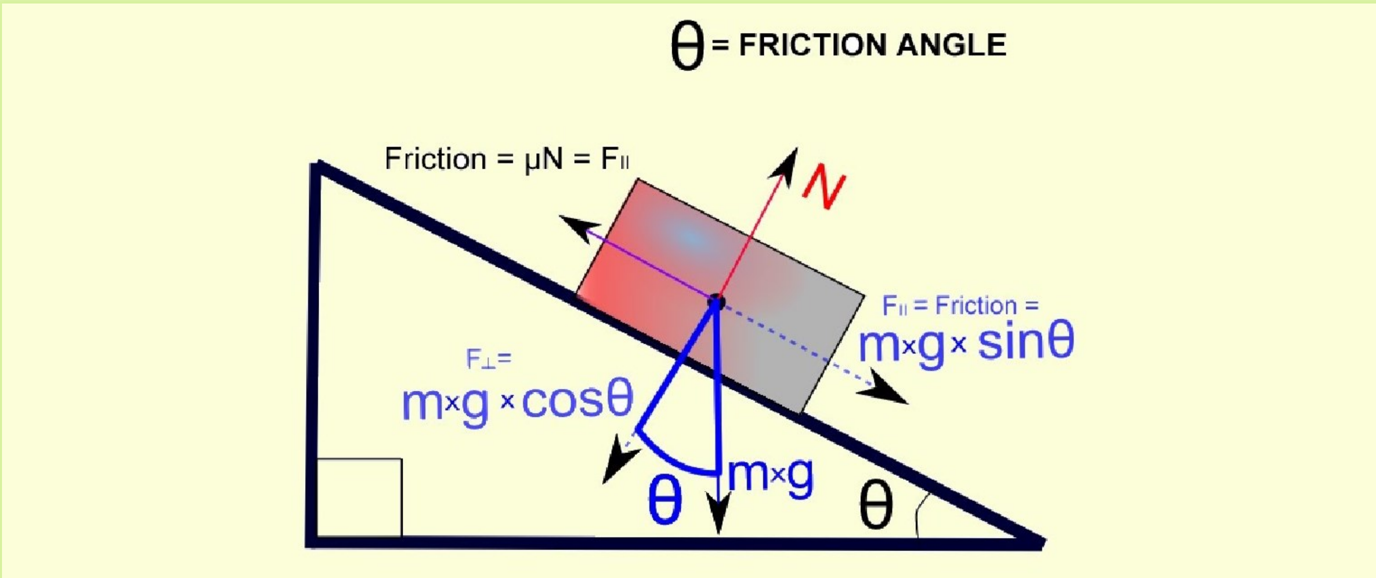
PHYSICS for JEE, NEET & BOARD

11th

LAWS OF MOTION



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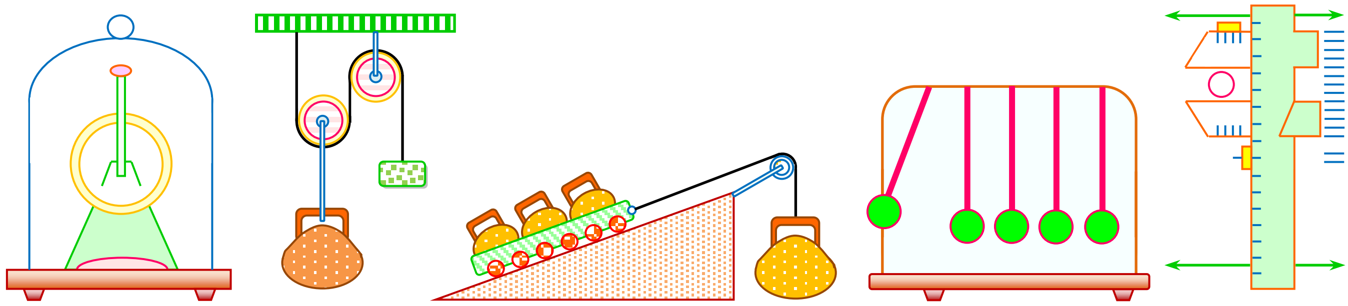


LAWS OF MOTION

1 INTRODUCTION

In the previous lesson, we described the motion of particles based on definitions of displacement, velocity and acceleration. We saw that uniform motion needed only the concept of velocity. Non-uniform motion required the additional concept of acceleration. So far, we have not asked the question; what causes the motion of bodies? Why do some objects accelerate at a higher rate than others? In this chapter, we will discuss the answers of these questions.

We use the concepts of force and mass to describe the change in motion of particles, then we will discuss the three basic laws of motion, which are based on experimental observation and were formulated more than three centuries ago by Newton.

This part of physics is classical mechanics, which describes the relationship between motion of a body and force acting on that body.

Classical mechanics deals only with objects that are very large compared with the dimensions of atoms (10^{-10}m) and move at speeds that are much less than the speed of light (3×10^8m/s).

2 FORCE AND INTERTIA

Everyone has a basic understanding of force from everyday experiences. Although forces are not visible, we do see and experiences their effects.

It can be defined as "Force is a push or pull which tries to change or changes the state of rest or uniform motion of a body" or we can say force is the cause of translatory motion.

It is our common observation that an object such as a chain lying in a room or a vehicle parked outside the house remains at rest unless a push or a pull is given to it. Such an object cannot move at its own. In other words, force has to be applied in order to move an object at rest. Also, if a body is moving along a straight line with some velocity, it is found that force is required to change the direction of motion or the magnitude of the velocity.

Thus force is an agent which

(i) changes or tends to change state of motion

(ii) changes or tends to change momentum

(iii) changes or tends to change shape and size of objects

Inertia: If you attempt to change the velocity of an object, the object resists this change.

Inertia is solely a property of an individual object; it is a measure of the response of an object to an external force. It can be defined in this way: ***It is the property of a body by virtue of which it opposes any change in its state of rest or uniform motion.***

Mass is used to measure inertia. The greater the mass of a body, the less that body changes its state of motion under the action of an applied force.

For example:consider two large, solid cylinders of equal size, one soft wood and the other steel. If you were to push the cylinders along the horizontal surface, the force required to give the steel cylinder some acceleration would be larger than the force needed to give the soft wood cylinder the same acceleration. Therefore, we say that the steel cylinder has more inertia than the soft wood.

Mass: "Mass is an inherent property of a body which is measure of the object's inertia (which is a tendency to resist an acceleration when a force acts on it) and is independent of body's surroundings and of the method used to measure it".

It is a scalar quantity. The SI unit of mass is the kilogram (kg). Mass is the quantity that obeys the rules of ordinary algebra.

For example: If you combine 3 kg mass, with a 5 kg mass, their total mass will be 8 kg.

Mass should not be confused with weight. Mass and weight are two different quantities. As we will see later, the weight of a body is equal to the magnitude of the force exerted by the earth on the body and varies with location.

For example; A person whose weight is 60 kg on Earth weighs only 10 kg on the Moon. On the other hand, the mass of a body is same everywhere, regardless of location. An object having a mass 6 kg on earth also has a mass of 6 kg. on the Moon.

3 NEWTONS LAWS OF MOTION

Till the mid of 17th century most of the philosophers thought that some influence was needed to keep a body moving. They thought that a body was in its 'natural state' when it was at rest and some external influence was needed to continuously move a body; otherwise it would naturally stop moving.

Confusions about these issues were solved in 1687 when Newton presented his three laws of motion. According to him influence is needed not for all kind of motion it is needed for accelerated motion only. Before going in details about these three laws, let us summarise these three laws first.

3.1 NEWTON'S FIRST LAW OF MOTION

Before we state Newton's first law, consider the following simple experiment. Suppose a book is lying on a table. Obviously, the book remains at rest in the absence of any influences. Now imagine that you push the book with a horizontal force large enough to overcome the force of friction between book and table. The book can then be kept in motion with constant velocity if the force you apply is equal in magnitude to the force of friction and in the direction opposite the friction force.

If the applied force exceeds the force of friction, the book accelerates. If you stop pushing, the book stops sliding after moving a short distance because the force of friction retards its motion. Now imagine pushing the book across a smooth, highly waxed floor. The book again comes to rest after you stop pushing, but not as quickly as before. Now imagine a floor so highly polished that friction is negligibly small; in this case, the book, once set in motion, slides until it hits the wall. Then forces that we have described are ***external* forces**, that is, forces exerted on the object by other objects.

Before about 1600, scientists felt that the natural state of matter was the state of rest. Galileo was the first to take a different approach to motion and the natural state of matter. He devised thought experiments, such as the one we just discussed for a book on a frictionless surface, and concluded that it is not the nature of an object to stop once set in motion: rather, it is its nature to resist changes in its motion. In his words, "Any velocity once imparted to a moving body will be maintained as long as the external causes of retardation are removed."

The New approach to motion was later formalized by Newton in a form that has come to be known as **Newton's first law of motion:**

An object at rest remains at rest and an object in motion will continue in motion with a constant velocity (that is, constant speed in a straight line) unless it experiences a net external force.

In simpler terms, we can say that *when the net force on a body is zero, its acceleration is zero*.

i.e., when $\Sigma\vec{F} = 0$,

then, $\vec{a} = 0$.

From the first law, we conclude that an isolated body (a body that does not interact with its environment) is either at rest or moving with constant velocity.

APPLICATIONS OF LAW OF INERTIA

- When a horse suddenly starts moving, the rider falls backwards. Because the lower part of the body of the rider (which is in contact with the horse) comes in motion but because of inertia the upper part tends to be at rest. Hence the rider falls backwards.
- We hit a carpet with a stick to remove the dust: On hitting with a stick, the carpet comes to motion; but because of inertia, the dust particles remain at rest. Due to this, dust particles get removed from the carpet.

3.2 NEWTON'S SECOND LAW OF MOTION

Newton's first law explains what happens to an object when the resultant of all external forces on it is zero: it either remains at rest or moves in a straight line with constant speed. Newton's second law answers the question of what happens to an object that has a non-zero resultant force acting on it.

Imagine you are pushing a block of ice across a frictionless horizontal surface. When you exert some horizontal force F, the block moves with some acceleration a. If you apply a force twice as large, the acceleration doubles. Likewise, if the applied force is increased to $3F$, the acceleration is tripled, and so on. From such observations we conclude that *the acceleration of an object is directly proportional to the resultant force acting on it.*

As stated in the preceding section, the acceleration of an object also depends on its mass. This can be understood by considering the following set of experiments. If you apply a force F to a block of ice on a frictionless surface, the block undergoes some acceleration a. If the mass of the block is doubled, the same applied force produces an acceleration $a/2$. If the mass is tripled, the same applied force produces an acceleration $a/3$, and so on. According to this observation, we conclude that *the acceleration of an object is inversely proportional to its mass.*

These observations are summarized in **Newton's second law:**

The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

Thus we can relate mass and force through the following mathematical statement of Newton's second law:

$$\sum \vec{F} = m\vec{a} \quad \ldots (2)$$

You should note that equation (2) is a *vector* expression and hence is equivalent to the following three component equations:

$$\sum F_x = ma_x$$
$$\sum F_y = ma_y$$
$$\sum F_z = ma_z$$

Linear Momentum

It is defined as the total quantity of motion contained in a body and is measured as the product of mass of body and its velocity. It is denoted by $\vec{p}$.

It is a vector quantity.

Suppose a particle of mass m moves with velocity $\vec{v}$.

$$\vec{p} = m\vec{v}$$

According to Newton's second law of motion, "the rate of change of linear momentum of a body is directly proportional to the external force applied on the body and this change always takes place in the direction of the applied force".

$$\vec{F} \propto \frac{d\vec{P}}{dt}$$

or, $$\vec{F} = k\frac{d\vec{P}}{dt}$$

where $\vec{P}$ represent the linear momentum of the body and $\vec{F}$ is force acting on it.
so,

$$(\ \vec{p} = m\vec{v}\ )$$

$$= km\frac{d\vec{v}}{dt}$$

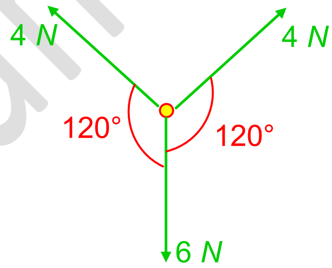
or, $\vec{F} = km\vec{a}$ [as $(d\vec{v}/dt) = \vec{a}$]

Now the units of force are so selected that k becomes unity,
i.e., if $m = 1$, $a = 1$ and $F = 1$,
so, $1 = k \times 1 \times 1$,
i.e., $k = 1$

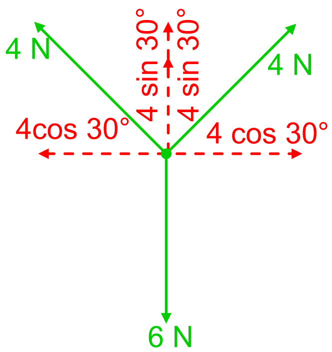
$$\vec{F} = \frac{d\vec{P}}{dt} = m\vec{a} \quad \dots (3)$$

Illustration 1

Question: **The diagram shows the forces that are acting on a particle. Find the acceleration of the particle.**



Solution: To check whether the particle will have any acceleration or not, let us see net force is zero or not. Resolving the forces in horizontal and vertical directions.



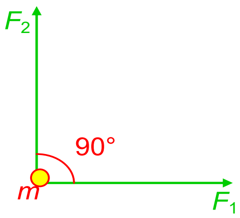
Net force in horizontal direction
$= 4\cos 30° - 4\cos 30°$
$= 0$

Net force in vertically downward direction
$= 6 - 4\sin 30° - 4\sin 30° = 2$ N

As net force is not zero, so the particle will have acceleration.

Illustration 2

Question: **Two forces $\vec{F}_1$ and $\vec{F}_2$ act on a 5.0 kg mass. If $F_1 = 20.0$ N and $F_2 = 15.0$ N, find the acceleration.**



Solution: Acceleration will be in the direction of net force and will have the magnitude given by

$$\sum \vec{F} = M\vec{a}$$

$$\vec{F} = \vec{F}_1 + \vec{F}_2$$

$$\therefore \quad |\vec{F}| = \sqrt{20^2 + 15^2} = 25 \text{ N}$$

$\therefore \qquad |\vec{a}| = \dfrac{|\vec{F}|}{5.0} = 5\,\text{ms}^{-2}$

If the resultants force is at angle α with $\vec{F}_1$.

$$\tan\alpha = \frac{15}{20} \Rightarrow \alpha = 37°$$

Therefore, acceleration is **5 ms^{-2} at an angle 37°** with the direction of $\vec{F}_1$.

Illustration 3

Question: **A force acts for 10 s on a body of mass 10 kg after which the force ceases and the body describes 50 m in the next 5s. Find the magnitude of the force.**

Solution: After the force ceases, the body covers 50 m in 5s.

Therefore, final velocity $v = \dfrac{\text{distance}}{\text{time}} = \dfrac{50}{5} = 10\ \text{ms}^{-1}$

Now, $v = u + at$

$\therefore \quad 10 = 0 + a \times 10$

or $\quad a = 1\ \text{ms}^{-2}$

$F = Ma = 10 \times 1 =$ **10 N**

Illustration 4

Question: **The driver of three wheeler moving with a speed of 36 km h^{-1} sees a child standing in the middle of the road and brings his vehicle to rest in 4s, just in time to save the child. What is the average retarding force on the vehicle? The mass of the three wheeler is 400 kg and the mass of the driver is 65 kg.**

Solution: $u = 36\ \text{kmh}^{-1} = 10$ m/s

Now, $v = u + at$

$\therefore \quad 0 = 10 + a \times 4$ or $a = -2.5\ \text{ms}^{-2}$

Mass of the driver and three wheeler, $M = 65 + 400 = 465$ kg

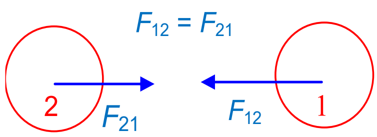
Therefore, $F = -465 \times 2.5 =$ **–1162.5 N**

3.3 NEWTON'S THIRD LAW OF MOTION

According to Newton's third law "for an every action there is an equal and opposite reaction and they act on two different bodies".

It means if two bodies interact, the force exerted on body 1 by body 2 is equal to and opposite the force exerted on body 2 by body 1:

$$\vec{F}_{12} = -\vec{F}_{21} \qquad \ldots (4)$$

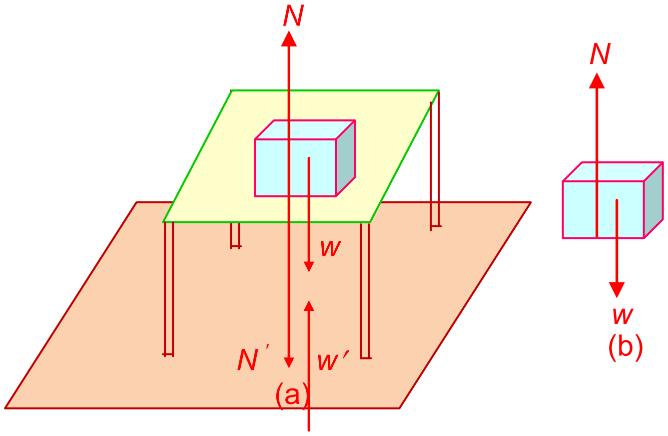


This law, which is illustrated in Figure, is equivalent to stating that *forces always occur in pairs* or that *a single isolated force cannot exist*. The force that body 1 exerts on body 2 is sometimes called the *action force,* while the force body 2 exerts on body 1 is called the *reaction force.* In reality, either force can be labelled the action or reaction force.

The action force is equal in magnitude to the reaction force and opposite in direction. In all cases, the action and reaction forces act on different objects.

For example, the weight of an object, *w*, is defined as the force the Earth exerts on the objects. If the object is a box resting on a table, as shown in figure the reaction force to *w* is the force the box exerts on the earth, *w*′. The box does not accelerate because it is held up by the table. The table therefore, exerts an upward action force, *N*, on the box, called the **normal force.** The normal force is the force that prevents the box form falling through the table and can have any value needed, up to the point of breaking of the table. The normal force balances the force of gravity acting on the box and provides equilibrium. The reaction to *N* is the force exerted by the box on the table, *N*′. Therefore, we conclude that

$w = -w'$ and $N = -N'$



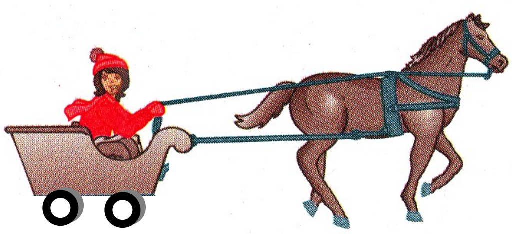
The forces N and N' have the same magnitude, which is the same as w unless the table has broken. Note that the forces acting on the box are w and N, as shown in Figure. The two reaction forces, w' and N', are exerted on objects other than the box. Remember, the two forces in an action-reaction pair always act on two different objects.

From the second law, we see that because the box is in equilibrium (a = 0), it follows that $w = N = mg$.

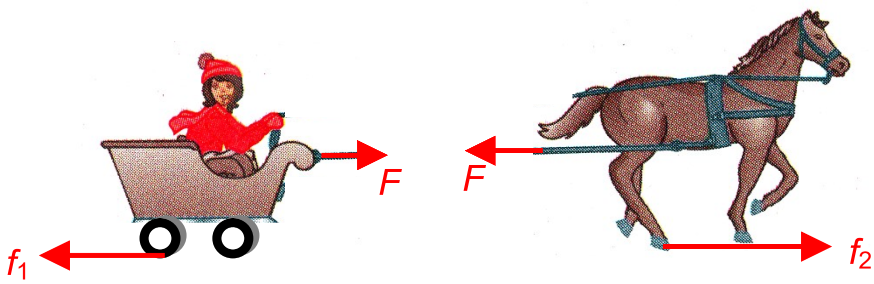
Another example, the force acting on a freely falling projectile is its weight, $w = mg$, which is the force exerted by the Earth on the projectile. The reaction to this force is the force of the projectile on the Earth, $w' = -w$. The reaction force, w' must accelerate the Earth toward the projectile just as the action force, w, accelerates the projectile toward the Earth. However, since the Earth has such a large mass, its acceleration due to this reaction force is negligibly small.

Illustration 5

Question: **A horse pulls a cart with a horizontal force, causing it to accelerate as shown in figure. Newton's third law says that the cart exerts an equal and opposite force on the horse. In view of this, how can the cart accelerate?**



Solution: The motion of any object is determined by the external forces that acts on it. If resultant of external force is non-zero, the object moves in the direction of resultant force. In this situation, the horizontal forces exerted on the cart are forward force exerted by the horse (F) and the backward contact force (f_1) due to roughness of surface. When forward force exerted on the cart exceeds the backward force, the resultant force on it is in the forward direction. This resultant force causes the cart to accelerate to the right. The horizontal force that acts on the horse are the forward contact force (f_2) due to roughness of surface and the backward force of the cart (F). The resultant of these two forces causes the horse to accelerate.



3.4 NEWTON'S SECOND LAW IS THE REAL LAW OF MOTION

Both Newton's first law and third law of motion follow from the second law of motion as explained below.

(a) To prove that Newton's first law is contained in second law. According to Newton's second law, the external force $(\vec{F})$ applied on a body and the acceleration $(\vec{a})$ produced are related to each other by the relation.

$$\vec{F} = M\vec{a}$$

If no external force acts on the body i.e. $\vec{F} = 0$, then

$$M\vec{a} = 0 \text{ or } \vec{a} = 0$$

we know, $\vec{v} = \vec{u} + \vec{a}t$

Setting $\vec{a} = 0$,

we have $\vec{v} = \vec{u}$

It implies that if no external force acts on the body, then a body initially at rest $(\vec{u} = 0)$ will remain at rest $(\vec{v} = 0)$ or if it is moving with some velocity $(\vec{u})$, it will keep on moving with the same velocity $(\vec{v} = \vec{u})$. This exactly is that Newton's first law of motion states. Hence, Newton's first law of motion is contained in the second law.

(b) To prove that Newton's third law is contained in second law. Consider a system of two bodies A and B mutually interacting with each other, such that no external force acts on the system. Suppose that the body A exerts force $\vec{F}_{AB}$ (action) on body B the body B exerts force $\vec{F}_{BA}$ (reaction) on the body A. If $\frac{d\vec{p}_1}{dt}$ and $\frac{d\vec{p}_2}{dt}$ are rate of change of momentum of the bodies A and B respectively due to the forces $\vec{F}_{BA}$ and $\vec{F}_{AB}$ acting on them, then

$$\vec{F}_{BA} = \frac{d\vec{p}_1}{dt} \text{ and } \vec{F}_{AB} = \frac{d\vec{p}_2}{dt}$$

Adding the two equations, we have

$$\vec{F}_{BA} + \vec{F}_{AB} = \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = \frac{d}{dt}(\vec{p}_1 + \vec{p}_2)$$

Since no external force acts on the system, the total of momentum of the system must be constant i.e. time rate of change of momentum of the system should be zero. Therefore, the above equation becomes

$$\vec{F}_{BA} + \vec{F}_{AB} = 0$$

or $$\vec{F}_{AB} = -\vec{F}_{BA}$$

i.e. action and reaction are equal and opposite. Hence Newton's third law of motion is contained in second law.

4 APPLYING NEWTON'S SECOND LAW

Before knowing how to apply Newton's laws of motion to solve questions and to know about the stepwise procedure to solve the same, let us know about the commonly used forces in such situations.

4.1 DIFFERENT TYPES OF FORCES

(i) Force arises from interaction of bodies either due to contact or from a distance. So it can be classified as **contact force** and **non-contact force**.

Contact Force is such type of force, which arises when a body is in physical contact with another.

Example: String tension, spring force, reaction force and friction force etc.

Non-Contact Force is such type of force, which does not involve physical contact between two bodies, but act through empty space. It is also known as field force.

The force of gravitational attraction between two objects is an example of this type of force. Magnet and electric charges can also interact in vacuum.

(ii) Force can also be classified as **conservative** or **non-conservative**. If under the action of a force work done in a round trip is zero or work is path independent, the force is said to be conservative otherwise not. Gravitational, electric and elastic forces are conservative while frictional and viscous forces are non-conservative. In presence of Non-conservative force there is loss of mechanical energy, which is usually converted into heat.

(iii) Force can also be classified as **internal** and **external**. Internal forces are those, which arise from the interactions with other particles that are parts of the system while external forces are those which originate beyond the system under consideration. Same force can be external or internal depending on the system. If we consider a body as the system the force of gravity of earth is external while if we consider body and earth as the system the force becomes internal. Total internal force acting on a system is always zero as these are parts of action-reaction pairs and cancel.

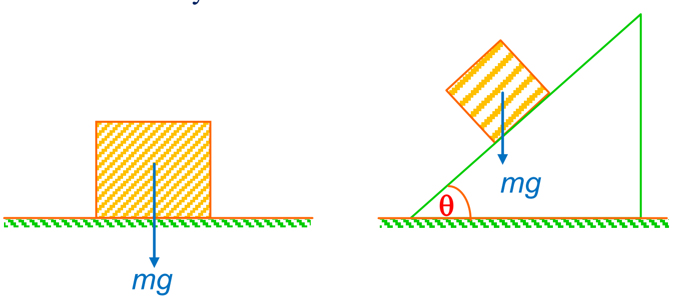
When we study different types of problems based on force, we should analyse different characteristic of forces, which helps in drawing the free body diagram and writing the equations of motion.

CHARACTERISTICS OF FORCE

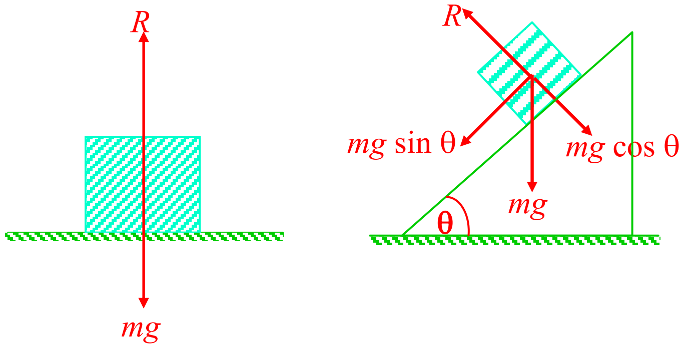
(i) ***Magnitude***
(ii) ***Direction***
(iii) ***Point of application***
(iv) ***Line of action***

(a) Weight (W): The weight of a body is the force by which it is pulled by the gravity of earth. If a body of mass 'm' is located at a point where acceleration due to gravity is $\vec{g}$, the weight $\vec{W} = m\vec{g}$

(i) Magnitude = mg
(ii) Direction will be towards the center of earth. To show the direction on the plane of paper we draw a line in the downward direction as shown below in the figure
(iii) Point of application is center of gravity of the block.
(iv) Line of action is vertically downward.



(b) Normal Reaction (*R*): When a body is pressed against a rigid surface, the body experiences a force, which is perpendicular to the surfaces in contact. This force is called 'normal force' or 'normal reaction'.



(a) (b)

(i) Magnitude of normal reaction is given by the action force perpendicular to the surface on which body is kept

in figure (a) $R = mg$

in figure (b) $R = mg\cos\theta$

(ii) Direction of normal reaction in figure (a) is perpendicular to the surface in upward direction and in figure (b) is perpendicular to the incline surface.

(iii) Point of application is on the surface of contact.

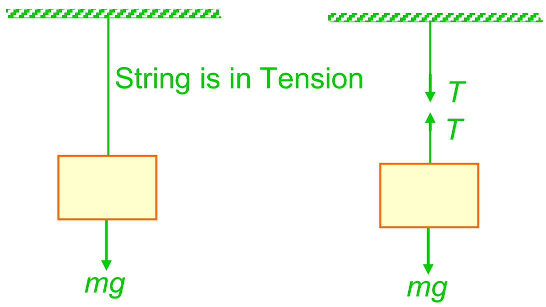
(iv) Line of action is perpendicular to the contact surface.

(c) Friction force (*f*) : If surfaces in contact are rough friction force also acts on bodies. Friction will be dealt in section-II. In this section we will assume all the surfaces to be smooth there will only be normal reaction between them.

(d) String Tension (*T*): String tension is an elastic force. Whenever a body is connected with another body or ceiling through a string, and string is in tension. It acts in opposite direction to the applied force on the string; it means it pulls another body (in contact) to which it is connected.

If string is inextensible bodies connected to its two ends move with the same magnitude of acceleration. If string is massless tension at each point of the string will be same.

In case of pulley and string, if pulley is massless and frictionless then tension on two sides of pulley will be same.



(e) Spring force

A spring is generally a helical metallic wire. When the two free ends of the spring are pulled away or pushed towards each other, the length of the spring is changed. The spring has tendency to come back to its original length. In other words, it develops an opposition to change in its length. This opposing force is called as ***the restoring force in a spring***.

Features of an ideal spring

1. The spring is massless

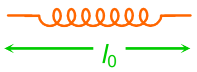
2. The deformation of the spring is uniform. It means change in length (decrease or increase) is uniform.

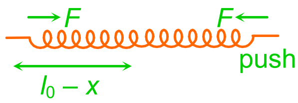
3. The elastic force developed in the spring is directly proportional to the change in the length of the spring.

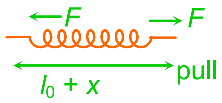
Electric force $\alpha \, . \, x$

$$F \alpha x$$

$$F = Kx$$

(a)


(b)


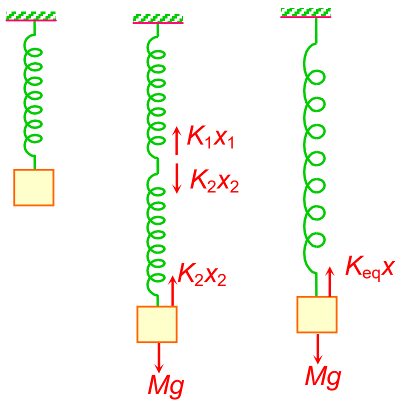
(c)


Where K is known as the spring constant or force constant

Series connection of springs

Two (or more than two) springs connected in series can be replaced by an other spring called as equivalent spring. The deformation in the equivalent spring due to the same block should be equal to sum of the deformation in the two (or more than two) springs due to same block.

Consider two springs having spring constant K_1 & K_2 respectively are connected in series. We have to find out the equivalent spring constant.



$$K_1x_1 = K_2x_2 \quad \text{... (i)}$$

$$K_2x_2 = Mg \quad \text{... (ii)}$$

$$x = x_1 + x_2 \quad \text{... (iii)}$$

$$K_{eq}(x_1 + x_2) = Mg \quad \text{... (iv)}$$

From equation 1 and 2 we get

$$x_1 = \frac{Mg}{K_1} \text{ and } x_2 = \frac{Mg}{K_2}$$

Put these values in equation (iv)

$$K_{eq}\left(\frac{Mg}{K_1} + \frac{Mg}{K_2}\right) = Mg$$

$$\frac{1}{K_1} + \frac{1}{K_2} = \frac{1}{K_{eq}}$$

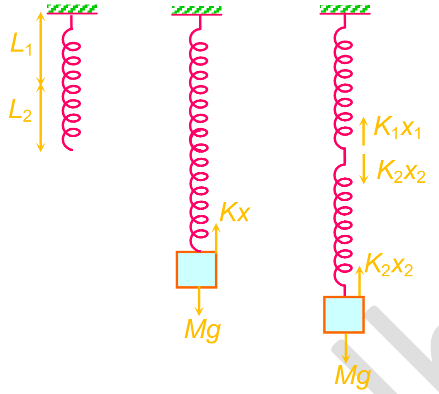
Cutting a spring

If a uniform spring is cut into two parts then what will be the force constant of each? Let us consider a spring of length *L*. After cutting the length of two springs are L_1 and L_2 respectively

$$L = L_1 + L_2 \qquad \ldots \text{(i)}$$

If we hang a mass *m* under it, then the total deformation of the spring is

$$x = \frac{Mg}{K}$$



If the spring has uniform deformation, then deformation per unit length of the spring is $\frac{x}{L}$

The deformation in the length $L_1 = \frac{xL_1}{L} = x_1$ (assume) and the deformation in the length $L_2 = \frac{xL_2}{L} = X_2$ (assume).

Now if join the pieces in series and hang the same mass under them, then we should get the deformation, which we obtained earlier when the two pieces formed the complete spring.

$$K_1x_1 = K_2x_2 = Mg$$

Now, $K_1\left(\frac{xL_1}{L}\right) = Mg$

or $K_1\left(\frac{Mg}{K}\right)\frac{L_1}{L} = Mg$ or $K_1 = \frac{KL}{L_1}$

Similarly $K_2 = \frac{KL}{L_2}$

$$K_1 = \frac{KL}{L_1} \text{ and } K_2 = \frac{KL}{L_2}$$

$$\frac{K_1}{K_2} = \frac{L_2}{L_1}$$

Examples

A spring of length *L* and spring constant *K* is cut into two parts of length $\frac{L}{3}$ and $\frac{2L}{3}$. Find out the spring constant of each.

Solution: $K_1 = \frac{KL}{L/3} = 3K$

$$K_2 = \frac{KL}{2L/3} = \frac{3K}{2}$$

4.2 PROBLEM SOLVING STRATEGY

In such questions you will be given a system of bodies under the action of forces and you will need to find out accelerations of different bodies and unknown forces on bodies. The following steps are needed by you to apply while solving such questions.

Step 1: Identify the system. Arigid body or two or more than two rigid bodies may be considered as system if they are moving with same acceleration (both in magnitude and direction). If internal forces between two bodies to be find out than they should be considered separately as system.

Step 2: Identify force acting on the system.

Step 3: Draw a free body diagram 9FBD) and resolve all the forces in the direction of motion and perpendicular to it. Apply Newton's second law in these mutually perpendicular directions separately, i.e.,

$$\sum F_x = ma_x : \quad \sum F_y = ma_{y:} \quad and \quad \sum F_z = ma_z$$

4.2.1 MAKING FREE BODY DIAGRAM (FBD)

It is a diagram that shows forces acting on the body making it free from other bodies applying forces on the body under consideration. Hence free body diagram will include the forces like weight of the body, normal force, tension in string and the applied force. The important thing while drawing **FBD** is the shape of the body should be taken under consideration and force should be shown in a particular way. For example weight should be applied from center of gravity of body, normal force(s) should be applied on the respective surface(s), tension should be applied on the side(s) of string(s).

Examples

(i) Free body diagram of a block resting on table

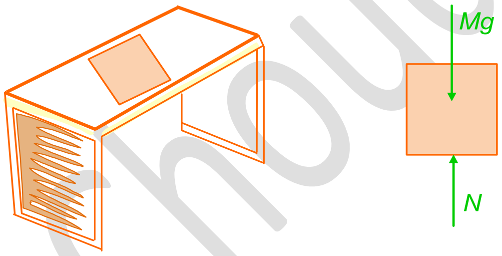


Fig. 5

(ii) Free body diagram of bodies in contact and moving together on smooth surface.

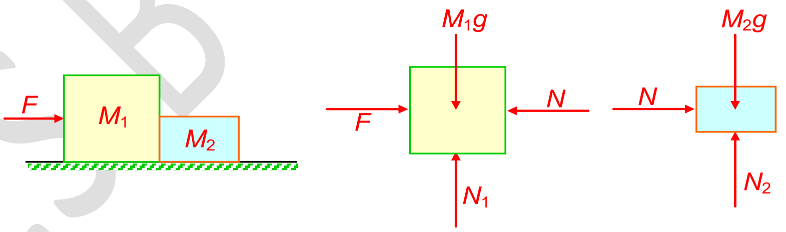


Fig. 6

Note that, normal force is taken normal to the surface of contact and towards the body under consideration

(iii) Free body diagram of bodies connected with strings and moving under the action of external force, on a smooth surface.

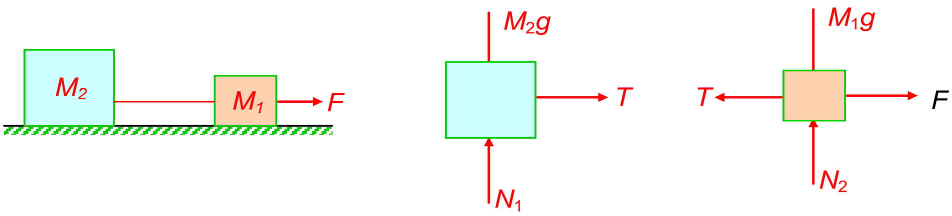


Fig. 7

Note that, tension is acting along the string and away from the body under consideration.

Step 3: Identify the direction of acceleration and resolve the forces along this direction and perpendicular to it.

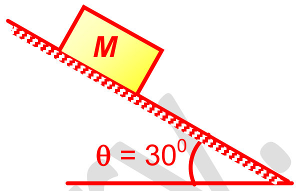
Step 4: Find net force in the direction of acceleration and apply $F = Ma$ to write equation of motion in that direction. In the direction of equilibrium take net force zero.

Step 5: If needed write relation between accelerations of bodies given in the situation

Step 6: Solve the written equations in steps 4 and 5 to find unknown accelerations and forces.

Illustration 6

Questions: **A block of mass $M = \sqrt{3}$ kg is placed on a frictionless, inclined plane of angle $\theta = 30^0$, as shown in the figure.**
Determine the acceleration of the block after it is released. What is force exerted by the incline on the block? [take g = 10 m/s^2]



Solution: When the block is released, it will move down the incline. Let its acceleration be a. As the surface is frictionless, so the contact force will be normal to the plane. Let it be N.

Here, for the block we can apply equation for motion along the plane and equation for equilibrium perpendicular to the plane.

i.e., $Mg \sin\theta = Ma$

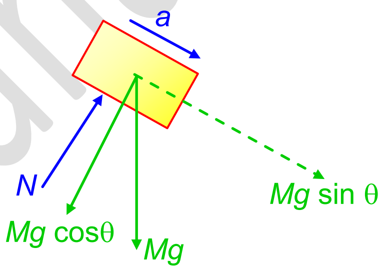
$\Rightarrow \quad a = g \sin\theta = 10 \times \sin 30^0 = 10 \times 1/2$

$a = \mathbf{5\ m/s^2}$

Also, $Mg \cos\theta - N = 0$

$\Rightarrow \quad N = Mg \cos\theta = \sqrt{3} \times 10 \times \frac{\sqrt{3}}{2}$

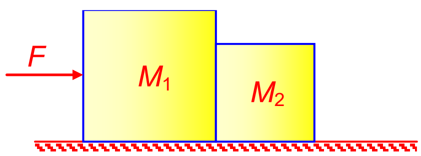
$\Rightarrow \quad N = \mathbf{15\ N}$



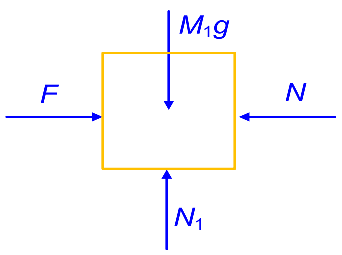
(FBD of Block)

Illustration 7

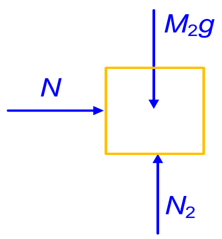
Question: ***One block pushes other***
Two blocks of masses M_1 = 2 kg and M_2 = 4kg are placed in contact with each other on a frictionless horizontal surface as shown in figure. A constant force F = 24 N is applied on M_1 as shown. Find magnitude of acceleration of the system. Also calculate the contact force between the blocks.



Solution: Here accelerations of both blocks will be same as they are rigid and in contact. As the surfaces are frictionless, contact force on any surface will be normal force only. Let the acceleration of each block is a and contact forces are N_1, N_2 and N as shown in free body diagrams of blocks.



(FBD of M_1)



(FBD of M_2)

Applying, Newton's Second Law for the blocks;

For M_1, $F - N = M_1 a$ … (i)

$M_1 g - N_1 = 0$ … (ii)

for M_2, $N = M_2 a$ …(iii)

$M_2 g - N_2 = 0$ … (iv)

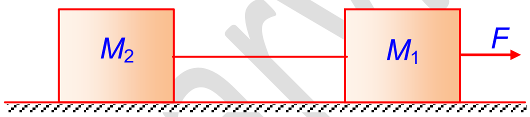
Solving (i) and (iii) $a = \dfrac{F}{M_1 + M_2} = \dfrac{24}{2+4} = $ **4 m/s²**

$$N = \frac{M_2 F}{M_1 + M_2} = \frac{4 \times 24}{2+4} = \mathbf{16\ N}$$

Illustration 8

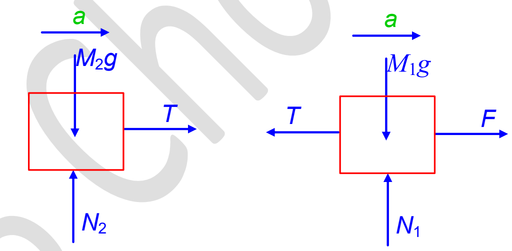
Question: ***Bodies connected with strings***

A light, inextensible string as shown in figure connects two blocks of mass M_1 = 2kg and M_2 = 8kg. *A force F* = 20 N *as shown acts upon M_1.* Find acceleration of the system and tension in string. (surface is frictionless)



***Solution*:** Here as the string is inextensible, acceleration of two blocks will be same. Also, string is mass less so tension throughout the string will be same. Contact force will be normal force only.

Let acceleration of each block is a, tension in string is T and contact force between M_1 and surface is N_1 and contact force between M_2 and surface is N_2.



(FBD of M_2) (FBD of M_1)

Applying Newton's second law for the blocks;

For M_1 $F - T = M_1 a$ … (i)

$M_1 g - N_1 = 0$ … (ii)

For M_2 $T = M_2 a$ … (iii)

$M_2 g - N_2 = 0$ … (iv)

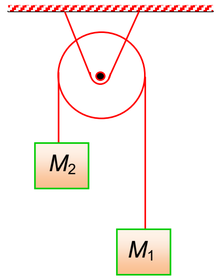
Solving (i) and (iii)

$$a = \frac{F}{M_1 + M_2} = \frac{20}{8+2} = \mathbf{2\ m/s^2}$$

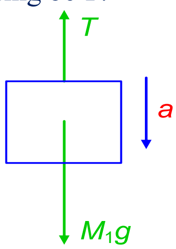
and $T = \dfrac{M_2 F}{M_1 + M_2} = \dfrac{8 \times 20}{2+8} = $ **16 N**

Illustration 9

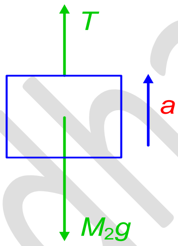
Question: ***Bodies connected with a string and the string passes over a pulley*. (Atwood's Machine)**
Two blocks of unequal masses M_1 = 3 kg and M_2 = 2kg are suspended vertically over a frictionless pulley of negligible mass as shown in figure. Find accelerations of each block and tension in the string. [take g = 10 m/s^2]



Solution: As the string is inextensible, the magnitude of acceleration of two blocks will be same. Pulley in question is massless and frictionless so tension in strings on two sides of pulley will be same.
Let acceleration of M_1 be 'a' (downward) then acceleration of M_2 will be 'a' (upward). Let the tension in string be T.



(FBD of block M_1)



(FBD of block M_2)

Applying Newton's second law for the blocks,

For M_1, $M_1g - T = M_1a$ … (i)

For M_2, $T - M_2g = M_2a$ … (ii)

Solving equation (i) & (ii), $a = \dfrac{(M_1 - M_2)g}{M_1 + M_2} = \dfrac{[3-2]}{3+2} \times 10$

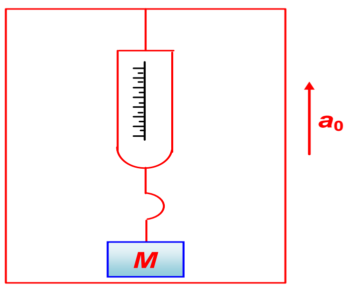
a = **2 m/s^2**

and $T = \dfrac{2M_1M_2}{M_1 + M_2} g = \dfrac{2 \times 3 \times 2}{3+2} \times 10$

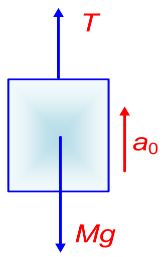
T = **24 N**

Illustration 10

Question: ***Weighing a body in an Elevator***
A block of mass M = 2 kg is suspended with the help of a spring balance. The spring balance is attached to the ceiling of an elevator moving with upward acceleration a_0 = 1 m/s^2 as shown in figure. What is reading of spring balance? [take g = 10 m/s^2]



Solution: A person outside the elevator will observe the block moving with the elevator upward with an acceleration a_0. Also spring balance will give the reading according to tension in spring. So calculating reading of spring balance means to find tension in the spring of spring balance.



(FBD of block)

Let tension in spring is T.

Applying Newton's Second law for the block,

$T - Mg = Ma_0 \Rightarrow T = M(g + a_0) = 2[10+1] \Rightarrow T = \mathbf{22\ N}$

This will be the reading of spring balance. Note that the reading given by spring balance is different from the weight of block.

4.3 CONSTRAINED MOTION

Till now we had seen the case when accelerations of the different parts of a system are same. There are situations in which the accelerations of different parts of the system may not be same. We get such situations in case of moveable pulleys or bodies in contact where each body is free to move.

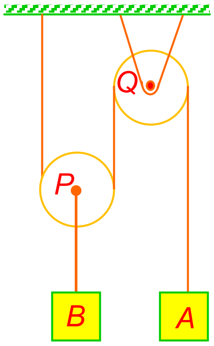


Fig. 8

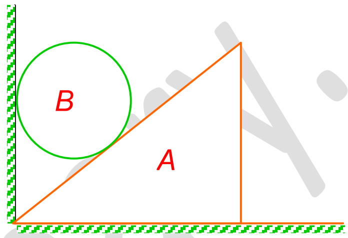


Fig. 9

For example in the figure 8, pulley *P* is movable which leads to different accelerations of block *B* and *A*.

In the figure 9, triangular wedge *A* and sphere *B* will not have same acceleration.

In such cases, a relationship between accelerations can be found by considering physical properties of system. We call such relations as constrained relation.

4.3.1 PULLEY CONSTRAIN

In a pulley-block system if two or more pulleys are involved in the system, the relation between the velocities and acceleration of different bodies can be find out by the following steps:-

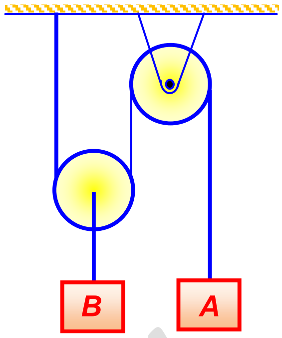
(i) Select a fixed pulley or support.

(ii) Assume distances (say x_A, x_B …. etc.) of all the blocks from this pulley. Write the equation for lengths of each string in terms of x_A, x_B, …..etc.

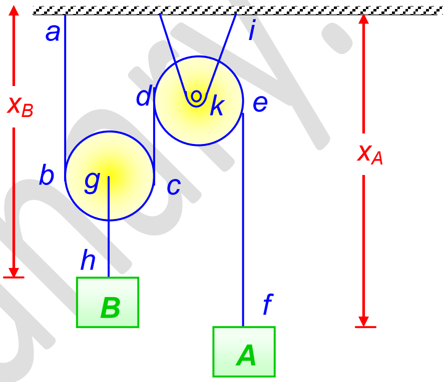
(iii) If string is not extensible and remain taut during the motion this length will remain constant. Differentiate once to get relation between velocities and twice to get relation between accelerations.

Illustration 11

Question: **Find the ratio of acceleration of blocks *A* and acceleration of block *B*.**



Solution: The physical property that we can use is the inextensibility of string.



i.e., $ab + \widehat{bc} + cd + \widehat{de} + ef$ = constant … (i)

Let at any moment *A* and *B* are at distances x_A and x_B from the support as shown in figure.

Let us take $gh = \ell_1$ and $ik = \ell_2$ and express the length in equation (i) in terms of x_A, x_B, ℓ_1 and ℓ_2

we get,

$$x_B - \ell_1 + \widehat{bc} + (x_B - \ell_1 - \ell_2) + \widehat{de} + (x_A - \ell_2) = \text{constant}$$

Here ℓ_1, ℓ_2, $\widehat{bc}$ and $\widehat{de}$ are constant

$\therefore \quad 2x_B + x_A$ = constant … (ii)

Let in time Δt, x_B change to $x_B + \Delta x_B$ and x_A changes to $x_A - \Delta x_A$

[*B* is assumed to move downward]

then, $2(x_B + \Delta x_B) + (x_A - \Delta x_A)$ = constant … (iii)

From (ii) and (iii)

$$2\Delta x_B - \Delta x_A = 0$$

Also, $\left(\dfrac{2\Delta x_B}{\Delta t}\right) - \left(\dfrac{\Delta x_A}{\Delta t}\right) = 0$

$\Rightarrow \quad 2V_B - V_A = 0$

Also, $\quad 2\Delta V_B - \Delta V_A = 0$

$\Rightarrow \quad \dfrac{2\Delta V_B}{\Delta t} - \dfrac{\Delta V_A}{\Delta t} = 0$

$\Rightarrow \quad 2a_B = a_A \Rightarrow \dfrac{a_A}{a_B} = \mathbf{2}$

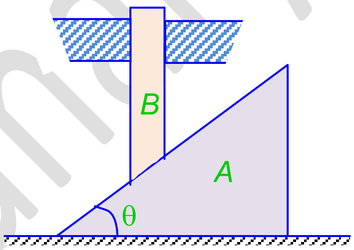
NOTE: Hence magnitude of acceleration of A is two times magnitude of acceleration of B.
Here we get the relation between the acceleration by using the inextensibility of string but after some practice such relation can easily be written by observation.
Let us think B moves by a distance x during an interval of time, this will cause movement of pulley g by x. an extra length of $2x$ of string will come to the left of pulley k. This must be coming from right side of pulleys. Hence displacement of A will be $2x$. On the basis of this discussion we can say if the acceleration of block B is a, then the acceleration of A will be $2a$.

4.3.2 WEDGE CONSTRAIN

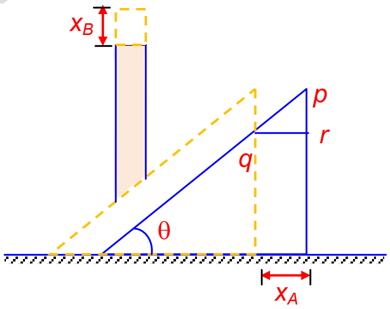
If a body moves such that it remains in contact with the wedge than there velocities and accelerations are related in some way.

***Illustration* 12**

Question: **In the arrangement shown A is a wedge and B is a rod. The rod is constrained to move vertical. The acceleration of wedge A is a_A and that of rod B is a_B. Find $\frac{a_A}{a_B}$. ($\theta = 45$)**



***Solution*:** Here the physical property that we can use is the rigidity of body. Let x_A and x_B are displacement of wedge and rod as shown in figure.
In Δpqr, $qr = x_A$ and $pr = x_B$



$\therefore \quad \tan\theta = \frac{x_B}{x_A}$

$\Rightarrow \quad x_B = x_A \tan\theta$

$\Rightarrow \quad V_B = V_A \tan\theta$

$\Rightarrow \quad a_B = a_A \tan\theta$

$\Rightarrow \quad \frac{a_A}{a_B} = \frac{1}{\tan 45^0}$

$\Rightarrow \quad \frac{a_A}{a_B} = \mathbf{1}$

5 EQUILIBRIUM OF CONCURRENT FORCES

If a number of forces act at the same point, they are called **concurrent forces**.
Consider that a body is under the action of a number of forces. Suppose that the body is in equilibrium under the action of these forces i.e. the body remains in its state of rest or of uniform motion along straight line, when acted upon these forces.

The condition that body may be in equilibrium or the number of forces acting on the body may be in equilibrium is that these should produce zero resultant force.

Consider three concurrent forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ acting at the point O shown in the figure. For $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ to be in equilibrium

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

The condition for equilibrium of three concurrent forces can also be expressed by making use of the triangle law of addition of vectors. Suppose the forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ are such that they can be represented by the three sides $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CA}$ of the triangle ABC taken in order.

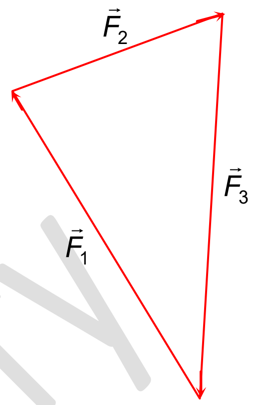
According to the triangle law of addition of vectors,

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

or $$\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = 0$$

or $$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$$

or $$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

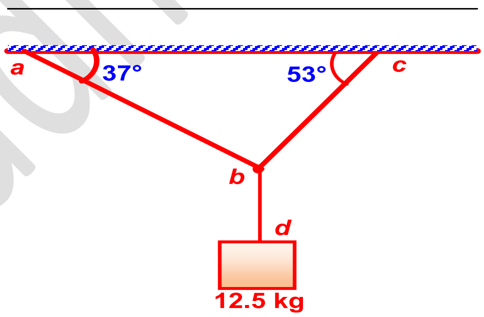


Therefore, the resultant of three concurrent forces will be zero and they will be in equilibrium, if they can be represented completely by the three sides of a triangle taken in order.

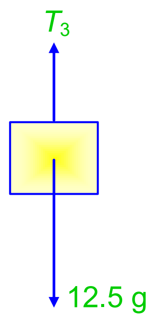
In case, a number of forces act at a point, then they will be in a equilibrium, if they can be represented completely by the sides of a closed polygon taken in order.

Illustration 13

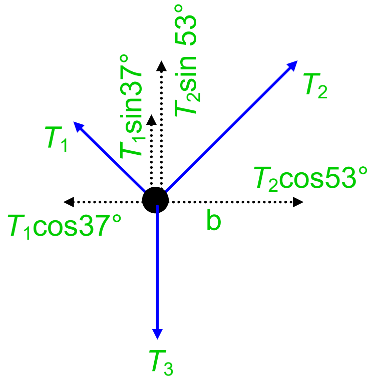
Questions: ***A body suspended with the help of strings.*** **A body of mass 12.5 kg is suspended with the help of strings as shown in figure. Find tension in the string connected with 12.5 kg block. Strings are light [$g = 10$ ms^{-2}]**



Solution: Let the tensions in strings *ab*, *bc* and *bd* are respectively T_1, T_2, and T_3. As the body is hanging in equilibrium, we can use the condition that net force on block is zero. This will give the value of T_3. To know the values of T_1 and T_2 we need to draw *FBD* of knot *b* also.



(FBD of hanging body)



(FBD of knot b)

For equilibrium of hanging body. T_3 = 12.5 g = **125 N**

PROFICIENCY TEST - I

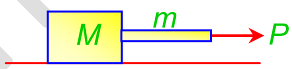
The following questions deal with the basic concepts of this section. Answer the following briefly. Go to the next section only if your score is at least 80%. Do not consult the Study Material while attempting these questions.

1. A body of mass 16 kg starts from rest. It acquires a velocity of 100 m/s, after it has traveled a distance of 200 metres. Calculate the average force exerted on the body.

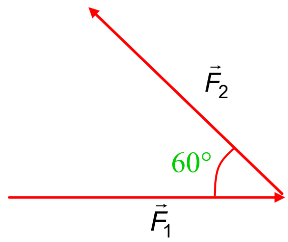
2. There is a block of mass 40 kg. A force of 60 newton is applied upon it. Calculate the time in which the block will acquire a velocity of 30 m/s.

3. A body of mass 45 kg is moving with a constant velocity of 10 m/sec. A constant force acts on the body for four seconds, and the speed of the body becomes 2 m/s in opposite direction. Calculate the acceleration produced and the force applied.

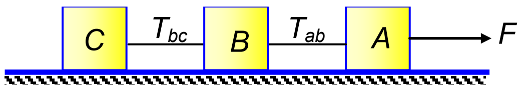
4. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. The force P is applied at one end of the rope. What is the force, which the rope exerts on the block?



5. Two forces $\vec{F}_1$ and $\vec{F}_2$ of equal magnitudes act as shown in figure on a 5.0 kg mass. If $|\vec{F}_1| = |\vec{F}_2| = 5$ N. Find the magnitude and direction of the acceleration with $\vec{F}_1$.

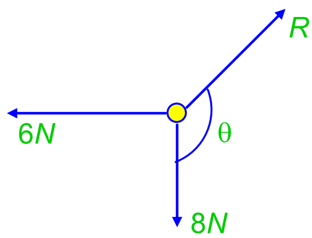


6. Three identical blocks, each of mass 0.6 kg. are connected by light strings as shown in the figure. They move on a smooth horizontal surface with an acceleration of 4.0 m/s^2 under the action of a force F. Calculate F and the tension in the two strings.



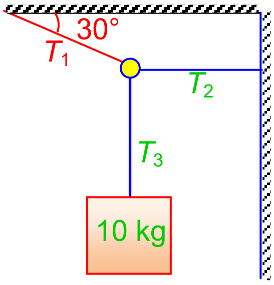
7. A force F_1 acting on a free mass m at rest produces in it acceleration of 1 m/s^2. Another force F_2 acting on the same mass at rest can produce in it a velocity of 10 m/s after 5 s. What would be the greatest acceleration of the mass m when both forces F_1 and F_2 act on it together?

8. The diagram shows the forces acting on a particle Find R and θ if the particle is moving with uniform velocity.



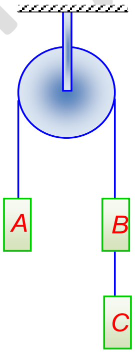
9. In a tug-of-war between two athletes, each pulls on the rope with a force of 200 N. What is tension in the rope?

10. Find tension T_1 and T_3 in the cords shown in the figure [$g = 10\ ms^{-2}$]



11. A uniform rope of length L and mass m is pulled by a constant force F. Find the tension in the rope at a distance x from the end where it is applied.

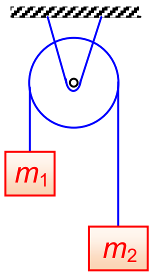
12. Three equal weights A, B & C each of mass 2kg are hanging on a string passing over a fixed frictionless pulley as shown in figure. Find the tension in string connecting B and C. ($g = 10\ m/s^2$)



13. A force acts for 10s on a body of mass 10^{-2} kg after which the force cases to act. The body traverses 0.5 m in the next 5s. What is the magnitude of the force ?

14. Two bodies of masses m_1 and m_2 are connected by a light string passing over a smooth light fixed pulley. The acceleration of the system is $g/7$. What would be the ratio of their masses?

15. Two masses $m_1 = 4$ kg and $m_2 = 6$ kg are connected by means of a light string which passes over a light frictionless pulley shown in the figure. Calculate acceleration of the two masses and tension in the string. Take $g = 10\ m/sec^2$.



ANSWERS TO PROFICIENCY TEST - I

1. 400 N
2. 20 sec.
3. $3 m/s^2$ opposite to initial velocity, 135 N
4. $\frac{MP}{m+M}$
5. 1 N, 60^0
6. 7.2 newton; $T_{ab} = 4.8$ N; $T_{bc} = 2.4$ N
7. $3\ m/s^2$
8. 10 N, 143^0
9. **200 N**
10. $T_1 = 200$ N, $T_3 = 100$ N
11. $\left(1-\frac{x}{L}\right)F$
12. $\frac{40}{3}$N
13. 10^{-4}N
14. 4 : 3
15. $2\ m/s^2$, 48 newton

6 REFERENCE FRAME

Reference frame is a co-ordinate system with respect to which motional quantities such as velocity displacement and acceleration are measured.

There are two types of reference frame:

(i) *Inertial reference frame*

(ii) *Non-Inertial reference frame*

6.1 INERTIAL FRAME

Newton's first law, sometimes called the *law of inertia*, defines a special set of reference frames called inertial frames. Newton's first law is valid only in **inertial frame** of reference.

An inertial frame of reference is an unaccelerated frame.

It means a frame of reference, which is either at rest or is moving with constant velocity, is called inertial frame of reference.

Example:

(i) A static lift.

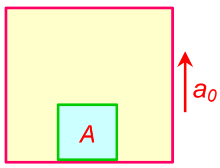
(ii) An elevator moving with constant velocity.

Any reference frame that moves with constant velocity relative to an inertial frame is itself an inertial frame. A reference frame that moves with constant velocity relative to the distant stars is the best approximation of an inertial frame.

The Earth is not an inertial frame because of its orbital motion about the Sun and rotational motion about its own axis. As the Earth travels in its nearly circular orbit around the Sun, it experiences a centripetal acceleration of about 4.4×10^{-3} m/s^2 toward the Sun. In addition, since the Earth rotates about its own axis once in every 24 hours, a point on the equator experiences an additional centripetal acceleration of 3.37×10^{-2} m/s^2 toward the center of the Earth. However, these accelerations are small compared with g i.e acceleration due to gravity and can often be neglected. In most situations we shall assume that any point stationary or moving uniformly with respect to earth constitutes the inertial frame.

Thus, if an object is moving with constant velocity, an observer in one inertial frame (say, one at rest relative to the object) will claim that the acceleration and the resultant force on the object are zero. An observer in *any other* inertial frame will also find that $a = 0$ and $F = 0$ for the object. According to the first law, a body at rest and one moving with constant velocity are equivalent. In other words, the absolute motion of an object has no effect on its behavior. Unless stated otherwise, we shall write the laws of motion relative to an observer "at rest" in an inertial frame.

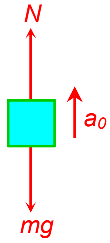
6.2 NON INERTIAL REFERENCE FRAME



A frame of reference having accelerated motion is called a non-inertial frame of reference. In an accelerated frame of reference, an object experiences a fictitious force. For example, a bus moving along a circular track is an accelerated frame of reference. It is our experience that the passengers sitting in such a bus are always thrown away from the centre. This force which tends to throw the passengers away from the centre is called centrifugal force and is a fictitious force. The motion of objects in accelerated frames of reference cannot be studied by simply applying Newton's laws of motion.

Newton's laws of motion can be applied in accelerated frames of reference if we agree to call ($-m\vec{a}_0$) a force acting on a particle *P*. Then while preparing the list of the forces acting on a particle *P*, we include all the (real) forces acting on *P* by all other sources and also include an imaginary force $-m\vec{a}_0$. Here $\vec{a}_0$ is the acceleration of the non-inertial frame under consideration. After applying this additional imaginary force (called pseudo force) $-m\vec{a}_0$ we can now use total force equals mass time acceleration even in non-inertial frame also. Now, with the help of a simple example let us see what problem arises if we don't apply the pseudo force $-m\vec{a}_0$ while using $\vec{F} = m\vec{a}$ (***second law***) in non-inertial frame.

Suppose a block A of mass m is placed on a lift ascending with an acceleration a_0. Let N be the normal reaction between the block and the floor of the lift. Free body diagram of A in ground frame of reference (inertial) is shown in figure.



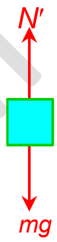
$\therefore \quad N - mg = ma_0$

or $\quad$ **$N = m(g + a_0)$** $\quad \ldots$ **(5)**

But if we draw the free body diagram of A with respect to the elevator (a non-inertial frame of reference) without applying the pseudo force, we get

$N' - mg = 0$ or $N' = mg$

Since, $N' \neq N$, either of the equations is wrong. But if we apply a pseudo force in non-inertial frame of reference, N' becomes equal to N as shown below.



Acceleration of block with respect to elevator is zero

$\therefore \quad N' - mg - ma_0 = 0 \quad$ or $\quad N' = m(g + a_0)$

$\therefore \quad N' = N$

Pseudo force is given by

$$\vec{F}_P = -m\vec{a}_0$$

Here $\vec{a}_0$ is the acceleration of the non-inertial frame of reference and m the mass of the body under consideration. In the whole chapter, we will show the pseudo force by $\vec{F}_P$.

Thus, we may conclude that pseudo force is not a real force. When we draw the free body diagram of a mass, with respect to an inertial frame of reference we apply only the real forces (forces which are actually acting on the mass), but when the free body diagram is drawn from a non-inertial frame of reference a pseudo force (in addition to all real forces) has to be applied to make the equation $\vec{F} = m\vec{a}$ to be valid in this frame also.

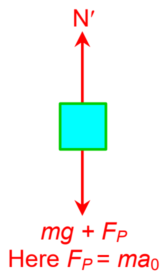
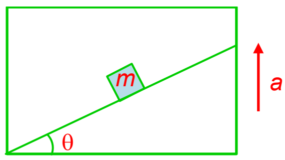


Illustration 14

Question: **In the adjoining figure, a wedge is fixed to an elevator moving upwards with an acceleration '*a*'. A block of mass '*m*' is placed over the wedge. Find the acceleration of the block with respect to wedge. Neglect friction.**



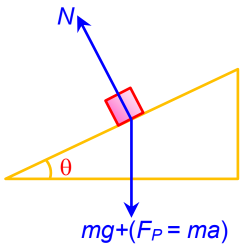
Solution: Since, acceleration of block w.r.t. wedge (an accelerating or non-inertial frame of reference) is to be find out.

$\therefore$ F.B.D. of block w.r.t. wedge.

The acceleration would had been $g\sin\theta$ (down the plane) if the lift were stationary or when only weight (i.e, mg) acts downwards.

Here, downward force is $m(g + a)$

$\therefore$ Acceleration of the block (of course w.r.t. wedge) will be $\mathbf{(g + a)\sin\theta}$ down the plane.



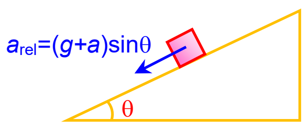
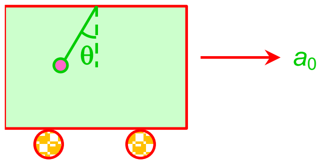


Illustration 15

Question: **A pendulum is hanging from the ceiling of a car having an acceleration a_0 = 10 m/s^2 with respect to the road. Find the angle made by the string with vertical at equilibrium.(g = 10 m/s^2)**



Solution: The situation is shown in figure. Suppose the mass of bob is m and the string makes an angle θ with vertical, the forces on the bob in the car frame (non-inertial frame) are indicated. The forces are

(i) tension in the string

(ii) mg vertically downwards

(iii) ma_0 in the direction opposite to the motion of car (pseudo force).

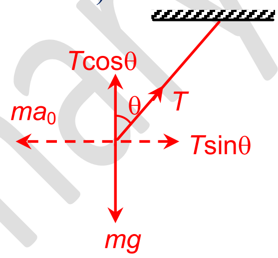
Writing the equation of equilibrium

$$T\sin\theta = ma_0$$

$$T\cos\theta = mg$$

$$\therefore\ \tan\theta = \frac{a_0}{g} = 45°$$

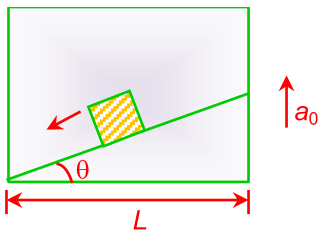
∴ the string is making an angle **45° with vertical at equilibrium**



FBD of bob w.r.t car

Illustration 16

Question: **A block slides down from top of a smooth inclined plane of elevation θ=45° fixed in an elevator going up with an acceleration a_0 = g. The base of incline has length L = 5m. Find the time taken by the block to reach the bottom.**



Solution: Let us solve the problem in the elevator frame. The free body force diagram is shown. The forces are

(i) N normal to the plane

(ii) mg acting vertically down

(iii) ma_0 (pseudo force).

If a is the acceleration of the body with respect to incline, taking components of forces parallel to the incline

$$mg\sin\theta + ma_0\sin\theta = ma$$

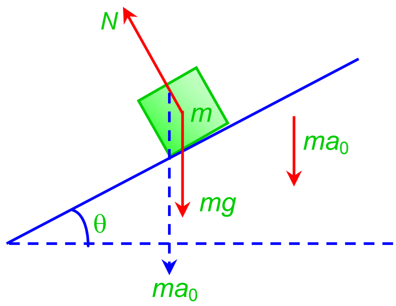
$$\therefore\quad a = (g + a_0)\sin\theta$$

This is the acceleration with respect to elevator.

The distance traveled is $\frac{L}{\cos\theta}$. If t is the time for reaching the bottom of incline

$$\frac{L}{\cos\theta} = 0 + \frac{1}{2}(g + a_0)\sin\theta . t^2$$

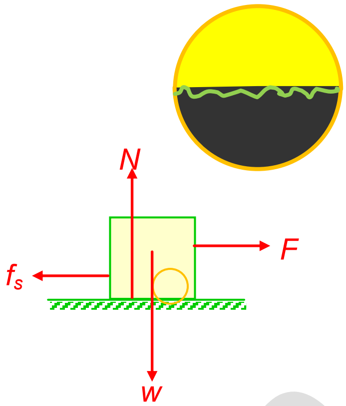
$$t = \left[\frac{2L}{(g + a_0)\sin\theta\cos\theta}\right]^{1/2} = \mathbf{1\ s}$$

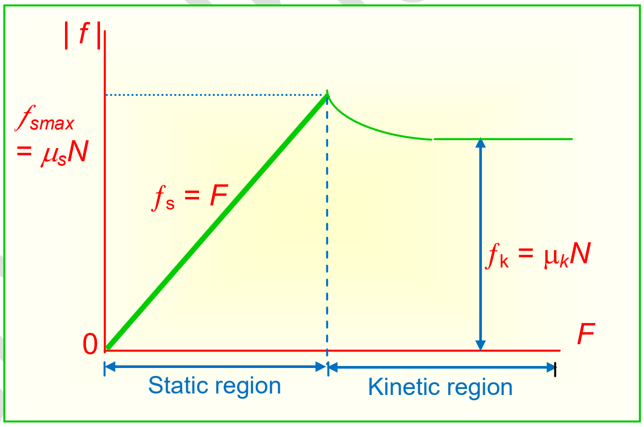


7 FORCES OF FRICTION

When a body is in motion either on a surface or through a viscous medium such as air or water, there is resistance to the motion because the body interacts with its surroundings. We call such resistance a force of friction. Forces of friction are very important in our everyday lives. They allow us to walk or run and are necessary for the motion of wheeled vehicles.

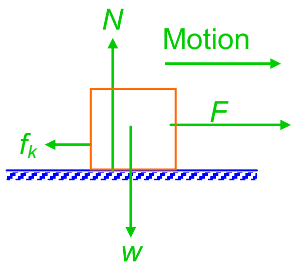
Consider a block on a horizontal table, as in Figure. If we apply an external horizontal force F to the block, acting to the right, the block remains stationary if F is not too large. The force that counteracts F and keeps the block from moving acts to the left and is called the ***frictional force***, f. As long as the block is not moving, $f = F$. Since the block is stationary, we call this frictional force the ***force of static friction, f_s***. Experiments show that this force arises from contacting points that protrude beyond the general level of the surfaces, even for surfaces that are apparently very smooth, as in Figure. (If the surfaces are clean and smooth at the atomic level, they are likely to weld together when contact is made.) The frictional force arises in part from one peak physically blocking the motion of a peak from the opposing surface and in part from chemical bonding of opposing points as they come into contact. If the surfaces are rough, bouncing is likely to occur, further complicating the analysis. Although the details of friction are quite complex at the atomic level, it ultimately involves the electrostatic force between atoms or molecules.





7.2 FORCE OF KINETIC FRICTION

If we increase the magnitude of F, as in Figure, the block eventually slips. When the block is on the verge of slipping, f_s is a maximum as shown by the graph in Figure. When F exceeds $(f_s)_{max}$, the block moves and accelerates to the right. When the block is in motion, the retarding frictional force becomes less than $(f_s)_{max}$, (fig.). When the block is in motion, we call the retarding force the ***force of kinetic friction, f_k***. The unbalanced force in the $+x$ direction, $F - f_k$, accelerates the block to the right. If $F = f_k$, the block moves to the right with constant speed. If the applied force F is removed, then the frictional force f acting to the left accelerates the block in the $-x$ direction and eventually brings it to rest.



Experimentally, one finds that, to a good approximation, both $(f_s)_{max}$ and f_k are *proportional to the normal force acting on the block*. The experimental observations can be summarized by the following **empirical laws of friction**:

(i) The direction of the force of static friction between any two surfaces in contact is opposite the direction of any applied force and can have values

$$f_s \leq \mu_s N \quad \dots (6)$$

where the dimensionless constant μ_s is called the **coefficient of static friction** and N is the magnitude of the normal force. The equality in Equation 6 holds when the block is on the verge of slipping, that is, when $f_s = (f_s)_{max} = \mu_s N$. The inequality holds when the applied force is less than this value.

(ii) The direction of the force of kinetic friction acting on an object is opposite the direction of its motion and is given by

$f_k = \mu_k N$

where μ_k is the **coefficient of kinetic friction.**

(iii) The values of μ_k and μ_s depend on the nature of the surfaces, but μ_k is generally less than μ_s. Typical values of μ range from around 0.05 to 1.5.

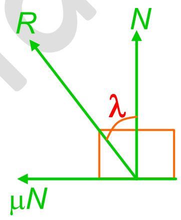
(iv) The coefficients of friction are nearly independent of the macroscopic area of contact between the surfaces.

Finally, although the coefficient of kinetic friction varies with speed, we shall neglect any such variations.

7.3 ANGLE OF FRICTION (λ)

At a point of rough contact, where slipping is about to occur the two forces acting on each object by the surface are the normal reaction N and frictional force $\mu_s N$.

The resultant of these two forces makes an angle λ with the normal where



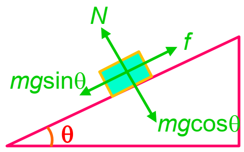
$$\tan\lambda = \frac{\mu_s N}{N} = \mu_s$$

or $\lambda = \tan^{-1}(\mu_s)$

This angle λ is called the angle of friction.

7.4 ANGLE OF REPOSE (α)

Suppose a block of mass m is placed on an inclined plane whose inclination θ can be increased or decreased. Let, μ_s be, the coefficient of friction between the block and the plane. At a general angle θ.



Normal reaction $N = mg\cos\theta$

Limiting friction $f_L = \mu_s N = \mu_s mg\cos\theta$

and the driving force (or pulling force) $mg\sin\theta$, act on the body we see that as θ is increased from 0° to 90°, normal reaction N and hence, the limiting friction, f_L is decreasing while the driving force F is increased. There is a critical angle called angle of repose (α) at which these two forces are equal. Now, if θ is further increased, then the driving force F becomes more than the limiting friction f_L and the block starts sliding.

Thus, $f_L = F$ at $\theta = \alpha$

or, $\mu_s mg\cos\alpha = mg\sin\alpha$

or, $\tan\alpha = \mu_s$

or, $\alpha = \tan^{-1}(\mu_s)$ … (7)

This angle α at which sliding starts is known as angle of repose

7.5 ROLLING FRICTION

When objects such as a wheel (disc or a ring), sphere or a cylinder rolls over a surface, the force of friction that comes into play is called rolling friction. It is found that it is much easier to moves a heavy load from one place to another by placing it over a cart with wheels than to move it by sliding it over the surface. It is because of the fact that rolling friction between two surfaces is less than the sliding friction between them.

Whenever a wheel moves on a level surface, the wheel causes a little depression. Due to this, a small hump is created just ahead of the wheel.

In climbing up the hump, the wheel encounters some opposition to the motion, which is called the rolling friction. In fact, the wheel continuously climbs up such humps due to the depressions caused on the level surface due to rolling of the wheel.

Illustration 17

Question: **A block of mass 1 kg is kept on a rough surface of coefficient of friction $\mu = 0.2$. Find the minimum horizontal force required to make it move.**

Solution: Limiting friction = μN

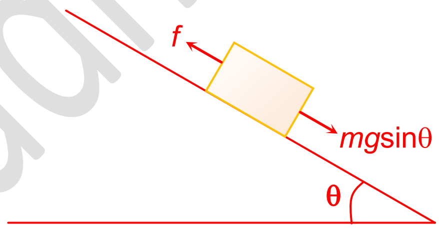
$N = mg = 10$ N

$f = 0.2 \times 10 = \mathbf{2N}$

Illustration 18

Question: **A block of mass 10 kg is moving uniformly on a ramp with inclination of $\theta = 30^0$ with horizontal. Find the force of friction acting on the block. (Take $g = 10$ ms^{-2})**

Solution: From the figure we see that two forces are acting along the plane and should balance each other as the block moves uniformly.



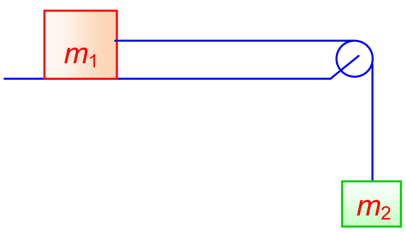
$f = mg\sin\theta$

$f = 10 \times 10 \times \sin 30^0$

$= 10 \times 10 \times \frac{1}{2} = \mathbf{50N}$

Illustration 19

Question: **A block of mass 12 kg is kept on a horizontal surface. It is connected to a string, which passes over a pulley. When a wt. of 8 kg is attached to the other end of the string, then the block just begins to move. Calculate the coefficient of static friction.**



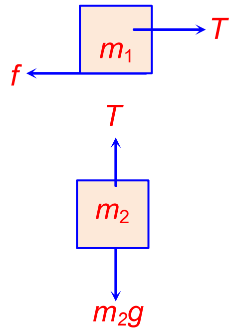
Solution: Let the tension in string be T

Then $T = f_{max}$

$T = \mu N = 120\,\mu$

$T = m_2 g = 80\text{N}$

$\mu = \frac{80}{120} = \mathbf{\frac{2}{3}}$



8 CIRCULAR MOTION

If a particle moves in such a way that its distance from a fixed point remains constant its path will be circular and its motion is called as circular motion. If we whirl a stone tied with a string at one end, its motion is circular motion. Motion of electron around the nucleus is treated as a circular motion. Motion of earth around the sun is treated as circular for some gravitational study.

8.1 ANGULAR KINEMATIC VARIABLES FOR CIRCULAR MOTION

Consider a particle moving on a circular path of center *O* and radius *R* as shown in figure. Let *OX* is a reference line (taken arbitrary) through *O*. At any moment *t* if particle is at *A*, then angle θ between *OA* and *OX* is called as its angular position. If during a time interval of Δ*t* particle moves from position *A* to *B*, the angle subtended by the arc *AB* on the centre Δθ is called as angular displacement. The rate at which particle subtend angle at the center is called as angular velocity represented by ω.

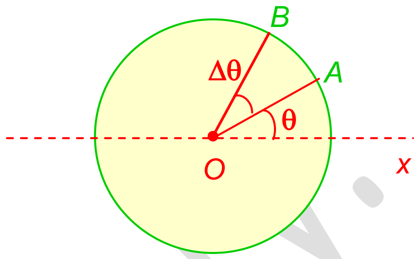


Fig. (6)

∴ Average angular velocity, $\bar{\omega} = \dfrac{\Delta\theta}{\Delta t}$ and

Instantaneous angular velocity, $\omega = \underset{\Delta t \to 0}{Lt} \dfrac{\Delta\theta}{\Delta t} = \dfrac{d\theta}{dt}$

The rate of change of angular speed is called as angular acceleration represented by α

∴ Average angular acceleration, $\bar{\alpha} = \dfrac{\Delta\omega}{\Delta t}$ and

Instantaneous angular acceleration, $\alpha = \underset{\Delta t \to 0}{Lt} \dfrac{\Delta\omega}{\Delta t} = \dfrac{d\omega}{dt}$

$$\alpha = \frac{d^2\theta}{dt^2} = \frac{\omega d\omega}{d\theta}$$

We have different kinds of motion in case of motion in one dimension such as motion with uniform velocity and motion with uniform acceleration. Also in those cases we have kinematic relation between the different linear variables. Similar derivation can be done in case of circular motion and such kinematic relations can be obtained for angular variables also.

For uniform circular motion;

Angular displacement, $\Delta\theta = \omega t$

For circular motion with uniform angular acceleration;

$$\left.\begin{aligned} \Delta\theta &= \omega_0 t + \frac{1}{2}\alpha t^2 \\ \omega &= \omega_0 + \alpha t \\ \omega^2 &= \omega_0^2 + 2\alpha\Delta\theta \end{aligned}\right\}$$

8.2 LINEAR KINEMATICS FOR CIRCULAR MOTION

In the previous article we have seen different angular kinematic variables and relation between them in different types of motion. Now we will define linear velocity and linear acceleration of a particle in circular motion.

As circular motion is the motion in a plane, we take two co-ordinate axes through center for the linear kinematics analysis.

Suppose a particle P is moving on a circular path of radius R. The axes are taken as shown in figure. At any moment its angular position is θ.

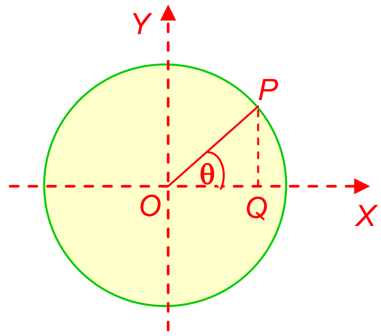


Fig. (7)

Position vector of particle, $\vec{r} = \vec{OQ} + \vec{QP}$

$$= R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$$

Also, unit vector along OP $\hat{e}_r$ can be written as,

$$\hat{e}_r = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

Unit vector along tangential direction $\hat{e}_t$ is given by

$$\hat{e}_t = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

Linear velocity and linear acceleration of particle in circular motion

In the previous discussion

Position vector of particle, $\vec{r} = R\cos\theta\hat{i} + R\sin\theta\,\hat{j}$

Velocity of particle, $\vec{v} = \dfrac{d\vec{r}}{dt}$

$$= R(-\sin\theta)\frac{d\theta}{dt}\hat{i} + R(\cos\theta)\frac{d\theta}{dt}\hat{j}$$

$$= R\omega\,[-\sin\theta\hat{i} + \cos\theta\hat{j}] \quad \left[\because \frac{d\theta}{dt} = \omega\right]$$

$$= R\omega\,\hat{e}_t$$

$\therefore$ $|\vec{v}| = R\omega$ and its direction is along the tangent to the path i.e., if a particle is moving on circular path its velocity is of magnitude equal to product of radius of circular path and its angular velocity and its direction is tangential to the circular path.

The acceleration of particle in circular motion $\vec{a} = \dfrac{d\vec{v}}{dt}$

From equation (2) $\vec{a} = R\left[\omega\dfrac{d}{dt}(-\hat{i}\sin\theta + \hat{j}\cos\theta) + \dfrac{d\omega}{dt}(-\hat{i}\sin\theta + \hat{j}\cos\theta)\right]$

$$= R\omega\left[-\hat{i}\cos\theta\frac{d\theta}{dt} - \hat{j}\sin\theta\frac{d\theta}{dt}\right] + R\frac{d\omega}{dt}\left[-\hat{i}\sin\theta + \hat{j}\cos\theta\right]$$

$$= -\omega^2 R\,\hat{e}_r + \frac{Rd\omega}{dt}\hat{e}_t$$

Therefore, acceleration of particle has two parts one along the tangent having magnitude $\dfrac{Rd\omega}{dt}$ or $R\alpha$ or $\dfrac{dv}{dt}$ is called as tangential acceleration. The other component of acceleration is $\omega^2 R = \dfrac{v^2}{R}$ along the radial direction and is called as radial acceleration or centripetal acceleration.

Hence total acceleration of particle $a = \sqrt{a_t^{\,2} + a_r^{\,2}} = \sqrt{(\omega^2 R)^2 + (\alpha R)^2}$

If direction of acceleration makes an angle β with radius than, $\tan\beta = \dfrac{a_t}{a_r}$

Illustration 20

Question: **The moon orbits the earth with a period of 11.574 days at a distance of $\sqrt{10} \times 10^8$ m from the centre of earth. Find its linear speed. ($\pi = \sqrt{10}$)**

Solution: The period of revolution of moon, $T = 11.574$ days $= 10^6$ s

Linear speed, $v = \omega R$

$$= \frac{2\pi}{T}R$$

$$= \frac{2 \times \sqrt{10} \times \sqrt{10} \times 10^8}{10^6}$$

$= \mathbf{2000\ ms^{-1}}$

Illustration 21

Question: **A particle moves in a circle of radius 20 cm. Its linear speed at any time is given by $v = 2t$ where v is in m/s and t is in seconds. Find the radial and tangential accelerations at $t = 3$ seconds.**

Solution: The linear speed at 3 seconds is

$v = 2 \times 3 = 6$ m/s

The radial acceleration at 3 seconds

$$= \frac{v^2}{r} = \frac{6 \times 6}{0.2} = \mathbf{180\ m/s^2}$$

The tangential acceleration is given by

$\frac{dv}{dt} = 2$, because $v = 2t$.

$\therefore$ tangential acceleration is **2 m/s²**.

8.3 UNIFORM CIRCULAR MOTION AND CENTRIPETAL FORCE

If a particle moves on a circular path with a constant speed, its motion is called as a uniform circular motion. In this motion angular speed of the particle is also constant. Linear acceleration in such motion will not have any tangential component, only radial or centripetal acceleration the particle possesses. Therefore in case of uniform circular motion the particle will have acceleration towards the center only and is called as centripetal acceleration having magnitude $\frac{v^2}{R}$ or $\omega^2 R$. The magnitude of acceleration remains constant but its direction changes with time.

If a particle moving on circular path is observed from an inertial frame it has an acceleration $\omega^2 R$ or $\frac{v^2}{R}$ acting towards center. Therefore from Newton's second law of motion, there must be a force acting on the particle towards the center of magnitude $m\omega^2 R$ or $\frac{mv^2}{R}$. This required force for a particle to move on circular path is called as **centripetal force.**

$\therefore$ centripetal force $= \frac{mv^2}{R}$

The term 'centripetal force' merely a force towards center, it tells nothing about its nature or origin. The centripetal force may be a single force due to a rope, a string, the force of gravity, friction and so forth or it may be resultant of several forces. Centripetal force is not a new kind of force, just as 'upward force' or a 'downward force' is not a new force. Therefore while analyzing motion of particle undergoing circular motion we need not to consider centripetal force as a force, we need to consider only external forces.

Illustration 22

Question: **A ball of mass 0.5 kg is attached to the end of a cord whose length is 1 m. The ball is whirled in a horizontal circle. If the cord can withstand a maximum tension of 50.0 N, what is the maximum speed of the ball can have before the cord breaks?**

Solution: **Because the centripetal force in this case is the force *T* exerted by the cord on the ball, we have**

$$T = m\frac{v^2}{r}$$

Solving for *v*, we have $v = \sqrt{\frac{Tr}{m}}$

The maximum speed that the ball can have corresponds to the maximum tension. Hence, we find

$$v_{\max} = \sqrt{\frac{T_{\max} r}{m}} = \sqrt{\frac{(50.0\,N)\,(1.m)}{0.500\,kg}} = \mathbf{10\ m/s}$$

8.4 CENTRIFUGAL FORCE

An observer in a rotating system is another example of a non-inertial observer. Suppose a block of mass *m* lying on a horizontal, frictionless turntable is connected to a string as in figure. According to an inertial observer, if the block rotates uniformly, it undergoes an acceleration of magnitude v^2/r, where *v* is its tangential speed. The inertial observer concludes that this centripetal acceleration is provided by the force exerted by the string *T*, and writes Newton's second law $T = mv^2/r$.

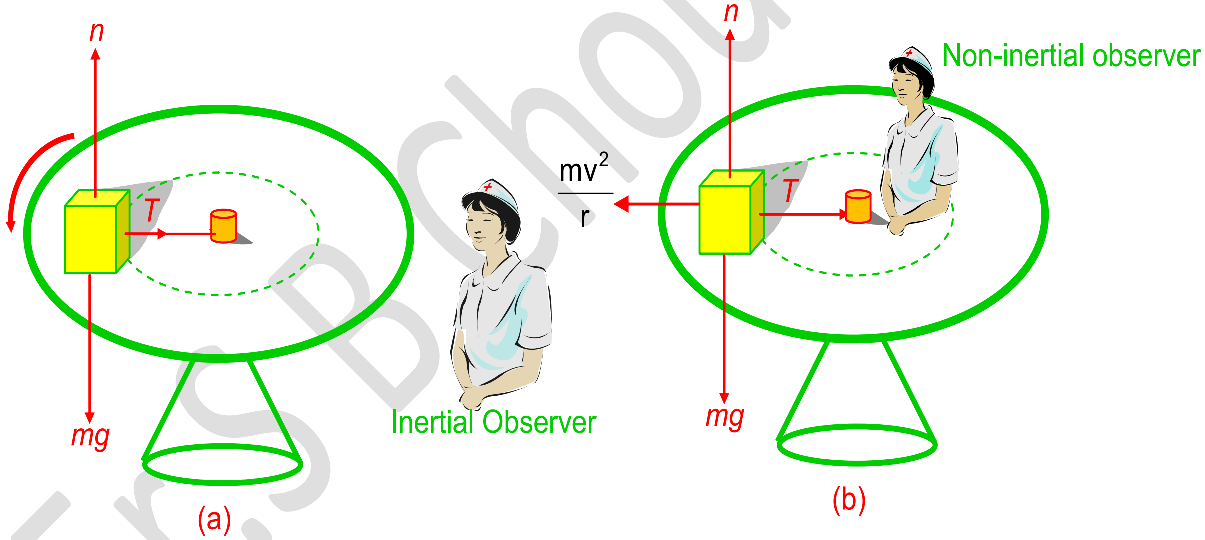


Fig. (12)

According to a non-inertial observer attached to the turntable, the block is at rest. Therefore, in applying Newton's second law, this observer introduces a fictitious outward force of magnitude mv^2/r. According to the **non-inertial observer**, this outward force balances the force exerted by the string and therefore $T - mv^2/r = 0$.

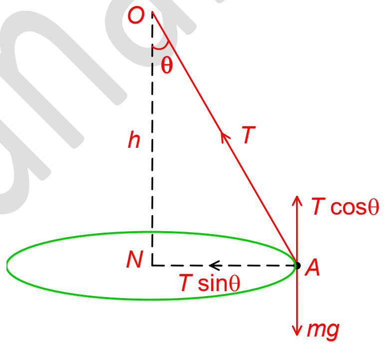
In fact, centrifugal force is a sufficient pseudo force only if we are analyzing the particles at rest in a uniformly rotating frame. If we analyze the motion of a particle that moves in the rotating frame we may have to assume other pseudo forces together with the centrifugal force. Such forces are called **Coriolis forces**. The Coriolis force is perpendicular to the velocity of the particle and also perpendicular to the axis of rotation of the frame. Once again it should be remembered that all these pseudo forces, centrifugal or Coriolis are needed only if the working frame is rotating. If we work from an inertial frame there is no need to apply any pseudo force. There should not be a misconception that centrifugal force acts on a particle because the particle describes a circle.

Therefore when we are working from a frame of reference that is rotating at a constant angular velocity ω with respect to an inertial frame. The dynamics of a particle of mass m kept at a distance r from the axis of rotation we have to assume that a force $m\omega^2 r$ acts radially outward on the particle. Only then we can apply Newton's laws of motion in the rotating frame. This radially outward pseudo force is called the centrifugal force.

You should be careful when using fictitious forces to describe physical phenomena. Remember that fictitious forces are used only in non-inertial frames of references. When solving problems, it is often best to use an inertial frame.

8.5 SOME IMPORTANT UNIFORM CIRCULAR MOTIONS

(i) Conical Pendulum: It consists of a string OA whose upper end O is fixed and a bob is tied at the free end. When the bob is drawn aside and given a horizontal push let it describe a horizontal circle with uniform angular velocity ω in such a way that the string makes an angle θ with vertical. As the string traces the surface of a cone of semi-vertical angle θ it is called conical pendulum. Let T be the tension in string, ℓ be the length and r be the radius of the horizontal circle described. The vertical component of tension balances the weight and the horizontal component supplies the centripetal force.



$$T\cos\theta = mg, \quad T\sin\theta = mr\omega^2$$

$$\therefore \quad \tan\theta = \frac{r\omega^2}{g}, \quad \omega = \sqrt{\frac{g\tan\theta}{r}}$$

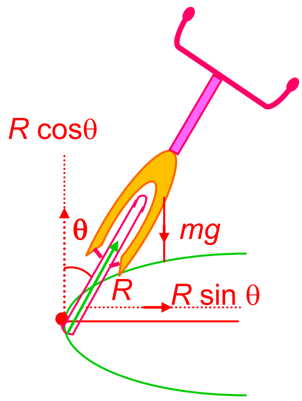
T being the period i.e., time for one revolution.

$$\therefore \quad \frac{2\pi}{T} = \sqrt{\frac{g\tan\theta}{\ell\sin\theta}}$$

$$T = 2\pi\sqrt{\frac{\ell\cos\theta}{g}} = 2\pi\sqrt{h/g}, \text{ where } h = \ell\cos\theta.$$

(ii) Motion of a cyclist on a circular path: Let a cyclist moving on a circular path of radius r bend away from the vertical by an angle θ.

R is the reaction from the ground. It can be resolved in the horizontal and vertical directions. The components are respectively equal to $R\sin\theta$ and $R\cos\theta$. The vertical component balances his weight mg. The horizontal component $R\sin\theta$ supplies the necessary force for making the circular path.



$$\therefore R\sin\theta = \frac{mv^2}{r}$$

$$R\cos\theta = mg$$

$$\therefore \tan\theta = v^2/rg$$

(iii) Banking of roads

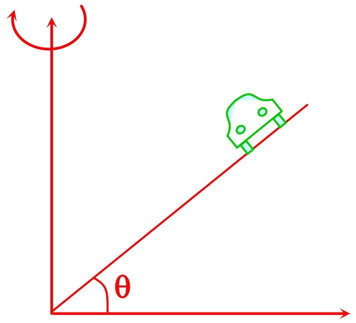
On highways or mountain roads the roads on a turn are raised so that the surface of the road becomes like the surface of a cone.

The car will neither slip inwards nor it will slip (or skid) outward if

$$V_{min} \le V \le V_{max}$$

$$\text{Where } V_{max} = R\omega_{min} = R\sqrt{\frac{g(\sin\theta + \mu\cos\theta)}{R(\cos\theta - \mu\sin\theta)}}$$

$$V_{min} = R\omega_{min} = R\sqrt{\frac{g}{R}\frac{(\sin\theta - \mu\cos\theta)}{(\cos\theta + \mu\sin\theta)}}$$



Angle of banking

The civil engineer designing the road sets the angle of banking for a particular speed (generally 45 km/hr) so that the car can turn without the help of friction meaning there is no tendency of slipping. To find such speed and angle combination put $\mu = 0$ in the above formula.

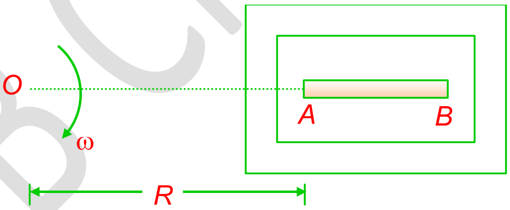
$$V = R\sqrt{\frac{g}{R}\frac{\sin\theta}{\cos\theta}}$$

$$V = \sqrt{Rg\tan\theta}$$

$$\tan\theta = \frac{V^2}{Rg}$$

Illustration 23

Question: **A table with smooth horizontal surface is fixed in a cabin that rotates with angular speed $\omega = 0.1$ rad/s in a circular path of radius $R = 2$m. A smooth groove AB of length $L = 1$cm ($<< R$) is made on the surface of table as shown in figure.**



A small particle is kept at the point A in the groove and is released to move, find the time taken by the particle to reach the point B.

Solution: Let us analyse the motion of particle with respect to table which is moving with cabin with an angular speed of ω. Along AB centrifugal force of magnitude $m\omega^2R$ will act at A on the particle which can be treated as constant from A to B as $L << R$.

$\therefore$ acceleration of particle along AB with respect to cabin $a = \omega^2 R$ (constant)

Required time 't' is given by

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow \quad L = 0 + \frac{1}{2}\times\omega^2 R t^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2L}{\omega^2 R}} = \mathbf{1s}$$

8.6 NON UNIFORM CIRCULAR MOTION

If the speed of the particle moving in a circle is not constant the acceleration has both radial and tangential components. The radial and tangential accelerations are

$$a_r = \omega^2 r = \frac{v^2}{r}$$

$$a_t = \frac{dv}{dt}$$

The magnitude of the resultant acceleration will be

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$

If the direction of resultant acceleration makes an angle β with the radius, then

$$\tan\beta = \frac{dv/dt}{v^2/r}$$

Now as acceleration of particle undergoing non-uniform circular motion is $a = \sqrt{(\omega^2 R)^2 + \left(R\frac{d\omega}{dt}\right)^2} = \sqrt{\left(\frac{v^2}{R}\right)^2 + \left(\frac{dv}{dt}\right)^2}$ in the direction $\tan^{-1}\left(\frac{dv/dt}{v^2/r}\right)$ with radius it need resultant force of $m\sqrt{\left(\frac{v^2}{R}\right)^2 + \left(\frac{dv}{dt}\right)^2}$ in the direction of acceleration.

Illustration 24

Question: **A car goes on a horizontal circular road of radius R = 1 m, the speed increasing at a rate $\frac{dv}{dt} = a = 3$ m/s². The friction coefficient between road and tyre is $\mu = \frac{1}{\sqrt{10}}$. Find the speed at which the car will skid.**

Solution: Here at any time t, the speed of car becomes V the net acceleration in the plane of road is $\sqrt{\left(\frac{v^2}{R}\right)^2 + a^2}$. This acceleration is provided by frictional force. At the moment car will slide,

$$M\sqrt{\left(\frac{v^2}{R}\right)^2 + a^2} = \mu Mg$$

$$\Rightarrow \quad v = [R^2(\mu^2 g^2 - a^2)]^{1/4}$$

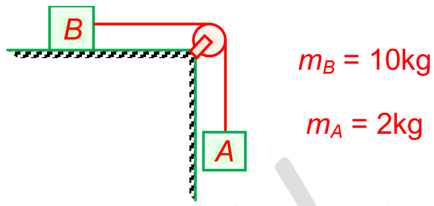
= **1 m/s**

Motion in a vertical circular is a common example of non-uniform circular motion that we will discuss in next lesson of 'Work, Energy and Power', as it needs same idea of energy and its conservation.

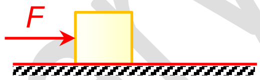
PROFICIENCY TEST–II

The following questions deal with the basic concepts of this section. Answer the following briefly. Go to the next section only if your score is at least 80%. Do not consult the Study Material while attempting these questions.

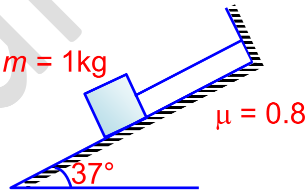
1. For the arrangement shown in figure. Find acceleration of blocks, tension in string and frictional force on blocks. The coefficient of friction between any block and surface is 0.5.



2. A body of mass M = 10 kg is found to be rest, when a force $F = 100\sqrt{3}$ N is applied on it (refer figure). What is net force exerted by the surface on the block?



3. For the arrangement shown in figure, find the tension in the string.

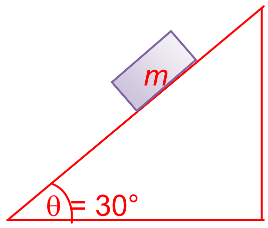


4. A car starts from rest on a half kilometer long bridge. The coefficient of friction between the tyre and the road is 1.0. Show that one cannot drive through the bridge in less than 10s.

5. A body of mass 2 kg is lying on a rough inclined plane of inclination 30°. Find the magnitude of the force parallel to the incline needed to make the blocks move
(a) up the incline
(b) down the incline
[coefficient of friction = 0.2]

6. A block slides down an inclined plane of slope of angle θ with a constant velocity. It is then projected up the plane with an initial velocity u. To what distance will it rise before coming to rest?

7. There is an inclined plane having an inclination of 30° with the horizontal. A body of mass 10 kg rest on it without applying any external force. Calculate the minimum force required to move the body up the plane (g = 10 m/s^2)



8. A scooter is moving on a straight horizontal surface with a velocity u, calculate the shortest distance in which the scooter can be stopped; if coefficient of friction between tires and road is μ.

9. A body of mass 80 kg is whirled in a circle with a velocity of 2 ms^{-1} using 0.6 m length of a string, which can withstand a tension of 15 N. Neglecting the force of gravity on the body, predict whether or not the string will break. Give reasons for your answer.

10. A mass of 100 kg is resting on a rough inclined plane of angle 30^0. If the coefficient of friction is $1/\sqrt{3}$, find the greatest and the least forces that acting parallel to the plane in both cases, just maintain the mass in equilibrium. (g = 10 m/s^2)

11. A body of mass 2 kg lying on a smooth surface is attached to a string 3 m long and then whirled round in a horizontal circle $60/\pi$ revolutions per minute. Find (i) angular velocity (ii) the linear velocity (iii) the centripetal acceleration and (iv) the tension in the string.

12. A bus moving with a velocity 12 m/s suddenly turns round a curve of radius 8 m. Find the force acting on a passenger of 70 kg due to this circular motion.

13. A certain string which is 1 m long will break if the load on it is more than 0.5 kg. A mass of 0.05 kg is attached to one end of it and the particle is whirled round in a horizontal circle by holding the free end of the string by one hand. Find the greatest angular velocity with which the string can be rotated.

14. A long smooth slab of length 9 m is having a block resting on its surface at one end. A force is applied to the slab which accelerates it horizontally with an acceleration of 2 ms^{-2}

(i) What is the pseudo force experienced by the block of mass 2 kg when seen from the slab?

(ii) What will be the speed of the block wrt the slab when it falls off the slab?

15. A lift falls down with acceleration of 5 ms^{-2}. A man in the lift experiences a loss of weight. What are the forces experienced by the man in the frame of lift? (mass of man = 60 kg and g = 10 m/s^2)

16. (a) A turn of radius is banked for vehicles moving at a speed of 36 km/hr. What is the angle of banking?

(b) In the above question, if the coefficient of friction is 0.4 then what is the minimum and maximum velocity for which no slipping takes place?

ANSWERS TO PROFICIENCY TEST–II

1. $a_A = a_B = 0$, $T = 20\text{N}$, $f_A = 0$, $f_B = 20$ N

2. 200 N

3. zero

5. (a) 13 N (b) zero

6 $\dfrac{u^2}{4g\sin\theta}$

7. 100 N

8. $\dfrac{u^2}{2\mu g}$

10. 100 kgf, zero

11. (i) 2 rad/s (ii) 6 m/s (iii) 12 m/s^2 (iv) 24 N

12. 1260 N

13. 10 rad/s

14. (i) 4 N (ii) 6 m/s

15. 600 N downward (weight), 300 N upward (pseudo force), 300 N upward (normal force)

16. (a) $R = 20$ m
(b) Between 14.7 km/hr and 54 km/hr

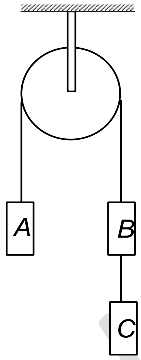
SOLVED OBJECTIVE EXAMPLES

Example 1:

Three equal weights *A*, *B* & *C* each of mass 2kg are hanging on a string passing over a fixed frictionless pulley as shown in figure. The tension in string connecting *B* and *C* is

(a zero **(b) 13 N**

(c) 3.3 N **(d) 19.6 N**



Solution:

For mass C, resultant force on $C = 2g - T_2$

$\therefore \quad 2g - T_2 = 2a$ … (i)

For mass B, $2g + T_2 - T_1 = 2a$ … (ii)

For mass A, $T_1 - 2g = 2a$ … (iii)

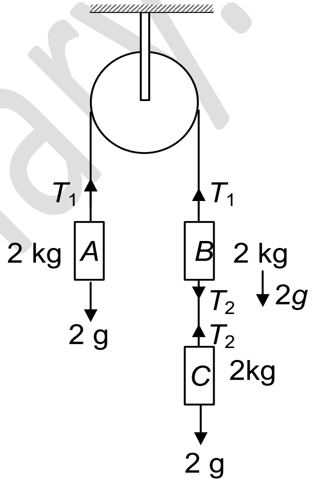
Adding (1), (2) and (3)

$$a = \frac{2g}{6} = \frac{g}{3}$$

Tension, $T_2 = 2g - 2a = 2g - \frac{2g}{3}$

$$= \frac{4g}{3} = \frac{4 \times 9.8}{3} = \frac{39.2}{3} = \mathbf{13\ N}$$

$\therefore$ **(b)**



Example 2:

A body of mass 1 kg is suspended from a spring balance graduated at $g = 10$ m/s². The spring balance is fixed in a lift, which is moving up with an acceleration of 5 m/s². What is the reading in the spring balance?

(a) 0.5 kg **(b) 1.5 kg** **(c) 1 kg** **(d) 3.5 kg**

Solution:

The tension in spring balance T is given by

$$T - mg = ma \Rightarrow T = m(g + a)\ \text{N}$$

$$= \frac{mg\left[1 + \frac{a}{g}\right]}{g}\text{kg} = \left(1 + \frac{5}{10}\right)\text{kg} = \mathbf{1.5\ kg}$$

$\therefore$ **(b)**

Example 3:

A force F_1 acting on a free mass m at rest produces in it acceleration of 1 m/s². Another force F_2 acting on the same mass at rest can produce in it a velocity of 10 m/s after 5 s. The greatest acceleration of the mass m when both forces F_1 and F_2 act on it together will be

(a) 2 m/s² **(b) 4 m/s²** **(c) 3 m/s²** **(d) 1 m/s²**

Solution:

F_1 produces an acceleration of 1 m/s².

F_2 produces a velocity 10 m/s after 5 s. If acceleration produced by F_2 is a then,

$v = at$, $10 = 5a$, $a = 2$ m/s²

Hence both together can produce a maximum acceleration of **3 m/s²**

$\therefore$ **(c)**

Example **4:**

A given object takes η times as much time to slide down a 45° rough incline as it takes to slide down a perfectly smooth 45° incline. The coefficient of kinetic friction between object and incline is given by

(a) $\mu = \frac{1}{1-\eta^2}$ (b) $\mu = 1 - \frac{1}{\eta^2}$ (c) $\mu = \sqrt{\frac{1}{1-\eta^2}}$ (d) $\mu = \sqrt{1 - \frac{1}{\eta^2}}$

Solution:

Acceleration without friction = $g \sin \alpha = \frac{g}{\sqrt{2}}$

With friction, the acceleration is $g(\sin \alpha - \mu \cos \alpha) = \frac{g}{\sqrt{2}}(1 - \mu)$

Since the body starts from rest, the distance is equal to $\frac{1}{2}at^2$.

Thus, $S = \frac{g}{\sqrt{2}}t_1^2 = \frac{g}{\sqrt{2}}(1-\mu)\, t_2^2$;

Given $t_2 = \eta\, t_1$ $\quad \frac{1-\mu}{1} = \frac{t_1^2}{t_2^2} = \frac{1}{\eta^2}$

$\therefore \mu = \left(1 - \frac{1}{\eta^2}\right)$

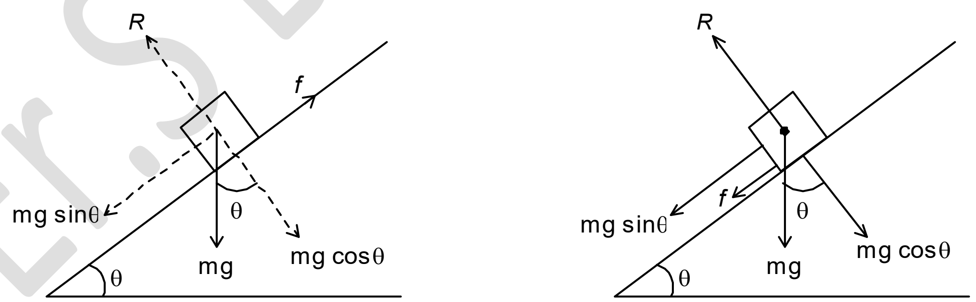
∴ **(b)**

Example **5:**

A block slides down an inclined plane of slope of angle θ with a constant velocity. It is then projected up the plane with an initial velocity *u*. The distance upto which it will rise before coming to rest is

(a) $\frac{u^2}{4g \sin\theta}$ (b) $\frac{u}{4g \sin\theta}$ (c) $\frac{u^2 \sin\theta}{4g}$ (d) $\frac{u \sin\theta}{4g}$

Solution:



The force diagram in the two cases are shown in Figure.

Case (i): When the block slides with constant velocity the acceleration is zero.

$\therefore \quad f = mg \sin\theta$

$\quad f = \mu R \quad = \mu\, mg \cos\theta$

$\therefore \quad \mu mg \cos\theta = mg \sin\theta$

$\quad \mu = \tan\theta$ ... (i)

Case (ii): The block is projected with velocity u and hence it experiences downward acceleration a. In this case

$$mg \sin\theta + \mu mg \cos\theta = ma$$

$mg \sin\theta + mg \tan\theta \cos\theta = ma$

$\therefore \quad a = 2g \sin\theta$ ... (ii)

Let x be the distance moved up before it comes to rest.

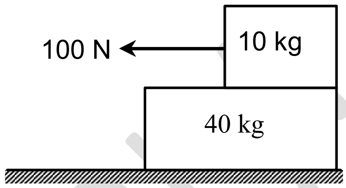
$0 - u^2 = -2 \times 2g \sin\theta\, S$

$\therefore \quad S = \dfrac{u^2}{4g\sin\theta}$

$\therefore$ **(a)**

***Example* 6:**

A 40 kg slab (*B*) rests on a smooth floor as shown in figure. A 10 kg block (*A*) rests on the top of the slab. The static coefficient of friction between slab and block is 0.6 while the kinetic friction coefficient is 0.4. The block (*A*) is acted upon by a horizontal force 100 N. If $g = 9.8$ m/s^2, the resulting acceleration of the slab (*B*) will be

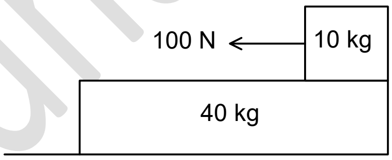


(a) 0.98 m/s^2 **(b) 1.47 m/s^2** **(c) 1.52 m/s^2** **(d) 6.1 m/s^2**

Solution:

For a force of 100 *N* on 10 kg block, relative motion will take place.

$\therefore$ The frictional force between 10 kg block and 40 kg block,



$f = \mu mg$

$= 0.4 \times 10 \times 9.8$ N

The acceleration of the slab of 40 kg is

$a = \dfrac{0.4 \times 10 \times 9.8}{40}$

$=$ **0.98 m/s^2**

$\therefore$ **(a)**

***Example* 7:**

On a slippery road with a coefficient of friction reduced to 0.2, the maximum speed at which a car can go round a curve of radius 100 m is

(a) 5 m/s **(b) 7 m/s** **(c) 14 m/s** **(d) 20 m/s**

Solution:

$v = \sqrt{\mu g r} = \sqrt{0.2 \times 100 \times 9.8} =$ **14 m/s**

$\therefore$ **(c)**

***Example* 8:**

Two particles move on a circular path (one just inside and the other just outside) with angular velocities ω and 5 ω starting from the same point. Then, which is incorrect

(a) they cross each other at regular intervals of time $\dfrac{2\pi}{4\omega}$ when their angular velocities are oppositely directed

(b) they cross each other at points on the path subtending an angle of 60° at the centre if their angular velocities are oppositely directed

(c) they cross at intervals of time $\dfrac{\pi}{3\omega}$ if their angular velocities are oppositely directed

(d) they cross each other at points on the path subtending 90° at the centre if their angular velocities are in the same sense

Solution:

If the angular velocities are oppositely directed, they meet at intervals of time $t = \dfrac{2\pi}{\omega_{rel}} = \dfrac{2\pi}{6\omega} = \dfrac{\pi}{3\omega}$

Angle subtended at the centre by the crossing points $\theta = \omega t = \dfrac{\pi}{3} = \mathbf{60^\circ}$

When their angular velocities are in the same direction,

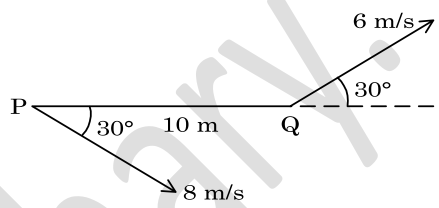
$t' = \dfrac{2\pi}{\omega_{rel}} = \dfrac{2\pi}{4\omega} = \dfrac{\pi}{2\omega}$ and $\theta' = \dfrac{\pi}{2\omega} \times \omega = \dfrac{\pi}{2} = \mathbf{90^\circ}$

∴ Option (b), (c) and (d) are correct and incorrect option is (a)

∴ **(a)**

Example 9:

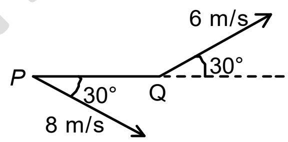
Two moving particles *P* and *Q* are 10 m apart at a certain instant. The velocity of *P* is 8 m/s making 30° with the line joining P and *Q* and that of *Q* is 6 m/s making 30° with *PQ* in the Figure.
Then the angular velocity of *Q* with respect to *P* in rad/s at that instant is



(a) 0 **(b) 0.1**
(c) 0.4 **(d) 0.7**

Solution:

Angular velocity of *Q* relative to *P*



$= \dfrac{\text{Projection of } V_{QP} \text{ perpendicular to the line } PQ}{\text{Separation between } P \text{ and } Q}$

$\dfrac{V_Q \sin\theta_2 - V_P \sin\theta_1}{PQ} = \dfrac{6\sin 30^\circ - (-8\sin 30^\circ)}{10} = \mathbf{0.7\ rad/s}$

∴ **(d)**

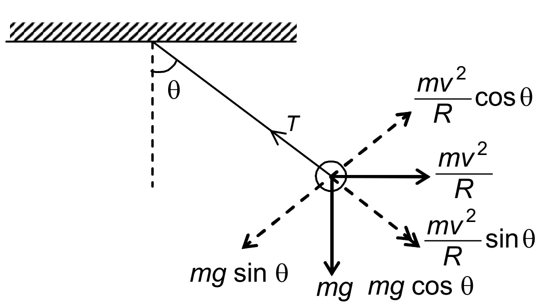
Example 10:

A car is moving in a circular horizontal track of radius 10 m with constant speed of 10 m/s. A plumb bob is suspended from roof by a light rigid rod of length 1 m. The angle made by the rod with the track is

(a) zero **(b) 30°** **(c) 45°** **(d) 60°**

Solution:

The different forces acting on the bob are shown in Figure. Resolving the forces along the length and perpendicular to the rod, we have



$mg\cos\theta + \dfrac{mv^2}{R}\sin\theta = T$

$mg\sin\theta = \dfrac{mv^2}{R}\cos\theta$

$\tan\theta = \dfrac{v^2}{Rg} = \dfrac{(10)^2}{(10)(10)} = 1$

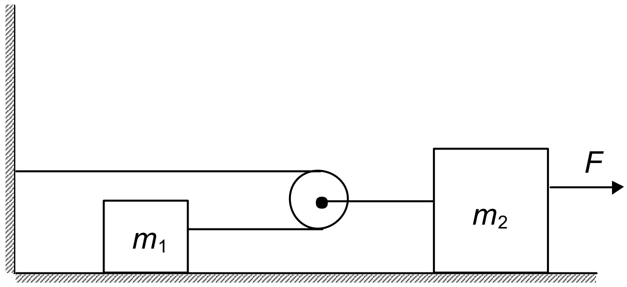
$\theta = \tan^{-1}(1) = \mathbf{45^\circ}$

∴ **(c)**

SOLVED SUBJECIVE EXAMPLES

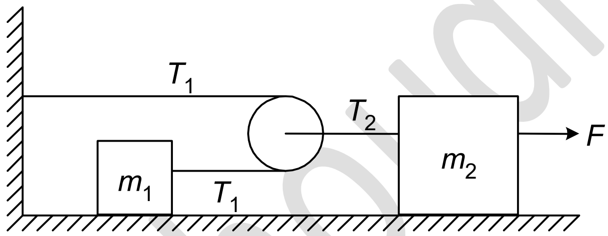
***Example* 1:**

In the system shown below, friction and mass of the pulley are negligible. Find the acceleration of m_2 if m_1 = 300 g, m_2 = 500 g and F = 3.4 N



Solution:

When the pulley moves a distance d, m_1 will move a distance $2d$. Hence m_2 will have twice as large an acceleration as m_2 has. Also because the total force on the pulley must be zero, $T_1 = (T_2/2)$.



For mass m_1, $T_1 = m_1\,(2a)$ … (i)

For mass m_2, $F - T_2 = m_2\,(a)$ … (ii)

Putting $T_1 = \dfrac{T_2}{2}$, (i) gives $T_2 = 4m_1\,a$

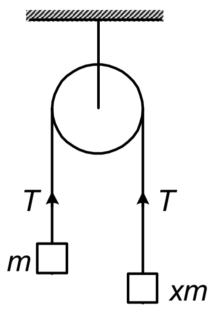
Substituting in equation (ii), $F = 4m_1a + m_2a = (4m_1 + m_2)a$

Hence $a = \dfrac{F}{(4m_1+m_2)} = \dfrac{3.4}{4(0.3)+0.5}$ = **2 m/s²**

***Example* 2:**

A light inextensible string passing over a smooth fixed pulley attaches two masses of magnitudes m and xm. Find the product of two possible values of x if the acceleration of the system is $g/4$.

Solution:



Two cases will arise according as $x < 1$ or $x > 1$

Case 1:

When $x < 1$, $xm < m$ and the mass m will fall while the mass xm will rise.

The equations of motion will be

for mass m, $mg - T = ma$ … (i)

for mass xm, $T - xmg = (xm)\,a$ … (ii)

Adding, $mg\,(1 - x) = (1 + x)\,ma$

or, $g(1 - x) = a\,(1 + x)$

It is given $a = \frac{g}{4}$. Putting this value,

$$(1-x) = \left(\frac{1+x}{4}\right)$$

or, $5x = 3$ Hence $x = \frac{3}{5}$

Case 2:

When $x > 1$, $xm > m$ and the mass xm will fall while mass m will rise. The equations of motion will be

for mass m, $T - mg = ma$ … (iii)

for mass xm, $xmg - T = (xm)a$ … (iv)

Adding, $(x-1)\, mg = (x+1)\, ma$

Putting $a = \frac{g}{4}$, $4(x-1) = (x+1)$

or, $3x = 5$ giving $x = \frac{5}{3}$

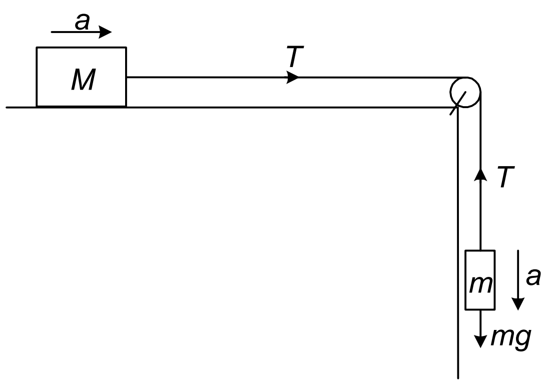
Thus the two possible values of x for which the acceleration of the system will be $\frac{g}{4}$ are $\frac{3}{5}$ and $\frac{5}{3}$.

Therefore their product is $\frac{3}{5} \times \frac{5}{3} = \mathbf{1}$

Example 3:

A mass of 2 kg hangs freely at the end of a string, which passes over a smooth pulley fixed at the edge of a smooth table. The other end of the string is attached to a mass *M* on the table. If the mass on the table is doubled the tension in the string increases by one-half. Find the mass *M*.

Solution:



The tension in the string is given by

$$T = \frac{mM}{m+M} g \quad \ldots \text{(i)}$$

In the second case M changes to $2M$ and T changes to $\frac{3}{2}T$

$$\therefore \quad \frac{3}{2}T = \frac{m(2M)}{m+(2M)} \cdot g \quad \ldots \text{(ii)}$$

Dividing (i) by (ii), we get

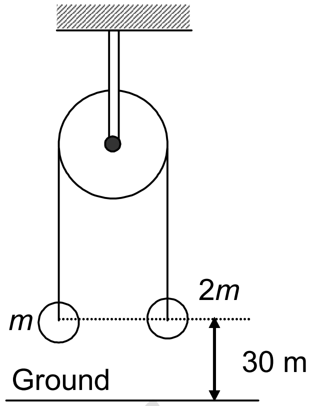
$$\frac{2}{3} = \frac{m+2M}{m+M} \times \frac{1}{2}$$

Substituting $m = 2$ kg, then

$M = \mathbf{1\ kg}$

Example 4:

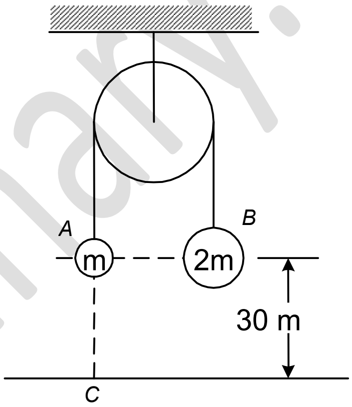
Two masses *m* and 2*m* are connected by a massless string, which passes over a pulley as shown in figure. The masses are held initially with equal lengths of the strings on either side of the pulley. Find the velocity of masses at the instant the lighter mass moves up a distance of 15m. The string is suddenly cut at that instant. Calculate the time taken by heavier mass to reach the ground. ($g = 10\ m/s^2$)



Solution:

The masses A and B of m and $2m$ respectively are initially along the horizontal position through the line AB.

When the masses are left free, B comes down, A moves up with acceleration a.



Now, $a = \dfrac{(2m-m)g}{2m+m} = \dfrac{g}{3}$

The initial velocities of both of them is zero.

When the lighter mass A moves up through a height 15 m, its velocity v is given by

$v = \sqrt{2 \times a \times S} = \sqrt{2 \times \dfrac{10}{3} \times 15} = \mathbf{10\ m/s}$

Both the masses A and B have the velocity of same magnitude 10 m/s. At this instant the string snaps.

Calculation of the time taken by *B* to reach the ground

$u = 10$ m/s

$a = 10\ m/s^2$

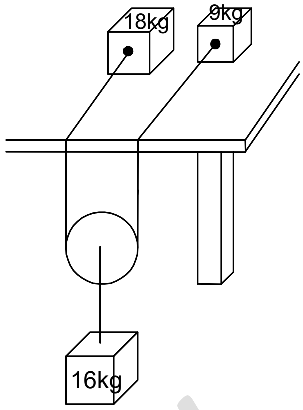
$S = (30–15) = 15$ m

$S = ut + \dfrac{1}{2} \times at^2 \Rightarrow 15 = 10 \times t + \dfrac{1}{2}\ 10 \times t^2$

or $t = \mathbf{2\ s}$

***Example* 5:**

Two particles of masses 9kg and 18 kg are placed on a smooth horizontal table. A string, which joins them, hangs over the edge supporting a light pulley, which carries a mass 16 kg.
The two parts of the string on the table are parallel and perpendicular to the edge of the table. The parts of the string outside the table are vertical. Find the acceleration of the particle of mass 16kg. [take g = 10 m/s^2]



Solution:

Let T be the tension in the string; a be the acceleration of the mass 18 kg; $2a$ be the acceleration of mass 9 kg.

$$T = m \cdot 2a$$

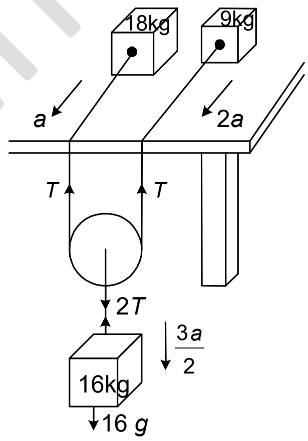
The mass 16 kg will come down with an acceleration $\frac{a+2a}{2} = \frac{3a}{2}$

$$16g - 2T = 16\left(\frac{3a}{2}\right)$$

$$16g - 4 \times 9a = 16\left(\frac{3a}{2}\right)$$

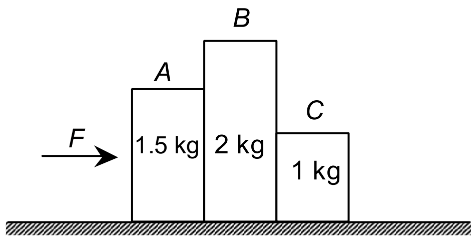
$$\Rightarrow \quad a = \frac{8}{3}\ \text{m/s}^2$$

$\therefore$ the acceleration of 16 kg mass $= \frac{3}{2}a$ $= \mathbf{4\ m/s^2}$



***Example* 6:**

In the system of three blocks *A*, *B* and *C* shown in figure, (i) how large a force *F* is needed to give the blocks an acceleration of 3 m/s^2, if the coefficient of friction between blocks and table is 0.3 (ii) how large a force does the block *A* exert on the block *B*? (g = 10 m/s^2)



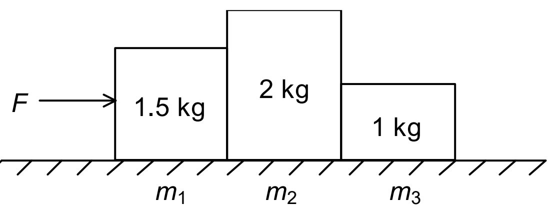
Solution:

(i) Let a be the acceleration of the system to right. All the three frictional forces $f_1 = \mu m_1 g$, $f_2 = \mu m_2 g$ and $f_3 = \mu m_3 g$ will be directed to the left as the motion of bodies is to the right. Hence, for the whole system

$$F - \mu m_1 g - \mu m_2 g - \mu m_3 g = (m_1 + m_2 + m_3)\, a$$

$$F = (m_1 + m_2 + m_3)\,(a + \mu g)$$

$$= (1.5 + 2 + 1)\,(3 + 0.3 \times 10) \ = \mathbf{27\ N}$$



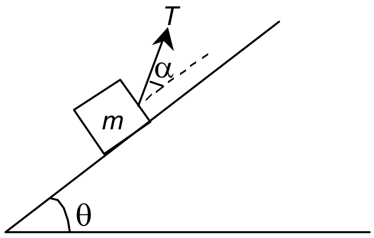
(ii) The force exerted by the 1.5 kg block on the 2 kg block $= F - m_1\,(a + \mu g)$

$= 27 - 1.5\,(3 + 0.3 \times 10)$

$= \mathbf{18\ N}$

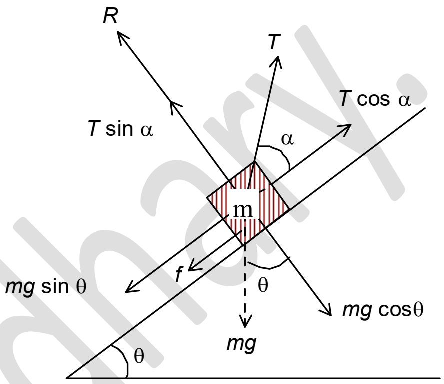
Example 7:

A block of mass $m = \sqrt{10}$ kg is pulled upward by means of a thread up an inclined plane forming an angle θ = 45° with the horizontal as shown in figure. The coefficient of friction is μ = 0.5. What is the value of the minimum tension? (g = 10 m/s^2)



Solution:

The different forces acting on the mass are shown in Figure. Let the mass move up the plane with an acceleration a. Writing the equation of motion



$R + T\sin\alpha = mg\cos\theta$

$R = mg\cos\theta - T\sin\alpha$ … (i)

$T\cos\alpha - mg\sin\theta - f = ma$ … (ii)

where f is the force of friction.

$f = \mu\ (mg\cos\theta - T\sin\alpha)$ … (iii)

Substituting the value of f from equation (iii) in equation (ii)

$T\cos\alpha - mg\sin\theta - \mu mg\cos\theta + \mu T\sin\alpha = ma$

$T(\cos\alpha + \mu\sin\alpha) = ma + mg\sin\theta + \mu mg\cos\theta$

$$T = \frac{ma + mg\sin\theta + \mu mg\cos\theta}{\cos\alpha + \mu\sin\alpha} \quad \text{… (iv)}$$

For T to be minimum (cos α + μ sin α) should be maximum.

$$\frac{d}{d\alpha}(\cos\alpha + \mu\sin\alpha) = 0$$

$$\frac{d^2}{d\alpha^2}(\cos\alpha + \mu\sin\alpha) = -ve$$

$$\frac{d}{d\alpha}(\cos\alpha + \mu\sin\alpha) = -\sin\alpha + \mu\cos\alpha = 0$$

$\mu = \tan\alpha$

It can be shown that $\frac{d^2}{d\alpha^2}$ is negative.

T will have minimum value when $a = 0$ and $\alpha = \tan^{-1}(\mu)$

From equation (iv)

$$T_{min} = \frac{mg\sin\theta + \mu mg\cos\theta}{\cos\alpha + \mu\sin\alpha}$$

$\cos\alpha + \mu\sin\alpha = \cos\alpha + \mu(\mu\cos\alpha)$

$= \cos\alpha + \mu^2\cos\alpha$

$$= \cos\alpha(1+\mu^2) = \frac{1+\mu^2}{\sec\alpha}$$

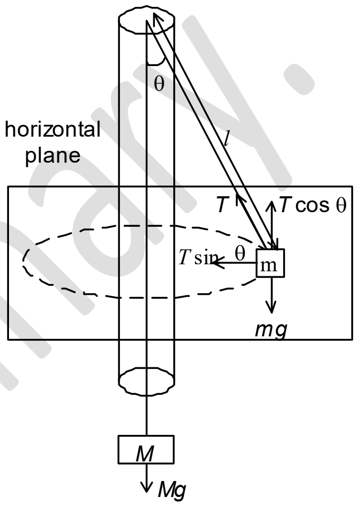
$$= \frac{1+\mu^2}{\sqrt{1+\tan^2\alpha}} = \frac{1+\mu^2}{\sqrt{1+\mu^2}} = \sqrt{1+\mu^2}$$

$$\therefore\ T_{min} = \frac{mg\sin\theta + \mu mg\cos\theta}{\sqrt{1+\mu^2}} = \mathbf{30\ N}$$

***Example* 8:**

A large mass M = 20 kg and a small mass m = 2kg hang at the two ends of the string that passes through a smooth tube as shown in Figure. The mass m moves around in a circular path, which lies in the horizontal plane. The length of the string from the mass m to the top of the tube is 10 cm and θ is the angle this length makes with vertical. What should be the frequency of rotation of mass m so that M remains stationary ? ($\sqrt{10} = \pi$)

Solution:



The forces acting on mass m and M are shown in Figure. When mass M is stationary

$T = Mg$ ...(i)

where T is tension in string.

For the smaller mass, the vertical component of tension T cosθ balances mg and the horizontal component T sin θ supplies the necessary centripetal force.

$T\cos\theta = mg$ ...(ii)

$T\sin\theta = mr\omega^2$ ...(iii)

ω being the angular velocity and r is the radius of horizontal circular path.

From (i) and (iii), $Mg\sin\theta = mr\omega^2$

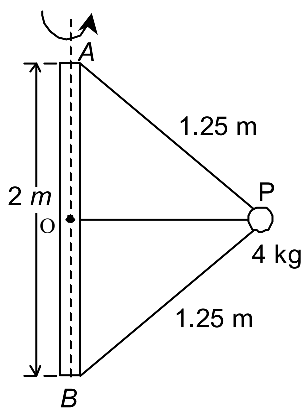
$$\omega = \sqrt{\frac{Mg\sin\theta}{mr}} = \sqrt{\frac{Mg\sin\theta}{m\ell\sin\theta}} = \sqrt{\frac{Mg}{m\ell}}$$

$$\text{Frequency of rotation} = \frac{1}{T} = \frac{1}{2\pi/\omega} = \frac{\omega}{2\pi}$$

$$\therefore \quad \text{Frequency} = \frac{1}{2\pi}\sqrt{\frac{Mg}{m\ell}} = \mathbf{5\ Hz}$$

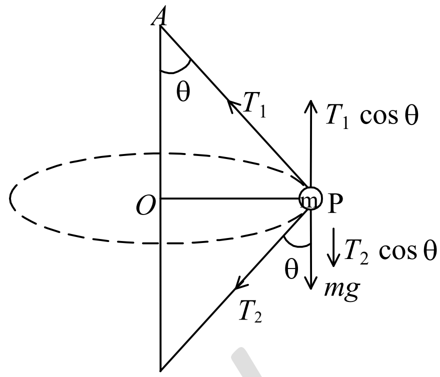
***Example* 9:**

The 4kg block in the Figure is attached to the vertical rod by means of two strings. When the system rotates about the axis of the rod, the two strings are extended as indicated in Figure. How many revolutions per minute must the system make in order that the tension in upper string is 60 N. What is tension in the lower string?



Solution:

The forces acting on block P of mass 4 kg are shown in the Figure. If θ is the angle made by strings with vertical, T_1 and T_2 tensions in strings for equilibrium in the vertical direction



$T_1 \cos\theta = T_2 \cos\theta + mg$

$(T_1 - T_2)\cos\theta = mg$

$\cos\theta = \frac{1}{1.25} = \frac{4}{5}$

$\left[\because \cos\theta = \frac{OA}{AP} = \frac{1}{1.25}\right]$

$\therefore\ T_1 - T_2 = \frac{mg}{\cos\theta} = \frac{5mg}{4} = \frac{5}{4} \times 4 \times 9.8 = 49\text{ N}$

Given $T_1 = 60$ N

$T_2 = T_1 - 49 = 60\text{ N} - 49\text{ N} = \mathbf{11\ N}$

The net horizontal force ($T_1 \sin\theta + T_2 \sin\theta$) provides the necessary centripetal force $m\omega^2 r$.

$\therefore (T_1 + T_2)\sin\theta = m\omega^2 r$

$\Rightarrow \omega^2 = \frac{(T_1 + T_2)\sin\theta}{mr}$

$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-(4/5)^2} = \frac{3}{5}$

$r = OP = \sqrt{(1.25)^2 - (1)^2} = 0.56$

$\therefore\ \omega^2 = \frac{(60+11)\frac{3}{5}}{(4 \times 0.56)} = 19$

$\omega = \sqrt{19} = 4.36$ rad/s

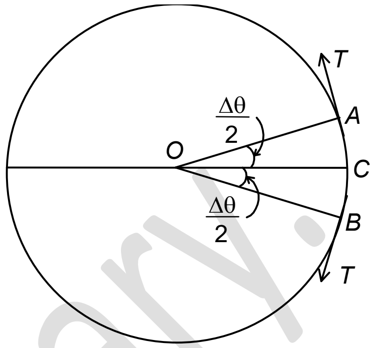
Frequency of revolution $= \frac{\omega}{2\pi} = \frac{4.36}{2 \times 3.14}$

= **42 rev/min**

***Example* 10:**

A metal ring of mass $m = 2\pi$ kg and radius $R = 1$ m is placed on a smooth horizontal table and is set rotating about its own axis in such a way that each part of ring moves with velocity $v = 5$ m/s. Find the tension in the ring.

Solution:



Consider a small part *ACB* of the ring that subtends an angle Δθ at the centre as shown in Figure. Let the tension in the ring be *T*.

The forces on this elementary portion *ACB* are

(i) tension *T* by the part of the ring left to *A*

(ii) tension *T* by the part of the ring right to *B*

(iii) weight $(\Delta m)\,g$

(iv) normal force N by the table.

As the elementary portion *ACB* moves in a circle of radius *R* at constant speed *v* its acceleration towards centre is $\frac{(\Delta m)v^2}{R}$.

Resolving the forces along the radius *CO*

$$T\cos\left(90^\circ - \frac{\Delta\theta}{2}\right) + T\cos\left(90^\circ - \frac{\Delta\theta}{2}\right) = \Delta m\frac{v^2}{R} \quad \text{...(i)}$$

$$2T\sin\frac{\Delta\theta}{2} = \Delta m\frac{v^2}{R} \quad \text{...(ii)}$$

Length of the part $ACB = R\Delta\theta$. The mass per unit length of the ring is $\frac{m}{2\pi R}$

$\therefore$ mass of this portion *ACB*, $\Delta m = \frac{R\Delta\theta m}{2\pi R} = \frac{m\Delta\theta}{2\pi}$

Putting this value of Δm in (ii),

$$2T\sin\frac{\Delta\theta}{2} = \frac{m\Delta\theta}{2\pi}\frac{v^2}{R}$$

$$\therefore T = \frac{mv^2}{2\pi R}\left(\frac{\frac{\Delta\theta}{2}}{\sin\left(\frac{\Delta\theta}{2}\right)}\right)$$

Since $\left(\frac{\frac{\Delta\theta}{2}}{\sin\left(\Delta\frac{\theta}{2}\right)}\right)$ is equal to 1,

$$T = \frac{mv^2}{2\pi R} = \mathbf{25\ N}$$

MIND MAP

Law 1. (Defines force and Inertia)
Everybody remains at rest or continues to move with uniform velocity unless an external force is applied to it.

Law 2. Gives relation between force and acceleration
i.e., $\sum \vec{F}_{ext} = M\vec{a}$

Law 3. When a body *A* exerts a force on another body *B*, *B* exerts an equal and opposite force on *A*. If one of these two forces is considered as action, then other will be reaction.

NEWTON'S LAWS OF MOTION

(Applicable for inertial frame only)

COMMONLY USED ASSUMPTIONS

Rigid body

Inextensible string

Massless string

Massless and frictionless pulley

COMMONLY USED FORCES

Normal force: Normal to the surfaces of contact and towards the body under consideration.
Weight of body: Equals to *Mg* and acts vertically downward.
Tension in string: along the string, away from the body under consideration.

Stepwise procedure to solve questions based on motion of connected bodies:

1. Identify the unknown forces and accelerations.
2. Draw FBD of bodies in the system.
3. Resolve forces in the direction of motion and perpendicular to it.
4. Apply $\Sigma\vec{F} = M\vec{a}$ in the direction of motion and $\Sigma\vec{F} = 0$ in the direction of equilibrium.
5. Write constraint relation if required and possible.
6. Solve the equations written in steps 4 and 5 to get the results.

MIND MAP

NON-INERTIAL FRAME

1. Non-inertial frame
- In this frame Newton's laws of motion is not applicable.
- To apply Newton's laws of motion in this frame, other than external forces, we need to consider pseudo force(s).

2. In non-inertial frame we can write Newton's Law as

$\vec{F}_{ext} + \vec{F}_{Pseudo} = M\vec{a}$

$\vec{F}_{Pseudo} = -M\vec{a}_{frame}$

FRICTION

1. It is a tangential force between the surfaces in contact that opposes relative motion or tendency of the relative motion between the contact surfaces.

2. Frictional force is of three types:
- Static frictional force: Self adjusting force having magnitude less than or equal to $\mu_s N$.
- Limiting frictional force: Maximum value of frictional force having value $\mu_s N$.
- Kinetic frictional force: Having value equal to $\mu_k N$.

3. Angle between the resultant reaction force in limiting friction and normal force is called as angle of friction θ, given by

$\tan\theta = \dfrac{f}{N}$

4. Angle made by an inclined plane with horizontal at which a body starts sliding down itself is called as angle of repose (α) given by,

$\alpha = \tan^{-1}(\mu_s)$

CIRCULAR MOTION

1. Angular kinematics relations
- Uniform circular motion, $\Delta\theta = \omega t$
- Uniformly accelerated circular motion,
- $\Delta\theta = \omega_o t + \frac{1}{2}\alpha t^2$, $\omega = \omega_o + \alpha t$
- $\omega^2 = \omega_o^2 + 2\alpha\Delta\theta$, $\Delta\theta_n = \omega_o + (2n-1)\frac{\alpha}{2}$

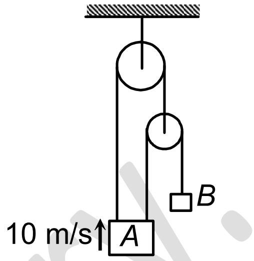
2. Relation between linear kinematic variable and angular kinematic variable
- Linear velocity, $V = \omega R$
- Tangential acceleration, $a_t = \alpha R$
- Radial or centripetal acceleration, $a_r = \omega^2 R$
- Total acceleration, $a = \sqrt{\alpha^2 R^2 + \omega^4 R^2}$ at an angle of $\tan^{-1}\left(\frac{a_t}{a_r}\right)$ with radius.

3. Centripetal force: $F_r = M\omega^2 R = \dfrac{MV^2}{R}$ towards the center.
- Centrifugal force: $F_r' = M\omega^2 R = \dfrac{MV^2}{R}$, away from the center (Applicable with respect to a rotating frame).

EXERCISE – I

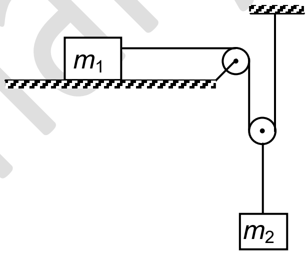
NEET-SINGLE CHOICE CORRECT

1. At a certain moment of time velocity of A is 10 m/s upward. The velocity of B at that time will be



(a) 30 m/s downward (b) 20 m/s downward
(c) 10 m/s down ward (d) 5 m/s down ward

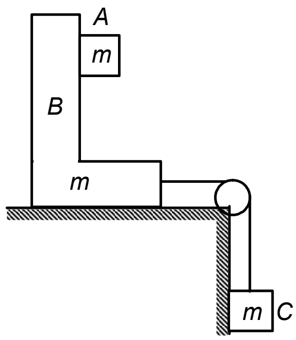
2. In the arrangement shown in figure if the surface is smooth, the acceleration of the block m_2 will be



(a) $\frac{m_2 g}{4m_1 + m_2}$ (b) $\frac{2m_2 g}{4m_1 + m_2}$

(c) $\frac{2m_2 g}{m_1 + 4m_2}$ (d) $\frac{2m_1 g}{m_1 + m_2}$

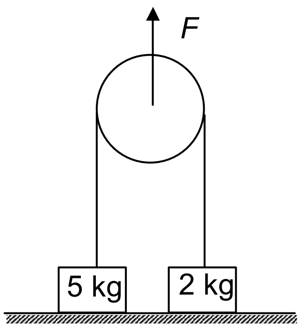
3. In the given figure, the acceleration of block A with respect to ground is (Neglect friction)



(a) $\frac{g}{3}$ (b) $\frac{g}{3}\sqrt{10}$

(c) $\frac{2g}{3}$ (d) g

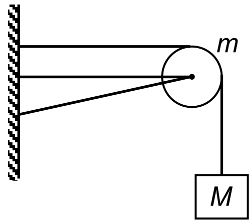
4. Two blocks of masses 5 kg and 2 kg are initially at rest on the floor as shown in figure. A light string, passing over a light frictionless pulley, connects them. An upward force F is applied on the pulley and maintained constant. Find the maximum value of F applied so that the accelerations of 5 kg is zero



($g = 10\ ms^{-2}$)

(a) 50 N (b) 100 N.
(c) 200 N. (d) none

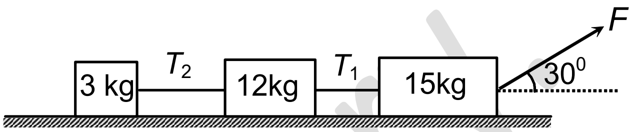
5. A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as shown. The force on pulley by the clamp is given by



(a) $\sqrt{2}$ Mg (b) $\sqrt{2}$ mg

(c) $\sqrt{(M+m)^2+m^2}\,g$ (d) $\sqrt{(M+m)^2+M^2}\,g$

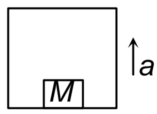
6. The ratio of T_1 and T_2 is (see figure)
(neglect friction)



(a) $\sqrt{3}:2$ (b) $1:\sqrt{3}$

(c) 1 : 5 (d) 5 : 1

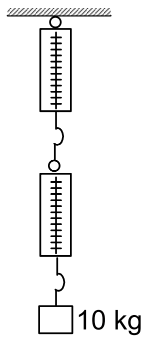
7. With what acceleration 'a' should the box of figure moving up so that the block of mass M exerts a force $7Mg/4$ on the floor of the box?



(a) g/4 (b) $g/2$

(c) $3g/4$ (d) $4g$

8. A block of mass 10 kg is suspended through two light spring balances as shown in figure.



(a) Both the scales will read 10 kg

(b) Both the scales will read 5 kg

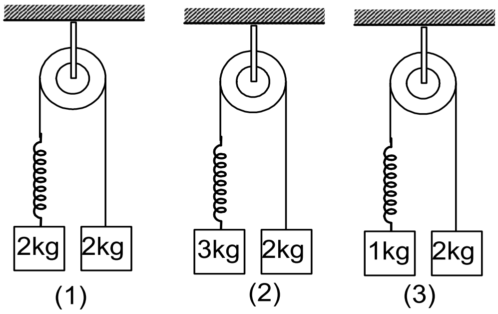
(c) The upper scale will read 10 kg and the lower zero

(d) The readings may be anything but their sum will be 10 kg

9. The over-bridge of a canal is in the form of a circular arc of radius R. What is the greatest speed at which a motor cyclist can cross the bridge without leaving the ground?

(a) $\sqrt{5gR}$ (b) $\sqrt{3gR}$ (c) $\sqrt{2gR}$ (d) $\sqrt{gR}$

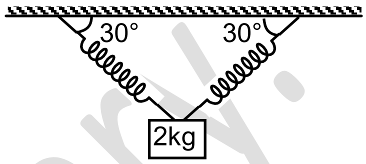
10. Same spring is attached with 2 kg, 3kg and 1 kg blocks in three different cases as shown in the figure. If x_1, x_2 and x_3 be the extensions in the spring in these three cases respectively then



(a) $x_1 = 0,\ x_3 > x_2$ (b) $x_2 > x_1 > x_3$

(c) $x_3 > x_1 > x_2$ (d) $x_1 > x_2 > x_3$

11. A block of mass 2 kg is hanging with two identical massless springs as shown in figure. The acceleration of the block just at the moment, the right spring breaks is ($g = 10\ m/s^2$)

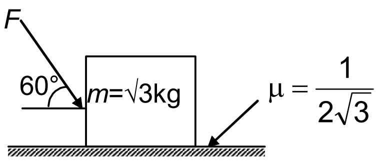


(a) 10 m/s^2 (b) 5 m/s^2 (c) 25 m/s^2 (d) 4 m/s^2

12. The force required to just move a body up an inclined plane is double the force required to just prevent it from sliding down. If ϕ is angle of friction and θ is the angle which incline makes with the horizontal, then

(a) $\tan\theta = \tan\phi$ (b) $\tan\theta = 2\tan\phi$ (c) $\tan\theta = 3\tan\phi$ (d) $\tan\phi = \tan\theta$

13. What is the maximum value of the force F such that the block shown in the arrangement does not move? ($g = 10\ m/s^2$)



(a) 20 N (b) 10 N

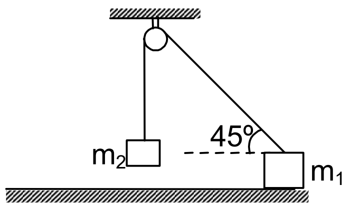
(c) 12 N (d) 15 N

14. A body is placed on a rough inclined plane of inclination θ. As the angle θ is increased from 0° to 90°, the contact force between the block and the plane

(a) remains constant (b) first remains constant then decreases

(c) first decreases then increases (d) first increases then decreases

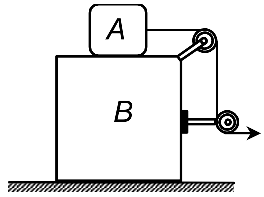
15. A block of mass m_1 rests on a rough horizontal plane. A light string attached to the body passes over a light pulley and carries at its other end a mass m_2 and the system just begins to move. The value of coefficient of friction between the surface and block is



(a) $\dfrac{m_2}{\sqrt{2}m_1 - m_2}$ (b) $\dfrac{m_2}{\sqrt{2}m_1 + m_2}$

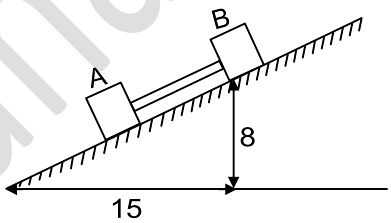
(c) $\frac{m_1}{\sqrt{2m_2+m_1}}$ (d) $\frac{m_1}{\sqrt{2m_2-m_1}}$

16. In the arrangement shown in figure $m_A = m_B = 2kg$. String is massless and pulley is frictionless. Block B is resting on a smooth horizontal surface, while friction coefficient between blocks A and B is $\mu = 0.5$. The maximum horizontal force F can be applied so that block A does not slip over the block B is. ($g = 10$ m/s^2)



(a) 25 N (b) 40 N
(c) 30 N (d) 20 N

17. Blocks A and B in the figure are connected by a bar of negligible mass. If masses of A and B are 170 kg each respectively and $\mu_A = 0.2$ and $\mu_B = 0.4$, where μ_A and μ_B are the coefficients of limiting friction between blocks and plane, calculate the force in the bar. ($g = 10$ m/s^2).



(a) 150 N (b) 75 N
(c) 200 N (d) 250 N

18. A hollow cylinder of radius R is rotated about its axis, which is kept vertical. Calculate the minimum frequency of revolution such that a body kept on the inside of the wall does not slip down. The coefficient of friction between body and surface of cylinder is μ.

(a) $\frac{1}{2\pi}\sqrt{\frac{g}{\mu R}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{\mu g}{R}}$ (c) $2\pi\sqrt{\frac{\mu g}{R}}$ (d) $2\pi\sqrt{\frac{R}{\mu g}}$

19. An insect is crawling up a hemispherical bowl of radius R. If the coefficient of friction is 1/3, the insect will be able to go up to height h equal to (take $3/\sqrt{10} = 0.95$)

(a) $\frac{R}{5}$ (b) $\frac{R}{10}$ (c) $\frac{R}{20}$ (d) $\frac{R}{30}$

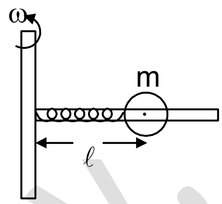
20. A block released on a rough inclined plane of inclination □ = 30º slides down the plane with an acceleration $g/4$, where g is the acceleration due to gravity. What is the coefficient of friction between the block and the inclined plane?

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{2\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

21. A uniform chain of length ℓ is placed on a rough table with length $n\ell$ hanging over the edge ($n < 1$). If the chain just begins to slide off the table by itself from this position, the coefficient of friction between chain and table is

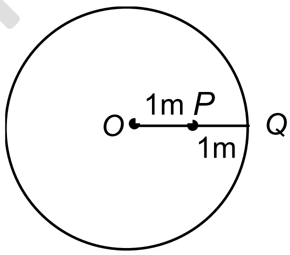
(a) $\frac{1}{n}$ (b) $\frac{n}{1-n}$ (c) $\frac{1}{n+1}$ (d) $\frac{1-n}{1+n}$

22. An undeformed spring of spring constant k has length ℓ_0. When the system rotates at an angular velocity ω as shown in figure the weight with mass m causes an extension of the spring. The length of the rotating spring is



(a) $\frac{\ell_0}{1-\frac{m\omega^2}{k}}$ (b) $\frac{\ell_0}{1+\frac{m\omega^2}{k}}$ (c) $\frac{\ell_0 k}{1+m\omega^2}$ (d) $\frac{\ell_0}{\frac{1}{k}+m\omega^2}$

23. Two spheres of equal masses are attached to a string of length 2 m as shown in figure. The string and the spheres are then whirled in a horizontal circle about O at a constant angular speed. The value of the ratio



$\left(\frac{\text{Tension in the string between P and Q}}{\text{Tension in the string between P and O}}\right)$ is

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) 2

24. A particle stays at rest as seen in a frame. We can conclude that

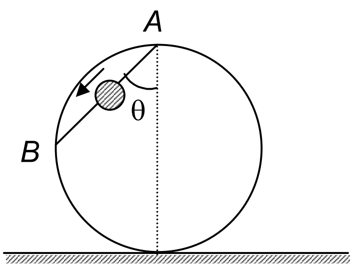
(a) the frame is inertial

(b) resultant force on the particle is zero

(c) the frame may be inertial but the resultant force on the particle is zero

(d) the frame may be noninertial but the resultant force on the particle is zero

25. A bead is free to slide down a smooth wire tightly stretched between points A and B on a vertical circle. If the bead starts from rest at A, the highest point on the circle



(a) its velocity v on arriving at B is proportional to sinθ

(b) its velocity v on arriving at B is proportional to tan θ

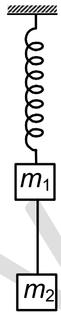
(c) time to arrive at B is proportional to cosθ

(d) time to arrive at B is independent of θ

EXERCISE – II

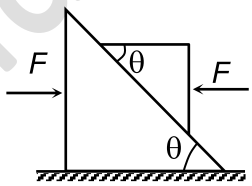
IIT-JEE- SINGLE CHOICE CORRECT

1. Consider the diagram a_1 and a_2 are accelerations of two blocks respectively just after cutting the spring. Similarly a_3 and a_4 are accelerations of two blocks just after cutting the string. Which one of the following option is correct?



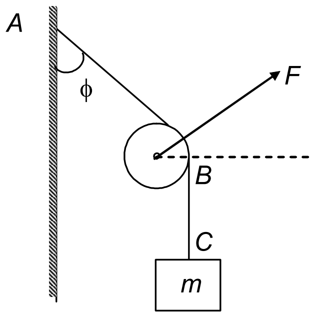
(a) $a_1 = g$ (b) $a_2 = g$
(c) $a_3 = m_1g/m_2$ (d) $a_4 = g$

2. In the figure shown, what should be the value of force F applied so that the whole system can remain in the state of rest? Both the wedges are of the same mass M and the angle inclination is θ. (Neglect friction)



(a) $Mg \tan \theta$ (b) $Mg/\tan\theta$
(c) $2Mg\tan\theta$ (d) $Mg\sin\theta$

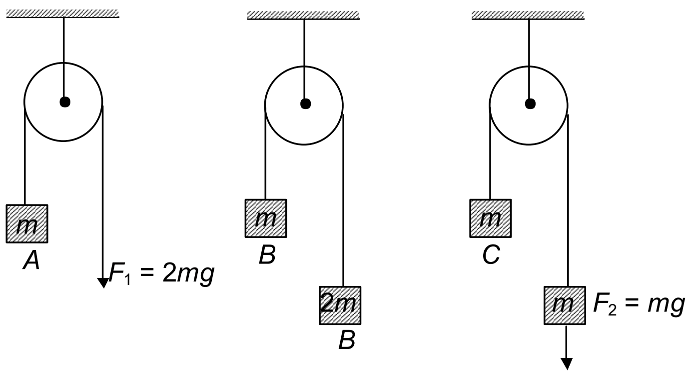
3. A wire ABC supports a body of mass m as shown in the figure. The wire passes over a fixed pulleys at B and is firmly attached to a vertical wall at A. The line AB makes an angle ϕ with the vertical, and the pulley at B exerts on the wire a force F, then the value of force F is



(a) $2\text{ mg} \sin \dfrac{\phi}{2}$ (b) $2\text{ mg} \cos \dfrac{\phi}{2}$
(c) $3\text{ mg} \cos \dfrac{\phi}{2}$ (d) none

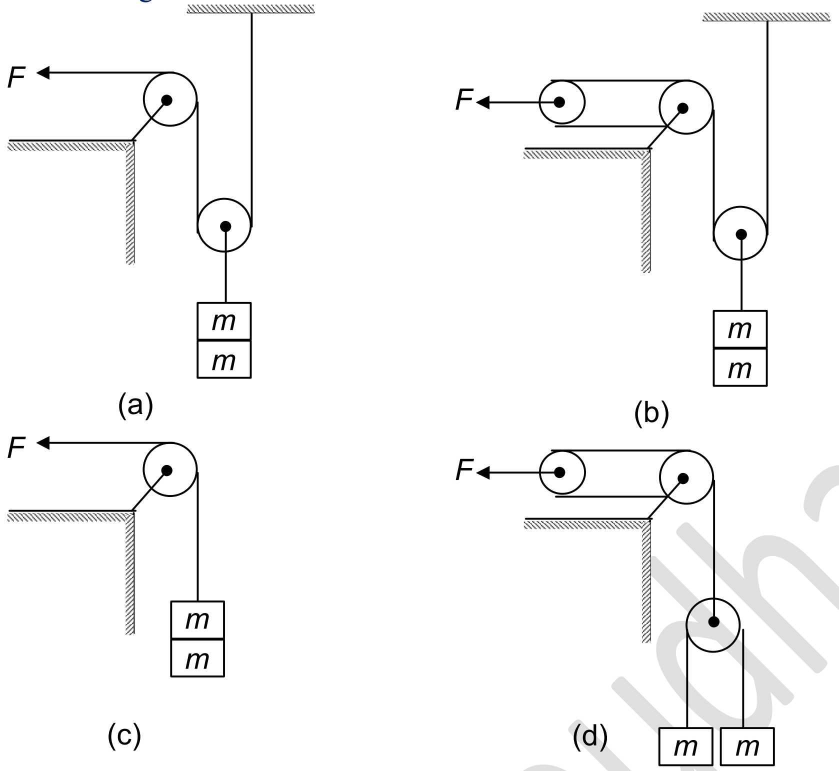
[

4. In the figure, the block A, B and C of mass m each, have acceleration a_1, a_2 and a_3 respectively. F_1 and F_2 are external forces of magnitudes $2\,mg$ and mg respectively. Then

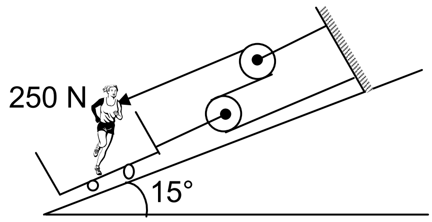


(a) $a_1 = a_2 = a_3$
(b) $a_1 > a_3 > a_2$
(c) $a_1 = a_2$, $a_2 > a_3$
(d) $a_1 > a_2$, $a_2 = a_3$

5. A man thinks about 4 arrangements as shown to raise two small bricks each having mass m. Which of the arrangement would take minimum time?

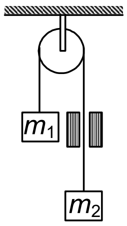


6. A trolley is being pulled up an incline plane by a man sitting on it (as shown in figure). He applies a force of 250 N. If the combined mass of the man and trolley is 100 kg, the acceleration of the trolley will be [sin 15° = 0.26]



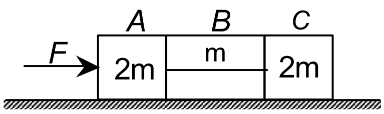
(a) 2.4 m/s^2 (b) 9.4 m/s^2
(c) 6.9 m/s^2 (d) 4.9 m/s^2

7. A weightless string passes through a slit over a pulley. The slit offers frictional force f to the string. The string carries two weights having masses m_1 and m_2 where $m_2 > m_1$, then acceleration of the weights is



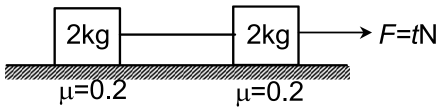
(a) $\dfrac{(m_2-m_1)g-f}{m_1+m_2}$ (b) $\dfrac{f-(m_2-m_1)g}{m_1+m_2}$ (c) $\dfrac{(m_1-m_2)g-f}{(m_1-m_2)}$ (d) $\dfrac{m_2g-f}{(m_1+m_2)}$

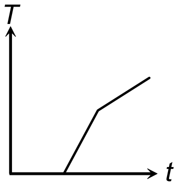
8. The system is pushed by a force F as shown in figure. All surfaces are smooth except between B and C. Friction coefficient between B and C is μ. Minimum value of F to prevent block B from downward slipping is

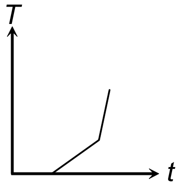


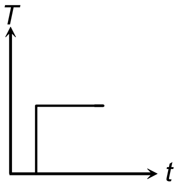
(a) $\left(\dfrac{3}{2\mu}\right)mg$ (b) $\left(\dfrac{5}{2\mu}\right)mg$ (c) $\left(\dfrac{5}{2}\right)\mu mg$ (d) $\left(\dfrac{3}{2}\right)\mu mg$

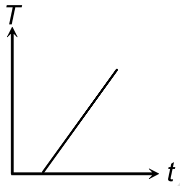
9. Two blocks each of mass m = 2kg placed on rough horizontal surface connected by massless string as shown in the figure. A variable horizontal force $F = t$ N (where t is time) is applied, then the tension T in string versus time graph is



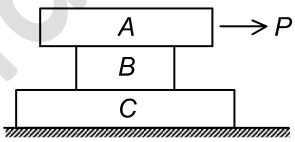
(a)


(b)


(c)


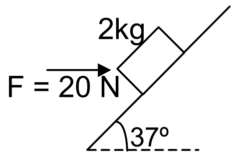
(d)


10. Find the least horizontal force P to start motion of any part of the system of the three blocks resting upon one another as shown in figure. The weight of blocks are A = 300N, B = 100 N and C = 200 N. Between A and B, $\mu = 0.3$, between B and C, $\mu = 0.2$ and between C and the ground $\mu = 0.1$.



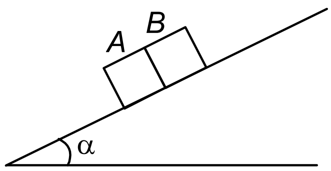
(a) 90 N (b) 60 N (c) 80 N (d) 100 N

11. A horizontal force F = 20 N is applied on a block of mass 2 kg placed an rough inclined plane with coefficient of friction μ = 0.2 as shown, then the acceleration of block is $\left(\tan 37^\circ = \frac{3}{4}, g = 10 m/s^2\right)$



(a) 2 m/s^2 (b) 1 m/s^2
(c) zero (d) 1.5 m/s^2

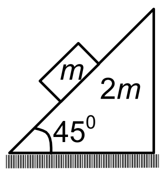
12. Two blocks A and B placed over an inclined plane of inclination α have masses m and M. The coefficients of friction between bodies and plane are respectively μ_1 and μ_2. Find the minimum value of α at which the blocks start moving if $\mu_1 > \mu_2$.



(a) $\tan \alpha = \frac{\mu_1 m + \mu_2 M}{(M+m)}$ (b) $\sin \alpha = \frac{\mu_1 m + \mu_2 M}{(M+m)}$

(c) $\cos \alpha = \frac{\mu_1 m + \mu_2 M}{(M+m)}$ (d) $\cot \alpha = \frac{\mu_1 m + \mu_2 M}{(M+m)}$

13. A block of mass m is placed on a wedge of mass $2m$ which rests on a rough horizontal surface. There is no friction between the block and the wedge. The minimum coefficient of friction between the wedge and the ground so that the wedge does not move is



(a) 0.167 (b) 0.2 (c) 0.23 (d) 0.33

14. A weight is moved from the bottom to the top of an inclined plane. If the force to drag it along the plane is to be smaller than to lift it the value of μ should be such that

(a) $\mu < \tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$ (b) $\mu < \tan\left(\frac{\theta}{2}-\frac{\pi}{4}\right)$ (c) $\mu < \tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$ (d) $\mu < \tan\theta$

15. A block, released from rest from the top of a smooth inclined plane of inclination □, has a speed *v* when it reaches the bottom. The same block, released from the top of a rough inclined plane of the same inclination □, has a speed *v/n* on reaching the bottom, where *n* is a number greater than unity. The coefficient of friction is given by

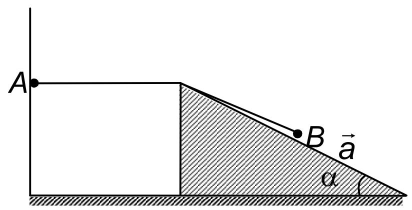
(a) $\mu = \left(1-\frac{1}{n^2}\right)\tan\theta$ (b) $\mu = \left(1-\frac{1}{n^2}\right)\cot\theta$

(c) $\mu = \left(1-\frac{1}{n^2}\right)^{1/2}\tan\theta$ (d) $\mu = \left(1-\frac{1}{n^2}\right)^{1/2}\cot\theta$

16. A boy of mass *m* is applying a horizontal force to slide a box of mass *M* on a rough horizontal surface. The coefficient of friction between the shoes of the boy and the floor is μ and that between the box and the floor is μ′. In which of the following cases it is certainly not possible to slide the box?

(a) $\mu < \mu',\ m < M$ (b) $\mu > \mu',\ m < M$ (c) $\mu < \mu',\ m > M$ (d) $\mu > \mu',\ m > M$

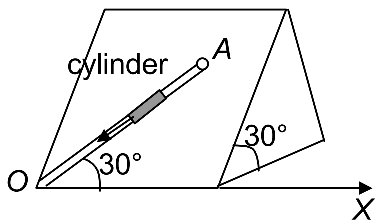
17. A weightless inextensible rope rests on a stationary wedge forming an angle α with the horizontal. One end of the rope is fixed to the wall at the point *A*. A small load is attached to the rope at the point *B*, the wedge starts moving to the right with constant acceleration '*a*'. What is the acceleration of the load when it is still on the wedge?



(a) 2a sin α (b) 2a cos α

(c) 2a sin $\frac{\alpha}{2}$ (d) 2a cos $\frac{\alpha}{2}$

18. An inclined plane makes an angle 30° with the horizontal. A groove *OA* = 5 m cut in the plane makes an angle 30° with *OX*. A short smooth cylinder is free to slide down under the influence of gravity. The time taken by the cylinder to reach from *A* to *O* is (g = 10 m/s^2)



(a) 4 s (b) 2 s

(c) $2\sqrt{2}$ s (d) 1 s

19. A particle of mass *m* is attached to one end of a string of length *l* while the other end is fixed to a point *h* meter above the horizontal table. The particle is made to revolve in a circle on the table so as to make *P* revolution per second. The maximum value of *P* if the particle is to be in contact with the table, will be

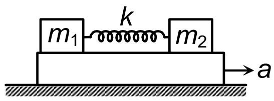
(a) $2\pi\sqrt{gh}$ (b) $\sqrt{g/h}$ (c) $2\pi\sqrt{h/g}$ (d) $\frac{1}{2\pi}\sqrt{g/h}$

20. A point moves along the arc of a circle of radius *R*. Its speed varies as $v = a\sqrt{s}$, where *a* is constant and *s* is the arc length travelled by the particle. The angle □□between the vector of total acceleration and the vector of velocity is given by

(a) $\tan^{-1}(R/S)$ (b) $\tan^{-1}(R/2S)$ (c) $\tan^{-1}(2S/R)$ (d) $\tan^{-1}(S/R)$

ONE OR MORE THAN ONE CHOICE CORRECT

1. Two blocks of masses m_1 and m_2 connected by a spring of spring constant k, are kept on a plank which in turn is kept over a frictionless horizontal surface as shown. Coefficients of friction between masses m_1 and m_2 and plank are μ_1 and μ_2. Plank is given a horizontal acceleration of magnitude a. Then



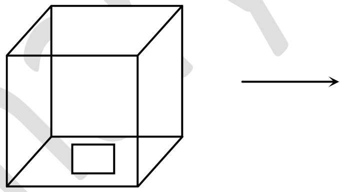
(a) elongation may take place in spring if $\mu_2 > \mu_1$

(b) compression may take place in spring $\mu_1 = \mu_2$

(c) elongation may take place in spring $\mu_2 < \mu_1$

(d) there will be no elongation or compression in the spring if $\mu_1 = \mu_2$

2. A solid block of mass 2 kg is resting inside a cube as shown in the figure. The cube is moving with the velocity $\vec{v} = 5t\hat{i} + 2\hat{j}$ m/s (t is time in second). The block is at rest with respect to the cube and coefficient of friction between the surfaces of cube and the block is 0.2. Then (g = 10 m/s^2)



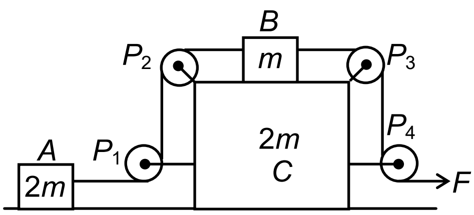
(a) force of friction acting on the block is 10 N.

(b) force of friction acting on the block is 4 N.

(c) the total force exerted by the block on the cube is 14 N.

(d) the total force exerted by the block on the cube is $10\sqrt{5}$ N.

3. In the figure shown all the surfaces are smooth. All the strings are either horizontal or vertical. A horizontal force of magnitude F Newton is acting at the end of the string towards P_4. Select the correct alternatives if m_1 = 1 kg



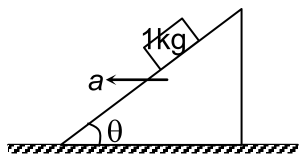
(a) acceleration of the block C is zero

(b) acceleration of the block C is $\frac{2F}{5}$ m/s^2

(c) net acceleration of the block B is $\frac{F}{3}$ m/s^2

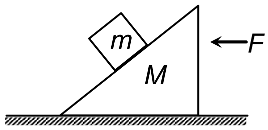
(d) net acceleration of the block B is $\frac{F}{5}$ m/s^2

4. A block of mass 1 kg is at rest relative to a smooth wedge moving leftwards with constant acceleration a = 5 m/s^2. Let N be the normal reaction between the block and the wedge. Then (g = 10 m/s^2)



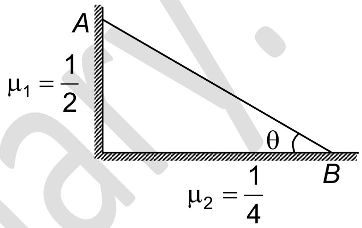
(a) $N = 5\sqrt{5}$ N (b) N = 15 N (c) $\tan\theta = \frac{1}{2}$ (d) $\tan\theta = 2$

5. A block of mass m is kept over a wedge of mass M. The coefficient of friction between block and wedge is μ and between wedge and ground is zero. A force F is applied on the wedge such that block remains at rest with respect to wedge. Then



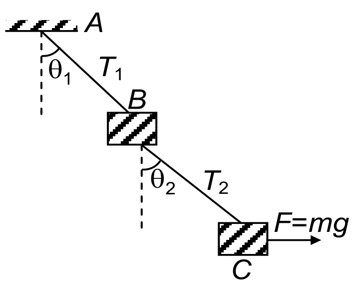
(a) $F_{max} = (M+m)g\left[\frac{\sin\theta+\mu\cos\theta}{\sin\theta-\mu\cos\theta}\right]$
(b) $F_{max} = (M+m)g\left[\frac{\sin\theta+\mu\cos\theta}{\cos\theta-\mu\sin\theta}\right]$
(c) $F_{min} = (M+m)g\left[\frac{\sin\theta-\mu\cos\theta}{\mu\cos\theta+\sin\theta}\right]$
(d) $F_{min} = (M+m)g\left[\frac{\sin\theta-\mu\cos\theta}{\mu\sin\theta+\cos\theta}\right]$

6. A uniform rod of weight W is made to lean between rough vertical wall and rough ground as shown. Friction coefficient between rod and wall is $\mu_1 = \frac{1}{2}$ and between the rod and the ground is $\mu_2 = \frac{1}{4}$. The correct options are



(a) the normal reaction between rod and wall is $\frac{\mu_2 W}{1+\mu_1\mu_2}$
(b) normal reaction between rod and ground is $\frac{W}{1+\mu_1\mu_2}$
(c) friction force between rod and wall is $\frac{\mu_2{}^2 W}{1+\mu_1\mu_2}$
(d) friction force between rod and ground is $\frac{\mu_1 W}{1+\mu_1\mu_2}$

7. The blocks B and C in figure have mass m each. The strings AB and BC are light, having tensions T_1 and T_2 respectively. The system is in equilibrium with a constant horizontal force mg acting on C.

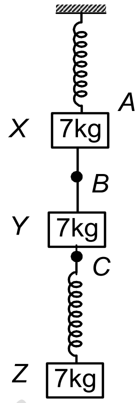


(a) $\tan\theta_1 = \frac{1}{2}$ (b) $\tan\theta_2 = 1$
(c) $T_1 = \sqrt{5}\ mg$ (d) $T_2 = \sqrt{2}\ mg$

8. Three identical cars A, B and C are moving at the same speed on three bridges. The car A goes on a plane bridge, B on a bridge convex upward and C goes on a bridge concave upward. Let F_A, F_B and F_C be the normal forces exerted by the cars on the bridges when they are at the middle of bridges.
(a) F_A is maximum of the three forces (b) F_B is minimum of the three forces
(c) F_C is maximum of the three forces (d) $F_A = F_B = F_C$

9. F, F_n and f denote the magnitudes of the contact force, normal force and friction force respectively exerted by one surface on the other when both are in contact. None of the force is zero. Then
(a) $F > F_n$ (b) $F > f$
(c) $F_n > f$ (d) $F_n - f < F < F_n + f$

10. The following figure shows an arrangement of three blocks X, Y and Z suspended by springs or strings as shown. Three points A, B and C are marked on the spring or strings. Let a_1, a_2 and a_3 be the instantaneous accelerations of blocks X, Y and Z respectively on cutting the arrangement from any of the points A, B or C. Select the correct alternative(s)



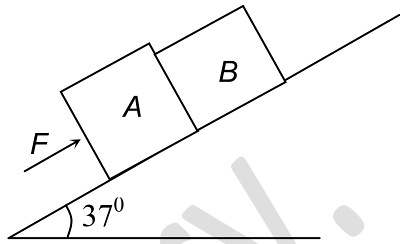
(a) $a_3 = 0$ when the arrangement is cut from A or B
(b) $a_1 = a_2 = 3g/2$ downward, when the arrangement is cut from A
(c) $a_1 = a_2 = g/2$ upward, when the arrangement is cut from C
(d) $a_1 = a_2 = a_3 = 0$ always

EXERCISE – III

MATCH THE FOLLOWING

Note: Each statement in column – I has one or more than one match in column –II.

1. Two blocks A and B of masses 5/3 kg and 5/6 kg are placed on an inclined plane of inclination 37^0 (with the horizontal) with initial normal reaction between the blocks is zero and coefficient of friction between the blocks and the surface is 0.9. A force F is applied on block A along the inclined plane as shown. (g = 10 m/s^2, tan 37^0 = 3/4)



	Column-I		Column-II
I.	The possible values of F to keep the blocks stationary is	**A.**	11 N
II.	Limiting values of contact force between the blocks	**B.**	3 N
III.	Contact force between the blocks when applied force F = 48 N	**C.**	16 N
IV.	The possible values of F to keep the blocks stationary if force is applied on block B down the inclined plane.	**D.**	zero
		E.	7 N

REASONING TYPE

Directions: Read the following questions and choose

(A) If both the statements are true and statement-2 is the correct explanation of **statement-1**.
(B) If both the statements are true but statement-2 is not the correct explanation of **statement-1**.
(C) If statement-1 is True and **statement-2** is False.
(D) If statement-1 is False **and statement-2** is True.

1. **Statement-1**: Two objects of equal mass rest on the opposite pans of an arm balance. Scale will remains balanced, when it is accelerated up or down in a lift.
Statement-2: Both masses experience unequal fictitious forces in magnitude as well as in direction
(a) (A) (b) (B) (c) (C) (d) (D)

2. **Statement-1**: A car accelerates on a horizontal road due to the force exerted by the engine of the car.
Statement-2: To accelerate a body force is always needed in the direction of required acceleration.
(a) (A) (b) (B) (c) (C) (d) (D)

3. **Statement-1**: Centrifugal force acts on a particle because the particle moves on a circle.
Statement-2: Centrifugal force is a pseudo force.
(a) (A) (b) (B) (c) (C) (d) (D)

4. **Statement-1**: If a string moves over a rough massive pulley without slipping, then tension in the string, on both sides of pulley will be different.
Statement-2: To rotate a massive pulley, there should be net torque acting on the pulley
(a) (A) (b) (B) (c) (C) (d) (D)

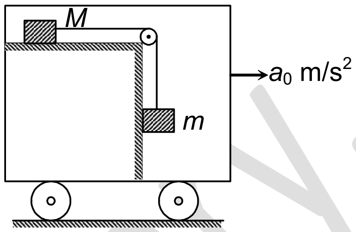
5. **Statement-1**: Value of frictional force as seen from an inertial frame, for a pair of solids may change if it is observed from a non inertial frame.

Statement-2: Coefficient of friction μ does not depends on frame of reference.

(a) (A) (b) (B) (c) (C) (d) (D)

LINKED COMPREHENSION TYPE

A wagon is going horizontally with an acceleration a_0 m/s^2. A system inside the wagon contains two blocks of mass M and m connected by a thread which passes over a pulley as shown in the figure (all the surfaces are smooth)



1. The tension in the string is given by

(a) $\frac{2mM(g+a_0)}{m+M}$ (b) $\frac{2mM(g-a_0)}{M+m}$ (c) $\frac{mM(g+a_0)}{M+m}$ (d) $\frac{mM(g+a_0)}{2m+M}$

2. Magnitude of normal reaction between the block m and vertical wall when acceleration of lift became $a_0/2$

(a) $\frac{Ma_0}{2}$ (b) $\frac{ma_0}{3}$ (c) ma_0 (d) $\frac{ma_0}{2}$

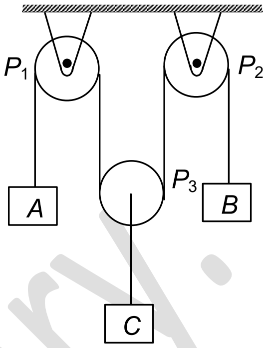
3. What is the total reaction force on the pulley as seen from the ground?

(a) $\frac{2mM(g+a_0)}{m+M}$ (b) $\frac{\sqrt{2}mM(g+a_0)}{M+m}$ (c) $\frac{mM(g+a_0)}{2M+m}$ (d) $\frac{mM(g+a_0)}{2m+M}$

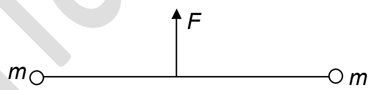
EXERCISE – IV

SUBJECTIVE PROBLEMS

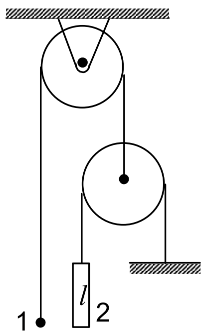
1. The diagram shows two blocks *A* and *B*, of masses $m_1 = 1/2$kg and $m_2 = 1/2$kg respectively connected by a light inextensible string passing over two fixed smooth pulleys P_1 and P_2 and under a light movable pulley P_3, which carries a block *C* of mass $m_3 = 1$ kg. Find
(a) the acceleration of block *A*,
(b) the acceleration of block *C*
(c) the tension in string connecting *A* and *B*. [take g = 10 m/s²]



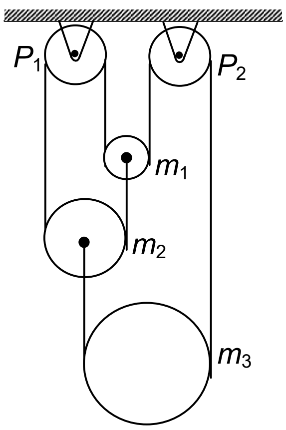
2. Two particles of equal masses $m = 1$kg are connected by a light string of length $L = 2\sqrt{5}$ m as shown in figure. A constant force *F* = 4 N is applied continuously at the middle of the string, always along the perpendicular bisector of the line joining the two particles. Find the acceleration of approach of the particles when the distance between them is $l = 2$m.



3. In the arrangement shown in the figure, the mass of ball 1 is $\eta = \frac{133}{68}$ times as great as rod 2. The length of the latter is l = 100 cm. The mass of the pulley and the threads, as well as the friction, is negligible. The ball is set on the same level as the lower end of rod and then released. How soon will the ball be opposite the upper end of the rod? [take g = 10 m/s²]



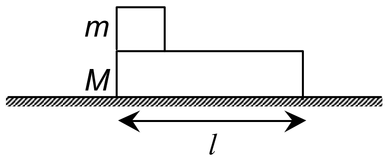
4. Figure shows three movable pulleys of masses $m_1 = 1$kg, $m_2 = 2$kg and $m_3 = 8$ kg connected by a single string. If the pulleys are frictionless, string is light inextensible and pulleys P_1 and P_2 are light, find tension in the string. [take $g = 10$ m/s²]



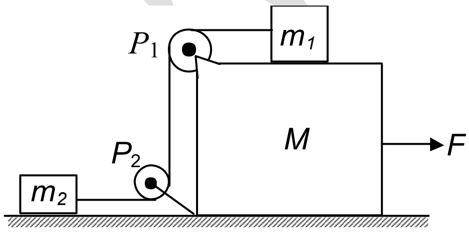
5. A particle is projected directly up a plane inclined at an angle α = 45° with horizontal with initial velocity u given by $u^2 = 2g\ell\,(\sin\alpha + \mu\cos\alpha)$ where μ is the coefficient of friction. If tan α > μ, find the velocity when it returns to the starting point and explain the difference between the initial and final kinetic energies. (Take $\mu = \frac{1}{2}$, $\ell = \frac{16}{5\sqrt{2}}$, g = 10 m/s²)

6. A chain of mass $m = 2\pi$ kg forming a circle of radius $R = 2$m is slipped on a smooth round cone with half angle $\theta = 45°$. Find the tension of chain if it rotates with constant angular velocity $\omega=\pi$ rad/s about a vertical axis coinciding with symmetry axis of the cone. ($\pi^2 = 10$)

7. Figure shows a small block of mass $m = 1$kg kept at the left end of a larger block of mass $M = 3$kg and length $l = 12$m. The system can slide on a horizontal road. The system is started towards right with an initial velocity v. The friction coefficient between the road and the bigger block is $\mu = 0.4$ and that between the blocks is $\mu/2$. Find the time elapsed before the smaller block separates from the bigger block.



8. In the figure masses m_1, m_2 and M are 20 kg, 5 kg and 50 kg respectively. The coefficient of friction between M and ground is zero. The coefficient of friction between m_1 and M and that between m_2 and ground is $\mu = 0.3$. The pulleys and the string are massless. The string is perfectly horizontal between P_1 and m_1 and also between P_2 and m_2. The string is perfectly vertical between P_1 and P_2. An external horizontal force F is applied to the mass M. The magnitude of the force of friction between m_1 and M be f_1 and that between m_2 and ground be f_2. For a particular F, it is found that $f_1=2f_2$. Find F, tension in the string T and acceleration a (in cm/s^2) of the masses. (Take $g = 10$ /s^2)



9. A cyclist rides along the circumference of a circle in a horizontal plane of radius $R = 100$ m, the friction coefficient being dependent only on distances r from the centre O of the plane as $\mu = \mu_o \left(1-\frac{r}{R}\right)$, where $\mu_o = 0.4$. Find the radius r of the circle with the centre at the point O along which the cyclist can ride with the maximum velocity. What is this maximum velocity?

10. Two block of mass $M_1 = 10$ kg and $M_2 = 5$ kg connected to each other by a massless inextensible string of length $l = 0.3$ m, are placed along a diameter of the table such that centre of the string is at the centre of the table. The coefficient of friction between the table and M_1 is $\mu = 0.5$ while there is no friction between M_2 and the table.
The table is rotating with an angular velocity of $\omega = 10$ rad/s about a vertical axis passing through its centre O such that the mass M_1 is at a distance $r = 0.124$ m from O. The masses are placed along the diameter of table on either side of the centre O. The masses are observed to be a rest with respect to an observer on the turn table
(a) calculate the frictional force on M_1
(b) The distances of M_1 and M_2 from axis of rotation be r_1 cm and r_2 cm such that the friction force on M_1 becomes zero. Find the values of r_1 and r_2.

ANSWERS

EXERCISE – I

NEET-SINGLE CHOICE CORRECT

1. (a)	2. (a)	3. (b)	4. (b)	5. (d)
6. (d)	7. (c)	8. (a)	9. (d)	10. (b)
11. (a)	12. (c)	13. (a)	14. (b)	15. (a)
16. (d)	17. (a)	18. (a)	19. (c)	20. (c)
21. (b)	22. (a)	23. (b)	24. (c)	25. (d)

EXERCISE – II

IIT-JEE-SINGLE CHOICE CORRECT

1. (c)	2. (a)	3. (a)	4. (b)	5. (a)
6. (d)	7. (a)	8. (b)	9. (a)	10. (b)
11. (c)	12. (a)	13. (b)	14. (c)	15. (a)
16. (a)	17. (a)	18. (b)	19. (d)	20. (c)

MORE THAN ONE CHOICE CORRECT

1. (a,d)	2. (a,d)	3. (a,c)	4. (a,c)	5. (b,d)
6. (a,b,c,d)	7. (a,b,c,d)	8. (b,c)	9. (a,b,d)	10. (a,b,c)

EXERCISE – III

MATCH THE FOLLOWING

1. I – A, B, C, D, E; II – A, D ; III – C ; IV – B, D

REASONING TYPE

1. (c)	2. (d)	3. (d)	4. (a)	5. (d)

LINKED COMPREHENSION TYPE

1. (c)	2. (d)	3. (b)

EXERCISE – IV

SUBJECTIVE PROBLEMS

1. (a) 0 m/s^2, $a_1 = g\left(1 - \dfrac{4}{1 + \dfrac{m_1}{m_2} + \dfrac{4m_1}{m_3}}\right)$

(b) 0 m/s^2, $a_2 = g\left(1 - \dfrac{4}{1 + \dfrac{m_2}{m_1} + \dfrac{4m_2}{m_3}}\right)$

(c) 5 N, $T = \dfrac{4g}{\dfrac{1}{m_1} + \dfrac{1}{m_2} + \dfrac{4}{m_3}}$

2. 2 m/s^2, $a = \dfrac{Fl}{\sqrt{L^2 - l^2}}$

3. $t = 3$ sec , $t = \sqrt{\dfrac{2l}{3g}\left(\dfrac{4+\eta}{2-\eta}\right)}$

4. 20 N, $T = \dfrac{4g}{\dfrac{1}{m_1} + \dfrac{1}{m_2} + \dfrac{4}{m_3}}$

5. 4 m/s, $v = \sqrt{2gl(\sin\theta - \mu\cos\theta)}$

6. 30 N, $T = \left(g\cot\theta + R\omega^2\right)\dfrac{m}{2\pi}$

7. 3 s, $t = \sqrt{\dfrac{2l}{a}}$ where $a = \mu g\left[\dfrac{m}{2M} + \dfrac{1}{2}\right]$

8. 60 N, $F = \left(2 + \dfrac{M}{m_1 + m_2}\right)\mu m_2 g$

18 N, $T = \mu m_2 g\left(1 + \dfrac{m_2}{m_1 + m_2}\right)$

60 cm/s^2, $a = \dfrac{\mu m_2 g}{m_1 + m_2}$

9. 50 m, $r = \dfrac{R}{2}$

10 m/s, $v_{max} = \dfrac{\sqrt{\mu_0 R g}}{2}$

10. (a) 36 N, $f_r = \omega^2\left[(m_1 + m_2)r - m_2 l\right]$

(b) 10 cm, $r_1 = \dfrac{m_2 l}{m_1 + m_2}$,

20 cm, $r_2 = \dfrac{m_1 l}{m_1 + m_2}$

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 1

Q.1 A wheel is subjected to uniform angular acceleration about its axis. Initially its angular velocity is zero. In the first 2 sec, it rotates through an angle θ_1 ; in the next 2 sec, it rotates through an additional angle θ_2. The ratio of θ_2 / θ_1 is-

(1) 1 (2) 2
(3) 3 (4) 5

Q.2 In applying the equation for motion with uniform angular acceleration $\omega = \omega_0 + \alpha t$, the radian measure -

(1) must be used for both ω and α
(2) may be used for both ω and α
(3) may be used for ω but not α
(4) cannot be used for both ω and α

Q.3 The linear and angular acceleration of a particle are 10 m/sec^2 and 5 rad/sec^2 respectively it will be at a distance from the axis of rotation -

(1) 50 m (2) 1/2 m
(3) 1 m (4) 2 m

Q.4 A tachometer is a device to measure -

(1) gravitational pull
(2) speed of rotation
(3) surface tension
(4) tension in a spring

Q.5 Two cars of masses m_1 and m_2 are moving along the circular path of radius r_1 and r_2. They take one round in the same time. The ratio of angular velocities of the two cars will be-

(1) $m_1 : m_2$ (2) $r_1 : r_2$
(3) 1 : 1 (4) $m_1 r_1 : m_2 r_2$

Q.6 A bottle of soda water is grasped by the neck and swing briskly in a vertical circle. Near which portion of the bottle do the bubbles collect ?

(1) near the near bottom
(2) in the middle of the bottle
(3) near the neck
(4) uniformly distributed in the bottle

Q.7 In circular motion, the centripetal acceleration is given by-

(1) $\mathbf{a} \times \mathbf{r}$ (2) $\boldsymbol{\omega} \times \mathbf{v}$ (3) $\mathbf{a} \times \mathbf{v}$(4) $\boldsymbol{\omega} \times \mathbf{r}$

Q.8 The ratio of angular speeds of minutes hand and hour hand of a watch is -

(1) 1 : 12 (2) 6 : 1
(3) 12 : 1 (4) 1 : 6

Q.9 A particle moves in a circle of radius 25 cm at two revolutions per second. The acceleration of particle in m/s^2 is -

(1) π^2 (2) $8\pi^2$
(3) $4\pi^2$ (4) $2\pi^2$

Q.10 A particle moves in circular path with uniform speed v. The change in its velocity on rotating through 60º is -

(1) $v\sqrt{2}$ (2) $\frac{v}{\sqrt{2}}$
(3) v (4) Zero

Q.11 In uniform circular motion-

(1) both velocity and acceleration are constant
(2) acceleration and speed are constant but velocity changes
(3) both acceleration and velocity change
(4) both acceleration and speed are constant

Q.12 When a body moves with a constant speed along a circle-

(1) no work is done on it
(2) no acceleration is produced in the body
(3) no force acts on the body
(4) its velocity remains constant

Q.13 What happens to the centripetal acceleration of a revolving body if you double the orbital speed v and halve the angular velocity ω ?

(1) the centripetal acceleration remains unchanged
(2) the centripetal acceleration is halved
(3) the centripetal acceleration is doubled
(4) the centripetal acceleration is quadrupled

Q.14 A body is moving with a constant speed v in a circle of radius r. Its angular acceleration is-

(1) Zero (2) $\frac{v}{r}$
(3) $\frac{v^2}{r^2}$ (4) $\frac{v^2}{r}$

Q.15 A string of length 1 m is fixed at one end and carries a mass of 100 gm at the other end. The string makes $(2/\pi)$ revolutions per second around vertical axis through the fixed end. Calculate the tension in the string-

(1) 1.0 N (2) 1.6 N
(3) 2 N (4) 4 N

Q.16 A body is revolving with a uniform speed V in a circle of radius r. The angular acceleration of the body is -

(1) $\frac{V}{r}$

(2) Zero

(3) $\frac{V^2}{r}$ along the radius and towards the centre

(4) $\frac{V^2}{r}$ along the radius and away from the centre

Q.17 A particle is acted upon by a constant force always normal to the direction of motion of the particle. It is therefore inferred that-

(a) Its velocity is constant

(b) It moves in a straight line

(c) Its speed is constant

(d) It moves in circular path

(1) a, d

(2) c, d

(3) a, b

(4) a, b, c

Q.18 A body of mass 2 kg is moving in a vertical of radius 2 m. The work done when it moves from the lowest point to the highest point is-

(1) 80 J (2) 40 J

(3) 20 J (4) 0

Q.19 A particle rests on the top of the hemisphere of radius R. The small horizontal velocity that must be imparted to the particle if it is to leave the hemisphere without sliding down. is-

(1) $v = (2gR)^{1/2}$ (2) $v = (gR/2)^{1/2}$

(3) $v = (gR)^{1/2}$ (4) $v = (2g/R)^{1/2}$

Q.20 A mass m is revolving in a vertical circle at the end of a string of length 20 cm. By how much does the tension of the string at the lowest point exceed the tension at the top most point?

(1) 2 m g (2) 4 m g

(3) 6 m g (4) 8 m g

Q.21 A car is travelling with linear velocity v on a circular road of radius r. If it is increasing it speed at the rate of 'a' metre/sec^2, then the resultant acceleration will be-

(1) $\sqrt{\left(\frac{v^2}{r^2}-a^2\right)}$ (2) $\sqrt{\left(\frac{v^4}{r^2}+a^2\right)}$

(3) $\sqrt{\left(\frac{v^4}{r^2}-a^2\right)}$ (4) $\sqrt{\left(\frac{v^2}{r^2}+a^2\right)}$

Q.22 On an unbanked road, a cyclist negotiating a bend of radius r at velocity v must lean inwards by an angle θ equal to -

(1) $\tan^{-1}(v^2/g)$ (2) $\tan^{-1}(g/v)$

(3) $\tan^{-1}(v^2/gr)$ (4) $\tan^{-1}(rg/v^2)$

Q.23 A string can bear a maximum tension of 100 Newton without breaking. A body of mass 1 kg is attached to one end of 1 m length of thin string and it is revolved in a horizontal plane. The maximum linear velocity which can be imparted to the body without breaking the string, will be -

(1) 10 m/s (2) 1 m/s

(3) 100 m/s (4) 1000 m/s

Q.24 The roadway of a bridge over a canal is in the form of a circular arc of radius 18 m. What is the greatest speed with which a motor cycle can cross the bridge without leaving ground.

(1) $\sqrt{98}$ m/s (2) $\sqrt{18\times9.8}$ m/s

(3) 18 × 9.8 m/s (4) 18 / 9.8 m/s

Q.25 A cyclist taking turn bends inwards while a car passenger take the same turn is thrown outwards. The reason is-

(1) car is heavier then cycle

(2) car has four wheels while cycle has only two

(3) difference in the speed of the two

(4) cyclist has to counteract the centrifugal force while in the case of car only the passenger is thrown by this force

Q.26 A cyclist turns around a curve at 15 miles/hour. If he turns at double the speed, the tendency to overturn is -

(1) doubled (2) quadrupled

(3) halved (4) unchanged

Q.27 A cyclist is moving on a circular track of radius 80 m with a velocity of 72 km/hr. He has to lean from the vertical approximately through an angle-

(1) $\tan^{-1}(1/4)$ (2) $\tan^{-1}(1)$

(3) $\tan^{-1}(1/2)$ (4) $\tan^{-1}(2)$

Q.28 A motor cyclist moving with a velocity of 72 km per hour on a flat road takes a turn on the road at a point where the radius of curvature of the road is 20 metres. The acceleration due to gravity is 10 m/s^2. In order to avoid skidding, he must not bend with respect to the vertical plane by an angle greater than-

(1) $\theta = \tan^{-1} 6$ (2) $\theta = \tan^{-1} 2$

(3) $\theta = \tan^{-1} 25.92$ (4) $\theta = \tan^{-1} 4$

Q.29 A particle of mass 'm' describes a circle of radius (r). The centripetal acceleration of the particle is $4/r^2$. The momentum of the particle -

(1) $\frac{2m}{r}$ (2) $\frac{2m}{\sqrt{r}}$

(3) $\frac{4m}{r}$ (4) $\frac{4m}{\sqrt{r}}$

Q.30 A particle move along a circular path of radius (r) with a uniform speed (v). The angle described by the particle in one second is given by -

(1) vr^{-1} (2) $v^{-1}r$
(3) vr^{-2} (4) v^2r

Q.31 A particle moving along a circular path with a speed (v) and its speed increases by 'g' per second. If the radius of the angular path be (r), than the net acceleration of the particle is -

(1) $\left(\frac{v^2}{r}+g\right)$ (2) $\left(\frac{v^2}{r}+g^2\right)$
(3) $\left(\frac{v^4}{r^2}+g^2\right)^{1/2}$ (4) $\left(\frac{v^2}{r}+g\right)^{1/2}$

Q.32 A particle is moving around a circular path with uniform angular speed (ω). The radius of the circular path is (r). The acceleration of the particle is -

(1) $\frac{\omega^2}{r}$ (2) $\frac{\omega}{r}$
(3) $v\omega$ (4) vr

Q.33 Let 'θ' denote the angular displacement of a simple pendulum oscillating in a vertical plane. If the mass of the bob is (m), then the tension in string is mg cos θ -

(1) always
(2) never
(3) at the extreme positions
(4) at the mean position

Q.34 The angular acceleration of particle moving along a circular path with uniform speed -

(1) uniform but non zero
(2) zero
(3) variable
(4) such as can not be predicted from given information

Q.35 A car moves on a circular road, describing equal angles about the centre in equal intervals of times which of the statements about the velocity of car are true -

(1) velocity is constant
(2) magnitude of velocity is constant but the direction of velocity change
(3) both magnitude and direction of velocity change
(4) velocity is directed towards the centre of circle

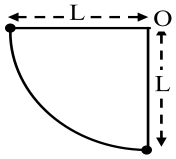
Q.36 A pendulum bob has a speed 3m/s while passing through its lowest position, length of the pendulum is 0.5 m then its speed when it makes an angle of 60º with the vertical is -

(1) 2 m/s (2) 1 m/s
(3) 4 m/s (4) 3 m/s

Q.37 An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 s. What is the linear speed of the motion -

(1) 2.3 cm/s (2) 5.3 cm/s
(3) 0.44 cm/s (4) none of these

Q.38 The mass of the bob of a simple pendulum of length L s m. If the bob is left from its horizontal position then the speed of the bob and the tension in the threads in the lowest position of the bob will be respectively -



(1) $\sqrt{2gL}$ and 3 mg (2) 3 mg and $\sqrt{2gL}$
(3) 2 mg and $\sqrt{2gL}$ (4) 2 gL ad 3 mg

Q.39 A stone of mass 1 kg is tied to the end of a string of 1 m long. It is whirled in a vertical circle. If the velocity of the stone at the top be 4 m/s. What is the tension in the string ?

(1) 6 N (2) 16 N
(3) 5 N (4) 10 N

Q.40 If the speed and radius both are trippled for a body moving on a circular path, then the new centripetal force will be -

(1) $F_2 = 2F_1$ (2) $F_2 = F_1$
(3) $F_2 = 3F_1$ (4) $F_2 = F_1/3$

Q.41 The blades of an aeroplane propeller are rotating at the rate of 600 revolutions per minute its angular velocity is -

(1) 10 π rad/s (2) 20 π rad/s
(3) 2 π rad/s (4) None of these

Q.42 A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 22 s than the acceleration of the stone is -

(1) 5 m/s^2 (2) 10 m/s^2
(3) 12.8 m/s^2 (4) None of these

Q.43 A particle moves on a circular path of radius (r) with speed (v) if its speed and radius both are doubled than centripetal force is -

(1) same (2) doubled
(3) quadrupled (4) eight times

Q.44 A particle moves in a circle of the radius 25 cm at two revolutions per second. The acceleration of the particle in m/sec^2 is -

(1) π^2 (2) $8\pi^2$
(3) $4\pi^2$ (4) $2\pi^2$

Q.45 When a body moves with a constant speed along a circle -
(1) no acceleration is produced in the body
(2) no force acts on the body
(3) its velocity remains constant
(4) no work gets done on it

Q.46 A particle moves in a circle describing equal angle in equal times, its velocity vector :
(1) remains constant
(2) changes in magnitude
(3) change in direction
(4) changes in magnitude and direction

Q.47 A mass of 2 kg is whirled in a horizontal circle by means of a string at an initial speed of 5 r.p.m. keeping the radius constant the tension in the string doubled the new speed is nearly -
(1) 7 r.p.m. (2) 14 r.p.m.
(3) 10 r.p.m. (4) 20 r.p.m.

Q.48 In a vertical circle of radius (r), at what point in its path a particle may has tension equal to zero-
(1) highest point
(2) lowest point
(3) at any point
(4) at a point horizontal from the centre of radius

Q.49 If the radius of circular path of two particles of same masses are in the ratio of 1 : 2 and have equal centripetal force their velocities should be in the ratio of -
(1) 1 : $\sqrt{2}$ (2) $\sqrt{2}$: 1
(3) 4 : 1 (4) 1 : 4

Q.50 A string of length 0.1 m cannot bear a tension more than 100 N. It is tied to a body of mass 100g and rotated in a horizontal circle. The maximum angular velocity is....
(1) 100 rad/s (2) 1000 rad/s
(3) 10000 rad/s (4) 0.1 rad/s

Q.51 The radius of the circular path of a particle is doubled but its frequency of rotation kept constant. If the initial centripetal force be F, then the final value of centripetal force will be -
(1) F (2) F/2 (3) 4F (4) 2F

Q.52 a 0.5 kg ball moves in a circle of radius 0.4 m at a speed of 4 ms^{-1}. The centripetal force on the ball is -
(1) 10 N (2) 20 N (3) 40 N (4) 80 N

Q.53 A car is travelling at 20 m/s on a circular road of radius 100 m. It is increasing in speed at the rate of 3 m/s^2. Its acceleration is -
(1) 3 m/s^2 (2) 4 m/s^2 (3) 5 m/s^2 (4) 7 ms^{-1}

Q.54 A weightless thread can bear tension upto 3.7 kg wt. A stone of mass 500 gm is tied to it and revolved in a circular path of radius 4m in a vertical plane. If g = 10 ms^{-2}, then the maximum angular velocity of the stone will be-

(1) 16 rad/s (2) $\sqrt{21}$ rad/s
(3) 2 rad/s (4) 4 rad/s

Q.55 A stone attached to one end of a string is whirled in a vertical circle. The tension in the string is maximum when -

(1) the string is horizontal
(2) the string is vertical with the stone at highest position
(3) the string is vertical with the stone at the lowest position
(4) the string makes an angle of 45º with the vertical

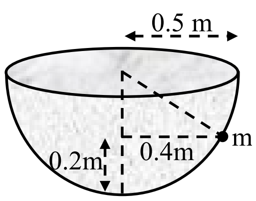
Q.56 A weightless thread can withstand tension upto 30 N. A stone of mass 0.5 kg is tied to it and is revolved in a circular path of radius 2m in a vertical plane. If g = 10 m/s^2, then the maximum angular velocity of the stone will be-

(1) 5 rad/s (2) $\sqrt{30}$ rad/s
(3) $\sqrt{60}$ rad/s (4) 10 rad/s

Q.57 A particle moving along a circular path. The angular velocity, linear velocity, angular acceleration and centripetal acceleration of the particle at any instant respectively are $\vec{\omega}, \vec{v}, \vec{\alpha}$ are $\vec{a}_c$. Which of the following relation is/are correct -

(a) $\vec{\omega} \perp \vec{v}$ (b) $\vec{\omega} \perp \vec{\alpha}$
(c) $\vec{v} \perp \vec{a}_c$ (d) $\vec{\omega} \perp \vec{a}_c$
(1) a, b, d (2) b, c, d
(3) a, b, c (4) a, c, d

Q.58 A small mass of 10 gm. Lies in a hemispherical bowl of radius 0.5 m at a height of 0.2 m from the bottom of the bowl. The mass will be in equilibrium of the bowl rotates at an angular speed of - (g = 10 m/sec^2)



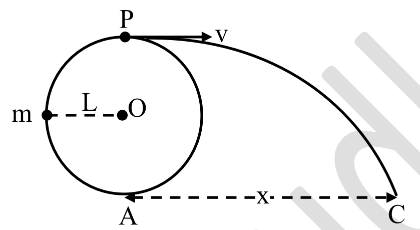
(1) $\frac{10}{\sqrt{3}}$ rad/s (2) $10\sqrt{3}$ rad/s
(3) 10 rad/s (4) $\sqrt{20}$ rad/s

Q.59 A body is revolving with a constant speed along a circle. If its direction of motion is reversed but the speed remains the same then -

(a) the centripetal force will not suffer any change in magnitude
(b) the centripetal force will have its direction reversed
(c) the centripetal force will not suffer any change in direction
(d) the centripetal force is doubled

(1) a, b (2) b, c
(3) c, d (4) a, c

Q.60 A body tied to a string of length L is revolved in a vertical circle with minimum velocity, when the body reaches the upper most point the string breaks and the body moves under the influence of the gravitational field of earth along a parabolic path. The horizontal range AC of the body will be -



(1) x = L (2) x = 2L
(3) $x = 2\sqrt{2L}$ (4) $x = \sqrt{2L}$

Q.61 A particle is moving in a vertical circle the tension in the string when passing through two position at angle 30º and 60º from vertical from lowest position are T_1 and T_2 respectively then -

(1) $T_1 = T_2$ (2) $T_1 > T_2$
(3) $T_1 < T_2$ (4) $T_1 \geq T_2$

Q.62 A body crosses the topmost point of a vertical circle with critical speed. What will be its centripetal acceleration when the string is horizontal -

(1) g (2) 2g (3) 3g (4) 6g

Q.63 A particle (P) is moving in a circle of radius (a) with a uniform speed (v). C is the centre of the circle and AB is a diameter. The angular velocity of particle when it is at point B about (A) and (C) are in the ratio -

(1) 1 : 1 (2) 1 : 2 (3) 2 : 1 (4) 4 : 1

Q.64 A small particle of mass 'm' starts sliding down from the top of a hemispherical bowl of radius 'r'. The particle and the surface of the hemisphere are frictionless. The height from the ground at which the particle leaves contact with the sphere is -

(1) 2r/3 (2)3r/2 (3) r/2 (4) r/3

Q.65 If a_r and a_t represent radial and tangential acceleration, the motion of a particle will be uniform circular motion if -

(1) $a_r = 0$ and $a_t = 0$ (2) $a_r = 0$ but $a_t \neq 0$
(3) $a_r \neq 0$ but $a_t = 0$ (4) $a_r \neq 0$ and $a_t \neq 0$

Q.66 A 1500 kg car moving on a flat road negotiates a curve whose radius is 35 m. If the coefficient of static friction between the tyres and the dry pavement is 0.5. Find the maximum speed, the car can have in order to makes the turn successfully -

(1) 13.1 m/s (2) 15.1 m/s (3) 20 m/s (4) 25 m/s

Q.67 Stone tied at one end of light string is whirled round a vertical circle. If the difference between the maximum and minimum tension experienced by the string wire is 2 kg, then the mass of the stone must be -

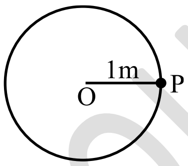
(1) 1 kg (2) 6 kg (3) 1/3 kg (4) 2 kg

Q.68 A body holds a pendulum in his hand while standing at the edge of a circular platform of radius r rotating at an angular speed ω. The pendulum will hang at an angle θ with the vertical such that -

(1) $\tan\theta = 0$ (2) $\tan\theta = \frac{\omega^2 r^2}{g}$

(3) $\tan\theta = \frac{r\omega^2}{g}$ (4) $\tan\theta = \frac{g}{\omega^2 r}$

Q.69 A mass tied to a string moves in a vertical circle with a uniform speed of 5 m/s as shown. At the point P the string breaks. The mass will reach a height above P of nearly -



(1) 1 m (2) 0.5 m (3) 1.27 m (4) 1.25 m

Q.70 A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle The motion of the particle takes place in a plane. It follows -

(1) its velocity is constant

(2) its K.E. is constant

(3) its acceleration is constant

(4) its moves in a straight line

Q.71 If the overbridge is concave instead of being convex, the thrust on the road at the lowest position will be -

(1) $mg + \frac{mv^2}{r}$ (2) $mg - \frac{mv^2}{r}$

(3) $\frac{m^2v^2g}{r}$ (4) $\frac{v^2g}{r}$

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 2

Q.1 A particle rests on the top of a hemisphere of radius R. The minimum linear horizontal velocity for the particle to leave the vertex without slipping will be-

(1) $v = \sqrt{2gR}$ (2) $v = \sqrt{\frac{Rg}{2}}$

(3) $v = \sqrt{gR}$ (4) $v = \sqrt{\frac{2g}{R}}$

Q.2 A particle of mass m is tied to a string of length ℓ and whirled into a horizontal plane. If tension in the string is T then the circular speed of the particle will be -

(1) $\sqrt{\frac{T\ell}{m}}$ (2) $\sqrt{\frac{2T\ell}{m}}$

(3) $\sqrt{\frac{3T\ell}{m}}$ (4) $\sqrt{\frac{T}{m\ell}}$

Q.3 A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is them rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other ends is -

(1) $\frac{ML\omega^2}{2}$ (2) $\frac{ML^2\omega}{2}$

(3) $ML\omega^2$ (4) $\frac{ML^2\omega^2}{2}$

Q.4 A motor cyclist moving with a velocity of 72 km/hr on a flat road takes a turn on the road at a point where the radius of curvature of the road is 20 meters. The acceleration due to gravity is 10 m/s^2. In order to avoid skidding, he must not bend with respect to the vertical plane by an angle greater than -

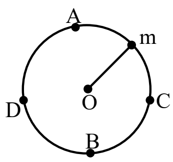
(1) $\theta = \tan^{-1}6$

(2) $\theta = \tan^{-1}2$

(3) $\theta = \tan^{-1} 25.92$

(4) $\theta = \tan^{-1}4$

Q.5 A mass is performing vertical circular motion (see figure). If the average velocity of the particle is increased, then at which point maximum breaking possibility of the string :



(1) A (2) B (3) C (4) D

Q.6 Two particle having mass 'M' and 'm' are moving in a circular path having radius R and r. If their time period are same then the ratio of angular velocity will be -

(1) $\frac{r}{R}$ (2) $\frac{R}{r}$

(3) 1 (4) $\sqrt{\frac{R}{r}}$

Q.7 In a circus stuntman rides a motorbike in a circular track of radius R in the vertical plane. The minimum speed at highest point of track will be -

(1) $\sqrt{2gR}$ (2) 2gR

(3) $\sqrt{3gR}$ (4) $\sqrt{gR}$

Q.8 If the equation for the displacement of a particle moving on a circular path is given by $(\theta) = 2t^3 + 0.5$, where θ is in radians and t in second, then the angular velocity of the particle after 2s from its start is -

(1) 8 rad/s (2) 12 rad/s

(3) 24 rad/s (4) 36 rad/s

Q.9 A particle moves along a circle of radius $(20/\pi)$m with constant tangential acceleration. If the velocity of the particle is 80 m/s at the end of the second revolution after motion has begun, the tangential acceleration is -

(1) 40 m/s^2 (2) 640 π m/s^2

(3) 160 π m/s^2 (4) 40 π m/s^2

Q.10 A stone is tied to a string of length 'ℓ' and is whirled in a vertical circle with the other end of the sting as the centre. At a certain instant of time, the stone is at its lowest position and has a speed 'u'. The magnitude of the change in velocity as it reached a position where the string is horizontal (g being acceleration due to gravity) is -

(1) $\sqrt{u^2 - g\ell}$ (2) $u - \sqrt{u^2 - 2g\ell}$

(3) $\sqrt{2g\ell}$ (4) $\sqrt{2(u^2 - g\ell)}$

Q.11 A stone tied to the end of a string of 1m long is whirled in a horizontal circle with a constant speed. If the stone makes 22 revolution in 44 second, what it the magnitude and direction of acceleration of the stone -

(1) $\pi^2 ms^{-2}$ and direction along the tangent to the circle

(2) $\pi^2 ms^{-2}$ and direction along the radius towards the centre

(3) $\pi^2/4$ ms^{-2} and direction along the radius towards the centre

(4) $\pi^2 ms^{-2}$ and direction along the radius away from the centre

Q.12 A car runs at a constant speed on a circular track of radius 100 m, taking 62.8 seconds for every circular lap. The average velocity and average speed for each circular lap respectively is -

(1) 0, 0 (2) 0, 10 m/s

(3) 10 m/s, 10 m/s (4) 10 m/s, 0

Q.13 A particle moves in a circle of radius 5 cm with constant speed and time period 0.2 π s. The acceleration of the particle is :

(1) 5 m/s^2 (2) 15 m/s^2

(3) 25 m/s^2 (4) 36 m/s^2

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 3

Q.1 If the radius of a circular path of a body revolving at a constant speed is doubled, the value of the centripetal force will be -
(1) halved (2) doubled
(3) four time (4) unchanged

Q.2 A bucket full of water is tied to a string of one meter length and is rotated in a vertical circle. What is the minimum speed at the highest point so that water in it does not spill ?
(1) $\sqrt{g}$ (2) $\sqrt{2g}$ (3) $\frac{\sqrt{g}}{2}$ (4) $\sqrt{5g}$

Q.3 Keeping the banking angle of the road constant, the maximum safe speed of passing vehicles is to be increased by 10%. The radius of curvature of the road will have to be changed from 20 m. to -
(1) 16 m (2) 18 m
(3) 24.20 m (4) 30.5 m

Q.4 A neutron star of enormous density is rotating at the rate of one rotation per second . If the radius of the star is 20 k, then the acceleration in m/s^2 units for any particle situated at the equator of the star will be - ($\pi^2 \approx 10$)
(1) 8×10^5 (2) 20×10^3
(3) 12×10^6 (4) 4×10^8

Q.5 A sphere of mass 10 kg is tied to a rope 4 m long. What will be the tension in the rope when the sphere is rotated along a circle in a horizontal plane with a velocity of 5 m/s (apporox.) -
(1) 10 kg wt (2) 14 kg wt
(3) 6 kg wt (4) 12 kg wt

Q.6 A body of mass 100 g is tied to one end of a 2m long string. The other end of the string is at the centre of the horizontal circle. The maximum revolution in one minute is 200. The maximum tensile strength of the string is approx -
(1) 8.76 N (2) 8.9 N
(3) 87 dyne (4) 87.64 N

Q.7 A body of mass 1 kg tied to one end of string is revolved in a horizontal circle of radius 0.1 m with a speed of 3 revolution/sec, assuming the effect of gravity is negligible, then linear velocity, acceleration and tension in the string will be -
(1) 1.88 m/s, 35.5 m/s^2, 35.5 N
(2) 2.88 m/s, 45.5 m/s^2, 45.5 N
(3) 3.88 m/s, 55.5 m/s^2, 55.5 N
(4) None of these

Q.8 A car moving on a horizontal road may be thrown out of the road in taking a turn -
(1) by the gravitational force
(2) due to lack of proper centripetal force
(3) due to rolling friction between the tyres and the road
(4) due to reaction of road

Q.9 A cyclist taking bends inwards while a car passenger taking same turn is thrown outwards. The reason is -
(1) car is heavier than cycle
(2) car has four wheels while cycle has only two
(3) difference in the speed of the two
(4) cyclist has to counteract the centrifugal force while in the case of car only the passenger is thrown by this force

Q.10 A particle of mass m is executing a uniform motion along a circular path of radius r. If the magnitude of its linear momentum is P, the radial force acting on the particle will be -
(1) pmr (2) rm/p
(3) mp^2/r (4) p^2/rm

Q.11 Sometimes a car overturns while turning at a curve. In such a case -
(1) the inner wheels leave the ground first
(2) the outer wheels leave the ground first
(3) both the inner and the outer wheels leave the ground simultaneously
(4) either of the wheels can leave the ground first

Q.12 The critical speed of a body at the highest point to complete a circle in a vertical plane should be-
(1) $\sqrt{rg}$ (2) $\sqrt{2rg}$ (3) $\sqrt{5rg}$ (4) $\sqrt{4rg}$

Q.13 Radius of the curved road on national highway is R. Width of the road is b. The outer edge of the road is raised by h with respect to inner edge so that a car with velocity v can pass safely over it. The value of h is -
(1) $\frac{v^2b}{Rg}$ (2) $\frac{v}{Rgb}$ (3) $\frac{v^2R}{bg}$ (4) $\frac{v^2b}{R}$

Q.14 The maximum speed of a car on a road turn of radius 30 m, if the coefficient of friction between the tyres and the road is 0.4, will be -
(1) 9.84 m/s (2) 10.84 m/s
(3) 7.84 m/s (4) 5.84 m/s

Q.15 A particle of mass (m) revolving in horizontal circle of radius (R) with uniform speed v. When particle goes from one end to other end of diameter than -
(1) K.E. change by $1/2\ mv^2$
(2) K.E. change by mv^2
(3) no change in momentum
(4) change in momentum is 2 mv

Q.16 Cream gets separated out of mild when it is churned, it is due to -
(1) gravitational force (2) centripetal force
(3) centrifugal force (4) frictional force

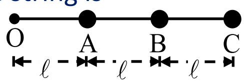
Q.17 A car when passes through a convex bridge exerts a force on it which is equal to -
(1) $Mg + \frac{Mv^2}{r}$ (2) $\frac{Mv^2}{r}$
(3) Mg (4) None of these

Q.18 The angular velocity of a wheel is 70 rad/s. If the radius of the wheel is 0.5m, then linear velocity of the wheel is -
(1) 70 m/s (2) 35 m/s
(3) 30 m/s (4) 20 m/s

Q.19 A cyclist on the ground goes round a circular path of circumference 34.3 m in $\sqrt{22}$ second. The angle made by him, with the vertical, will be -
(1) 45º (2) 40º
(3) 42º (4) 48º

Q.20 A motor cycle driver doubles its velocity when he is having a turn. The force exerted outwardly will be -
(1) double (2) half
(3) 4 times (4) 1/4 times

Q.21 There identical particles are joined together by a thread as shown in figure. All the three particle are moving in a horizontal plane. If the velocity of the outermost particle is v_0, then the ratio of tensions in the three section of the string is -



(1) 3 : 5 : 7 (2) 3 : 4 : 5
(3) 7 : 11 : 6 (4) 3 : 5 : 6

Q.22 A car is moving on a circular path and takes a turn. If R_1 and R_2 be the reactions on the inner and outer wheels respectively then -
(1) $R_1 < R_2$ (2) $R_1 = R_2$
(3) $R_1 \geq R_2$ (4) $R_1 > R_2$

Q.23 A motorcycle is travelling on a curved track of radius 500 m if the coefficient of friction between road and tyres is 0.5. The speed avoiding skidding, will be -
(1) 50 m/s (2) 75 m/s
(3) 25 m/s (4) 35 m/s

Q.24 A 500 kg car takes a round turn of radius 50 m with a velocity of 36 km/hr. The centripetal force is
(1) 250 N (2) 1000 N
(3) 750 N (4) 1200 N

Q.25 A stone of mass 0.2 kg is tied to one end of a thread of length 0.1 m whirled in a vertical circle. When the stone is at the lowest point of circle, tension in thread is 52 N, then velocity of the stone will be -
(1) 4 m/s (2) 5 m/s
(3) 6 m/s (4) 7 m/s

Q.26 A stone is attached to the end of a string and whirled in horizontal circle, then -
(1) its linear and angular momentum are constant
(2) only linear momentum is constant
(3) its angular momentum is constant but linear momentum is variable
(4) both are variable

Q.27 Angular velocity of minute hand of a clock is -
(1) $\frac{\pi}{30}$ rad/s
(2) 8π rad/s
(3) $\frac{2\pi}{1800}$ rad/s
(4) $\frac{\pi}{1800}$ rad/s

Q.28 A car moving with speed 30 m/s on a circular path of radius 500 m. Its speed is increasing at the rate of $2 m/s^2$. The acceleration of the car is-
(1) $9.8 m/s^2$ (2) $1.8 m/s^2$
(3) $2 m/s^2$ (4) $2.7 m/s^2$

Q.29 A hollow sphere has radius 6.4 m. Minimum velocity required by a motor cyclist at bottom to complete the circle will be -
(1) 17.7 m/s (2)10.2 m/s
(3) 12.4 m/s (4) 16.0 m/s

Q.30 If a particle is rotating in a circle, what will happen ?
(1) no force is acting on particle
(2) velocity of particle is constant
(3) particle has no acceleration
(4) no work is done

Q.31 For a particle in a non-uniform accelerated circular motion -
(1) velocity is radial and acceleration is transverse only
(2) velocity is transverse and acceleration is radial only
(3) velocity is radial and acceleration has both radial and transverse components
(4) velocity is transverse and acceleration has both radial and transverse components

Q.32 A mass m is attached to the end of a rod of length ℓ. The mass goes around a vertical circular path with the other end hinged at the centre. What should be the minimum velocity of mass at the bottom of the circle so that the mass completes the circle ?
(1) $\sqrt{4g\ell}$ (2) $\sqrt{3g\ell}$
(3) $\sqrt{5g\ell}$ (4) $\sqrt{g\ell}$

Q.33 A fighter plane is moving in a vertical circle of radius 'r'. Its minimum velocity at the highest point of the circle will be -
(1) $\sqrt{3gr}$ (2) $\sqrt{2gr}$
(3) $\sqrt{gr}$ (4) $\sqrt{\frac{gr}{2}}$

Q.34 When a ceiling fan is switched off its angular velocity reduced to 50% while it makes 36 rotations. How many more rotation will it make before coming to rest (Assume uniform angular retardation)
(1) 18 (2) 12
(3) 36 (4) 48

Q.35 A particle is kept at rest at the top of a sphere of diameter 42 m. When disturbed slightly, it slides down. At what height 'h' from the bottom, the particle will leave the sphere -
(1) 14 m (2) 28 m
(3) 35 m (4) 7m

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 4

These questions of two statements each, printed as Assertion and Reason. While answering these Questions you are required to choose any one of the following four responses.
(A) If both Assertion & Reason are true & the Reason is a correct explanation of the Assertion.
(B) If both Assertion and Reason are true but Reason is not a correct explanation of the Assertion.
(C) If Assertion is true but the Reason is false.
(D) If both Assertion & Reason are false.

Q.1 **Assertion :** In a uniform circular motion, the linear speed and angular speed of the body are constant.
Reason : A body can move on a circular path without having acceleration.
(1) A (2) B (3) C (4) D

Q.2 **Assertion :** The resultant acceleration of an object in circular motion is towards the centre if the speed is constant.
Reason : A vector is necessarily changed if it is rotated through an angle.
(1) A (2) B (3) C (4) D

Q.3 **Assertion :** Work done by the centripetal force in moving a body along a circle is always zero.
Reason : In circular motion the displacement of the body is along the force.
(1) A (2) B (3) C (4) D

Q.4 **Assertion :** Centripetal and centrifugal forces cancel each other.
Reason : This is because they are always equal and opposite.
(1) A (2) B (3) C (4) D

Q.5 **Assertion :** A cyclist bends inwards from his vertical position, while turning to secure the necessary centripetal force.
Reason : Friction between the tyres and road provides him the necessary centripetal force.
(1) A (2) B (3) C (4) D

Q.6 **Assertion :** Work done by centripetal force is zero.
Reason : Centripetal force acts perpendicular to the displacement.
(1) A (2) B (3) C (4) D

Q.7 **Assertion :** When a particle moves in a circle with a uniform speed its acceleration is constant but the velocity changes.
Reason : Angular displacement is not a axial vector.
(1) A (2) B (3) C (4) D

Q.8 **Assertion :** The total displacement moved by a point located on the periphery of a wheel of radius R in one revolution is $2\pi R$. Wheel is rolling.
Reason : In rolling motion of a wheel, every point on its periphery comes in contact with the surface once in one revolution.
(1) A (2) B (3) C (4) D

Q.9 **Assertion :** The tendency of skidding or overturning is quadrupled, when a cyclist doubles his speed of turning.
Reason : Angle of bending measured from ground., decreases as velocity of vehicle increases.
(1) A (2) B (3) C (4) D

Q.10 **Assertion :** A particle moves along circular path with constant speed then acceleration must be present.
Reason : A particle moves with variable velocity then acceleration must be present.
(1) A (2) B (3) C (4) D

Q.11 **Assertion :** In uniform circular motion speed of particle must be constant.
Reason : In uniform circular motion no force or acceleration is acting on particle acting parallel or in anti parallel to the direction of velocity.
(1) A (2) B (3) C (4) D

Q.12 **Assertion :** When the direction of motion of a particle moving in a circular path is reversed the direction of radial acceleration still remains the same (at the given point).
Reason : Particle revolves on circular path in any direction such as clockwise or anticlockwise the direction of radial acceleration is always towards the centre of the circular path.
(1) A (2) B (3) C (4) D

Q.13 **Assertion :** For uniform circular motion it is necessary the speed of the particle is constant.
Reason : There is no tangential force or tangential acceleration acting on particle in uniform circular motion.
(1) A (2) B (3) C (4) D

Q.14 **Assertion :** A car moving on a horizontal road may be thrown out of the road in taking a turn due to lack of proper centripetal force.
Reason : If a particle moves in a circle describing equal angles in equal intervals of time. Then the velocity vector changes its magnitude.
(1) A (2) B (3) C (4) D

Q.15 **Assertion :** Acceleration of the particle in uniform circular motion remain constant.
Reason : Velocity of the particle doesn't change in circular motion.
(1) A (2) B (3) C (4) D

Q.16 **Assertion :** A cyclist always bends inwards while negotiating a curve.
Reason : By bending, cyclist lowers his centre of gravity.
(1) A (2) B (3) C (4) D

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 1 (ANSWERS)

Ques.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	3	2	4	2	3	3	2	3	3	3	3	1	1	1	2	2	2	1	3	3
Ques.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans.	2	3	1	2	4	2	3	2	2	1	3	3	3	2	2	1	2	1	1	3
Ques.	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Ans.	2	3	2	3	4	3	1	1	1	1	4	2	3	4	3	1	4	1	4	2
Ques.	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**									
Ans.	2	3	2	1	3	1	3	3	4	2	1									

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 2 (ANSWERS)

Ques.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
Ans.	3	1	1	2	2	3	4	3	1	4	2	2	1

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 3 (ANSWERS)

Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	1	1	3	1	3	4	1	2	4	4	1	1	1	2	4	3	4	2	1	3
Q.No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**					
Ans.	4	1	1	2	2	3	4	4	1	4	4	3	3	2	3					

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 4 (ANSWERS)

Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Ans.	3	3	3	4	3	1	4	1	1	1	1	1	1	3	4	3