v}$ is tangential, hence $\Delta\vec{v}$ is perpendicular to tangential direction. It means direction of acceleration is radially inwards.

Illustration 23

Question: **A particle moves in a circle of radius 0.5 m with a linear speed of 2 m/s. Find its angular speed.**

Solution: The angular speed is

$$\omega = \frac{v}{r} = \frac{2}{0.5} = \mathbf{4\ rad/s}$$

Illustration 24

Question: **The speed of a particle moving in a circle of radius $r = 2$ m varies with time t as $v = t^2$ where t is in second and v in m/s. Find the radial, tangential and net acceleration at $t = 2$s.**

Solution: Linear speed of particle at $t = 2$s is $v = (2)^2 = 4$ m/s

∴ radial acceleration $a_r = \frac{v^2}{r} = \frac{(4)^2}{2} = 8\ \text{m/s}^2$

The tangential acceleration is $a_t = \frac{dv}{dt} = 2t$

∴ tangential acceleration at $t = 2s$ is

$a_t = (2)(2) = 4\ \text{m/s}^2$

∴ Net acceleration of particle at $t = 2$ s is

$$a = \sqrt{(a_r)^2 + (a_t)^2} = \sqrt{(8)^2 + (4)^2}$$

or $a = \sqrt{\mathbf{80}}\ \mathbf{m/s}^2$

Illustration 25

Question: **A stone of mass 0.3 kg tied to the end of a string in a horizontal plane is whirled round in a circle of radius 1 m with a speed of 40 rev min^{-1}. Calculate the tension in the string. What is the maximum speed with which the stone can be whirled around if the string can withstand a maximum tension of 200 N? Givne $\pi^2 = 9.87$.**

Solution: $M = 0.3$ kg , $R = 1$ m

Angular frequency = 40 rev min^{-1}

Angular speed , $\omega = \frac{40 \times 2\pi}{60}$ rad s^{-1} = 1.333 π rad s^{-1}

The centripetal force is provided by the tension in the string.

$$\text{Tension} = \frac{Mv^2}{R} = MR\omega^2$$

$$= 0.3\,\text{kg} \times 1\,\text{m} \times (1.333\pi)^2\ \text{N} = 5.26\ \text{N}$$

The string can withstand a maximum tension of 200 N. If v_{max} be the maximum speed of the stone

Then

$$200 = \frac{Mv^2_{max}}{R} = \sqrt{\frac{200 \times 1}{0.3}} = \text{ms}^{-1} = \mathbf{25.82\ ms^{-1}}.$$

PROFICIENCY TEST–II

The following questions deal with the basic concepts of this section. Answer the following briefly. Go to the next section only if your score is at least 80%. Do not consult the Study Material while attempting these questions.

1. A projectile is projected at an angle of 30° from the horizontal with a velocity of 98 m/s. Calculate
(a) time to reach the maximum height, (b) maximum height,
(c) time of flight, (d) horizontal range.

2. A cricketer throws a ball with a velocity of 60 m/s at an angle of 60°. He then runs to catch the same ball. What should be his minimum velocity so that he may catch the ball before it strikes the ground?

3. A man can throw a ball to a maximum horizontal distance of 100 m Then what would be maximum vertical distance upto which he can throw the ball assuming the ball to have same initial velocity.

4. What should be the angle of projection so that horizontal range of a projectile is equal to its maximum vertical height.

5. A particle is projected at an angle of 30° with the horizontal. What should be the other angle of projection with the same speed for which the horizontal range will be same?

6. Two trains *A* and *B* have lengths 100 m and 80 m respectively. They move in opposite directions along parallel tracks at 72 km/hr and 54 km/hr respectively. What is the time taken by one train to cross the other?

7. A monkey is climbing a vertical tree with a velocity of 12 m/s while a dog runs towards the tree chasing the monkey with a velocity of 16 m/s. Find the magnitude of velocity of the dog relative to the monkey.

8. A 100 m long train running with uniform velocity overtakes a man running in the same direction on the platform at a speed of 5 m/s in 10 seconds. Find the velocity of the train.

9. A man can swim at a speed of 3 km/h in still water. He wants to cross a 500 m wide river flowing at 2 km/h. He keeps himself always at an angle of 120^0 with the river flow while swimming. Find the time he takes to cross the river and at what point on the opposite bank will he arrive?

10. A man is walking on a level road at a speed of 3.0 km/h. Raindrops fall vertically with a speed of 4.0 km/h. Find the velocity of the raindrops with respect to the man.

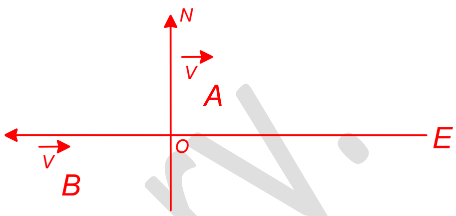
11. When two bodies move uniformly towards each other, the distance between them decreases by 6 metres/second. If both the bodies move in the same direction with the same speeds, the distance between them increases by 4 metres/second. What are the speeds of the two bodies.

12. A wheel of radius 2 metre is making 60 revolutions per minute. Calculate the linear velocity of a point of rim

13. A body of mass 1.5 kg is making 2 revolutions per second in a circle of radius 0.25 m. Calculate (1) linear velocity (2) centripetal acceleration (3) centripetal force.

14. Two stones are projected from a point *P* in the same direction with a velocity of 20 m/s one at angle α above the horizontal and the other at angle α to the vertical. Find the horizontal distance of each stone from P one second after projection and the horizontal separation between the stones. (Take $\cos\alpha = 4/5$; $\sin\alpha = 3/5$)

15. A ship '*A*' steams due north at 16 km/hr and a ship '*B*' due west at 12 km/hr. At a certain moment position of *B* is $10\hat{i}$ and of *A* is $0\hat{i}$. The distances are measured in kilometer. Find the velocity of *A* relative to *B*. Find also the nearest distance of approach of the ships.



ANSWERS TO PROFICIENCY TEST–II

1. (a) 5 s,
(b) 122.5 metre,
(c) 10 s,
(d) $490\sqrt{3}$ metre

2. 30 m/s.

3. 50 metre

4. $\tan^{-1}(4)$

5. 60°

6. 5.14 sec

8. 15 m/s.

9. $\frac{1}{3\sqrt{3}}$ h and $\frac{1}{6\sqrt{3}}$ km

10. 5.0 km/h

11. $u = 5$ m/s, $v = 1$ m/s.

12. 12.6 m/s

13. (1) 3.14 m/s
(2) 39.5 m/s^2,
(3) 59.2 N

14. 16 m; 12 m; 4 m

15. 20 km/h, 8 km

SOLVED OBJECTIVE EXAMPLES

***Example* 1:**

A particle is travelling with velocity of 2 m/s and moves in a straight line with a retardation of 0.1 m/s^2. The time at which the particle is 15m from the starting point is

(a) 10 s **(b) 20 s** **(c) 25 s** **(d) 40 s**

Solution:

$S = ut + \frac{1}{2}at^2; 15 = 2t + \frac{1}{2}\times(-0.1)t^2$

$\Rightarrow$ $20 \times 15 = 40t - t^2$ or $t^2 - 40t + 300 = 0$

$(t - 30)(t - 10) = 0;\ t = 30$ s

or $t = 10$ s

The particle is at *a* distance 15 m from starting point at t = **10 s** and also t = **30 s**.

$\therefore$ **(a)**

***Example* 2:**

A particle moves along a straight line according to the law $S^2 = at^2 + 2bt + c$. The acceleration of the particle varies as

(a) S^{-3} **(b) $S^{2/3}$** **(c) S^2** **(d) $S^{5/2}$**

Solution:

$S = (at^2 + 2bt + c)^{1/2}$

Differentiating, $\frac{dS}{dt} = \frac{1}{2}(at^2 + 2bt + c)^{-1/2} \times (2at + 2b) = \frac{at + b}{\sqrt{at^2 + 2bt + c}}$

$$\frac{d^2S}{dt^2} = \frac{\left(\sqrt{at^2 + 2bt + c}\right)\times a - \dfrac{(at+b)(at+b)}{\sqrt{at^2 + 2bt + c}}}{(at^2 + 2bt + c)}$$

$$= \frac{a(at^2 + 2bt + c) - (at + b)^2}{\sqrt{at^2 + 2bt + c} \times (at^2 + 2bt + c)} = \frac{(ac - b^2)}{S \cdot S^2}$$

$\therefore$ $\frac{d^2S}{dt^2} \propto \frac{1}{S^3}$ $\Rightarrow$ **acceleration $\propto S^{-3}$**

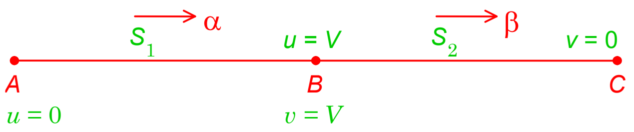
$\therefore$ **(a)**

***Example* 3:**

A car accelerates from rest at a constant rate α for sometime, after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t, the maximum velocity acquired by car is

(a) $\frac{\alpha\beta t}{\alpha + \beta}$ **(b) $\frac{(\alpha + \beta)}{\alpha\beta}t$** **(c) $\frac{\alpha^2 + \beta^2}{\alpha\beta}t$** **(d) $\frac{\alpha^2 - \beta^2}{\alpha\beta}t$**

Solution:



For motion from A to B, $V = \alpha t_1$ or $t_1 = \frac{V}{\alpha}$

For motion from B to C, $0 = V - \beta t_2$ or $t_2 = \frac{V}{\beta}$

$\therefore \quad t = t_1 + t_2 = \frac{V}{\alpha} + \frac{V}{\beta} = \frac{V(\alpha+\beta)}{\alpha\beta}$

or, $\quad V = \frac{\alpha\beta}{(\alpha+\beta)} t$

$\therefore$ **(a)**

Example 4:

Two particles P and Q start simultaneously from A with velocities 15 m/s and 20 m/s respectively. They move in the same direction with different accelerations. When P overtakes Q at B, velocity of P is 30 m/s. The velocity of Q at B is

(a) 30 m/s (b) 25 m/s (c) 20 m/s (d) 15 m/s

Solution:

As displacement (in uniformly accelerated motion) = average velocity × time

The average velocity is the same, when overtaking takes place.

$15 + 30 = 20 + v$

or, $v =$ **25 m/s**

$\therefore$ **(b)**

Example 5:

A stone A is dropped from rest from a height h above the ground. A second stone B is simultaneously thrown vertically up with velocity v. The value of v which would enable the stone B to meet the stone A midway between their initial positions is

(a) $2gh$ (b) $2\sqrt{gh}$ (c) $\sqrt{gh}$ (d) $\sqrt{2gh}$

Solution:

Time of travel of each stone = t

Distance travelled by each stone = $\frac{h}{2}$

For stone A, $\frac{h}{2} = \frac{1}{2} g t^2 \quad i.e., t = \sqrt{\frac{h}{g}}$

For stone B, $\frac{h}{2} = ut - \frac{1}{2} g t^2 = u\sqrt{\frac{h}{g}} - \frac{1}{2} g \left(\frac{h}{g}\right)$

$\Rightarrow \frac{h}{2} = u\sqrt{\frac{h}{g}} - \frac{h}{2}$

or, $u\sqrt{\frac{h}{g}} = h$

$\therefore \quad u = h\sqrt{\frac{g}{h}} = \sqrt{gh}$

$\therefore$ **(c)**

Example 6:

A body is dropped from rest from a height h. It covers a distance $9h/25$ in the last second of fall. The height h is

(a) 102.5 m (b) 112.5 m (c) 122.5 m (d) 132.5 m

Solution:

t is time to reach ground.

$h = \frac{1}{2} a t^2; \left(1 - \frac{9}{25}\right) h = \frac{1}{2} a (t-1)^2$

$\left(1-\frac{9}{25}\right)=\frac{(t-1)^2}{t^2}; \frac{16}{25}=\frac{(t-1)^2}{t^2}$ or $\frac{4}{5}=\frac{t-1}{t}$ $\therefore$ $t = 5$ sec

$h = \frac{1}{2} \times 9.8 \times 5^2 =$ **122.5 m**

$\therefore$ **(c)**

Example 7:

A particle *P* is projected vertically upward from a point *A*. Six seconds later, another particle *Q* is projected vertically upward from *A*. Both *P* and *Q* reach A simultaneously. The ratio of maximum heights reached by *P* and *Q* = 64 : 25. Find the velocity of the projection of *Q* in m/s

(a) 7 *g* **(b) 6 *g*** **(c) 5 *g*** **(d) 4 *g***

Solution:

$\frac{1}{2}g(t+3)^2 : \frac{1}{2}gt^2 = 64 : 25$

or $(t+3)^2 : t^2 = 64 : 25$; or $(t+3) : t = 8 : 5$

$5t + 15 = 8t$ or $3t = 15$; $t = 5$ seconds

$v = g \times t = g \times 5 =$ **5g m/s**

$\therefore$ **(c)**

Example 8:

A stone is dropped from rest from the top of a cliff. A second stone is thrown vertically down with a velocity of 30 m/s two seconds later. At what distance from the top of a cliff do they meet?

(a) 60 m **(b) 120 m** **(c) 80 m** **(d) 44 m**

Solution:

The two stones meet at distance S from top of cliff t seconds after first stone is dropped.

For 1st stone $S = \frac{1}{2}gt^2$; For 2nd stone $S = u(t-2) + \frac{1}{2}g(t-2)^2$

i.e., $\frac{1}{2}gt^2 = ut - 2u + \frac{1}{2}gt^2 - 2gt + 2g$

$0 = (u - 2g)t - 2(u - g)$; $t = \frac{2(u-g)}{u-2g} = \frac{2(30-10)}{30-20} = 4$ s

Distance S at which they meet $= \frac{1}{2} \times gt^2 = \frac{1}{2} \times 10 \times 16$

= **80 m from top of cliff**

$\therefore$ **(c)**

Example 9:

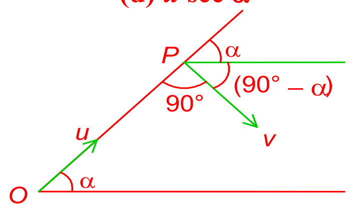
A particle is projected from a point *O* with velocity *u* in a direction making an angle α upward with the horizontal. At *P*, it is moving at right angles to its initial direction of projection. Its velocity at *P* is

(a) *u* tan α **(b) *u* cot α** **(c) *u* cosec α** **(d) *u* sec α**

Solution:

$v \cos(90 - \alpha) = v \sin \alpha = u \cos \alpha$; $v =$ ***u* cot α**

$\therefore$ **(b)**



Example 10:

In the previous example the time of flight from O to P is

(a) $\frac{u \operatorname{cosec} \alpha}{g}$ (b) $\frac{u \sin \alpha}{g}$ (c) $\frac{u \tan \alpha}{g}$ (d) $\frac{u \sec \alpha}{g}$

Solution:

$v = g't = (g \cos\alpha)t = u \cot \alpha$

$t = \frac{u\cot\alpha}{g\cos\alpha} = \frac{u}{g} \cdot \frac{1}{\sin\alpha} = \frac{u\operatorname{cosec}\alpha}{g}$

$\therefore$ **(a)**

Example 11:

A particle is projected from O and is moving freely under gravity and strikes the horizontal plane through O at a distance R from it. Then which of the following is incorrect?

(a) There will be two angles of projection if $Rg < u^2$

(b) There will be more than two angles of projection if $Rg = u^2$

(c) The two possible angles of projection are complementary

(d) The products of the times of flight for two directions of projection is $2R/g$

Solution:

The range $R = \frac{2u^2 \sin\alpha\cos\alpha}{g}$

$\sin 2\alpha = \frac{Rg}{u^2} \Rightarrow Rg = u^2$, then $\sin 2\alpha = 1$, so $\alpha = 45^0$.

$\sin 2\alpha_1 = \sin 2\alpha_2 \Rightarrow \frac{Rg}{u^2}$ $\sin 2\alpha_1 - \sin 2\alpha_2 = 0$

or, $2 \cos (\alpha_1 + \alpha_2) \sin (\alpha_1 - \alpha_2) = 0$

If $Rg < u^2$, $\alpha_1 + \alpha_2 = \pi/2$; if $Rg = u^2$, $\alpha_1 = \alpha_2 = \pi/2$

$t_1 t_2 = \frac{2u\sin\alpha_1}{g} \times \frac{2u\sin\alpha_2}{g} = \frac{4u^2 \sin\alpha_1 \cos\alpha_1}{g^2} = \frac{2R}{g}$ since $\alpha_1 + \alpha_2 = \pi/2$

$\therefore$ **(b)**

Example 12:

The velocity of a particle P moving freely under gravity is 4.9 m/s, the direction being 30° with the downward normal

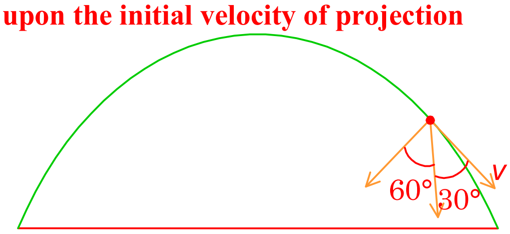
(a) its acceleration normal to the direction of motion at P = 9.8 m/s^2

(b) the radius of curvature of P of the parabolic trajectory of particle is 4. 9 m

(c) the particle has no acceleration normal to the direction of motion

(d) the radius of curvature at P of the path depends upon the initial velocity of projection

Solution:



The component of acceleration perpendicular to velocity is $g \cos 60°$ and this provide the necessary centripetal acceleration.

$g \cos 60° = \frac{g}{2} = 4.9$ m/s^2

The radius of curvature is $\frac{v^2}{r} = 4.9$;

$r = \frac{v^2}{4.9}$ = **4.9 m**

$\therefore$ **(b)**

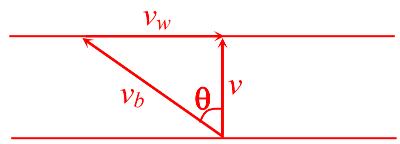
Example **13:**

A boat which has a speed of 5 km/hr in still water crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water in kilometres per hour is

(a) 1 **(b) 3** **(c) 4** **(d) $\sqrt{41}$**

Solution:

For shortest possible path



$$v = \frac{1\text{km}}{15\,\text{min}} = 4\,\text{km/h}$$

$$v_w^2 + v^2 = v_b^2$$

$$v_w = \sqrt{v_b^2 - v^2} = \sqrt{5^2 - 4^2} = 3\text{kmh}^{-1}$$

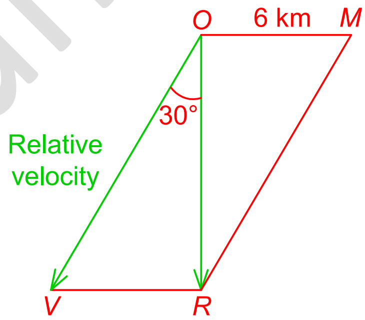
∴ **(b)**

Example **14:**

A man running at 6 km/hr on a horizontal road in vertically falling rain observes that the rain hits him at 30° from the vertical. The actual velocity of rain has magnitude

(a) 6 km/hr **(b) $6\sqrt{3}$ km/hr** **(c) $2\sqrt{3}$ km/hr** **(d) 2 km/hr**

Solution:



Velocity of rain = Velocity of man + Relative velocity of rain

OR gives the actual velocity.

$$\tan 30° = \frac{VR}{OR} = \frac{1}{\sqrt{3}} = \frac{6}{OR}$$

or, $OR = 6\sqrt{3}$ **km/hr**

∴ **(b)**

Example **15:**

A boat which has a speed of 5 km/hr in still water crosses a river of width 3 km along the shortest possible path in *t* min. The river flows at the rate of 3 km/hr. The time taken *t* is

(a) 20 min **(b) 25 min** **(c) 45 min** **(d) 55 min**

Solution:

$$t = \frac{AB}{\sqrt{5^2 - 3^2}} = \frac{3}{4}$$ = **45 minutes**

∴ **(c)**

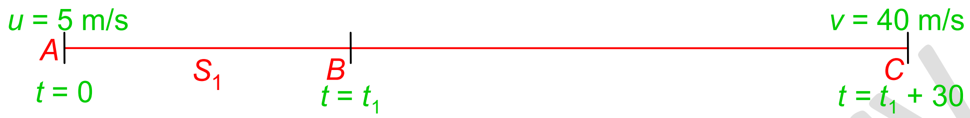
SOLVED SUBJECTIVE EXAMPLES

***Example* 1:**

A particle moves along a straight path *ABC* with a uniform acceleration of 0.5 m/s^2. While it crosses *A* its velocity is found to be 5 m/s. It reaches *C* with a velocity 40 m/s, 30 seconds after it has crossed B in its path. Find the distance *AB*.

Solution:

The velocity while it crosses the point *A* is 5 m/s



Considering the displacement *AC*,

initial velocity u = 5 m/s

final velocity v = 40 m/s

acceleration a = 0.5 m/s^2

$\therefore$ time of motion $t = \frac{v-u}{a} = \frac{40-5}{0.5} = 70$ s

For the displacement *AB*,

initial velocity u = 5 m/s

acceleration a = 0.5 m/s^2

time of motion t = 70 – 30 = 40 s

$$\therefore \quad AB = S = ut + \frac{1}{2}at^2 = (5 \times 40) + \left(\frac{1}{2} \times 0.5 \times 40^2\right) = 200 + 400 = \mathbf{600\ m}$$

***Example* 2:**

A particle moving with uniform acceleration in a straight line covers a distance of 3 m in the 8th second and 5 m in the 16th second of its motion. What is the displacement (in cm) of the particle from the beginning of the 6th second to the end of 15th second?

Solution:

The distance traveled during the nth second of motion of a body is given by

$$S_n = u + \frac{1}{2}a\,(2n-1)$$

For the motion during the 8th second,

$$3 = u + \frac{1}{2}a\,(16-1) = u + \frac{15a}{2} \quad \text{... (i)}$$

For the motion during the 16th second,

$$5 = u + \frac{1}{2}a\,(32-1) = u + \frac{31a}{2} \quad \text{... (ii)}$$

Subtracting equations (i) from (ii)

$8a = 2$

or acceleration $a = \frac{1}{4}$ ms^2

From equation (1), $u = 3 - \left(\frac{15}{2} \times \frac{1}{4}\right) = \frac{9}{8}$ ms^2

Now, the velocity at the end of 5 s (velocity at the beginning of 6th second)

$v_1 = u + 5a$

The velocity at the end of 15th *s*, $v_2 = u + 15a$

Average velocity during this interval of 10 seconds = $\frac{v_1 + v_2}{2}$

$$= \frac{(u+5a)+(u+15a)}{2} = u + 10a$$

Distance travelled during this interval

S = average velocity x time = $(u + 10a) \times t$

$$= \left(\frac{9}{8} + \frac{10}{4}\right) \times 10 = \frac{290}{8} = 36.25 \text{ m} = \mathbf{3625 \text{ cm}}$$

***Example* 3:**

An automobile can accelerate or decelerate at a maximum value of $\frac{5}{3}$ m/s^2 and can attain a maximum speed of 90 km/hr. If it starts from rest, what is the shortest time in which it can travel one kilometre, if it is to come to rest at the end of the kilometre run?

Solution:

In order that the time of motion be shortest, the automobile should attain the maximum velocity with the maximum acceleration after the start, maintain the maximum velocity for as along as possible and then decelerate with the maximum retardation possible, consistent with the condition that, the automobile should come to rest immediately after covering a distance of 1 km.

Let t_1 be the time of acceleration, t_2 be the time of uniform velocity and t_3 be the time of retardation.

Now, maximum velocity possible = 90 km/hr = 90 × $\frac{5}{18}$ m/s = 25 m/s

$$t_1 = \frac{v-u}{a} = \frac{25-0}{\frac{5}{3}} = 15 \text{ s}$$

Similarly, the time of retardation is also given by

$$t_3 = \frac{0-25}{-\frac{5}{3}} = 15 \text{ s}$$

During the period of acceleration, the distance covered

= average velocity x time

$$= \frac{25+0}{2} \times 15 = 187.5 \text{ m}$$

During the period of retardation, the distance covered is the same and hence

= 187.5 m

∴ the total distance covered under constant velocity = 1000 – 375 = 625 m

Time of motion under constant velocity, $t_2 = \frac{625}{25} = 25s$

∴ the shortest time of motion = $t_1 + t_2 + t_3$ = 15 + 25 + 15 = **55 seconds**

***Example* 4:**

A stone is dropped into a well and the sound of the splash is heard $3\frac{1}{8}$ seconds later. If the velocity of sound in air is 352.8 m/s, find the depth of the well (in cm). g = 9.8 m/s^2.

Solution:

Let x metres be the depth of the well and t the time taken by the stone to reach the surface of water.

In this case $u = 0$, $a = 9.8$ m/s^2

Now in the relation, $S = ut + \frac{1}{2}at^2$,

we have

$$x = 0 + \frac{1}{2}(9.8)\,t^2$$

or $\quad x = 4.9t^2 \quad$ ... (i)

Time taken by the sound to travel the distance x up (Motion of sound wave is not affected by gravity) is

$$\left(3\frac{1}{8} - t\right) \text{ seconds}$$

Distance traveled = Velocity of sound × Time taken

$$x = 352.8\left(\frac{25}{8} - t\right) \quad \text{... (ii)}$$

From equations (i) and (ii),

$$4.9t^2 = 352.8\left(\frac{25}{8} - t\right)$$

or, $\quad 4.9t^2 + 352.8t - 1102.5 = 0$

or, $\quad t^2 + 72t - 225 = 0$

Solving, $t = 3$ or -75 s.

Since the negative value of t has no meaning, $t = 3$ s.

This gives $x = 4.9t^2 = 4.9 \times 9 = 44.1$ m

Hence the depth of the well = 44.1 m = **4410 cm**

***Example* 5:**

A circus artist maintains four balls in motion making each in turn rise to a height of 5 m from his hand. With what velocity (in m/s) does he project them and the height (in cm) of the other three balls at the instant when the fourth one is just leaving his hand? (take $g = 10$ m/s^2.)

Solution:

Obviously, to maintain proper distances, the artists must throw the balls after equal intervals of time. Let the interval of time be t, so that when the fourth ball is just leaving his hand, the first ball would have travelled for time $3t$, the second for time $2t$ and the third for time t. The second obviously would just have reached the maximum height of 5 m.

If v be the initial velocity of throw of each ball, then for the second ball we have,

$$v_2 = 0 = v - g\,(2t) \quad \text{... (i)}$$

and $\quad s_2 = 5 = v\,(2t) - \frac{1}{2}g(2t)^2 \quad$ ... (ii)

These gives, $v = 2gt$

and $\quad v \cdot 2t = 2t^2 + 5$

or $\quad v = 20t$

and $\quad v \cdot 2t = 20t^2 + 5$

Solving for t, we get $20t^2 = 5$ or $t = \frac{1}{2}$ second.

Therefore, $v = 20 \times \frac{1}{2} = 10$ m/s. Thus each ball is thrown up with initial velocity of **10 m/s**.

For the first ball, which would have come down for time $(3t - 2t) = t$, we have

$$S = 0 + \frac{1}{2} g \cdot t^2$$

$$= \frac{1}{2} \cdot 10 \cdot \left(\frac{1}{2}\right)^2 = \frac{5}{4} = \textbf{125 cm}$$

Therefore, it will be at a height of (5 – 1.25) = 3.75 m = **375 cm** from the hand and going downwards.

For the third ball, which will have risen up for time *t*,

$$S_3 = vt - \frac{1}{2}gt^2 = 10\left(\frac{1}{2}\right) - \frac{1}{2} \cdot 10 \cdot \left(\frac{1}{2}\right)^2 = 5 - 1.25 = \textbf{375 cm}$$

***Example* 6:**

A stone is projected from the point on the ground in such a direction so as to hit a bird on the top of a telegraph post of height *h* and then attain the maximum height 2*h* above the ground. If at the instant of projection the bird were to fly away horizontally with uniform speed $v = 2(\sqrt{2} - 1)$ m/s. Find the horizontal velocity of the stone if the stone still hits the bird while descending.

Solution:

The situation is shown in Figure. Let θ be the angle of projection and *u* the velocity of projection.

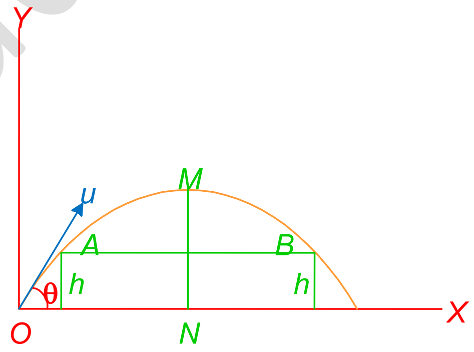
Maximum height $MN = 2h$

$$MN = 2h = \frac{u^2 \sin^2 \theta}{2g}$$

$$\therefore u \sin \theta = 2\sqrt{gh} \qquad \text{... (i)}$$

Let *t* be the time taken by stone to attain the vertical height *h* above the ground.

$$\therefore \qquad h = (u \sin \theta)t - \frac{1}{2}gt^2$$



$$t^2 - \left(\frac{2u\sin\theta}{g}\right)t + \frac{2h}{g} = 0$$

$$t = \frac{u\sin\theta}{g} \pm \sqrt{\frac{u^2\sin^2\theta}{g^2} - \frac{2h}{g}}$$

Substituting the value of u sin θ from (i),

$$t = \frac{2\sqrt{gh}}{g} \pm \sqrt{\frac{4gh}{g^2} - \frac{2h}{g}} = \sqrt{\frac{4h}{g}} \pm \sqrt{\frac{2h}{g}}$$

$$t_1 = \sqrt{\frac{4h}{g}} - \sqrt{\frac{2h}{g}} \qquad t_2 = \sqrt{\frac{4h}{g}} + \sqrt{\frac{2h}{g}}$$

where t_1 and t_2 are time to reach *A* and *B* respectively shown in the figure. If *v* is the horizontal velocity of bird, then

$AB = vt_2$.

AB is also equal to $u \cos \theta (t_2 - t_1)$, where $u \cos \theta$ is constant horizontal velocity of stone.

$$t_2 - t_1 = 2\sqrt{\frac{2h}{g}}$$

$\therefore \quad u\cos\theta \cdot 2\sqrt{\dfrac{2h}{g}} = vt_2$

$$\frac{v}{u\cos\theta} = \frac{2\sqrt{\frac{2h}{g}}}{t_2} = \frac{2\sqrt{\frac{2h}{g}}}{\sqrt{\frac{2h}{g}}\cdot(\sqrt{2}+1)}$$

$$\frac{v}{u\cos\theta} = \frac{2}{\sqrt{2}+1}$$

$\therefore \quad u\cos\theta = $ **1 m/s**

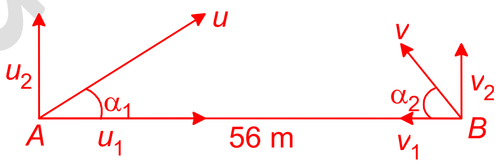
***Example* 7:**

Two particles are projected at the same instant from two points *A* and *B* on the same horizontal level where *AB* = 56 m, the motion taking place in a vertical plane through *AB*. The particle from *A* has an initial velocity of 39 m/s at an angle $\sin^{-1}\left(\frac{5}{13}\right)$ with *AB* and the particle from *B* has an initial velocity of 25 m/s at an angle $\sin^{-1}\left(\frac{3}{5}\right)$ with BA. Show that the particles would collide in mid-air and find when and where the impact occurs.

Solution:

AB = 56 m.

At *A*, a particle is projected with velocity u = 39 m/s. u_1 and u_2 are its horizontal and vertical components respectively. The angle u makes with *AB* is α_1.



Given that $\sin\alpha_1 = \frac{5}{13} \therefore \cos\alpha_1 = \frac{12}{13}$.

Similarly, for the particle projected from *B*, with velocity v = 25 m/s, v_1 and v_2 are the horizontal and vertical components respectively.

$\sin\alpha_2 = \frac{3}{5} \quad \therefore \cos\alpha_2 = \frac{4}{5}$.

Now $u_2 = u\sin\alpha_1 = 39 \times \frac{5}{13} = 15$ m/s.

$v_2 = v\sin\alpha_2 = 25 \times \frac{3}{5} = 15$ m/s.

The vertical components of the velocities are the same at the start. Subsequently at any other instant t their vertical displacement are equal and have a value

$h = 15t - 5t^2$

which means that the line joining their positions at the instant t continues to be horizontal and the particles come closer to each other.

Their relative velocity in the horizontal direction

$= 39\cos\alpha_1 + 25\cos\alpha_2$

$= 39 \times \frac{12}{13} + 25 \times \frac{4}{5} = 36 + 20 \quad =$ **56 m/s**

Time of collision $= \frac{AB}{56} = \frac{56}{56} =$ **1 s**, after they were projected.

Height at which the collision occurs $= ut - \frac{1}{2}at^2 = 15(1) - \frac{1}{2}(10)(1)^2 =$ **10 m**

The horizontal distance of the position of collision from $A = 39 \times \frac{12}{13} \times 1\text{ s} =$ **36 m**

Example 8:

A shell is fired from a gun from the bottom of a hill along its slope. The slope of the hill is $\alpha = 30°$ and the angle of barrel to the horizontal is $\beta = 60°$. The initial velocity of shell is 21 m/s. Find the distance from the gun to the point at which the shell falls.

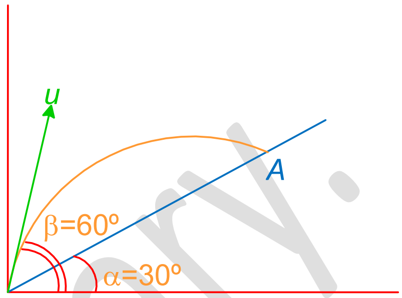
Solution:

We can write the equation of motion as

$$x = ut\cos\beta$$

$$y = ut\sin\beta - \frac{gt^2}{2}$$

$$OA = \ell$$



At the moment the shell falls to the ground

$$x = \ell\cos\alpha = \ell\cos 30°$$

$$y = \ell\sin\alpha = \ell\sin 30°$$

$$\ell\cos\alpha = ut\cos\beta \quad \text{... (i)}$$

$$\ell\sin\alpha = ut\sin\beta - \frac{gt^2}{2} \quad \text{... (ii)}$$

$$\therefore \quad t = \frac{\ell\cos\alpha}{u\cos\beta}$$

$$\ell\sin\alpha = \frac{\ell\cos\alpha\sin\beta}{\cos\beta} - \frac{g\ell^2\cos^2\alpha}{2u^2\cos^2\beta} \Rightarrow \ell = \frac{2u^2\sin(\beta-\alpha)\cos\beta}{g\cos^2\alpha}$$

Substituting $u = 21$ m/s, $\alpha = 30^\circ$, $\beta = 60^\circ$ and $g = 9.8$ m/s^2, we get $\boldsymbol{\ell = 30}$ **m**

Example 9:

A man can swim at a velocity V_1 relative to water in a river flowing with speed V_2. Show that it will take him $\frac{V_1}{\sqrt{V_1^2 - V_2^2}}$ times as long to swim a certain distance upstream and back as to swim the same distance and back perpendicular to the direction of the stream ($V_1 > V_2$). (given $\sqrt{24}V_1 = \sqrt{25}V_2$)

Solution:

Suppose the man swims a distance x up and the same distance down the stream.

Velocity of man upstream relative to the ground = $V_1 - V_2$.

Time taken for this, $t_1 = \frac{x}{V_1 - V_2}$

Velocity of man downstream relative to the ground = $V_1 + V_2$

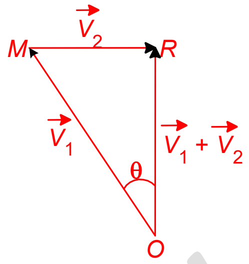
Time taken for this, $t_2 = \frac{x}{V_1 + V_2}$

Total time taken $t_1 + t_2 = \frac{x}{(V_1 - V_2)} + \frac{x}{(V_1 + V_2)} = \frac{2V_1 x}{(V_1^2 - V_2^2)}$

Next the man intends crossing the river perpendicular to the direction of the stream. If he wants to cross the river straight across he must swim in a direction OM such that the vector sum of velocity of man + velocity of river will give him a velocity relative to the ground in a direction perpendicular to the direction of the stream. In the Figure the velocity relative to the ground is $\vec{OR}$ and the magnitude of $\vec{OR} = \sqrt{V_1^2 - V_2^2}$

Now the man swims a distance x up and x down perpendicular to the river flow. Time taken for this,

$$t = \frac{2x}{\sqrt{V_1^2 - V_2^2}}$$



Then the ratio, $\frac{t_1 + t_2}{t} = \frac{2V_1 x}{(V_1^2 - V_2^2)} \div \frac{2x}{\sqrt{V_1^2 - V_2^2}}$

$$= \frac{2V_1 x}{(V_1^2 - V_2^2)} \times \frac{\sqrt{V_1^2 - V_2^2}}{2x} = \frac{V_1}{\sqrt{V_1^2 - V_2^2}} = \mathbf{5}$$

***Example* 10:**

A man walking eastward at 6 km/hr finds that the wind seems to blow directly from north. On doubling his velocity, the wind appears to come *N* 30° *E*. Find the speed of the wind.

Solution:

Actual velocity of the man = 6 km/hr eastward.

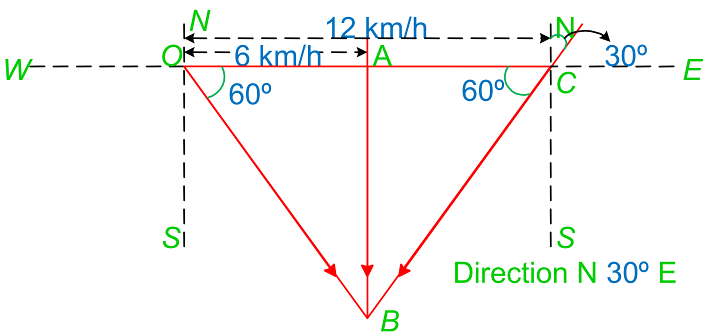
The direction of the relative velocity of the wind in this case is North to South.

If $\vec{OA}$ represents the velocity of man and $\vec{AB}$ represents the relative velocity of the wind, then

velocity of man + relative velocity of wind = velocity of wind = $\vec{OB}$ (say)

It is also given that when the velocity of the man is doubled (i.e., 12 km/hr) the wind seems to blow from a direction *N* 30°E. Representing this by vector $\vec{OC}$ = New velocity of man = 12 km/hr.

The direction of the relative velocity of the wind in this case is *CB*. The two directions of the relative velocity meet at *B*. Hence $\vec{OB}$ should give the real velocity of the wind. From the geometry of the Figure, it is clear that *OBC* is an equilateral triangle. Hence the magnitude of the real velocity of the wind = **12 km/hr**.



MIND MAP

KINEMATICS

1. Relation between kinematic variables for motion in one dimension

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \frac{vdv}{dx}$$

2. Equations of motion in one dimension

- Motion with uniform velocity $S = vt$
- Motion with uniform acceleration,

$$S = ut + \frac{1}{2}at^2$$

$$v = u + at$$

$$v^2 = u^2 + 2as$$

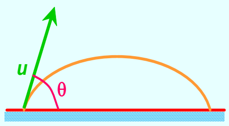
$$S_n = u + (2n-1)\frac{a}{2}$$

3. Graphical representation of motion

- Slope of tangent to position time graph gives velocity.
- Slope of tangent to v–t curve gives acceleration.
- Area enclosed between v–t curve and time axis between an interval of time gives displacement.
- Slope of tangent to a–t curve gives rate of change of acceleration
- Area enclosed between a–t curve and time axis between an interval of time gives change in velocity.

4. Projectile on horizontal plane

- Time of flight, $T = \frac{2u\sin\theta}{g}$
- Range $R = \frac{u^2\sin 2\theta}{g}$



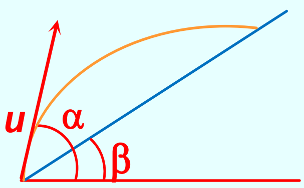
- Maximum Height, $H = \frac{u^2\sin^2\theta}{2g}$
- Equation of trajectory ,

$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2}\sec^2\theta$$

- For maximum range, $\theta = 45°$
- For a given speed and given range, there are two possible angles of projection; θ and $(90° - \theta)$.

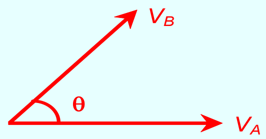
5. Projectile on inclined plane

- Time of flight, $T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$
- Range, $R = \frac{2u^2\sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta}$



6. Relative velocity

- $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$



- $|\vec{V}_{AB}| = \sqrt{V_A^2 + V_B^2 - 2V_AV_B\cos\theta}$
- If relative velocity makes an angle α with V_A then,

$$\tan\alpha = \frac{V_B\sin\theta}{V_A - V_B\cos\theta}$$

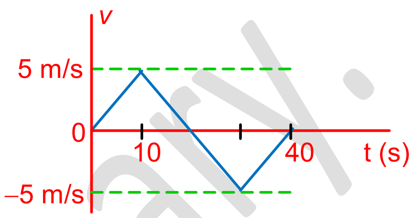
EXERCISE – I

NEET-SINGLE CHOICE CORRECT

1. On a displacement-time graph two straight lines make angles 30° and 60° with the time-axis. The ratio of the velocities represented by them is

(a) $1 : \sqrt{3}$ (b) 1 : 3 (c) $\sqrt{3} : 1$ (d) 3 : 1

2. The velocity-time plot is shown in figure. Find the average speed in time interval $t = 0$ to $t = 40$s during the period

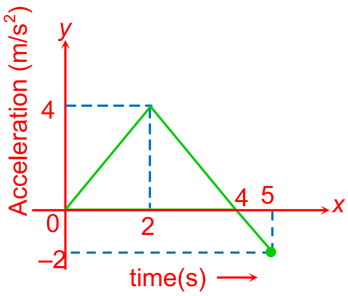


(a) zero (b) 2.5 m/s
(c) 5 m/s (d) none of these

3. A block is released from rest on a smooth inclined plane. If S_n denotes the distance traveled by it from $t = (n–1)$ s to $t = n$ s, then the ratio $\frac{S_n}{S_{n+1}}$ is

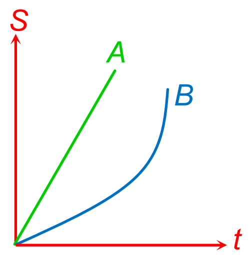
(a) $\frac{2n}{2n-1}$ (b) $\frac{2n-1}{2n+1}$ (c) $\frac{2n+1}{2n-1}$ (d) $\frac{2n}{2n+1}$

4. Figure shows the graph of acceleration of particle as a function of time. The maximum speed of the particle is (particle starts from rest)



(a) 7 m/s (b) 8 m/s
(c) 4 m/s (d) 16 m/s

5. If the displacement (s) and time (t) graph of two moving particles *A* and *B* in straight line is shown in the figure, then which of following is incorrect.



(a) *A* is moving with constant velocity
(b) *B* is moving with increasing speed
(c) *A* is moving with non-zero constant acceleration
(d) acceleration of *B* may be constant for some time

6. A body covers one-third of the distance with a speed v_1, the second one-third of the distance with a speed v_2 and the remaining distance with a speed v_3. The average speed is

(a) $\frac{v_1 + v_2 + v_3}{3}$ (b) $\frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$

(c) $\frac{v_1v_2 + v_2v_3 + v_3v_1}{3}$ (d) $\frac{v_1v_2v_3}{3}$

7. A particle is falling freely under gravity. In first t second it covers s_1 and in the next t seconds it covers s_2, then t is given by

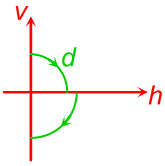
(a) $\sqrt{\frac{s_2 - s_1}{2g}}$ (b) $\sqrt{\frac{s_2 - s_1}{g}}$ (c) $\sqrt{\frac{s_2 s_1}{g}}$ (d) $\sqrt{\frac{s_2^2 - s_1^2}{g}}$

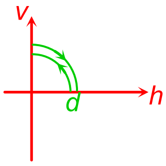
8. Two cars A and B are traveling in the same direction with velocities v_A and v_B ($v_A > v_B$) when the car A is at a distance s behind the car B, the driver of the car A applies brakes producing a uniform retardation a and there will be no collision when

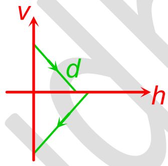
(a) $s < \frac{(v_A - v_B)^2}{2a}$ (b) $s = \frac{(v_A - v_B)^2}{2a}$ only

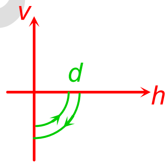
(c) $s \geq \frac{(v_A - v_B)^2}{2a}$ (d) $s \leq \frac{(v_A - v_B)^2}{2a}$

9. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $d/2$. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as

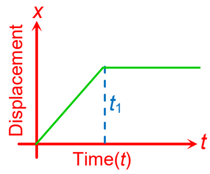
(a) 

(b) 

(c) 

(d) 

10. The $x - t$ graph shown in figure represents



(a) Constant velocity
(b) Velocity of the body is continuously changing
(c) constant acceleration
(d) The body travels with constant speed upto time t_1 and then stops

11. A velocity of 5 km/hr to the East is changed into 5 km/hr to the North. The change in velocity is

(a) zero (b) $5\sqrt{2}$ km/hr due North-East

(c) $5\sqrt{2}$ km/hr due North-West (d) 10 km/hr due East

12. A particle is projected such that the horizontal range and vertical height are the same. Then the angle of projection with horizontal is

(a) $\tan^{-1}(4)$ (b) $\tan^{-1}\left(\frac{1}{4}\right)$ (c) $\pi/4$ (d) $\pi/3$

13. A cricket ball is hit with a velocity of 25 m s^{-1} at angle of 60° above the horizontal. How far above the ground, ball passes over a fielder 50 m from the bat (consider the ball is struck very close to the ground) Take $\sqrt{3} = 1.7$ and $g = 10$ m s^{-2}.

(a) 6.8 m (b) 7 m (c) 5 m (d) 10 m

14. A body has an initial velocity of 3 ms^{-1} and has an acceleration of 1 ms^{-2} normal to the direction of the initial velocity. Then its velocity, 4 second after the start is

(a) 7 ms^{-1} along the direction of initial velocity
(b) 7 ms^{-1} along the normal to the direction of the initial velocity
(c) 7 ms^{-1} mid-way between the two directions
(d) 5 ms^{-1} at an angle of $\tan^{-1}\frac{4}{3}$ with the direction of the initial velocity

15. Two particles are projected simultaneously in the same vertical plane from the same point, with different speeds u_1 and u_2, making angles θ_1 and θ_2 respectively with the horizontal, such that $u_1 \cos\theta_1 = u_2 \cos\theta_2$. The path followed by one w.r.t other (as long as both are in flight), is

(a) a horizontal straight line
(b) a vertical straight line
(c) a parabola
(d) a straight line making an angle $|\theta_1 - \theta_2|$ with the horizontal

16. A particle is projected with velocity u at angle θ with the horizontal. The time after which the acceleration vector and velocity vector becomes perpendicular to each other during the path is

(a) $\frac{u\sin\theta}{g}$ (b) $\frac{u\cos\theta}{g}$ (c) $\frac{2u\sin\theta}{g}$ (d) $\frac{2u\cos\theta}{g}$

17. An aircraft flying horizontally at 360 km/hr releases a bomb (with zero velocity relative to the aircraft) at a stationary tank 200 m away from a point just below the aircraft (at the time of release). What must be the height of the aircraft above the tank if the bomb is to hit the tank?

(a) 100 m (b) 9.8 m (c) 19.6 m (d) 98 m

18. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$m/s. The cartesian equation of its path is ($g = 10$ m/s^2) (here $\hat{i}$ & $\hat{j}$ are the unit vectors along horizontal and vertical direction)

(a) $y = 2x - 5x^2$ (b) $y = x - 5x^2$ (c) $4y = 2x - 5x^2$ (d) $y = 2x - 25x^2$

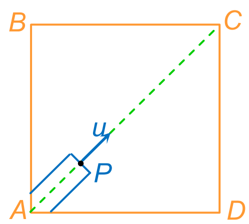
19. A shot is fired from a gun with a muzzle velocity of 98 m/s at an angle of elevation 45° from the ground. Its range is found to be 900 m. The range decreased by air resistance is

(a) 8 m (b) 80 m (c) 98 m (d) 9.8 m

20. The velocity of projection of a particle if it does not rise more than 3 m in a range of 600 m is

(a) 383.4 m/s (b) 273 m/s (c) 343 m/s (d) 3.83 m/s

21. A large rectangular box falls vertically with an acceleration a. A toy gun fixed at A and aimed towards C fires a particle P. Which of the following statement is false?



(a) P will hit C if $a = g$
(b) P will hit the roof BC if $a > g$
(c) P will hit the wall CD if $a < g$
(d) may be either (a), (b) or (c), depending on the projection speed of P.

22. A particle starts from the origin of coordinates at time $t = 0$ and moves in the xy plane with a constant acceleration α in the y-direction. Its equation of motion is $y = \beta x^2$. Its velocity component in the x-direction is

(a) variable (b) $\sqrt{\frac{2\alpha}{\beta}}$ (c) $\frac{\alpha}{2\beta}$ (d) $\sqrt{\frac{\alpha}{2\beta}}$

23. A man who can swim at a speed v relative to the water wants to cross a river of width d, flowing with a speed u. The point opposite to him across the river is P. Which of the following statement is false?

(a) The minimum time in which he can cross the river is d/v

(b) he can reach the point P in time dv.

(c) he can reach the point P in time $\dfrac{d}{\sqrt{v^2 - u^2}}$

(d) he cannot reach P if $u > v$.

24. A disc of radius 1 m is rotating about its centre with angular velocity of 20π rad/s. The angular displacement (in radian) of the disc before it comes to rest under constant angular retardation of 2π rad/s^2 is

(a) 100π (b) 100 (c) 10π (d) $100\pi^2$

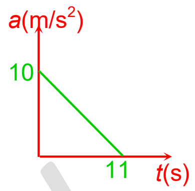
25. A particle moves one quarter of a circular path of radius 20 m in 10 s. The magnitude of average velocity of the particle is

(a) 2.83 m/s (b) 2.73 m/s (c) 2.93 m/s (d) 2.63 m/s

EXERCISE – II

IIT-JEE- SINGLE CHOICE CORRECT

1. A particle starting from rest undergoes a rectilinear motion with acceleration a. The variation of a with time t is shown in the figure. The maximum velocity attained by the particle during the motion is

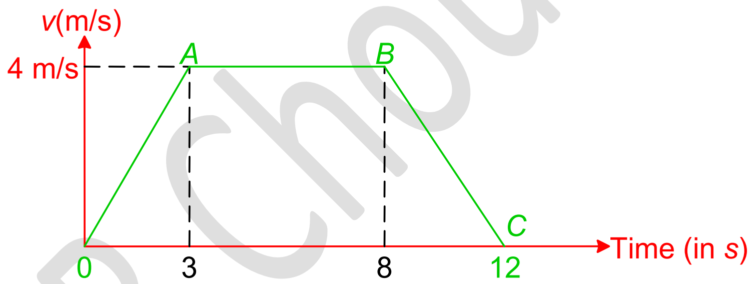


(a) 55 m/s (b) 550 m/s
(c) 110 m/s (d) 650 m/s

2. A bullet is fired into a fixed target looses half of its velocity after penetrating 3 cm. How much further it will penetrate before coming to rest assuming that it faces constant resistance to motion?
(a) 0.5 cm (b) 1 cm (c) 1.5 cm (d) none

3. A trolley runs down a slope from rest with constant acceleration. In the first second of its motion it travels 1.6 m. Its acceleration (in m/s^2) is
(a) 3.2 (b) 1.6 (c) 0.8 (d) 2.4

4. From the velocity-time graph given of a particle moving in a straight line, one can conclude that

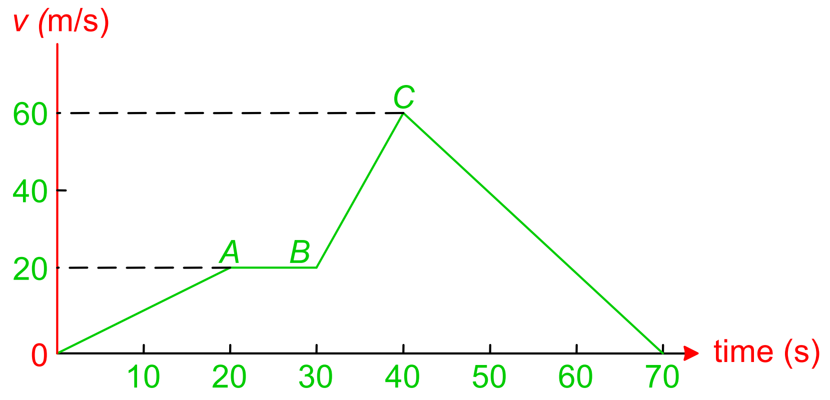


(a) its average velocity during the 12 seconds interval is 24/7 m/s
(b) its velocity for the first 3 seconds is uniform and is equal to 4 m/s
(c) the body has zero acceleration between t = 3 s and t = 8 s
(d) the body has a uniform velocity from t = 8 s to t = 12 s

5. Two trains each of length 90 m moving in opposite directions along parallel tracks meet when their speeds are 60 km/hr and 40 km/hr. If their accelerations are 0.3 m/s^2 and 0.15 m/s^2 respectively, find the time they take to pass each other.
(a) 8 s (b) 4 s (c) 2 s (d) 6.17 s

6. A particle moves with constant acceleration for 6 seconds after starting from rest. The distances travelled during the consecutive 2 seconds interval are in the ratio
(a) 1 : 1 : 1 (b) 1 : 2 : 3 (c) 1 : 3 : 5 (d) 1 : 5 : 9

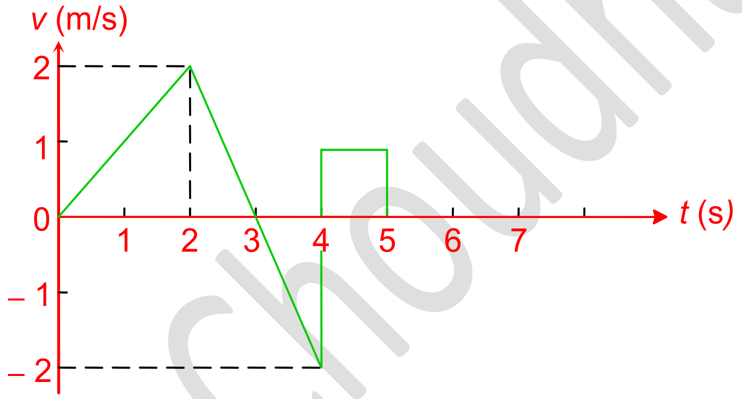
7. The velocity-time graph of a body is given below.



The maximum acceleration in m/s^2 is

(a) 4 (b) 3 (c) 2 (d) 1

8. The velocity-time graph of a body moving along a straight line is as follows:



The displacement of the body in 5 s is

(a) 5 m (b) 2 m (c) 4 m (d) 3 m

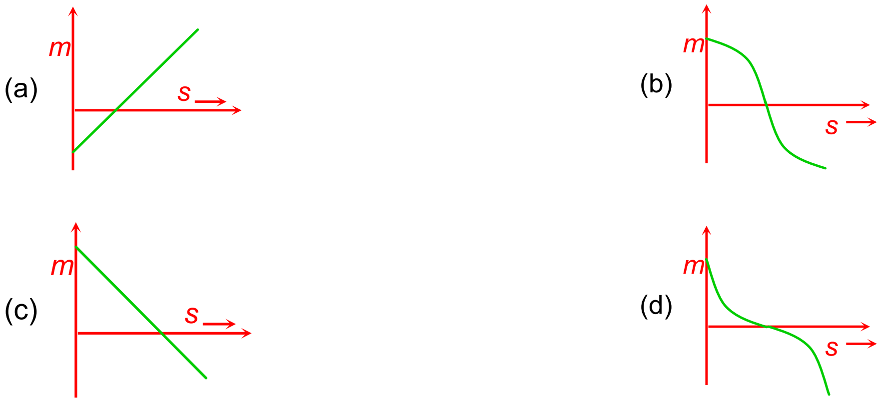
9. A person is standing in a stationary lift drops a coin from a certain height h. It takes time t to reach the floor of the lift. If the lift is rising up with a uniform acceleration a, the time taken by the coin (dropped from the same height h) to reach the floor will be

(a) t (b) $t\sqrt{\frac{a}{g}}$ (c) $t\left(1+\frac{a}{g}\right)^{-\frac{1}{2}}$ (d) $t\left(1-\sqrt{\frac{a}{g}}\right)^{\frac{1}{2}}$

10. A particle is moving in straight line. The velocity v of the particle varies with time t as $v = t^2 - 4t$, then the distance traveled by the particle in $t = 0$ to $t = 6$s (where t in second and v is in m/s).

(a) $\frac{64}{3}$ m (b) zero (c) $\frac{32}{3}$ m (d) none

11. The graph which represents the variation of slope m of the trajectory of a projectile with horizontal displacement s is



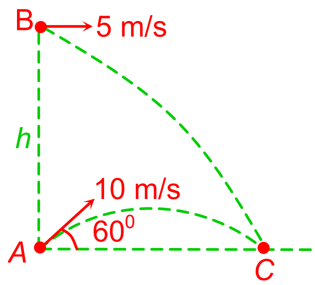
12. A man on the observation platform of a train moving with constant velocity drops a ball while leaning over the railing. The path of the ball as seen by an observer standing on the ground nearby is a

(a) straight line vertically down (b) horizontal straight line
(c) parabola (d) circle

13. Velocity and acceleration of a particle at some instant of time are $\vec{v} = (3\hat{i} + 4\hat{j})$ m/s and $\vec{a} = -(6\hat{i} + 8\hat{j})$ m/s^2 respectively. At the same instant particle is at origin. Maximum x-co-ordinate of particle will be

(a) 1.5 m (b) 0.75 m (c) 2.25 m (d) 4.0 m

14. A particle A is projected from the ground with an initial velocity of 10 m/s at an angle of 60^0 with horizontal. From what height h should an another particle B be projected horizontally with velocity 5 m/s so that both the particles collides at point C if both are projected simultaneously (g = 10 m/s^2)

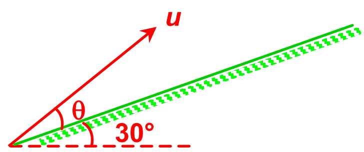


(a) 10 m (b) 30 m
(c) 15 m (d) 25 m

15. A particle has displacement of 40 m along OX in 5 s and 30 m perpendicular to OX (parallel to OY) in the next 5 s. The magnitude of the average velocity in m/s of the particle during the 10 s interval is

(a) 6 (b) 7 (c) 8 (d) 5

16. A particle is projected up an inclined as shown in figure. For maximum range over the inclined plane, the value of θ should be



(a) 45° (b) 15°
(c) 30° (d) 60°

17. A ball rolls off the top of a staircase with a horizontal velocity u ms^{-1}. If the steps are h m high and w m wide the ball will hit the edge of the nth step if

(a) $n = \dfrac{gw^2}{2hu^2}$ (b) $n = \dfrac{2hu^2}{gw^2}$ (c) $n = \dfrac{2u^2}{gw^2h}$ (d) $n = \dfrac{2hw^2u^2}{g}$

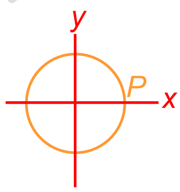
18. A large number of bullets are fired in all directions with the same sped v. The maximum area on the ground on which these bullets will spread is

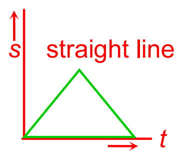
(a) $\dfrac{\pi v^2}{g}$ (b) $\dfrac{\pi v^4}{g^2}$ (c) $\dfrac{\pi^2 v^4}{g^2}$ (d) $\dfrac{\pi^2 v^2}{g^2}$

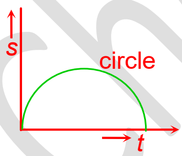
19. When a man moves down the fixed inclined plane with a constant speed 5 m/s which makes an angle of 37^0 with the horizontal, he finds that the rain is falling vertically downward. When he moves up the same inclined plane with the same speed, he finds that the rain makes an angle $\theta = \tan^{-1}\left(\dfrac{7}{8}\right)$ with the horizontal. The speed of the rain is

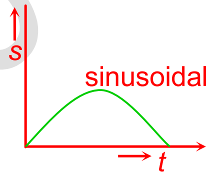
(a) $\sqrt{116}$ m/s (b) $\sqrt{32}$ m/s (c) 5 m/s (d) $\sqrt{73}$ m/s

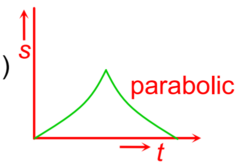
20. A particle moves on a circular path with uniform angular velocity. The magnitude of displacement(s) of the particle from point P can be shown on the graph as



(a)


(b)


(c)


(d)


ONE OR MORE THAN ONE CHOICE CORRECT

1. The displacement-time graph of a particle having on straight line is a straight line. Therefore

(a) its acceleration is non-zero (b) its velocity is constant
(c) its displacement is constant
(d) both its velocity and acceleration are uniform

2. A train moving with a constant speed along a straight track takes a bend in a curve with the same speed. Due to this

(a) its velocity is changed in magnitude (b) its velocity is not changed
(c) particle is accelerated (d) its velocity is changed

3. A ball is thrown upward from the ground with velocity u. It is at a height 100 m at two times t_1 and t_2 respectively. If g = 10 m/s^2, then

(a) $t_1\, t_2 = 20$ (b) $t_1 + t_2 = 20$ (c) $t_1 t_2 = \dfrac{u}{5}$ (d) $t_1 + t_2 = \dfrac{u}{5}$

4. The maximum horizontal range and maximum height attained by a projectile are R and H respectively. If a constant horizontal acceleration $a = g/4$ is imparted to the projectile due to wind, then

(a) its horizontal range is R (b) its horizontal range is $R + H$
(c) its maximum height is $\dfrac{H}{2}$ (d) its maximum height is H

5. A particle is projected from a point P with a velocity v at an angle θ with horizontal. At a certain point Q, it moves at right angles to its initial direction. Then
(a) velocity of particle at Q is $v \sin\theta$
(b) velocity of particle at Q is $v \cot\theta$
(c) time of flight from P to Q is (v/g) cossec θ
(d) time of flight from P to Q is $(v/g) \sec\theta$

6. A particle is projected vertically upward and it will reach at a height from the ground in t_1 sec and time to reach the ground from that height is t_2 then
(a) Initial velocity of projection is $\dfrac{g(t_1+t_2)}{2}$
(b) velocity at half of its maximum height will be $\dfrac{g(t_1+t_2)}{2\sqrt{2}}$
(c) maximum distance travelled by particle in vertical direction is $\dfrac{g}{2}(t_1+t_2)^2$
(d) all of the above

7. Velocity of a particle moving in a curvilinear path varies with time as $\vec{v}=(2t\,\hat{i}+t^2\hat{j})$m/s. Here, t is in second. At $t = 1$s
(a) acceleration of particle is 8 m/s²
(b) tangential acceleration of particle is $\dfrac{6}{\sqrt{5}}$ m/s²
(c) radial acceleration of particle is $\dfrac{2}{\sqrt{5}}$ m/s²
(d) radius of curvature to the path is $\dfrac{5\sqrt{5}}{2}$ m

8. During uniform circular motion of a particle, which of the following is incorrect.
(a) distance-time graph is a straight line
(b) distance-time graph is a parabola
(c) displacement-time graph is a straight line
(d) displacement-time graph is a parabola

9. A car is moving rectilinearly on a horizontal path with acceleration a_0. A person sitting inside the car observes that an insect S is crawling up the screen with an acceleration a. If θ is the inclination of the screen with the horizontal, the acceleration of the insect.
(a) parallel to screen is $a + a_0 \cos\theta$
(b) along the horizontal is $a_0 - a\cos\theta$
(c) perpendicular to screen is $a_0 \sin\theta$
(d) perpendicular to screen is $a_0 \tan\theta$

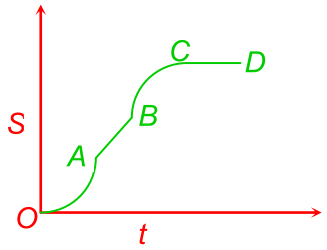
10. River is flowing with a velocity $\vec{v}_R = 4\hat{i}$ m/s. A boat is moving with a velocity of $\vec{v}_{BR} = (-2\hat{i}+4\hat{j})$ m/s relative to river. The width of the river is 100 m along y-direction. Choose the correct alternative(s)
(a) the boat will cross the river in 25s
(b) absolute velocity of boat is $2\sqrt{5}$ m/s
(c) drift of the boat along the river current is 50 m
(d) the boat can never cross the river.

EXERCISE – III

MATCH THE FOLLOWING

Note: Each statement in column – I has one or more than one match in column –II.

1. The displacement versus time graph of a particle is as shown in figure.



Column-I	Column-II
I. In path *OA*	**A.** Velocity increases with time.
II. In path *AB*	**B.** Magnitude of acceleration is a non-zero.
III. In path *BC*	**C.** Velocity is a non-zero constant
IV. In path *CD*	**D.** acceleration is zero
	E. velocity is zero

REASONING TYPE

Directions: Read the following questions and choose

(A) If both the statements are true and statement-2 is the correct explanation of **statement-1**.
(B) If both the statements are true but statement-2 is not the correct explanation of **statement-1**.
(C) If statement-1 is True and **statement-2** is False.
(D) If statement-1 is False **and statement-2** is True.

1. **Statement-1**: A body having uniform speed in circular path has a constant acceleration.
Statement-2: Direction of acceleration is always towards the centre.
(a) (A) (b) (B) (c) (C) (d) (D)

2. **Statement-1**: Two bodies of masses *M* and *m* ($M > m$) are allowed to fall from the same height if the force of air resistance for each be the same then both the bodies will reach the earth simultaneously.
Statement-2: Acceleration due to gravity does not depend on the mass of the body.
(a) (A) (b) (B) (c) (C) (d) (D)

3. **Statement-1**: During the motion, the magnitude of displacement may be equal to distance.
Statement-2: When the particle is moving on the straight line then displacement will be equal to distance.
(a) (A) (b) (B) (c) (C) (d) (D)

4. **Statement-1**: Net acceleration of a particle, moving on a circle with constant tangential acceleration, remains constant.

Statement-2: $a_N = \sqrt{a_r^2 + a_T^2}$, where a_N is magnitude of net acceleration, a_r is magnitude of centripetal acceleration and a_T is magnitude of tangential acceleration.

(a) (A) (b) (B) (c) (C) (d) (D)

5. **Statement-1**: If $\vec{v}$ is the velocity of the particle in motion at time *t*, then $\left|\int \vec{v}\, dt\right| \le \int |\vec{v}|\, dt$

Statement-2: $\left|\int \vec{v} dt\right|$ = magnitude of displacement. $\int |\vec{v}| dt$ = distance covered.

(a) (A) (b) (B) (c) (C) (d) (D)

LINKED COMPREHENSION TYPE

Two particles are moving on a circle of radius *a* with uniform speed *v*, one is moving on a peripheral of the circle while other is oscillating on its diameter between the ends of the diameter. Let us suppose that *T* is time period of one rotation for first particle and assume both will starts from same point on circle then

1. What is the magnitude of the relative velocity at $t = 0$

(a) v (b) $\sqrt{2}v$ (c) $2v$ (d) $\sqrt{3}v$

2. What will be the magnitude of the relative velocity at $t = T/6$

(a) $v\sqrt{2-\sqrt{3}}$ (b) $v\sqrt{2+\sqrt{3}}$ (c) $v\sqrt{4-\sqrt{3}}$ (d) $v\sqrt{2-\sqrt{2}}$

3. What will the magnitude of the relative velocity at $t = \frac{T}{3}$.

(a) $v\sqrt{2+\sqrt{3}}$ (b) $v\sqrt{2-\sqrt{3}}$ (c) $v\sqrt{4-\sqrt{3}}$ (d) $v\sqrt{2-\sqrt{2}}$

EXERCISE – IV

SUBJECTIVE PROBLEMS

1. A train passes a station A at 40 km/hr and maintains this speed for 7 km and is then uniformly retarded, stopping at B, which is 8.5 km from A. A second train starts from A at the instant the first train passes and being accelerated for part of the journey and uniformly retarded for the rest stops at B at the same time as the first train. What is the greatest speed of the second train?

2. A particle is moving in a straight line and is observed to be at a distance 'a' from a marked point initially, to be at a distance 'b' after an interval of n seconds, to be at a distance 'c' after $2n$ seconds and to be at a distance 'd' after $3n$ seconds. Prove that if the acceleration is uniform, $d - a = 3(c - b)$ and that the acceleration is equal to $\frac{c+a-2b}{n^2}$.

3. A particle projected with velocity u strikes at right angles a plane through the point of projection inclined at an angle β to the horizon. Show that the height of the point struck above the horizontal plane through the point of projection is $\frac{2u^2}{g}\left(\frac{\sin^2\beta}{1+3\sin^2\beta}\right)$ and that the time of flight up to that instant is, $t=\frac{2u}{g\sqrt{1+3\sin^2\beta}}$.

4. A particle is projected with a velocity $2\sqrt{ag}$ so that it just clears two walls of equal height 'a', which are at a distance $2a$ apart. Show that the time of passing between the walls is $2\sqrt{a/g}$.

5. A bottle was released from rest from a height of h=60 m above the ground. Simultaneously, a stone was thrown from a point on the ground s= 60 m distant horizontally from the bottle, with a velocity u at an angle of projection of θ, in a vertical plane containing the bottle. If the stone strikes the bottle t = 3s after the instant of projection, find the velocity u and the angle θ of projection.

6. An aeroplane flies horizontally at height h at a constant speed v. An anti-aircraft gun fires a shell at the plane when it is vertically above the gun. Show that the minimum muzzle velocity of the shell required to hit the plane is $\sqrt{v^2+2gh}$ at an angle $\tan^{-1}(\sqrt{2gh}/v)$.

7. A projectile is projected with a velocity u at an angle α to the horizontal, in the vertical plane. If after time t, it is moving in a direction making an angle β with the horizontal, prove that $gt\cos\beta = u\sin(\alpha-\beta)$

8. t(=2s) seconds after the projection, a projectile is moving in a direction at an angle θ (=30°) to the horizontal. After one more second, it is moving horizontally. Determine the magnitude and direction of the initial velocity.

9. Two particles start simultaneously from the same point and move along two straight lines, one with uniform velocity u and the other with constant acceleration f. Show that their relative velocity is least after time $\left(\frac{u\cos\alpha}{f}\right)$ and that the least relative velocity is $u\sin\alpha$, where α is the angle between the lines.

10. Two trains each having a speed of 30 km/hr are headed at each other on the same straight track. A bird that can fly at 60 km/hr flies off one train when they are 60 km apart and heads directly for the other train. On reaching the other train it flies directly back to the first and so forth. Find (a) how many trips can the bird make from one train to the other before the trains collide? (b) what is the total distance travelled by the bird?

ANSWERS

EXERCISE – I

NEET-SINGLE CHOICE CORRECT

1. (b)	2. (b)	3. (b)	4. (b)	5. (c)
6. (b)	7. (b)	8. (c)	9. (a)	10. (d)
11. (c)	12. (a)	13. (c)	14. (d)	15. (b)
16. (a)	17. (c)	18. (a)	19. (b)	20. (a)
21. (d)	22. (d)	23. (b)	24. (a)	25. (a)

EXERCISE – II

IIT-JEE-SINGLE CHOICE CORRECT

1. (a)	2. (b)	3. (a)	4. (c)	5. (d)
6. (c)	7. (a)	8. (d)	9. (c)	10. (a)
11. (c)	12. (c)	13. (b)	14. (c)	15. (d)
16. (c)	17. (b)	18. (b)	19. (b)	20. (c)

ONE OR MORE THAN ONE CHOICE CORRECT

1. (b,c)	2. (c,d)	3. (a,d)	4. (b,d)	5. (b,c)
6. (a,b)	7. (b,c,d)	8. (b,c,d)	9. (b,c)	10. (a,b,c)

EXERCISE – III

MATCH THE FOLLOWING

1. I – A, B; II – C, D; III – B; IV – D, E

REASONING TYPE

1. (d)	2. (d)	3. (c)	4. (d)	5. (a)

LINKED COMPREHENSION TYPE

1. (b)	2. (a)	3. (a)

EXERCISE – IV

SUBJECTIVE PROBLEMS

1. 68 km/hr

5. 28.2 m/s, θ = 45°; $u = \frac{h\sqrt{2}}{t}, \theta = 45^0$

8. $2g\sqrt{3}$, 60°; $2g\sqrt{3},\ \theta = \tan^{-1}\left[\frac{2\sin\theta + t}{2\cos\theta}\right]$

10. (a) Infinity (b) 60 km

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 1

Q.1 A runner completes one round of a circular path of radius r in 40 seconds. His displacement after 2 minutes 20 seconds will be -

(1) Zero (2) $2\pi r$

(3) 2r (4) $7\pi r$

Q.2 A particle covers half of the circle of radius r. Then the displacement and distance of the particle are respectively -

(1) $2\pi r$, 0 (2) 2r, πr

(3) $\frac{\pi r}{2}$, 2r (4) πr, r

Q.3 A car travels from place A to the place B at 20 km/hour and returns at 30 km/hour. The average speed of the car for the whole journey is-

(1) 25 km/hour (2) 24 km/hour

(3) 50 km/hour (4) 5 km/hour

Q.4 A car travels a distance of 2000 m. If the first half distance is covered at 40 km/hour and the second half at velocity v and if the average velocity is 48 km/hour, then the value of v is -

(1) 56 km/hour (2) 60 km/hour (3) 50 km/hour (4) 48 km/hour

Q.5 A car travels the first half of the journey at 40 km/hour and the second half at 60 km/hour. The average speed of a car is -

(1) 40 km/hour (2) 48 km/hour

(3) 52 km/hour (4) 60 km/hour

Q.6 If a body starts from rest, the time in which it covers a particular displacement with uniform acceleration is-

(1) inversely proportional to the square root of the displacement

(2) inversely proportional to the displacement

(3) directly proportional to the displacement

(4) directly proportional to the square root of the displacement

Q.7 A truck travelling due north at 20m/s turns west and travels with the same speed. What is the change in velocity ?

(1) 40 m/s north-west (2) $20\sqrt{2}$ m/s north-west

(3) 40 m/s south-west (4) $20\sqrt{2}$ m/s south-west

Q.8 A car covers half of the distance with speed 60 km/hr and rest of the half with speed 30 km/hr. The average speed of the car is -

(1) 45 km/hr (2) 40 km/hr (3) 20.0 km/hr (4) 50 km/hr

Q.9 A motor car covers 1/3rd part of total distance with v_1 = 10 km/hr, second 1/3rd part with v_2 = 20 km/hr and rest 1/3rd part with v_3 = 60 km/hr. What is the average speed of the car?

(1) 18 km/hr (2) 45 km/hr

(3) 6 km/hr (4) 22.5 km/hr

Q.10 A car travels first 1/3 of the distance AB at 30 km/hr, next 1/3 of the distance at 40 km/hr, last 1/3 of the distance at 24 km/hr. Its average speed in km/hr for the whole journey is -

(1) 40 (2) 35
(3) 30 (4) 28

Q.11 A train travels from one station to another at a speed of 40 km/hour and returns to the first station at the speed of 60 km/hour. Calculate the average speed and average velocity of the train -

(1) 48 km/hr, zero (2) 84 km/hr, 10 km/hr
(3) 84 km/hr, zero (4) 48 km/hr, 10 km/hr

Q.12 A passenger travels along a straight line with velocity v_1 for first half time and with velocity v_2 for next half time, then the mean velocity v is given by -

(1) $v = \frac{v_1 + v_2}{2}$ (2) $v = \sqrt{v_1 v_2}$ (3) $v = \sqrt{\frac{v_2}{v_1}}$ (4) $\frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}$

Q.13 At an instant t, the co-ordinates of a particle are $x = at^2$, $y = bt^2$ and $z = 0$, then its velocity at the instant t will be -

(1) $t\sqrt{a^2+b^2}$ (2) $2t\sqrt{a^2+b^2}$
(3) $\sqrt{a^2+b^2}$ (4) $2t^2\sqrt{a^2+b^2}$

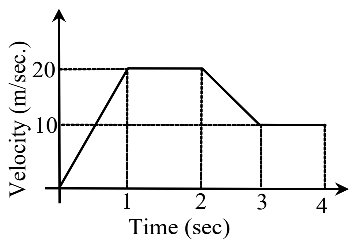
Q.14 The displacement y (in meters) of a body varies with time (in seconds) according to the equation $y = -\frac{2}{3}t^2 + 16t + 2$. How long does the body come to rest ?

(1) 8 seconds (2) 10 seconds
(3) 12 seconds (4) 14 second

Q.15 The initial velocity of a particle (at t = 0) is u and the acceleration of particle at time t is given by f = at, where a is a constant. Which of the following relation for velocity v of particle after time t is true?

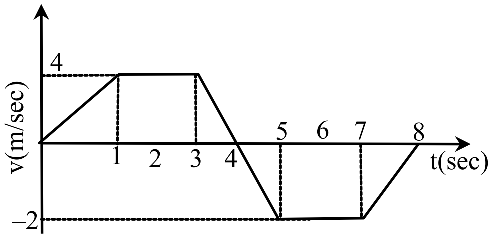
(1) $v = u + at^2$ (2) $v = u + at^2/2$
(3) $v = u + at$ (4) None of these

Q.16 The variation of velocity of a particle moving along straight line is shown in figure. The distance traversed by the body in 4 seconds is -



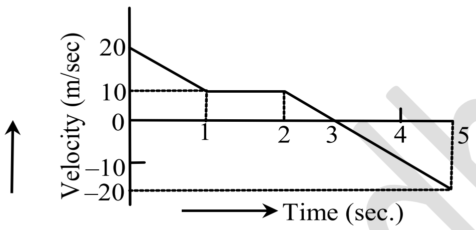
(1) 70 m (2) 60 m
(3) 40 m (4) 55 m

Q.17 The v-t graph of a linear motion is shown in adjoining figure. The distance from origin after 8 seconds is -



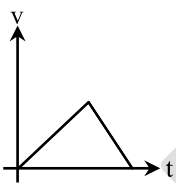
(1) 18 meters (2) 16 meters
(3) 8 meters (4) 6 meters

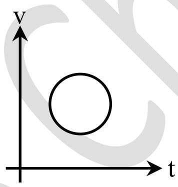
Q.18 From figure the distance travelled in 5 second is –

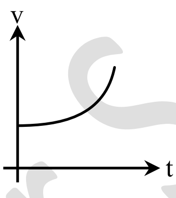


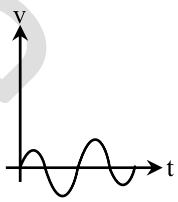
(1) 10m (2) 30m
(3) 50m (4) zero

Q.19 Which one of the following curves do not represent motion in one dimension-

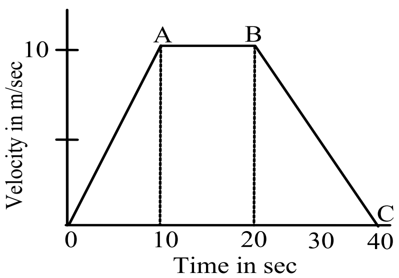
(1)


(2)


(3)


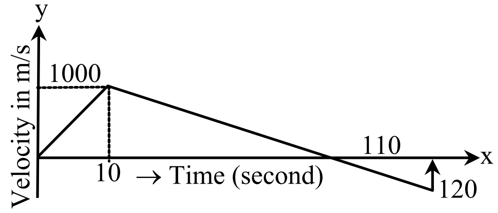
(4)


Q.20 The adjoining curve represents the velocity-time graph of a particle, its acceleration values along OA, AB and BC in metre/sec^2 are respectively-



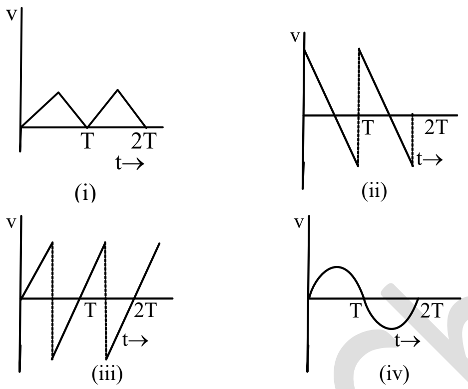
(1) 1, 0, –0.5
(2) 1, 0, 0.5
(3) 1, 1, 0.5
(4) 1, 0.5, 0

Q.21 Adjacent graph shows the variation of velocity of a rocket with time. Find the time of burning of fuel from the graph-



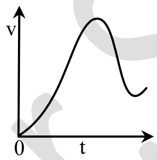
(1) 10 sec
(2) 110 sec
(3) 120 sec
(4) cannot be estimated from the graph

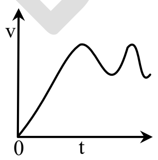
Q.22 A ball is dropped from certain height on a glass floor so that it rebounds elastically to the same height. If the process continues, the velocity -time graph for such a motion would be -

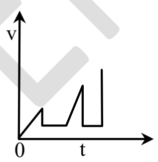


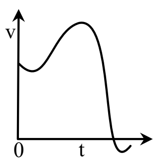
(1) (i) (2) (ii)
(3) (iii) (4) (iv)

Q.23 The following figures show some velocity v versus time t curves. But only some of these can be realised in practice. This are-

(a) 

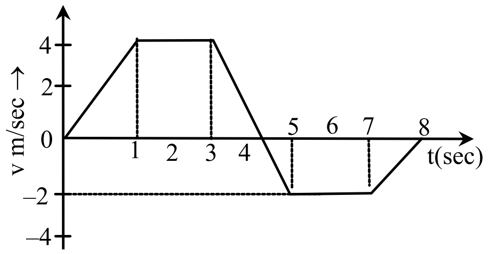
(b) 

(c) 

(d) 

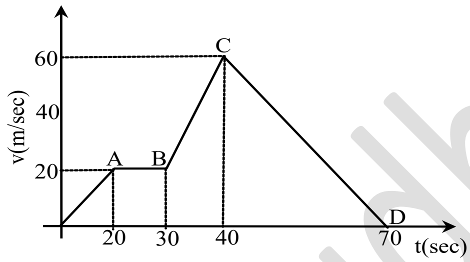
(1) only a, b and d (2) only a , b , c
(3) only b and c (4) all of them

Q.24 The velocity-time graph of a linear motion is shown below. The displacement from the origin after 8 seconds is -



(1) 18m (2) 16 m (3) 6m (4) 6 cm

Q.25 The following shows the time-velocity graph for a moving object. The maximum acceleration will be-



(1) 1 m/sec^2 (2) 2m/sec^2
(3) 3m/sec^2 (4) 4m/sec^2

Q.26 A particle moves from the position of rest and attains a velocity of 30 m/sec after 10sec. The acceleration will be-
(1) 9 m/sec^2 (2) 18 m/sec^2
(3) 3 m/sec^2 (4) 4 m/sec^2

Q.27 The relation between time t and displacement x is expressed by $x = 2 - 5t + 6t^2$. What will be the initial velocity of the particle ?
(1) –5m/sec (2) –3m/sec
(3) 6m/sec (4) 3m/sec

Q.28 A particle, after starting from rest , experiences, constant acceleration for 20 seconds. If it covers a distance of S_1, in first 10 seconds and distance S_2 in next 10 sec, then -
(1) $S_2 = S_1/2$ (2) $S_2 = S_1$
(3) $S_2 = 2S_1$ (4) $S_2 = 3S_1$

Q.29 A body sliding on a smooth inclined plane requires 4sec to reach the bottom after starting from rest at the top. How much time does it take to cover one fourth the distance starting from the top -
(1) 1sec (2) 2 sec
(3) 0.4sec (4) 1.6 sec

Q.30 A moving train is stopped by applying brakes. It stops after travelling 80m. If the speed of the train is doubled and retardation remains the same, it will cover a distance -
(1) same as earlier
(2) double the distance covered earlier
(3) four times the distance covered earlier
(4) half the distance covered earlier

Q.31 If u is the initial velocity of a body and a the acceleration , the value of distance travelled by n^{th} second is -

(1) $u + \frac{1}{2}a(2n+1)$ (2) $u + \frac{1}{2}a(2n-1)$

(3) $u - \frac{1}{2}a(2n+1)$ (4) $u - \frac{1}{2}a(2n-1)$

Q.32 A body starts from rest, the ratio of distances travelled by the body during 3^{rd} and 4^{th} seconds is -

(1) 7/5 (2) 5/7 (3) 7/3 (4) 3/7

Q.33 A body starting from rest and has uniform acceleration 8 m/s^2. The distance travelled by it in 5^{th} second will be -

(1) 36m (2) 40m

(3) 100m (4) 200m

Q.34 Which one of the following equations represent the motion of a body with finite constant acceleration. In these equations y denotes the position of the body at time t and a, b, and c are the constant of the motion -

(1) $y = a/t + bt$ (2) $y = at$

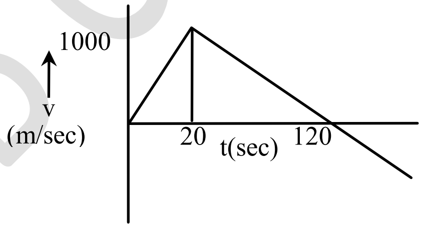
(3) $y = at + bt^2$ (4) $y = at + bt^2 + ct^3$

Q.35 A particle travels for 40 seconds under the influence of a constant force. If the distance travelled by the particle is S_1 in the first twenty seconds and S_2 in the next twenty second, then-

(1) $S_2 = S_1$ (2) $S_2 = 2S_1$

(3) $S_2 = 3S_1$ (4) $S_2 = 4S_1$

Q.36 A rocket is projected vertically upwards and its time-velocity graph is shown in the figure. The maximum height attained by the rocket is -



(1) 1km (2) 10km

(3) 100km (4) 60km

Q.37 An object is released from some height. Exactly after one second, another object is released from the same height. The distance between the two objects exactly after 2 seconds of the release of second object will be -

(1) 4.9 m (2) 9.8 m

(3) 19.6 m (4) 24.5 m

Q.38 A stone is thrown vertically upwards from the top of a tower with a velocity u and it reaches the ground with a velocity 3u. The height of the tower is -

(1) $3u^2/g$ (2) $4u^2/g$ (3) $6u^2/g$ (4) $9u^2/g$

Q.39 A stone is dropped from a bridge and it reaches the ground in 4 seconds. The height of the bridge is -

(1) 78.4m (2) 64m (3) 260m (4) 2000m

Q.40 Two bodies of different masses m_a and m_b are dropped from two different heights, viz a and b. The ratio of times taken by the two to drop through these distances is -

(1) a : b (2) $\frac{m_a}{m_b} : \frac{b}{a}$

(3) $\sqrt{a} : \sqrt{b}$ (4) $a^2 : b^2$

Q.41 A body thrown up with a finite speed is caught back after 4 sec. The speed of the body with which it is thrown up is -

(1) 10 m/sec (2) 20 m/sec

(3) 30 m/sec (4) 40 m/sec

Q.42 A stone is thrown vertically upwards with an initial velocity of 30 m/s. The time taken for the stone to rise to its maximum height is -

(1) 0.326s (2) 3.26s

(3) 30.6s (4) 3.06s

Q.43 A body is thrown upward and reaches its maximum height. At that position -

(1) its velocity is zero and its acceleration is also zero

(2) its velocity is zero but its acceleration is maximum

(3) its acceleration is minimum

(4) its velocity is zero and its acceleration is the acceleration due to gravity

Q.44 Two trains each of length 50 m are approaching each other on parallel rails. Their velocities are 10 m/sec and 15 m/sec. They will cross each other in -

(1) 2 sec (2) 4 sec

(3) 10 sec (4) 6 sec

Q.45 A train is moving in the north at a speed 10 m/sec. Its length is 150 m. A parrot is flying parallel to the train in the south with a speed of 5 m/s. The time taken by the parrot to cross the train will be -

(1) 12 sec (2) 8 sec

(3) 15 sec (4) 10 sec

Q.46 A horse rider is moving towards a big mirror with velocity v. The velocity of his image with respect to him is -

(1) 0 (2) 4 v

(3) 2 v (4) v

Q.47 A particle moves with constant speed v along a regular hexagon ABCDEF in same order (i.e. A to B , B to C, C to D, D to E, E to F, F to A...). Then magnitude of average velocity for its motion from A to C is -

(1) v (2) v/2 (3) $\sqrt{3}$ v/2 (4) None of these

Q.48 A particle moves with a velocity v in a horizontal circular path. The change in its velocity for covering 60º will be -

(1) v $\sqrt{2}$ (2) v/$\sqrt{2}$ (3) v$\sqrt{3}$ (4) v

Q.49 A body is dropped from a height h from the state of rest. It covers a distance of 9h/25 in the last second. What is the height from which the body falls? (in meter)

(1) 12.5 (2) 1.25 (3) 125 (4) Zero

Q.50 If the position vector of a particle is $\vec{r} = 3t\hat{i} - 4\hat{j} + \hat{k}$, the particle will be -

(1) moving with uniform velocity
(2) stationary
(3) moving with uniform acceleration
(4) insufficient data

Q.51 A particle is executing a circular motion of radius R with a uniform speed v. After completing half the circle, the change in velocity and in speed will be respectively -

(1) zero, zero (2) 2v, zero
(3) 2v, 2v (4) zero, 2v

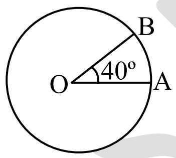
Q.52 The numerical ratio of displacement to distance is-

(1) always < 1 (2) always = 1
(3) always > 1 (4) ≤ 1

Q.53 A man walks 30 m towards north, then 20 m towards east and in the last $30\sqrt{2}$ m towards south-west. The displacement from origin is-

(1) 10 m towards west
(2) 10 m towards east
(3) $60\sqrt{2}$ m towards north west
(4) $60\sqrt{2}$ m towards east north

Q.54 A person is moving in a circle of radius r with constant speed v. The change in velocity in moving from A to B is-



(1) 2 v cos 40º (2) 2 v sin 40º
(3) 2 v cos 20º (4) 2 v sin 20º

Q.55 A man walks for some time 't' with velocity (v) due east. Then he walks for same time 't' with velocity (v) due north. The average velocity of the man is-

(1) 2v (2) $\sqrt{2}$ v (3) v (4) $\frac{v}{\sqrt{2}}$

Q.56 A stone is dropped from a height h. Simultaneously, another stone is thrown up from the ground which reaches a height 4h. The two stones will cross each other after time-

(1) $\sqrt{\frac{h}{8g}}$ (2) $\sqrt{8gh}$ (3) $\sqrt{2gh}$ (4) $\sqrt{\frac{h}{2g}}$

Q.57 Acceleration is defined as the rate of change of velocity. Unit of rate of change of acceleration is-

(1) m/s^2 (2) m/s^3
(3) m/s (4) m^2/s^3

Q.58 A particle moves along the side AB, BC, CD of a square of side 25 cm with a velocity of 15 m/s. Its average velocity is-

(1) 15 m/s (2) 10 m/s
(3) 7.5 m/s (4) 5 m/s

Q.59 A train accelerates from rest at a constant rate α for distance x_1 and time t_1. After that retards rest at to constant rate β for distance x_2 in time t_2 and comes to the rest, which of the following relation is correct -

(1) $\frac{x_1}{x_2} = \frac{\alpha}{\beta} = \frac{t_1}{t_2}$ (2) $\frac{x_1}{x_2} = \frac{\beta}{\alpha} = \frac{t_1}{t_2}$

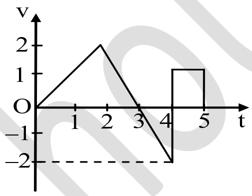
(3) $\frac{x_1}{x_2} = \frac{\alpha}{\beta} = \frac{t_2}{t_1}$ (4) $\frac{x_1}{x_2} = \frac{\beta}{\alpha} = \frac{t_2}{t_1}$

Q.60 The length of a second's hand of a watch is 1cm. The change in velocity of its tip in 15 second is-

(1) zero (2) $\frac{\pi}{30\sqrt{2}}$ cm/s

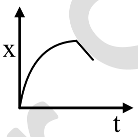
(3) $\frac{\pi}{30}$ cm/s (4) $\frac{\pi}{30} \times \sqrt{2}$ cm/s

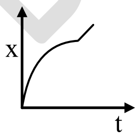
Q.61 The velocity verses time graph of a body moving along a straight line is as shown in figure. The ratio of displacement and distance covered by body in 5 second is-

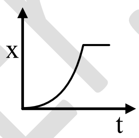


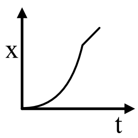
(1) 2 : 3 (2) 3 : 5
(3) 1 : 1 (4) 1.5 : 5

Q.62 A car starts from rest accelerates uniform for 4 second and then moves with uniform velocity which of the x-t graph represent the motion of the car-

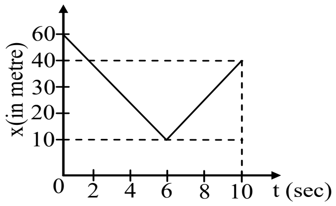
(1) 

(2) 

(3) 

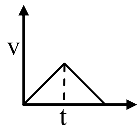
(4) 

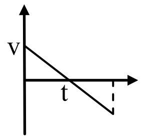
Q.63 The figure shows the displacement time graph of a particle moving on a straight line path. What is the average velocity of the particle over 10 seconds-

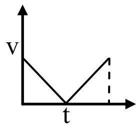


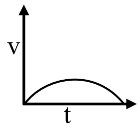
(1) 2 m/s (2) 4 m/s
(3) 6 m/s (4) 8 m/s

Q.64 A body is projected vertically upward from the surface of the earth, then the velocity time graph is-

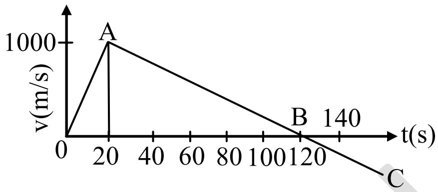
(1) 

(2) 

(3) 

(4) 

Q.65 A rocket is launched upward from the earth surface whose velocity time graph shown in figure. Then maximum height attained by the rocket is-



(1) 1 km (2) 10 km
(3) 100 km (4) 60 km

Q.66 In the above question **(Q.65)** covered height by the rocket before retardation is-
(1) 1 km (2) 10 km
(3) 20 km (4) 60 km

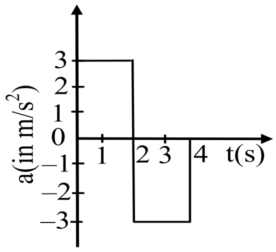
Q.67 In above question **(Q.65)** mean velocity of rocket during it attained the maximum height-
(1) 100 m/s (2) 50 m/s
(3) 500 m/s (4) 25/3 m/s

Q.68 In above question **(Q.65)** the retardation of rocket is-
(1) 50 m/s^2 (2) 100 m/s^2
(3) 500 m/s^2 (4) 10 m/s^2

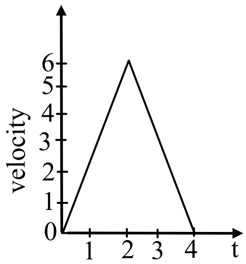
Q.69 In above question **(Q.65)** the acceleration of rocket is-
(1) 50 m/s^2 (2) 100 m/s^2
(3) 10 m/s^2 (4) 1000 m/s^2

Q.70 In above question **(Q.65)** the rocket goes up and get down on the following parts respectively-
(1) OA and AB (2) AB and BC
(3) OA and ABC (4) OAB and BC

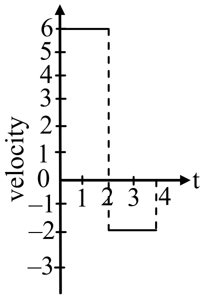
Q.71 For motion of a particle acceleration time graph is shown in figure then the velocity time curve for the duration of 0-4 sec is-



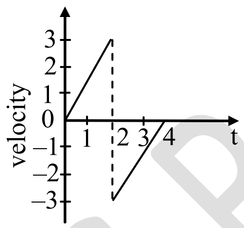
(1)



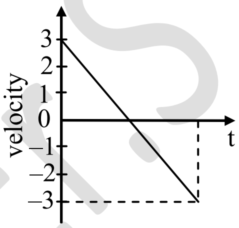
(2)



(3)

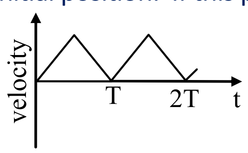


(4)

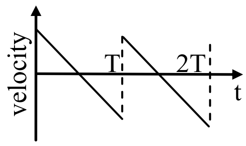


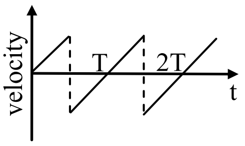
Q.72 A ball is dropped from the certain height on the surface of glass. It is collide elastically the comes back to initial position. If this process it repeated then the velocity time graph is-

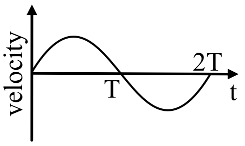
(1)



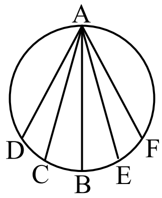
(2)



(3) 

(4) 

Q.73 A disc in which several grooves are cut along the chord drawn from a point 'A', is arranged in a vertical plane, several particles starts slipping from 'A' along the grooves simultaneously. Assuming friction and resistance negligible, the time taken in reaching the edge of disc will be-



(1) Maximum in groove AB
(2) Maximum in groove AD
(3) Same in all groove
(4) According to the heights of B, C, D, E, F

Q.74 A body slides on an inclined plane. If height of inclined plane is 'h' and length is 'ℓ' and angle of inclination is θ then time is taken for travelling from upper point to lower point-

(1) $\sqrt{\frac{2h}{g}}$ (2) $\sqrt{\frac{2\ell}{g}}$
(3) $\frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$ (4) $\sin\theta\sqrt{\frac{2h}{g}}$

Q.75 A river is flowing at the rate of 6 km/hr. A swimmer swims across with a velocity of 9 km/hr with respect to water. The resultant velocity of the man will be in (km/hr) -

(1) $\sqrt{117}$ (2) $\sqrt{340}$ (3) $\sqrt{17}$ (4) $3\sqrt{40}$

Q.76 If x denotes displacement in time t and x = a cos t, then acceleration is-

(1) a cos t (2) – a cos t
(3) a sin t (4) – a sin t

Q.77 If displacement of a particle is zero, the distance covered-

(1) must be zero
(2) may or may not be zero
(3) cannot be zero
(4) depends upon the particle

Q.78 If the distance covered is zero, the displacement-

(1) must be zero
(2) may or may not be zero
(3) cannot be zero
(4) depends upon the particle

Q.79 The numerical value of the ratio of velocity to speed is-

(1) always less than one

(2) always equal to one

(3) always more than one

(4) equal to or less than one

Q.80 A body displaces through 30m eastwards and then moves 20m northwards. The total displacement is-

(1) $10\sqrt{13}$ m (2) $10\sqrt{5}$ m

(3) $10\sqrt{7}$ m (4) 50 m

Q.81 The magnitude of average velocity is equal to the average speed when a particle moves-

(1) one a curved path

(2) in the same direction

(3) with constant acceleration

(4) with constant retardation

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 2

Q.1 The velocity of a particle moving with constant acceleration at an instant t_0 is 10 m/s. After 5 seconds of that instant the velocity of the particle is 20 m/s. The velocity at 3 second before t_0 is-

(1) 8 m/s (2) 4 m/s
(3) 6 m/s (4) 7 m/s

Q.2 Six particles situated at the corners of a regular hexagon of side (1) move at a constant speed (v) each particle maintains a direction towards the particle at the next corner. The time taken by the particles to meet each other is-

(1) $\frac{a}{v}$ (2) $\frac{2a}{v}$ (3) $\frac{a\sqrt{3}}{v}$ (4) $\frac{a}{\sqrt{3}\,v}$

Q.3 A stone is dropped into a well in which the level of water is h below the top of the well. If v is velocity of sound, the time T after which the splash is heard is given by -

(1) $T = \frac{2h}{v}$ (2) $T = \sqrt{\frac{2h}{g}} + \frac{h}{v}$
(3) $T = \sqrt{\frac{2h}{v}} + \frac{h}{g}$ (4) $T = \sqrt{\frac{h}{2g}} + \frac{2h}{v}$

Q.4 The engine of a train passes an electric pole with a velocity 'u' and the last compartment of the train crosses the same pole with a velocity 'v'. Then the velocity with which the mid-point of the train passes the pole is-

(1) u (2) v
(3) $\frac{u+v}{2}$ (4) $\sqrt{\frac{u^2+v^2}{2}}$

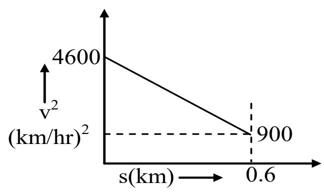
Q.5 A particle starts from rests and travels a distance S with uniform acceleration, then it travels a distance 2S with uniform speed, finally it travels a distance 3S with uniform retardation and comes to rest. If the complete motion of the particle is a straight line then the ratio of its average velocity to maximum velocity is-

(1) $\frac{6}{7}$ (2) $\frac{4}{5}$ (3) $\frac{3}{5}$ (4) $\frac{2}{5}$

Q.6 A stone falls from a balloon that is descending at a uniform rate of 12 m/s. The displacement of the stone from the point of release after 10 sec is-

(1) 490 m (2) 510 m
(3) 610 m (4) 725 m

Q.7 A graph between the square of the velocity of a particle and the distance (s) moved is shown in figure. The acceleration of the particle in kilometers per hour square is-



(1) 2250 (2) 3084

(3) –2250 (4) – 3084

Q.8 A person walks up a stalled escalator in 90 sec. When standing on the same escalator now moving, he is carried in 60 sec. The time he would take to walk up the moving escalator will be-

(1) 27 s (2) 72 s
(3) 18 s (4) 36 s

Q. 9 Position of a particle moving along x-axis is given by $x = 2 + 8t - 4t^2$. The distance travelled by the particle from t = 0 t = 2 is-

(1) 0 (2) 8
(3) 12 (4) 16

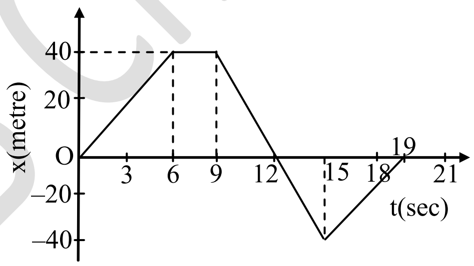
Q.10 A particle is thrown up vertically with a speed 'v_1', in air. It takes time t_1 in upward journey and $t_2(>t_1)$ in the downward journey and returns to the starting point with a speed v_2. Then-

(1) $v_1 = v_2$
(2) $v_1 < v_2$
(3) $v_1 > v_2$
(4) Data is insufficient

Q.11 A particle of mass 3 kg moves under the force of $4\hat{i}+8\hat{j}+10\hat{k}$ N. If the particle starts from rest and was at origin initially. Its new co-ordinates after 3 second is-

(1) (4, 8, 10) (2) (6, 12, 15)
(3) (2, 4, 5) (4) (3, 6, 7.5)

Q.12 A person walks along an east-west street and a graph of his displacement from home is shown in figure. His average velocity for the whole time interval is-



(1) 0 m/s (2) 23 m/s
(3) 8.4 m/s (4) None of above

Q.13 A body is falling from height 'h' it takes t_1 time to reach the ground. The time taken to cover the first half of height is-

(1) $t_2 = \frac{t_1}{\sqrt{2}}$ (2) $t_1 = \frac{t_2}{\sqrt{2}}$

(3) $t_2 = \sqrt{3}\,t_1$ (4) None of these

Q.14 From a tower of height 'h' a particle is projected horizontally with velocity 'u' and another particle thrown down with the same velocity 'u'. If the time taken by these be t_1 and t_2 respectively-

(1) $t_1 = t_2$ (2) $t_1 > t_2$
(3) $t_1 < t_2$ (4) $t_1 = 3t_2$

Q.15 Two balls are dropped from different heights at different instants. Second ball is dropped 2 sec after the first ball. If both balls reach the ground simultaneously after 5 sec of dropping the first ball, the difference of initial heights of the two balls will be - (g = 9.8 m/s^2)

(1) 58.8 m (2) 78.4 m

(3) 98.0 m (4) 117.6 m

Q.16 Two balls are thrown simultaneously, (a) vertically upwards with a speed of 20 m/s from the ground and (b) vertically downwards from a height of 40 m with the same speed and along the same line of motion. At which point the balls will collide- (take g = 10 m/s^2)

(1) 15 m above from the ground

(2) 15 m below from the top of the tower

(3) 20 m above from the ground

(4) 20 m below from the top of the tower

Q.17 A body is dropped from the top of a tower covers $\frac{7}{16}$ of total height in the last second of its fall. The time of fall is -

(1) 2 s (2) 4 s (3) < 1 s (4) > 5 s

Q.18 A ball is dropped from the top of a tower which is 'h' meter high, reach the ground in 't' sec. At time $\frac{t}{2}$ meter the ball will be-

(1) At the height of $\frac{h}{2}$ meter from the earths surface

(2) At the height of $\frac{h}{4}$ meter from the earth's surface

(3) At the height of $\frac{3h}{4}$ meter from the earth's surface

(4) At the height of h meter from the earth's surface

Q.19 Which of the following statements is wrong about a ball thrown vertically up ?

(1) It is moving with constant acceleration

(2) It may have different velocities at the same height

(3) It may have two positions at the same time

(4) It have same speed at the same heights

Q.20 A body is released from the top of an inclined plane of inclination (θ). It reaches the bottom with velocity (v). If keeping the length same the angle of inclination is doubled, what will be the velocity of the body on reaching the ground-

(1) v (2) 2v

(3) $[2\cos\theta]^{1/2}$ v (4) $[2\sin\theta]^{1/2}$ v

Q.21 A particle falling with constant acceleration covers 3m distance in third second of its motion. The distance covered in first 5 second shall be-

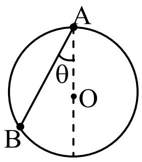
(1) 5 m (2) 15 m (3) 30 m (4) 45 m

Q.22 A particle slides from rest from the topmost point of a vertical circle of radius (r) along a smooth chord making an angle (θ) with the vertical the time of decent is-

(1) Least for $\theta = 0º$ (2) Maximum for $\theta = 0º$

(3) Least for $\theta = 45º$ (4) Independent of θ

Q.23 A bead is free to slide down a smooth wire tightly stretched between points A and B on a vertical circle of radius R. If the bead starts from rest at 'A', the highest point on the circle, its velocity when it arrives at B is-



(1) $2\sqrt{gR}$ (2) $2\sqrt{gR}\cos\theta$

(3) $\frac{2\sqrt{R}}{g}$ (4) None of these

Q.24 Two cars are moving in the same directions with the same speed of 30 km/hr. They are separated by 5 km. What is the speed of car moving in the opposite direction if it meets the two cars at an interval of 4 minute.-

(1) 45 km/hr. (2) 60 km/hr.

(3) 105 km/hr. (4) None

Q.25 A stone is thrown upwards and it rises to a height of 200m. The relative velocity of the stone with respect to the earth will be maximum at-

(1) Height of 100 m (2) Height of 150 m

(3) Highest point (4) The ground

Q.26 A bus starts from rest moving with acceleration $2 m/s^2$. A cyclist 96 m behind the bus starts simultaneously towards the bus at 20 m/s. After what time he will be able to overtake the bus-

(1) 8 s (2) 10 s (3) 12 s (4) 14 s

Q.27 A train moves in north direction with a speed of 54 km/hr and a monkey running on the roof of the train, against its motion with a velocity of 18 km/hr with respect to the train then the velocity of monkey as observed by a man standing on the ground-

(1) 5 m/s due south (2) 25 m/s due south

(3) 10 m/s due south (4) 10 m/s due north

Q.28 A bird is flying with a speed of 40 km/hr in the north direction. A train is moving with a speed of 40 km/hr in the west direction. A passenger sitting in the train will see the bird moving with velocity-

(1) 40 km/hr in NE direction

(2) $40\sqrt{2}$ km/hr in NE direction

(3) 40 km/hr in NW direction

(4) $40\sqrt{2}$ km/hr in NW direction

Q.29 A man is walking on a road with a velocity 3km/hr. Suddenly rain starts falling. The velocity of rain is 10 km/hr in vertically downward direction the relative velocity of the rain with respect to man is-

(1) $\sqrt{13}$ km/hr (2) $\sqrt{7}$ km/hr

(3) $\sqrt{109}$ km/hr (4) 13 km/hr

Q.30 A boy is running on the plane road with velocity (v) with a long hollow tube in his hand. The water is falling vertically downwards with velocity (u). At what angle to the vertical, he must incline the tube so that the water drops enters in it without touching its side -

(1) $\tan^{-1}\frac{v}{u}$ (2) $\sin^{-1}\frac{v}{u}$

(3) $\tan^{-1}\frac{u}{v}$ (4) $\cos^{-1}\frac{v}{u}$

Q.31 A man is walking at a speed 3 m/s rain drops are falling vertically with a speed 3 m/s –
(i) What is the velocity of rain drop with respect to the man ?
(ii) At what angle from vertical, the man should hold his umbrella ?
(1) 2.42 m/s, 30º in forward direction
(2) 4.24 m/s, 45º in forward direction
(3) 1.24 m/s, 60º in forward direction
(4) None of these

Q.32 A boat takes 2 hours to go 8 km and come back in still water lake. With water velocity of 4 km/hr, the time taken for going upstream of 8 km and coming back is-
(1) 140 min (2) 150 min
(3) 160 min (4) 170 min

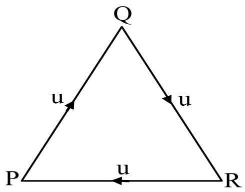
Q.33 A man wishes to swim across a river 0.5 km wide. If he can swim at the rate of 2 km/h in still water and the river flows at the rate of 1 km/h. The angle (with respect to the flow of the river) along which he should swim so at to reach a point exactly opposite his starting point, should be-
(1) 60º (2) 120º
(3) 145º (4) 90º

Q.34 A boat moves relative to water with a speed which is $\frac{1}{n}$ times the river flow speed. At what angle to the stream direction be boat move to minimize drifting -

(1) $\frac{\pi}{2}$ (2) $\sin^{-1}\frac{1}{n}$

(3) $\frac{\pi}{2}+\sin^{-1}\frac{1}{n}$ (4) $\frac{\pi}{2}+\sin^{-1}(n)$

Q.35 Three person P, Q and R of same mass travel with same speed u along an equilateral triangle of side 'd' such that each one faces the other always. After how much time will they meet each other-



(1) $\frac{d}{u}$ seconds

(2) $\frac{2d}{3u}$ seconds

(3) $\frac{2d}{\sqrt{3}u}$ seconds

(4) $d\sqrt{3}u$ seconds

Q.36 A particle moving in a straight line covers half the distance with speed of 3 m/s. The other half of the distance is covered in two equal time intervals with speed of 4.5 m/s and 7.5 m/s respectively. The average speed of the particle during this motion is-

(1) 4.0 m/s (2) 5.0 m/s
(3) 5.5 m/s (4) 4.8 m/s

Q.37 The relation between time t and distance x is $t = \alpha x^2 + \beta x$, where α and β are constant. The retardation is-.

(1) $2\alpha v^2$ (2) $2\alpha v^3$
(3) $2\alpha\beta v^3$ (4) $2\beta^2 v^3$

Q.38 The velocity-time relation of an electron starting from rest is given by u = kt, where k = 2m/s^2. The distance traversed in 3 sec is-

(1) 9 m (2) 16 m
(3) 27 m (4) 36 m

Q.39 The velocity acquired by a body moving with uniform acceleration is 30 m/s in 2 seconds and 60 m/s in 4 seconds. The initial velocity is-

(1) zero (2) 2 m/s
(3) 4 m/s (4) 10 m/s

Q.40 A particle moves with uniform acceleration and v_1, v_2 and v_3 denote the average velocities in the three successive intervals of time t_1, t_2 and t_3. Which of the following relations is correct ?

(1) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_2 + t_3)$
(2) $(v_1 - v_2) : (v_2 - v_3) = (t_1 + t_2) : (t_2 + t_3)$
(3) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_1 - t_3)$
(4) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_2 - t_3)$

Q.41 A car accelerates from rest at a constant rate of 2m/s^2 for some time. Then, it retards at a constant rate of 4 m/s^2 and comes to rest. If it remains in motion for 3 second the maximum speed attained by the car is-

(1) 2 m/s (2) 3 m/s
(3) 4 m/s (4) 6 m/s

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 3

Q.1 The position x of a particle varies with time (t) as $x = at^2 - bt^3$. The acceleration at time t of the particle will be equal to zero, where t is equal to-

(1) $\frac{2a}{3b}$ (2) $\frac{a}{b}$

(3) $\frac{a}{3b}$ (4) zero

Q.2 A particle moves along a straight line such that its displacement at any time t is given by
$s = t^3 - 6t^2 + 3t + 4$ metres.
The velocity when the acceleration is zero is-

(1) 3 m/s (2) –12 m/s

(3) 42 m/s (4) – 9 m/s

Q.3 The displacement x of a particle moving in one dimension under the action of constant force is related to time t by the equation $t = \sqrt{x} + 3$, where x is in metres and t is in seconds. Find the displacement of the particle when its velocity is zero-

(1) zero (2) 12 m

(3) 6 m (4) 18 m

Q.4 A car moves along a straight line whose equation of motion is given by $s = 12t + 3t^2 - 2t^3$, where s is in metres and t is in seconds. The velocity of the car at start will be-

(1) 7 m/s (2) 9 m/s

(3) 12 m/s (4) 16 m/s

Q.5 The x and y co-ordinates of a particle at any time t are given by $x = 7t + 4t^2$ and $y = 5t$, where x and y are in m and t in s. The acceleration of the particle at 5s is-

(1) zero (2) 8 m/s^2

(3) 20 m/s^2 (4) 40 m/s^2

Q.6 The deceleration experienced by a moving motor boat, after its engine is cut off is given by $\frac{dv}{dt} = -kv^3$, where k is constant. If v_0 is the magnitude of the velocity at cut off, the magnitude of the velocity at a time t after the cut-off is-

(1) $\frac{v_0}{2}$ (2) v_0

(3) $v_0 e^{-k/1}$ (4) $\frac{v_0}{\sqrt{(2v_0^2kt+1)}}$

Q.7 The displacement of a particle is given by $y = a + bt + ct^2 - dt^4$
The initial velocity and acceleration are respectively-

(1) b, – 4d

(2) –b, 2c

(3) b, 2c

(4) 2c, –4d

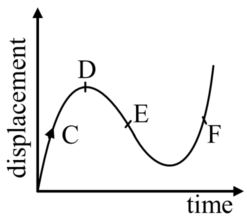
Q.8 The displacement of a particle is represented by the following equation :
$s = 3t^3 + 7t^2 + 5t + 8$
where s is in metre and t in second. The acceleration of the particle at t = 1 is-
(1) 14 m/s^2 (2) 18 m/s^2
(3) 32 m/s^2 (4) zero

Q.9 The relation 3t = $\sqrt{3x}+6$ describes the displacement of a particle in one direction where x is in meters and t in seconds. The displacement, when velocity is zero, is-
(1) 24 m (2) 12 m
(3) 5 m (4) zero

Q.10 A force $\vec{F} = 6t^2\hat{i} + 4t\hat{j}$ is acting on a particle of mass 3kg then what will be velocity of particle at t = 3 second and if at t = 0, particle is at rest-
(1) $18\hat{i}+6\hat{j}$ (2) $18\hat{i}+12\hat{j}$
(3) $12\hat{i}+6\hat{j}$ (4) none

Q.11 A person travels along a straight road for the first half time with a velocity v_1 and the second half time with a velocity v_2. Then the mean velocity $\bar{v}$ is given by-
(1) $\bar{v} = \frac{v_1+v_2}{2}$ (2) $\frac{2}{\bar{v}} = \frac{1}{v_1} + \frac{1}{v_2}$
(3) $\bar{v} = \sqrt{v_1 v_2}$ (4) $\bar{v} = \sqrt{\frac{v_2}{v_1}}$

Q.12 The displacement-time graph of a moving particle is shown. The instantaneous velocity of the particle is negative at the point-



(1) D (2) F
(3) C (4) E

Q.13 A particle is moving eastward with a velocity of 5m/s in 10 s, the velocity changes to 5 m/s northward. The average acceleration in this time is-
(1) zero
(2) $\frac{1}{\sqrt{2}}$ m/s^2 towards north-west
(3) $\frac{1}{\sqrt{2}}$ m/s^2 towards north-east
(4) $\frac{1}{2}$ m/s^2 towards north-west

Q.14 A particle is moving with a velocity of 10 m/s towards east. After 10 s its velocity changes to 10 m/s towards north. Its acceleration is-
(1) zero
(2) $\sqrt{2}$ m/s^2 towards N-W
(3) $\frac{1}{\sqrt{2}}$ m/s^2 towards N-W
(4) $\frac{1}{\sqrt{2}}$ m/s^2 towards N-E

Q.15 If a car at rest accelerates uniformly to a speed of 144 km/h in 20 second, it coveres a distance of-
(1) 20 m (2) 400 m
(3) 1440 m (4) 2980 m

Q.16 A body starts from rest, what is the ratio of the distance travelled by the body during the 4th and 3rd second ?
(1) $\frac{7}{5}$ (2) $\frac{5}{7}$ (3) $\frac{7}{3}$ (4) $\frac{3}{7}$

Q.17 A car moving with a speed of 40 km/hr can be stopped by applying brakes after at least 2m. If the same car is moving with a speed of 80 km/hr., what is the minimum stopping distance ?
(1) 2 m (2) 4 m
(3) 6 m (4) 8 m

Q.18 A body is thrown vertically upwards from the top A of a tower. It reaches the ground in t_1 seconds. If it thrown vertically downwards from A with the same speed it reaches the ground in t_2, seconds. If it is allowed to fall freely from A, then the time it takes to reach the ground is given by-
(1) $t = \frac{t_1 + t_2}{2}$ (2) $t = \frac{t_1 - t_2}{2}$
(3) $t = \sqrt{t_1 t_2}$ (4) $t = \sqrt{\frac{t_1}{t_2}}$

Q.19 With what speed should a body be thrown upwards so that the distances traversed in 5th second and 6th second are equal ?
(1) 58.4 m/s (2) 49 m/s
(3) $\sqrt{98}$ m/s (4) 98 m/s

Q.20 The water drops fall at regular intervals from a tap 5m above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant ?
(1) 1.25 m (2) 2.50 m
(3) 3.75 m (4) 4.00 m

Q.21 A man is slipping on a frictionless inclined plane and a bag falls down from the same height. Then the speed of both is related as-
(1) $v_B > v_m$
(2) $v_B < v_m$
(3) $v_B = v_m$

(4) v_B and v_m can't related

Q.22 If a ball is thrown vertically upwards with 40 m/s, its velocity after two second will be-

(1) 10 m/s (2) 20 m/s
(3) 30 m/s (4) 40 m/s

Q.23 A particle is thrown vertically upward. Its velocity at half of the height is 10 m/s. The maximum height attained by it is ($g = 10\ m/s^2$)-

(1) 8 m (2) 20 m
(3) 10 m (4) 16 m

Q.24 A boat which has a speed of 5 km per hour in still water crosses a river of width 1km along the shortest possible path in fifteen minutes. The velocity of the river water in km per hour is-

(1) 1 (2) 2
(3) 3 (4) $\sqrt{41}$

Q.25 A train of 150 m length is going towards north direction at a speed of 10 m/s. A parrot files at a speed of 5m/s towards south direction parallel to the railway track. The time taken by the parrot to cross the train is equal to-

(1) 12 s (2) 8 s
(3) 15 s (4) 10 s

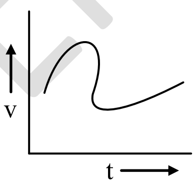
Q.26 Two trains, each 50 m long, are travelling in opposite directions with velocity 10 m/s and 15 m/s. The time of crossing is-

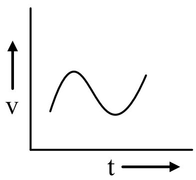
(1) 2 s (2) 4 s
(3) $2\sqrt{3}$ s (4) $4\sqrt{3}$ s

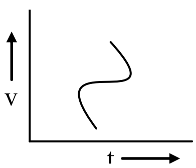
Q.27 A boat is sailing at a velocity $3\hat{i}+4\hat{j}$ with respect to ground and water in river is flowing with a velocity $-3\hat{i}-4\hat{j}$. Relative velocity of the boat with respect to water is-

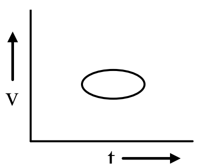
(1) $8\hat{j}$ (2) $5\sqrt{2}$
(3) $6\hat{i}+8\hat{j}$ (4) $-6\hat{i}-8\hat{j}$

Q.28 Which of the following velocity-time graph shows a realistic situation for a body in motion ?

(1)


(2)


(3)


(4)


Q.29 If a ball is thrown vertically upwards with speed u, the distance covered during the last 't' seconds of its ascent is-

(1) ut (2) $\frac{1}{2}gt^2$

(3) $ut - \frac{1}{2}gt^2$ (4) $(u + gt)t$

Q.30 A man throws ball with the same speed vertically upwards one after the other at an interval of 2 seconds. What should be the speed of the throw so that more than two balls are in the sky at any time ? (Given $g = 9.8\ m/s^2$)

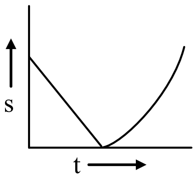
(1) More than 19.6 m/s

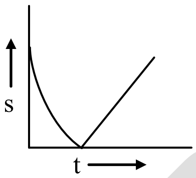
(2) At least 9.8 m/s

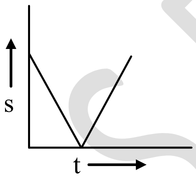
(3) Any speed less than 19.6 m/s

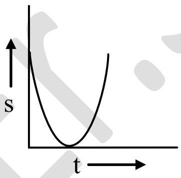
(4) Only with speed 19.6 m/s

Q.31 A ball is thrown vertically upwards. Which of the following plots represents the speed-time graph of the ball during its flight if the air resistance is not ignored-

(1) 

(2) 

(3) 

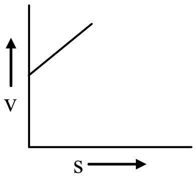
(4) 

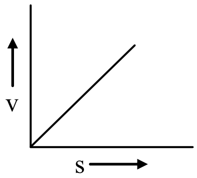
Q.32 There different objects of masses m_1, m_2 and m_3 are allowed to fall from rest and from the same points 'O' along three different frictionless paths. The speeds of the three objects on reaching the ground, will be in the ratio of-

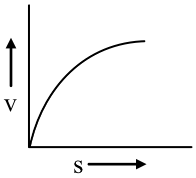
(1) $m_1 : m_2 : m_3$ (2) $m_1 : 2m_2 : 3m_3$

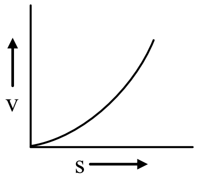
(3) 1 : 1 : 1 (4) $\frac{1}{m_1} : \frac{1}{m_2} : \frac{1}{m_3}$

Q.33 A body starting from rest moves along a straight line with a constant acceleration. The variation of speed (v) with distance (s) is represented by the graph

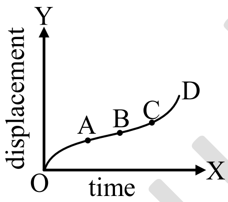
(1) 

(2) 

(3) 

(4) 

Q.34 The graph between the displacement x and time t for a particle moving in a straight line is shown in figure. During the interval OA, AB, BC and CD, the acceleration of the particle is-



	OA	AB	BC	CD
(1)	+	0	+	+
(2)	–	0	0	+
(3)	+	0	–	+
(4)	–	0	–	0

Q.35 Velocity-time curve for a body projected vertically upwards-

(1) Parabola (2) Ellipse
(3) Hyperbola (4) Straight line

Q.36 A particle moves along a straight line OX. At a time t(in seconds) the distance x (in metres) of the particle from O is given by $x = 40 + 12t - t^3$. How long would the particle travel before coming to rest-

(1) 24 m (2) 40 m
(3) 56 m (4) 16 m

Q.37 Two bodies, A (of mass 1 kg) and B (of mass 3 kg), are dropped from heights of 16 m and 25m respectively. The ratio of the time taken by them to reach the ground is-

(1) $\frac{5}{4}$ (2) $\frac{12}{5}$ (3) $\frac{5}{12}$ (4) $\frac{4}{5}$

Q.38 When a ball is thrown up vertically with velocity v_0, it reaches a maximum height of 'h'. If one wishes to triple the maximum height then the ball should be thrown with velocity-

(1) $\sqrt{3}\, v_0$ (2) $3v_0$
(3) $9v_0$ (4) $3/2v_0$

Q.39 A particle is moving along x-axis has acceleration f, at time t, given by f = $f_0\left(1-\frac{t}{T}\right)$, where f_0 and T are constants. The particle at t = 0 has zero velocity. In the time interval between t = 0 and the instant when f = 0, the particle velocity (v_x) is-

(1) $\frac{1}{2}f_0T$ (2) f_0T

(3) $\frac{1}{2}f_0T^2$ (4) f_0T^2

Q.40 A car moves from X to Y with a uniform speed v_u and returns to Y with a uniform speed v_d. The average speed for this round trip is-

(1) $\frac{v_u+v_d}{2}$ (2) $\frac{2v_dv_u}{v_d+v_u}$

(3) $\sqrt{v_uv_d}$ (4) $\frac{v_dv_u}{v_d+v_u}$

Q.41 The position x of a particle with respect to time t along x-axis is given by $x = 9t^2 - t^3$ where x is in metres and t in seconds. What will be the position of this particle when it achieves maximum speed along the +x direction ?

(1) 24 m (2) 32 m
(3) 54 m (4) 81 m

Q.42 A particle starts its motion from rest under the action of a constant force. If the distance covered in first 10 seconds is S_1 and that covered in the first 20 seconds is S_2, then-

(1) $S_2 = S_1$ (2) $S_2 = 2S_1$
(3) $S_2 = 3S_1$ (4) $S_2 = 4S_1$

Q.43 A bus is moving with a speed of 10 m/s on a straight road. A scooterist wishes to overtake the bus in 100 s. If the bus is at a distance of 1 km from the scooterist, with what speed should the scooterist chase the bus-

(1) 10 m/s (2) 20 m/s
(3) 40 m/s (4) 25 m/s

Q.44 A boy standing at the top of a tower of 20 m height drops a stone. Assuming g = 10 ms^{-2}, the velocity with which it hits the ground is :

(1) 5.0 m/s (2) 10.0 m/s
(3) 20.0 m/s (4) 40.0 m/s

Q.45 A particle covers half of its total distance with speed v_1 and the rest half distance with speed v_2. Its average speed during the complete journey is :

(1) $\frac{v_1+v_2}{2}$ (2) $\frac{v_1v_2}{v_1+v_2}$

(3) $\frac{2v_1v_2}{v_1+v_2}$ (4) $\frac{v_1^2v_2^2}{v_1^2+v_2^2}$

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 4

Q.1 The displacement of a particle starting from rest (at t = 0) is given by $s = 6t^2 - t^3$. The time at which the particle will attain zero velocity again, is-
(1) 4s (2) 8s
(3) 12s (4) 16s

Q.2 The velocity of a body depends on time according to the equation $v = 20 + 0.1\ t^2$. The body is undergoing-
(1) uniform acceleration
(2) uniform retardation
(3) non-uniform acceleration
(4) zero acceleration

Q.3 A particle moves along the x-axis such that is coordinate(x) varies with time (t), according to the expression : $x = 2 - 5t + 6t^2$, where x is in meters and t is in seconds. The initial velocity of the particle is-
(1) –5 m/s (2) –3 m/s
(3) 6 m/s (4) 2 m/s

Q.4 If for a particle position $x \propto t^2$ then-
(1) velocity is constant
(2) acceleration is constant
(3) acceleration is variable
(4) none of these

Q.5 Relation between displacement x and time t is $x = 2 - 5t + 6t^2$, the initial acceleration will be-
(1) $-3\ m/s^2$ (2) $12\ ms^2$
(3) $2\ ms^2$ (4) $-5\ ms^2$

Q.6 the displacement 'x' of a particle moving along a straight line at time t is given by $x = a_0 + a_1t + a_2t^2$. The acceleration of the particle is-
(1) a_1 (2) a_2
(3) $2a_2$ (4) $3a_2$

Q.7 A body is moving according to the equation $x = at + bt^2 - ct^3$. Then its instantaneous speed is given by-
(1) $a + 2b + 3ct$ (2) $a + 2bt - 3ct^2$
(3) $2b - 6ct$ (4) none of these

Q.8 If the distance covered by a particle is given by the relation $x = at^2$. The particle is moving with-
(1) constant acceleration
(2) zero acceleration
(3) variable acceleration
(4) none of these

Q. 9 Starting from rest, the acceleration of a particle is $a = 2(t - 1)$. The velocity of the particle at t = 5s is-

(1) 15 m/s (2) 25 m/s
(3) 5 m/s (4) None of these

Q.10 A 100 m long train crosses a man travelling at 5 km/hr, in opposite direction, in 7.2 seconds then the velocity of train is-
(1) 40 km/hr (2) 25 km/hr
(3) 20 km/hr (4) 45 km/hr

Q.11 A body goes to 10 km north and 20 km east. The displacement from initial point is-
(1) 22.36 km (2) 2 km
(3) 5 km (4) 20 km

Q.12 A scooter going due east at 10 m/s turns right through an angle of 90º. If the speed of the scooter remains unchanged in taking this turn, the change in the velocity of the scooter is-
(1) 20.0 m/s in south-western direction
(2) zero
(3) 10.0 m/s in south-east direction
(4) 14.14 m/s in south-western direction

Q.13 A body is imparted motion from rest to move in a straight line. It is then obstructed by an opposite force, then-
(1) the body may necessarily change direction
(2) the body is sure to slow down
(3) the body will necessarily continue to move in the same direction at the same speed
(4) none of the above

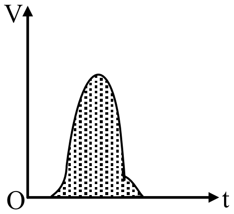
Q.14 A boy walks to his school at a distance of 6 km with a speed of 2.5 km/h, and walks back with a constant speed by 4 km/h. His average speed for trip expressed in km/h is-
(1) $\frac{24}{13}$ (2) $\frac{40}{13}$
(3) 3 (4) 4.8

Q.15 Which of the following four statements is false ?
(1) A body can have zero velocity and still be accelerated
(2) A body can have a constant velocity and still have a varying speed
(3) A body can have a constant speed and still have a varying velocity
(4) The direction of the velocity of a body can change when its acceleration is constant

Q.16 One car moving on a straight road covers one thirds of the distance with 20 km/hr and the rest with 60 km/hr. The average speed is-
(1) 40 km/hr (2) 80 km/hr
(3) $46\frac{2}{3}$ km/hr (4) 36 km/hr

Q.17 A car travels first half distance between two places with a speed of 30 km/hr and remaining half with a speed of 50 km/h. The average speed of the car is-
(1) 37.5 km/hr. (2) 10 km/hr.
(3) 42 km/hr. (4) 40 km/hr.

Q.18 Figure below shows the velocity time graph of a one dimensional motion. Which of the following characteristic of the particle is represented by the shaded area ?



(1) Speed (2) Distance covered
(3) Acceleration (4) Momentum

Q.19 In a straight line motion of the distance travelled is proportional to the square root of the time taken. The acceleration of the particle is proportional to-
(1) v (2) v^2
(3) v^3 (4) $\sqrt{v}$

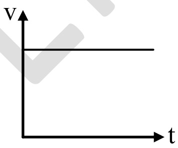
Q.20 A body starts from rest is moving under a constant acceleration up to 20 sec. If it moves S_1 distance in first 10 sec., and S_2 distance in next 10 sec then S_2 will be equal to-
(1) S_1 (2) $2S_1$
(3) $3S_1$ (4) $4S_1$

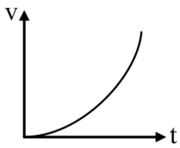
Q.21 If a body starts from rest and travels 120 m in the 8^{th} second, then acceleration is-
(1) 16 m/s^2 (2) 10 m/s^2
(3) 0.227 m/s^2 (4) 0.03 m/s^2

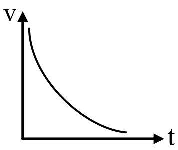
Q.22 If a train travelling at 72 km/h is to be brought to rest in a distance of 200 m, then its retardation should be-
(1) 20 m/s^2 (2) 2 m/s^2
(3) 10 m/s^2 (4) 1 m/s^2

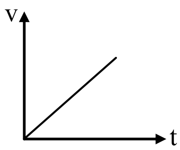
Q.23 A body travels 200 cm in the first two seconds and 220 cm in the next 4 sec with same acceleration. The velocity of the body at the end of the 7^{th} second is-
(1) 5 cm/s (2) 10 cm/s
(3) 15 cm/s (4) 20 cm/s

Q.24 Which of the following velocity-time graphs represent uniform motion ?

(1) 

(2) 

(3) 

(4) 

Q.25 If a body starts from rest and travels 120 cm in the 6^{th} second then what is the acceleration ?
(1) 0.20 m/s^2 (2) 0.027 m/s^2
(3) 0.218 m/s^2 (4) 0.003 m/s^2

Q.26 A car moving with a velocity of 10 m/s can be stopped by the application of constant force F in a distance of 20 m. If the velocity of the car is 30 m/s. It can be stopped by this force in-

(1) $\frac{20}{3}$ m (2) 20 m
(3) 60 m (4) 180 m

Q.27 A particle travels 10 m in first 5 seconds, 10m in next 3 seconds. Assuming constant acceleration what is the distance travelled in next 2 second-

(1) 8.3 m (2) 9.3 m
(3) 10.3 m (4) None of these

Q.28 A body starts from rest and has an acceleration 20 cm/s^2. What is the distance covered by the body in first 8 seconds ?

(1) 160 cm (2) 640 cm
(3) 1280 cm (4) 1640 cm

Q.29 Initially a body is at rest. If its acceleration is 5m/s^2 then the distance travelled in the 18^{th} second is-

(1) 86.6 m (2) 87.5 m
(3) 88 m (4) 89 m

Q.30 The initial velocity of a particle $\vec{u} = 4\hat{i} + 3\hat{j}$. It is moving with uniform acceleration, $\vec{a} = 0.4\hat{i} + 0.3\hat{j}$. Its velocity after 10 second is-

(1) 3 unit (2) 4 unit
(3) 5 unit (4) 10 unit

Q.31 A body is dropped from a tower with zero velocity, reaches ground in 4s. The height of the tower is about-

(1) 80 m (2) 20 m
(3) 160 m (4) 40 m

Q.32 A ball is thrown upward with a velocity of 100 m/s. It will reach the ground after-

(1) 10 s (2) 20 s
(3) 5 s (4) 40 s

Q.33 A particle is dropped vertically from rest, from a height. The time taken by it to fall through successive distance of 1 km each will then be-

(1) all equal, being equal to $\sqrt{\frac{2}{g}}$ second

(2) in the ratio of the square roots of the integers $1 : \sqrt{2} : \sqrt{3}$

(3) in the ratio of the difference in the square roots of the integers i.e., $\sqrt{1}$, $(\sqrt{2}-\sqrt{1})$, $(\sqrt{3}-\sqrt{2})$, $(\sqrt{4}-\sqrt{3})$ ....

(4) in the ratio of the reciprocals of the square roots of the integers, i.e., $\frac{1}{\sqrt{1}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}$ ....

Q.34 A ball of mass m_1 and another ball of mass m_2 are dropped from equal height. If times taken by the balls are t_1 and t_2 respectively, then-

(1) $t_1 = \frac{t_2}{2}$ (2) $t_1 = t_2$

(3) $t_1 = 4t_2$ (4) $t_1 = \frac{t_2}{4}$

Q.35 If an iron ball and a wooden ball of the same radius are released from a height h in vacuum then time taken by both of them to reach ground will be-

(1) unequal (2) exactly equal

(3) roughly equal (4) zero

Q.36 A body falls freely from rest. It covers as much distance in the last second of its motion as covered in the first three seconds. The body has fallen for a time of-

(1) 3s (2) 5s (3) 7s (4) 9s

Q.37 A body projected vertically upwards with a velocity u returns to the staring point in 4 second. If g = 10 m/s^2, the value of u is (m/s)-

(1) 5 (2) 20

(3) 40 (4) 10

Q.38 A body released from a height falls freely towards earth. Another body is released from the same height exactly one second later. The separation between the two bodies two second after the release of the second body is-

(1) 9.8 m (2) 49 m

(3) 24.5 m (4) 19.6 m

Q.39 A body is falling freely under gravity. The ratio of distance covered by it in 1^{st}, 2^{nd} and 3^{rd} second respectively is-

(1) 1 : 4 : 9 (2) 1 : 2 : 3

(3) 1 : 3 : 5 (4) 1 : 2 : 5

Q.40 A balloon starts rising from the ground with an acceleration of 1.25 m/s^2. After 8s, a stone is released from the balloon. The stone will- (taking g = 10 m/s^2)

(1) have a displacement of 50 m

(2) cover a distance of 40 m in reaching the ground

(3) reach the ground in 4s

(4) begin to move down after being released

Q.41 A ball is thrown vertically upwards. Assuming the air resistance to be constant and considerable-

(1) the time of ascent $\geq$ the time of descent

(2) time time of ascent < the time of descent

(3) the time of ascent > the time of descent

(4) the time of ascent = the time of descent

Q.42 Two particles of masses m_1 and m_2 are dropped from heights h_1 and h_2. They reach the earth after times t_1 and t_2 respectively. Then-

(1) $\frac{t_1}{t_2}=\sqrt{\frac{h_1}{h_2}}$ (2) $\frac{t_1}{t_2}=\sqrt{\frac{h_2}{h_1}}$

(3) $\frac{t_2}{t_1}=\frac{h_2}{h_1}$ (4) None of these

Q.43 A river is flowing from east to west at a speed of 5m/min. A man on south bank of river, capable of swimming 10 m/min in still water, wants to swim across the river in shorter time; he should swim-

(1) due north
(2) due north-east
(3) due north-east with double the speed of river
(4) none of the above

Q.44 A motorcycle is moving with a velocity 80 km/hr ahead of a car moving with a velocity of 65 km/hr in the same direction. What is the relative velocity of the motorcycle with respect to the car-

(1) 15 km/hr (2) 20 km/hr
(3) 25 km/hr (4) 145 km/hr

Q.45 When a train is stopped by applying break it stops after travelling a distance of 50 metres. If speed of train is doubled and same retarding force is applied then it stops after travelling a distance of-

(1) 50 m (2) 100 m
(3) 200 m (4) 400 m

Q.46 A particle moves in east direction with 15 m/sec in 2 sec then moves in northward with 5 m/s for 8 sec than average speed of the particle is-

(1) 1 m/s (2) 5 m/s
(3) 7 m/s (4) 10 m/s

Q.47 A car is moving with velocity v. If car stop after applying break at a distance of 20m. If velocity of car is doubled, then how much distance it will cover (travel) after applying brake-

(1) 40 m (2) 80 m
(3) 160 m (4) 320 m

Q.48 A body starts falling from height 'h' and travels distance h/2 during last second of motion then time of travel is – (in second)

(1) $\sqrt{2}-1$ (2) $2+\sqrt{2}$
(3) $\sqrt{2}+\sqrt{3}$ (4) $\sqrt{3}+2$

Q.49 A particle starts from rest with constant acceleration. The ratio of space-average velocity to the time average velocity is-

(1) $\frac{1}{2}$ (2) $\frac{3}{4}$

(3) $\frac{4}{3}$ (4) $\frac{3}{2}$

Q.50 A particle of mass 4 kg is acted upon by steady force of 4N. Distance travelled the particle in 4 sec is -

(1) 16 m (2) 2 m
(3) 8 m (4) 4 m

Q.51 A car covers $\frac{1}{3}$ distance with speed 20 km/hr and $\frac{2}{3}$ distance with 60 km/hr. Average speed is-

(1) 40 km/hr (2) $50\sqrt{2}$ km/hr
(3) 36 km/hr (4) 80 km/hr

Q.52 A stone falls freely such that the distance covered by it in the last second of its motion is equal to the distance covered by it in the first 5 seconds. It remained in air for-

(1) 12 s (2) 13 s
(3) 25 s (4) 26 s

Q.53 A particle moves for 20 seconds with velocity 3m/s and then moves with velocity 4 m/s for another 20 seconds and finally moves with velocity 5 m/s for next 20 seconds. What is the average velocity of the particle ?

(1) 3 m/s (2) 4 m/s
(3) 5 m/s (4) zero

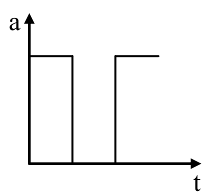
Q.54 A body starts from rest and with a uniform acceleration of 10 m/s^2 for 5 seconds. During the next 10 seconds it moves with uniform velocity. The total distance travelled by the body is-

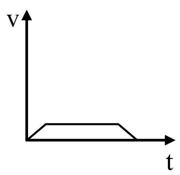
(1) 100 m (2) 125 m
(3) 500 m (4) 625 m

Q.55 An object travels 10 km at a speed of 100 m/s and another 10 km at 50 m/s. The average speed over the whole distance is-

(1) 75 m/s (2) 55 m/s
(3) 66.7 m/s (4) 33.3 m/s

Q.56 Acceleration-time graph of a body is shown. The corresponding velocity-time graph of the same body is-



(1)


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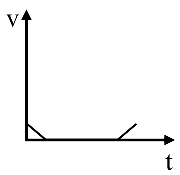

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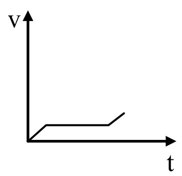
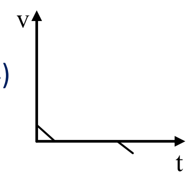
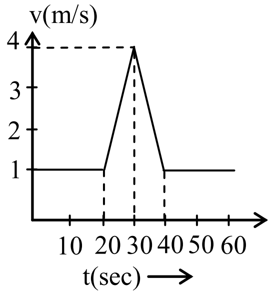


(4)


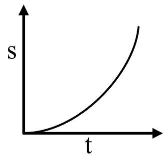
Q.57 Velocity-time (v-t) graph for a moving object is shown in the figure. Total displacement of the object during the time interval when there is non-zero acceleration and retardation is-

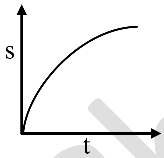


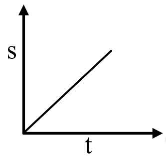
(1) 60 m (2) 50 m
(3) 30 m (4) 40 m

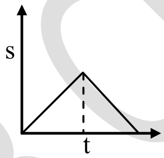
Q.58 The area under acceleration-time graph gives-
(1) Distance travelled
(2) Change in acceleration
(3) Force acting
(4) Change in velocity

Q.59 Which graph represents the uniform acceleration ?

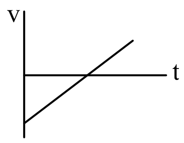
(1) 

(2) 

(3) 

(4) 

Q.60 Which of the following situation is represented by the velocity-time graph show in the diagram ?



(1) A stone thrown up vertically, returning back to the ground
(2) A car decelerating at constant rate and then accelerating at the same rate
(3) A ball falling from a height and then bouncing back
(4) None of the above

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 5

These questions of two statements each, printed as Assertion and Reason. While answering these Questions you are required to choose any one of the following four responses.

(A) If both Assertion & Reason are true & the Reason is a correct explanation of the Assertion.

(B) If both Assertion and Reason are true but Reason is not a correct explanation of the Assertion.

(C) If Assertion is true but the Reason is false.

(D) If Assertion & Reason both are false.

Q.1 **Assertion :** A body can have acceleration even if its velocity is zero at a given instant of time.
Reason : A body is momentarily at rest when it reverses its direction of motion.
(1) A (2) B (3) C (4) D

Q.2 **Assertion :** If the displacement of the body is zero, the distance covered by it may not be zero.
Reason : The displacement depends only on end points ; the distance (path length) depends on the actual path.
(1) A (2) B (3) C (4) D

Q.3 **Assertion :** The magnitude of average velocity of the object over an interval of time is either smaller than or equal to the average speed of the object over the same interval.
Reason : Path length (distance) is either equal or greater than the magnitude of displacement.
(1) A (2) B (3) C (4) D

Q.4 **Assertion :** An object can have constant speed but variable velocity.
Reason : Speed is a scalar but velocity is a vector quantity.
(1) A (2) B (3) C (4) D

Q.5 **Assertion :** The speed of a body can be negative.
Reason : If the body is moving in the opposite direction of positive motion, then its speed is negative.
(1) A (2) B (3) C (4) D

Q.6 **Assertion :** A positive acceleration can be associated with a 'slowing down' of the body.
Reason : The origin and the positive direction of an axis are a matter of choice.
(1) A (2) B (3) C (4) D

Q.7 **Assertion :** A negative acceleration of a body can be associated with a "speeding up" of the body.
Reason : Increase in speed of a moving body is independent of its direction of motion.
(1) A (2) B (3) C (4) D

Q.8 **Assertion :** When a body is starts from rest subjected to a uniform acceleration, it always move in a straight line.
Reason : Straight line motion is the natural tendency of the body.
(1) A (2) B (3) C (4) D

Q.9 **Assertion :** The distance traversed, during equal intervals of time, by a body falling from rest are in ratio 1 : 3 : 5 : 7..... [Galileo's law of odd numbers]

Reason : A particle in one-dimensional motion with zero speed may have non-zero velocity.

(1) A (2) B (3) C (4) D

Q.10 **Assertion :** When a particle moves with constant velocity its average velocity, its instantaneous velocity and its speed are all equal in magnitude.

Reason : The average velocity of a particle moving on a straight line is zero in a time interval. It is possible that the instantaneous velocity is never zero in the interval.

(1) A (2) B (3) C (4) D

Q.11 **Assertion :** The average velocity of a particle is zero in a time interval. It is possible that the instantaneous acceleration is never zero in the interval.

Reason : The magnitude of average velocity in an interval is equal to its average speed in that interval.

(1) A (2) B (3) C (4) D

Q.12 **Assertion :** An object may have varying speed without having varying velocity.

Reason : If the velocity is zero at an instant, the acceleration should also be zero at that instant.

(1) A (2) B (3) C (4) D

Q.13 **Assertion :** Average speed of a particle in a given time interval is never less than the magnitude of the average velocity.

Reason : The magnitude of the velocity (instantaneous velocity) of a particle is equal to its speed.

(1) A (2) B (3) C (4) D

Q.14 **Assertion :** In successive time intervals if the average velocities of a particle are equal then the particle must be moving with constant velocity.

Reason : When a particle moves with uniform velocity, its displacement always increases with time.

(1) A (2) B (3) C (4) D

Q.15 **Assertion :** In a free fall, the initial velocity of a body may or may not be zero.

Reason : A heavy body falls at a faster rate as compared to a lighter body.

(1) A (2) B (3) C (4) D

Q.16 **Assertion :** If a body is dropped from the top of a tower of height h and another body is thrown up simultaneously with velocity u from the foot of the tower, then both of them would meet after a time $\frac{h}{u}$.

Reason : For a body projected upwards, the distance covered by the body in the last second of its upward journey is always 4.9 m irrespective of velocity of projection. ($g = 9.8\ m/sec^2$)

(1) A (2) B (3) C (4) D

Q.17 **Assertion :** An observer is moving due east and wind appears him to blow from north. Then the actual direction of air blow is towards south-east.

Reason : Because $\vec{V}_R = \vec{V}_A - \vec{V}_M$ where $\vec{V}_R$ is the relative velocity of wind with respect to man, $\vec{V}_A$ = actual velocity of wind (w.r.t. ground) and $\vec{V}_M$ = velocity of man w.r.t. ground.

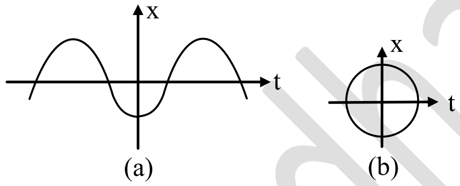
(1) A (2) B (3) C (4) D

Q.18 **Assertion :** The distance and displacement both are equal when the particle moves in a straight line.

Reason : Because in straight line motion distance travelled = | displacement |.

(1) A (2) B (3) C (4) D

Q.19 **Assertion :** Graph (a) represent one dimensional motion of a particle. While graph (b) can not represent 1 - D motion of the particle.



Reason : In 1 - D motion, there is only one value of displacement at one particular time.

(1) A (2) B (3) C (4) D

Q.20 **Assertion :** In general | Displacement | $\leq$ distance.

Reason : The instantaneous speed is equal to the magnitude of the instantaneous velocity.

(1) A (2) B (3) C (4) D

Q.21 **Assertion :** In a free fall, the initial velocity of a body may not be zero.

Reason : Free falls means the vertical acceleration of the body is equal to g.

(1) A (2) B (3) C (4) D

Q.22 **Assertion :** Distance is a actual length of the path but displacement is a shortest distance between initial and final position.

Reason : Distance is a scalar quantity and it is always positive but displacement is a vector quantity. It may be positive, negative or zero.

(1) A (2) B (3) C (4) D

Q.23 **Assertion :** Rest of a body is a relative term.

Reason : Motion of a body may be absolute term.

(1) A (2) B (3) C (4) D

Q.24 **Assertion :** If a body moves on a straight line, magnitude of its displacement and distance covered by it will be same.

Reason : Along the straight line body can move only in one direction.

(1) A (2) B (3) C (4) D

Q.25 **Assertion :** Separation between two bodies moving in the same direction on the ground with same acceleration, will not increase.

Reason : Because they are relatively at rest.

(1) A (2) B (3) C (4) D

Q.26 **Assertion :** To cross the river in minimum time, swimmer should swimming in perpendicular direction to the water current.

Reason : Because in this case river flow helps to cross the river.

(1) A (2) B (3) C (4) D

Q.27 **Assertion :** A body moving with constant acceleration always travels equal distance in equal time intervals.

Reason : Motion of the body with constant acceleration is an uniform motion.

(1) A (2) B (3) C (4) D

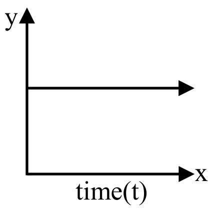
Q.28 **Assertion :** A body, whatever its motion is always at rest in a frame of reference which is fixed to the body itself.

Reason : The relative velocity of a body with respect to itself is zero.

(1) A (2) B (3) C (4) D

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 6

Q.1 In the graph shown in fig. the time is plotted along x-axis. Which quantity associated with the projectile motion is plotted along the y-axis-



(1) kinetic energy (2) momentum
(3) horizontal velocity (4) none of the above

Q.2 A particle of mass m is projected with a velocity v making an angle 45° with the horizontal. The magnitude of the angular momentum of the projectile about the point of projection when the particle is at its maximum height h, is -

(1) zero (2) $\frac{mv^3}{4\sqrt{2}g}$

(3) $\frac{mv^3}{\sqrt{2}g}$ (4) $m^2\sqrt{2gh^3}$

Q.3 In case of a projectile fired at an angle equally inclined to the horizontal and vertical with velocity (u). The horizontal range is -

(1) $\frac{u^2}{g}$ (2) $\frac{u^2}{2g}$

(3) $\frac{2u^2}{g}$ (4) $\frac{u^2}{g^2}$

Q.4 A shell is fired vertically upwards with a velocity v_1 from the deck of a ship travelling at a speed of v_2. A person on the shore observes the motion of the shell as parabola. Its horizontal range is given by -

(1) $\frac{2v_1^2 v_2}{g}$ (2) $\frac{2v_1 v_2^2}{g}$

(3) $\frac{2v_1 v_2}{g}$ (4) $\frac{2v_1^2 v_2^2}{g}$

Q.5 The range of a projectile when fired at 75° with the horizontal is 0.5 km. what will be its range when fired at 45° with same speed -
(1) 0.5 km (2) 1.0 km
(3) 1.5 km (4) 2.0 km

Q.6 A particle is projected with a velocity u making an angle θ with the horizontal. At any instant, its velocity v is at right angles to its initial velocity u; then v is -
(1) u cos θ (2) u cos θ
(3) u cot θ (4) u sec θ

Q.7 The speed at the maximum height of a projectile is $\frac{\sqrt{3}}{2}$ times of its initial speed 'u' of projection. Its range on the horizontal plane-

(1) $\frac{\sqrt{3}u^2}{2g}$ (2) $\frac{u^2}{2g}$

(3) $\frac{3u^2}{2g}$ (4) $\frac{3u^2}{g}$

Q.8 What is the ratio of P.E. w.r.t. ground and K.E. at the top most point of the projectile motion -

(1) $\cos^2\theta$ (2) $\sin^2\theta$

(3) $\tan^2\theta$ (4) $\cot^2\theta$

Q.9 A ball is thrown at an angle θ with the horizontal and the range is maximum. The value of $\tan\theta$ is -

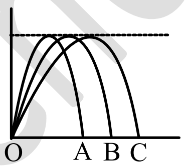
(1) 1 (2) $\sqrt{3}$ (3) $\frac{1}{\sqrt{3}}$ (4) 2

Q.10 A student is able to throw a ball vertically to maximum height of 40 m. The maximum distance to which the student can throw the ball in the horizontal direction -

(1) $40\,(2)^{1/2}$ m (2) $20(2)^{1/2}$ m

(3) 20 m (4) 80 m

Q.11 Three projectile A,B and C are thrown from the same point in the same plane. Their trajectories are shown in the figure. Then which of the following statement is true.



(1) The time of flight is the same for all the three
(2) The launch speed is greatest for particle C
(3) The horizontal velocity component is greatest for particle C
(4) All of the above

Q.12 A projectile is thrown with an initial velocity of $\vec{v} = a\hat{i} + b\hat{j}$, if the range of projectile is double of maximum height reached by it then -

(1) a = 2b (2) b = a

(3) b = 2a (4) b = 4a

Q.13 The equation of a projectile is $y = \sqrt{3}\,x - \frac{gx^2}{2}$ the angle of projection is -

(1) 30° (2) 60° (3) 45° (4) none

Q.14 The equation of projectile is $y = 16x - \frac{x^2}{4}$ the horizontal range is-

(1) 16 m (2) 8 m

(3) 64 m (4) 12.8 m

Q.15 A marble is dropped vertically, Another identical marble B is projected horizontally from the same point at the same instant -

(1) A will reach the ground earlier than B
(2) B will reach the ground earlier than A
(3) Both A and B will reach the ground at the same instant
(4) None of the above

Q.16 If air resistance is not considered in projectiles, the horizontal motion takes place with -

(1) Constant velocity
(2) Constant acceleration
(3) Constant retardation
(4) Variable velocity

Q.17 If a projectile is fired at an angle θ with the vertical with velocity u, then maximum height attained is given by -

(1) $\frac{u^2 \cos\theta}{2g}$ (2) $\frac{u^2 \sin^2\theta}{2g}$

(3) $\frac{u^2 \sin^2\theta}{g}$ (4) $\frac{u^2 \cos^2\theta}{2g}$

Q.18 If R is the maximum horizontal range of a particle, then the greatest height attained by it is -

(1) R (2) 2R (3) $\frac{R}{2}$ (4) $\frac{R}{4}$

Q.19 A ball is projected upwards. Its acceleration at the highest point is -

(1) zero
(2) directed upwards
(3) directed downwards
(4) such as cannot be predicted

Q.20 A bullet is fired from a cannon with velocity 500 ms^{-1}. If the angle of projection is 15° and g = 10 ms^{-2}, then the range is -

(1) 25×10^3 m (2) 12.5×10^3 m
(3) 50×10^2 m (4) 25×10^2 m

Q.21 If a projectile is fired at an angle θ with the horizontal with velocity u, then the time of flight is given by -

(1) $\frac{u\cos\theta}{2}$ (2) $\frac{u\sin\theta}{g}$

(3) $\frac{2u\sin\theta}{g}$ (4) $\frac{2u\cos\theta}{g}$

Q.22 The horizontal range of a projectile is maximum when angle of projection is -

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{6}$

(3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

Q.23 Two stone are projected with the same speed but making different angles with the horizontal. Their ranges are equal. If the angle of projection of one is $\frac{\pi}{3}$ and its maximum height is y_1 then the maximum height of the other will be -

(1) $3y_1$ (2) $2y_1$
(3) $\frac{y_1}{2}$ (4) $\frac{y_1}{3}$

Q.24 A projectile is thrown from a point in a horizontal plane such that its horizontal and vertical velocity component are 9.8 m/s and 19.6 m/s respectively. Its horizontal range is -

(1) 4.9 m (2) 9.8 m
(3) 19.6 m (4) 39.2 m

Q.25 A particle is projected with a velocity v, so that its range on a horizontal plane is twice the greatest height attained. If g is acceleration due to gravity then its range is -

(1) $\frac{4v^2}{5g}$ (2) $\frac{4g}{5v^2}$
(3) $\frac{4v^3}{5g^2}$ (4) $\frac{4v}{5g^2}$

Q.26 A projectile is thrown into space so as to have the maximum possible horizontal range equal to 400 m. Taking the point of projection as the origin, the coordinates of the point where the velocity of the projectile is minimum are -

(1) (400, 100) (2) (200, 100)
(3) (400, 200) (4) (200, 200)

Q.27 A particle is fired with velocity u making angle θ with the horizontal. What is the change in velocity when it is at the highest point ?

(1) $u \cos \theta$ (2) u
(3) $u \sin \theta$ (4) $(u \cos \theta - u)$

Q.28 In the above, the change in speed is -

(1) $u \cos \theta$ (2) u
(3) $u \sin \theta$ (4) $(u \cos \theta - u)$

Q.29 An arrow is shot into the air. Its range is 200 metres and its time of flight is 5s. If the value of g is assumed to be 10 ms^{-2}, then the horizontal component of the velocity of arrow is -

(1) 25 m/s (2) 40 m/s
(3) 31.25 m/s (4) 12.5 m/s

Q.30 In the Q. 29, the maximum height attained by the arrow is -

(1) 25 m (2) 40 m
(3) 31.25 m (4) 12.5 m

Q.31 In the Q.29, the vertical component of the velocity is -

(1) 25 m/s (2) 40 m/s
(3) 12.5 m/s (4) 31.25 m/s

Q.32 In the Q.29, the angle of projection with the horizontal is -

(1) $\tan^{-1}\frac{4}{5}$ (2) $\tan^{-1}\frac{5}{4}$

(3) $\tan^{-1}\frac{5}{8}$ (4) $\tan^{-1}\frac{8}{5}$

Q.33 The ceiling of a hall is 40 m high. For maximum horizontal distance, the angle at which the bell can be thrown with a speed of 56 ms^{-1} without hitting the ceiling of the hall is-

(1) 25° (2) 30° (3) 45° (4) 60°

Q.34 For the top of a tower 19.6 m high, a ball is thrown horizontally. If the line joining the point of projection to the point where it hits the ground makes an angle of 45° with the horizontal, then the initial velocity of the ball is-

(1) 9.8 ms^{-1} (2) 4.9 ms^{-1}

(3) 14.7 ms^{-1} (4) 2.8 ms^{-1}

Q.35 When a particle is thrown horizontally the resultant velocity of the projectile at any time t is given by -

(1) gt (2) $\frac{1}{2}gt^2$ (3) $\sqrt{u^2+g^2t^2}$ (4) $\sqrt{u^2-g^2t^2}$

Q.36 A ball is projected upwards from the top of a tower with a velocity of 50 ms^{-1} making an angle of 30° with the horizontal. The height of the tower is 70m. After how much time from the instant of throwing will the ball reach the ground ?

(1) 2s (2) 5s (3) 7s (4) 9s

Q.37 A ball is thrown at different angles with the same speed u and from the same point and it has the same range in both the case. If y_1 and y_2 be the heights attained in the two cases, then $y_1 + y_2$ = ...

(1) $\frac{u^2}{g}$ (2) $\frac{2u^2}{g}$ (3) $\frac{u^2}{2g}$ (4) $\frac{u^2}{4g}$

Q.38 Two balls A and B are thrown with speed u and $\frac{u}{2}$ respectively. Both the balls cover the same horizontal distance before returning to the plane of projection. If the angle of projection of ball B is 15° with the horizontal, then the angle of projection of A is -

(1) $\sin^{-1}\frac{1}{8}$ (2) $\frac{1}{2}\sin^{-1}\frac{1}{8}$

(3) $\frac{1}{3}\sin^{-1}\frac{1}{8}$ (4) $\frac{1}{4}\sin^{-1}\frac{1}{8}$

Q.39 At what angle with the horizontal should a ball be thrown so that its range R is related to the time of flight as $R = 5T^2$. Take $g = 10\ ms^{-2}$ -

(1) 30° (2) 45°

(3) 60° (4) 90°

Q.40 A projectile is projected with initial velocity $(6\hat{i}+8\hat{j})$ m/s. If $g = 10\ ms^{-2}$, then horizontal range is -

(1) 4.8 metre (2) 9.6 metre

(3) 19.2 metre (4) 14.0 metre

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 7

Q.1 The maximum range of a gun or horizontal range is 16 km. If g = 10 ms^{-2}, the muzzle velocity of the shell must be –
(1) 1600 ms^{-1} (2) 400 ms^{-1}
(3) $200\sqrt{2}$ ms^{-1} (4) $160\sqrt{10}$ ms^{-1}

Q.2 If a body A of mass M is thrown with velocity u at an angle of 30° to the horizontal and another body B of the same mass is thrown with the same speed at an angle of 60° to the horizontal, the ratio of horizontal range of A to B will be –
(1) 1 : 3 (2) 1 : 1
(3) 1 : $\sqrt{3}$ (4) $\sqrt{3}$: 1

Q.3 Galileo writes that for angles of projection of a projectile at angle (45° + α) and (45° – α), the horizontal ranges described by the projectile are in the ratio of -
(1) 2 : 1 (2) 1 : 2
(3) 1 : 1 (4) 2 : 3

Q.4 During projectile motion, the quantities that remain unchanged are -
(1) force and vertical velocity
(2) acceleration and horizontal velocity
(3) kinetic energy and acceleration
(4) acceleration and momentum

Q.5 A boy aims at a bird from a point at a horizontal distance of 100 m. The gun can impart a velocity of 500 ms^{-1} to the bullet. At what height above the bird must he aim his gun in order to hit (g = 10 ms^{-2})
(1) 20 cm (2) 40 cm
(3) 50 cm (4) 100 cm

Q.6 A ball whose kinetic energy is E, is thrown at an angle of 45° with the horizontal, its kinetic energy at the highest point of its flight will be –
(1) E (2) $\frac{E}{\sqrt{2}}$
(3) $\frac{E}{2}$ (4) zero

Q.7 A bullet is fired from a gun with velocity 500 ms^{-1}, then the maximum range is –
(1) 25×10^3 m (2) 12.5×10^3 m
(3) 50×10^2 m (4) 25×10^2 m

Q.8 A body is projected at such an angle that the horizontal range is three times the greatest height. The angle of projection is –
(1) 25° (2) 33° (3) 42° (4) 53°

Q.9 A projectile can have the same range R for two angles of projection. If t_1 and t_2 be the times of flight in the two cases –

(1) $t_1t_2 \propto R^2$ (2) $t_1t_2 \propto R$

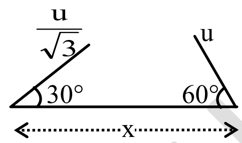
(3) $t_1t_2 \propto \frac{1}{R}$ (4) $t_1t_2 \propto \frac{1}{R^2}$

Q.10 The angle which the velocity vector of a projectile, thrown with a velocity v at an angle θ to the horizontal, will make with the horizontal after time t of its being thrown up is-

(1) θ (2) $\tan^{-1}\left(\frac{\theta}{t}\right)$

(3) $\tan^{-1}\left(\frac{v\cos\theta}{v\sin\theta - gt}\right)$ (4) $\tan^{-1}\left(\frac{v\sin\theta - gt}{v\cos\theta}\right)$

Q.11 Two particles separated at a horizontal distance x as shown in fig. they projected at the same line as shown in fig. with different initial speeds. The time after which the horizontal distance between them become zero –



(1) $\frac{x}{u}$ (2) $\frac{u}{2x}$

(3) $\frac{2u}{x}$ (4) none of these

Q.12 Two projectiles of same mass and with same velocity are thrown at an angle 60° and 30° with the horizontal, then which quantity will remain same-

(1) time of flight

(2) range of projectile

(3) max height acquired

(4) all of them

Q.13 A body is thrown horizontally with a velocity $\sqrt{2gh}$ from the top of a tower of height h. It strikes the level ground through the foot of the tower at a distance x from the tower. The value of x is -

(1) h (2) $\frac{h}{2}$

(3) 2h (4) $\frac{3}{4}$h

Q.14 At the uppermost point of a projectile its velocity and acceleration are at an angle of-

(1) 180° (2) 90°

(3) 60° (4) 45°

Q.15 For angles of projection of a projectile at angles (45°– θ) and (45° + θ), the horizontal ranges described by the projectile are in the ratio of -

(1) 1 : 1 (2) 2 : 3

(3) 1 : 2 (4) 2 : 1

Q.16 A particle starting from the origin (0,0) moves in a straight line in the (x, y) plane. Its coordinates at a later time are $(\sqrt{3}, 3)$. The path of the particle makes with the x-axis an angle of -

(1) 0° (2) 30°
(3) 45° (4) 60°

Q.17 A missile is fired for maximum range with an initial velocity of 20 m/s. If g = 10 m/s^2, the range of the missile is :

(1) 20 m (2) 40 m
(3) 50 m (4) 60 m

Q.18 A projectile is fired at an angle of 45º with the horizontal. Elevation angle of the projectile at its highest point as seen from the point of projection, is :

(1) 45º (2) 60º
(3) $\tan^{-1}\frac{1}{2}$ (4) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 8

Q.1 A body is thrown with a velocity of 9.8 ms^{-1} making an angle of 30° with the horizontal. It will hit the ground after a time-

(1) 3 s (2) 2 s

(3) 1.5 s (4) 1s

Q.2 An aeroplane moving horizontally with a speed of 180 km/hr. drops a food packet while flying at a height of 490 m. The horizontal range of the packet is -

(1) 180 m (2) 980 m

(3) 500 m (4) 670 m

Q.3 Three particles A, B and C are projected from the same point with the same initial speeds making angles 30°, 45° and 60° respectively with the horizontal. Which of the following statements is correct ?

(1) A, B and C have unequal ranges

(2) Ranges of A and C are equal and less than that of B

(3) Ranges of A and C are equal and greater than that of B

(4) A, B and C have equal ranges

Q.4 A plane is flying horizontally at 98 ms^{-1} and releases an object which reaches the ground in 10 s. The angle made by it while hitting the ground is -

(1) 55° (2) 45°

(3) 60° (4) 75°

Q.5 At the top of the trajectory of a projectile, the acceleration is -

(1) maximum (2) minimum

(3) zero (4) g

Q.6 If the range of a gun which fires a shell with muzzle speed v, is R, then the angle of elevation of the gun is -

(1) $\cos^{-1}\frac{v^2}{Rg}$ (2) $\cos^{-1}\frac{Rg}{v^2}$

(3) $\frac{1}{2}\sin^{-1}\frac{v^2}{Rg}$ (4) $\frac{1}{2}\sin^{-1}\frac{Rg}{v^2}$

Q.7 A stuntman plans to run across a roof top and then horizontally off it to land on the roof of next building. The roof of the next building is 4.9 metre below the first one and 6.2 metre away from it. What should be his minimum roof top speed in m/s, so that he can successfully make the jump ?

(1) 3.1 (2) 4.0 (3) 4.9 (4) 6.2

Q.8 The maximum range of a projectile fired with some initial velocity is found to be 1000 metre. The maximum height (H) reached by this projectile is -

(1) 250 metre (2) 500 metre

(3) 1000 metre (4) 2000 metre

Q.9 Range of a projectile is R, when the angle of projection is 30°. Then, the value of the other angle of projection for the same range, is -

(1) 45° (2) 60° (3) 50° (4) 40°

Q.10 The number of bullets are fired in all possible direction with the same initial velocity u. The maximum area of ground covered by bullets is

(1) $\pi\left(\frac{2u^2}{g}\right)^2$ (2) $3\pi\left(\frac{u}{g}\right)^2$

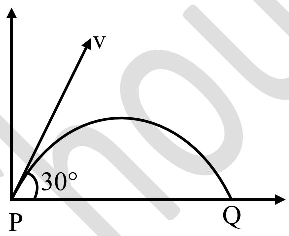
(3) $5\pi\left(\frac{u}{2g}\right)^2$ (4) $\pi\left(\frac{u^2}{g}\right)^2$

Q.11 A body is thrown with some velocity from the ground. Maximum height when it is thrown at 60° to horizontal is 90 m. What is the height reached when it is thrown at 30° to the horizontal -

(1) 90 m (2) 45 m

(3) 30 m (4) 15 m

Q.12 A projectile of mass m is fired with velocity v from a point as shown in figure, neglecting air resistance, what is the change in momentum when leaving P and arriving at Q ?



(1) 0 (2) mv

(3) 4 mv (4) $\frac{2mv}{\sqrt{3}}$

Q.13 A particle is projected at an angle of 45° from 8m before the foot of a wall, just touches the top of the wall and falls on the ground on the opposite side at a distance 4 m from it. The height of wall is -

(1) $\frac{2}{3}$m (2) $\frac{4}{3}$m

(3) $\frac{8}{3}$m (4) $\frac{3}{4}$m

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 9

These questions of two statements each, printed as Assertion and Reason. While answering these Questions you are required to choose any one of the following four responses.

(A) If both Assertion & Reason are true & the Reason is a correct explanation of the Assertion.

(B) If both Assertion and Reason are true but Reason is not a correct explanation of the Assertion.

(C) If Assertion is true but the Reason is false.

(D) If Assertion & Reason both are false.

Q.1 **Assertion :** A body dropped from a given height and another body projected horizontally from the same height strike the ground simultaneously.

Reason : Because horizontal velocity has no effect in the vertical direction.

(1) A (2) B (3) C (4) D

Q.2 **Assertion :** A projectile is thrown with an initial velocity of $(a\hat{i}+b\hat{j})$ m/sec. If range of projectile is maximum then a = b.

Reason : In projectile motion, angle of projection is equal to 45° for maximum range condition.

(1) A (2) B (3) C (4) D

Q.3 **Assertion :** If the position vector of a particle moving in space is given by $\vec{r} = 2t\hat{i} - 4t^2\hat{j}$, then the particle moves along a parabolic trajectory.

Reason : Because $\vec{r} = x\hat{i} + y\hat{j}$ and $\vec{r} = 2t\hat{i} - 4t^2\hat{j} \Rightarrow y = -x^2$

(1) A (2) B (3) C (4) D

Q.4 **Assertion :** In projectile motion, when horizontal range is n times the maximum height, the angle of projection is given by $\tan\theta = \frac{4}{n}$

Reason : In the case of horizontal projection the vertical velocity increases with time.

(1) A (2) B (3) C (4) D

Q.5 **Assertion :** Path of projected ball becomes parabolic in gravitational field ($\theta \neq 90°$).

Reason : Gravitational force always act perpendicular to velocity during the motion.

(1) A (2) B (3) C (4) D

Q.6 **Assertion :** In projectile motion, the acceleration is constant in both magnitude and direction but the velocity changes in both magnitude and direction.

Reason : When a force or acceleration is acting in an oblique direction to the direction of velocity then both magnitude and direction of the velocity may be changed.

(1) A (2) B (3) C (4) D

Q.7 **Assertion :** In projectile motion the projectile hits the ground with the same velocity with which it was thrown.

Reason : In projectile motion horizontal velocity remains same but vertical velocity continuously change and particle strikes the ground with same vertical velocity with which the particle was thrown in vertical direction.

(1) A (2) B (3) C (4) D

Q.8 **Assertion :** In projectile motion the vertical velocity of the particle is continuously decreased during its ascending motion .
Reason : In projectile motion downward constant acceleration is present in vertical direction.
(1) A (2) B (3) C (4) D

Q.9 **Assertion :** The path of one projectile as seen from another projectile is a straight line.
Reason : Two projectiles projected at angles α and $90° - \alpha$ have same range .
(1) A (2) B (3) C (4) D

Q.10 **Assertion :** In case of projectile motion acceleration, horizontal component of velocity and mechanical energy remains unchanged but speed, vertical component of velocity, momentum, K.E. and P.E. change.
Reason : In the presence of air resistance, the range and maximum height attained reduce, but time of flight increases.
(1) A (2) B (3) C (4) D

Q.11 **Assertion :** In the projectile motion projected body behave just like a freely falling body.
Reason : There is no change in linear momentum in projectile motion.
(1) A (2) B (3) C (4) D

Q.12 **Assertion :** Projectile motion is uniformly accelerated motion. (Neglect the effect of air)
Reason : In projectile motion, speed remains constant.
(1) A (2) B (3) C (4) D

Q.13 **Assertion :** Horizontal range is same for angle of projection θ and $(90 - \theta)$ if speed of projection is same.
Reason : Horizontal range is independent of angle of projection.
(1) A (2) B (3) C (4) D

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 1 (ANSWERS)

Ques.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	3	2	2	2	2	4	4	2	1	3	1	1	2	3	2	4	1	3	2	1
Ques.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans.	1	3	1	3	4	3	1	4	2	3	2	2	1	3	3	4	4	2	1	3
Ques.	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Ans.	2	4	4	2	4	3	3	4	3	1	2	4	1	4	4	1	2	4	2	4
Ques.	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
Ans.	2	4	1	2	4	2	3	4	1	4	1	3	3	3	1	2	2	1	4	1
Ques.	**81**																			
Ans.	2																			

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 2 (ANSWERS)

Ques.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	2	2	2	4	3	3	4	4	2	3	2	1	1	2	2	1	2	3	3	3
Ques.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans.	2	4	2	1	4	1	4	2	3	1	2	3	2	4	2	1	2	1	1	2
Ques.	**41**																			
Ans.	3																			

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 3 (ANSWERS)

Ques.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	3	4	1	3	2	4	3	3	4	1	1	4	2	2	2	1	4	3	2	3
Ques.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans.	3	2	3	3	4	2	3	2	2	1	4	3	3	2	4	4	4	1	1	2
Ques.	**41**	**42**	**43**	**44**	**45**															
Ans.	3	4	2	3	3															

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 4 (ANSWERS)

Ques.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	1	3	1	2	2	3	2	1	1	4	1	4	2	2	2	4	1	2	3	3
Ques.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans.	1	4	2	1	3	4	1	2	2	4	1	2	3	2	2	2	2	3	3	3
Ques.	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Ans.	2	1	1	1	3	3	2	2	2	3	3	2	2	4	3	3	2	4	1	3

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 5 (ANSWERS)

Ques.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	1	1	1	1	4	1	1	1	3	2	3	4	2	4	3	2	4	4	1	2
Ques.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**												
Ans.	1	2	3	4	4	3	4	1												

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 6 (ANSWERS)

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	3	2	1	3	2	3	1	3	1	4	4	3	2	3	3	1	4	4	3	2
Ques.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	3	4	4	4	1	2	3	4	2	3	1	3	2	1	3	3	3	2	2	2

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 7 (ANSWERS)

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Ans.	2	2	3	2	1	3	1	4	2	4	1	2	3	2	1	4	2	3

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 8 (ANSWERS)

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13
Ans.	4	3	2	2	4	4	4	1	2	4	3	2	3

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 9 (ANSWERS)

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13
Ans.	1	1	1	2	3	1	4	1	2	2	3	3	3