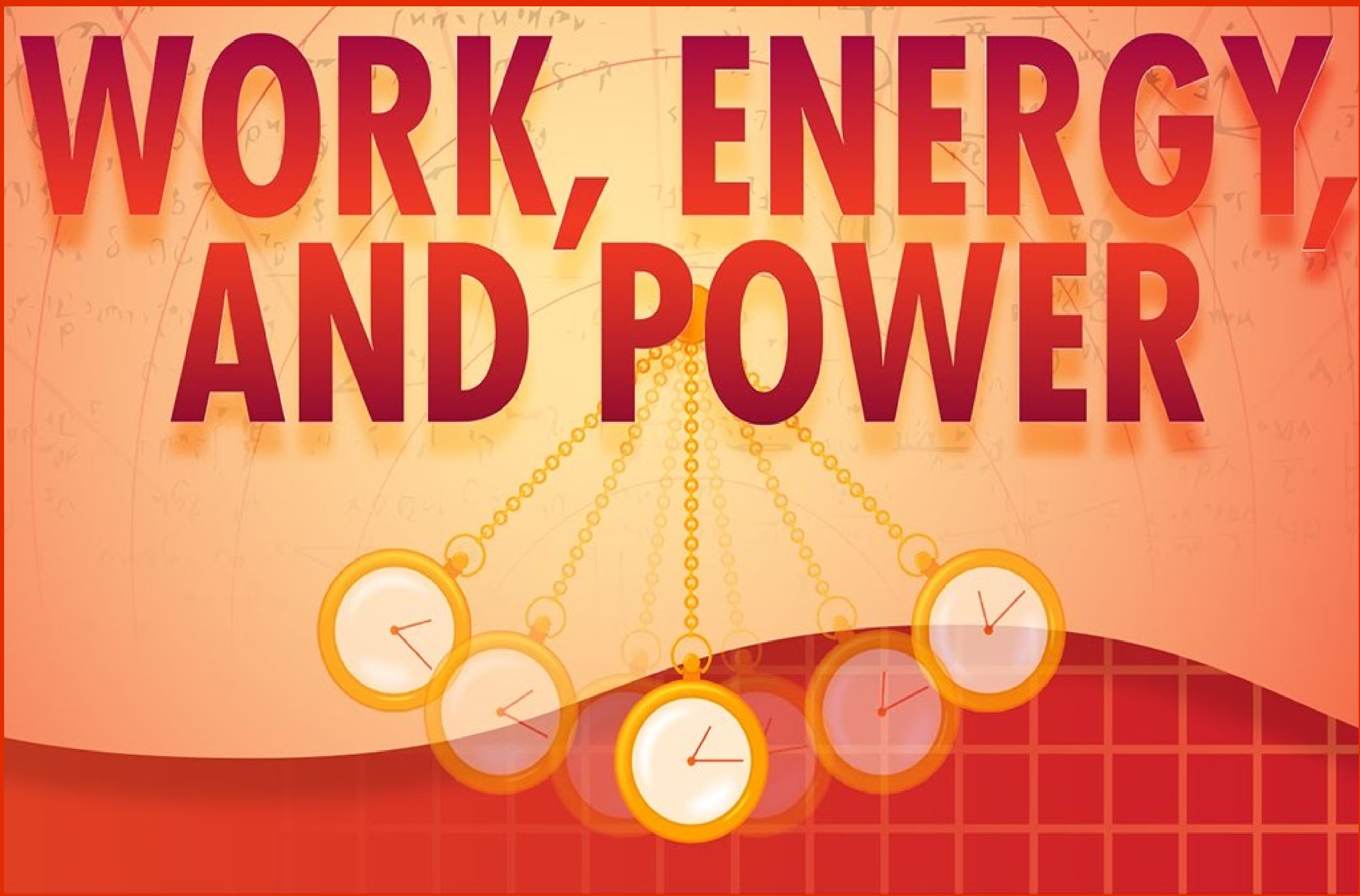
PHYSICS

for JEE, NEET & BOARD

11th



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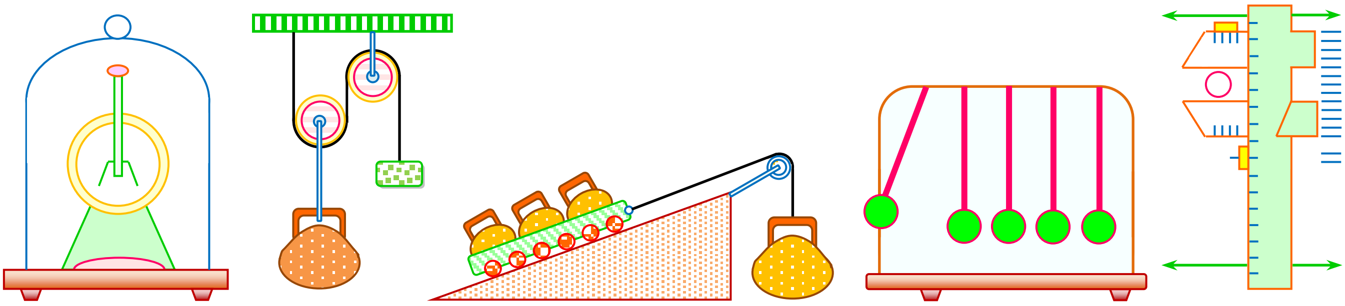


PHYSICS BOOKLET FOR JEE NEET & BOARDS

India's First Colour Smart Book

WORK, ENERGY AND POWER

1 WORK

Almost all the terms we have used so far, velocity, acceleration, force etc. have same meaning in Physics as they have in our everyday life. We, however, encounter a term 'work' whose meaning in Physics is distinctly different from its everyday meaning. e.g. consider a person holding a 100 kg weight on his head moves through a distance of 100 m. In everyday language we will say he is doing work. But according to definition of work in Physics, you will say the work done by the man is zero.

1.1 WORK DONE BY A CONSTANT FORCE

Consider an object undergoes a displacement S along a straight line while acted on a force F that makes an angle θ with S as shown.

The work done W by the agent is the product of the component of force in the direction of displacement and the magnitude of displacement.

i.e., $W = (F\cos\theta)S$ … (1)

$= F\,S\cos\theta$

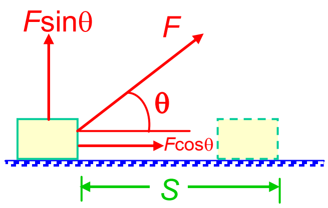


Fig. 1

Work done is a scalar quantity and its S.I. unit is N-m or joule (J). We can also write work done as a scalar product of force and displacement.

$$W = \vec{F}.\vec{S}. \quad \text{… (2)}$$

From this definition, we conclude the following points:

(i) force does no work if point of application of force does not move ($S = 0$).

(ii) work done by a force is zero if displacement is perpendicular to the force ($\theta = 90°$).

(iii) if angle between force and displacement is acute ($\theta < 90°$), we say that work done by the force is positive.

(iv) if angle between force and displacement is obtuse ($\theta > 90°$), we say that work done by the force is negative.

Illustration 1

Question: **A person slowly lifts a block of mass m = 10kg through a vertical height h =1m and then walks horizontally a distance d = 2m while holding the block. Determine work done by the person. (g = 10 m/s^2)**

Solution: The man slowly lifts the block, therefore he must be applying a force equal to the weight of the block, mg, the work done during vertical displacement is mgh, since the force is in the direction of displacement. The work done by the person during the horizontal displacement of the block is zero. Since the applied force during this process is perpendicular to displacement. Therefore total work done by the man is mgh = **100 J**

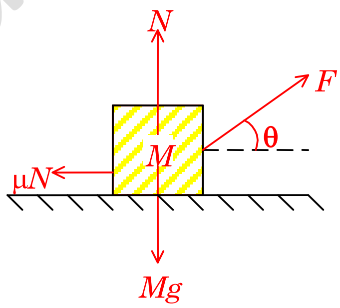
Illustration 2

Question: **A block of mass M = 11 kg is pulled along a horizontal surface by applying a force at an angle $\theta = 45^0$ with horizontal. Coefficient of friction between block and surface is μ =0.1 If the block travels with uniform velocity, find the work done by this applied force during a displacement d= 1m of the block.**

Solution: The forces acting on the block are shown in Figure. As the block moves with uniform velocity the forces add up to zero.

$\therefore F\cos\theta = \mu N$ ...(i)

$F\sin\theta + N = Mg$ ...(ii)



Eliminating N from equations (i) and (ii),

$F\cos\theta = \mu(Mg - F\sin\theta)$

$$F = \frac{\mu Mg}{\cos\theta + \mu\sin\theta}$$

Work done by this force during a displacement d

$$W = F \cdot d\cos\theta = \frac{\mu Mgd\cos\theta}{\cos\theta + \mu\sin\theta} = \mathbf{10\ J}$$

Illustration 3

Question: **A particle moving in the xy plane undergoes a displacement $\vec{s}$ = (8.0$\hat{i}$ + 6.0$\hat{j}$) m while a constant force F = (4.0$\hat{i}$ + 3.0$\hat{j}$) N acts on the particle.**

(a) Calculate the magnitude of the displacement and that of the force.

(b) Calculate the work done by *the force*.

Solution:

(a) $$s = \sqrt{x^2 + y^2} = \sqrt{(8.0)^2 + (6.0)^2} = \mathbf{10\ m}$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(4.0)^2 + (3.0)^2} = \mathbf{5\ N}$$

(b) Work done by force, $W = \vec{F} \cdot \vec{s}$

$= (4.0\hat{i} + 3.0\hat{j}) \cdot (8.0\hat{i}) + 6.0\hat{j})$ N.m

$= 32 + 0 + 0 + 18 = 50$ N.m = **50J**

1.2 WORK DONE BY A VARYING FORCE

Consider a particle being displaced along the x-axis under the action of a varying force, as shown in the figure. The particle is displaced in the direction of increasing x from $x = x_i$ to $x = x_f$. In such a situation, we cannot use $W = (F\cos\theta)s$ to calculate the work done by the force because this relationship applies only when F is constant in magnitude and direction. However, if we imagine that the particle undergoes a very small displacement Δx, shown in the figure 2(a), then the x component of the force, F_x, is approximately constant over this interval, and we can express the work done by the force for this small displacement as

$$W_1 = F_x\,\Delta x$$

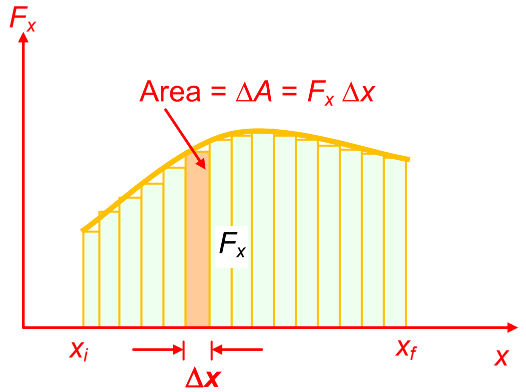


Fig. 2 (a)

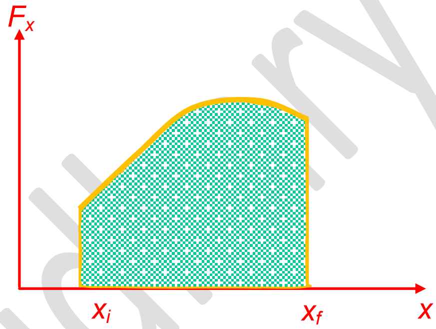


Fig. 2 (b)

This is just the area of the shaded rectangle in the figure 2(a). If we imagine that the F_x versus x curve is divided into a large number of such intervals, then the total work done for the displacement from x_i to x_f is approximately equal to the sum of a large number of such terms.

$$W \cong \sum_{x_i}^{x_f} F_x \Delta x$$

If the displacements are allowed to approach zero, then the number of terms in the sum increases without limit but the value of the sum approaches a definite value equal to the area under the curve bounded by F_x and the x axis.

$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x\, dx$$

This definite integral is numerically equal to the area under the F_x versus x curve between x_i and x_f. Therefore, we can express the work done by F_x for the displacement of the object from x_i to x_f as

$$W = \int_{x_i}^{x_f} F_x\, dx \qquad \dots (3)$$

This equation reduces to equation (1) when $F_x = F\cos\theta$ is constant.

If more than one force acts on a particle, the total work done is just the work done by the resultant force. For systems that do not act as particles, work must be found for each force separately. If we express the resultant force in the x-direction as $\sum F_x$, then the net work done as the particle moves from x_i to x_f is

$$W_{net} = \int_{x_i}^{x_f} \left(\sum F_x\right) dx \qquad \dots (4)$$

Illustration 4

Question: **A force $F = (4.0\,x\,\hat{i} + 3.0y\hat{j})$ N acts on a particle which moves in the x-direction from the origin to $x = 5.0$ m. Find the work done on the object by the force.**

Solution: Here the work done is only due to x component of force because displacement is along x-axis.

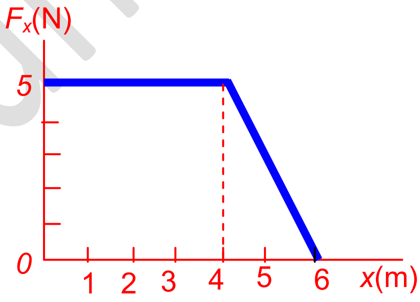
i.e., $W = \int_{x_1}^{x_2} F_x dx$

$$= \int_0^5 4x\,dx = \left[2x^2\right]_0^5 = \mathbf{50\ J}$$

Illustration 5

Question: **Force acting on a particle varies with x as shown in figure. Calculate the work done by the force as the particle moves from $x = 0$ to $x = 6.0$ m.**

Solution: The work done by the force is equal to the area under the curve from $x = 0$ to $x = 6.0$ m. This area is equal to the area of the rectangular section from $x = 0$ to $x = 4.0$ m plus the area of the triangular section from $x = 4.0$ m to $x = 6.0$ m. The area of the rectangle is (4.0) (5.0) N.m = 20J, and the area of the triangle is $\frac{1}{2}$(2.0) (5.0) N.m = 5.0 J. Therefore, the total work done is **25J.**



1.3 WORK DONE BY A SPRING

A common physical system for which the force varies with position is a spring-block system as shown in the figure 3. If the spring is stretched or compressed by a small distance from its unstreached configuration, the spring will exert a force on the block given by

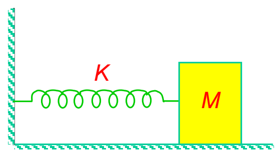


Fig. 3

$F = -kx$, where x is compression or elongation in spring. k is a constant called spring constant whose value depends inversely on un-stretched length and the nature of material of spring.

Important: The negative sign indicates that the direction of the spring force is opposite to x, the displacement of the free end.

Consider a spring block system as shown in the figure 3 and let us calculate work done by spring when the block is displaced by x_0.

At any moment if elongation in spring is x, then force on block by the spring is kx towards left. Therefore, work done by the spring when block further displaces by dx towards right.

$dW = -kxdx$ [Negative sign shows force is opposite to displacement]

$\therefore$ Total work done by the spring, $W = -\int_0^{x_0} kx\ dx$

$$= -\frac{1}{2}kx_0^2$$

Similarly, work done by the spring when it is given a compression x_0 is $-\frac{1}{2}kx_0^2$. We can also say that work done by an agent in giving an elongation or compression of x_0 is $\frac{1}{2}kx_0^2$.

2 POWER

From a practical viewpoint, it is interesting to know not only the work done on an object but also the rate at which the work is being done. ***The time rate of doing work is called power***.

If an external force is applied to an object (which we assume as a particle), and if the work done by this force is ΔW in the time interval Δt, then the average power during this interval is defined as

$$\overline{P} = \frac{\Delta W}{\Delta t} \qquad \ldots (5)$$

The work done on the object contributes to increasing the energy of the object. A more general definition of power is the time rate of energy transfer. The instantaneous power is the limiting value of the average power as Δt approaches zero.

i.e., $$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} \qquad \ldots (6)$$

where we have represented the infinitesimal value of the work done by dW (even though it is not a change and therefore not a differential). We find from equation (2) that $dW = \vec{F}.d\vec{s}$. Therefore, the instantaneous power can be written as

$$P = \frac{dW}{dt} = \vec{F}.\frac{d\vec{s}}{dt} = \vec{F}.\vec{v} \qquad \ldots (7)$$

where we have used the fact that $\vec{v} = \frac{d\vec{s}}{dt}$.

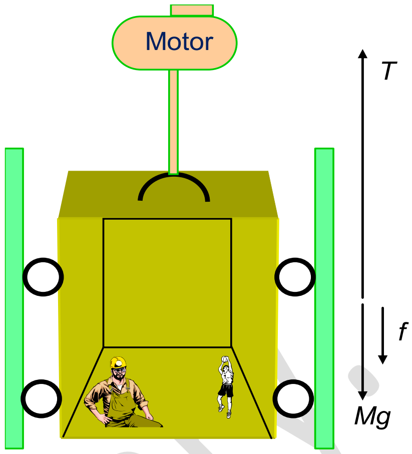
The SI unit of power is 'joule per second (J/s), also called watt (W)' (after James Watt);

1W = 1J/s = 1 kg.m^2/s^3.

Illustration 6

Question: **An elevator has a mass of 1000 kg and carries a maximum load of 800 kg. A constant frictional force of 4000 N retards its upward motion, as shown in the figure.**

What must be the minimum power delivered by the motor to lift the elevator at a constant speed of 3.00 m/s?



Solution:

The motor must supply the force T that pulls the elevator upward. From Newton's second law and from the fact that $a = 0$ since v is constant, we get

$T - f - Mg = 0$, Where M is the total mass (elevator plus load), equal to 1800 kg.

$\Rightarrow T = f + Mg$

$= 4.00 \times 10^3 \text{N} + (1.80 \times 10^3 \text{ kg}) (10 \text{ m/s}^2) = \mathbf{22 \times 10^3 \text{ N}}$

Using equation (7) and the fact that T is in the same direction as v, we have gives

$P = Tv$

$= (22 \times 10^3 \text{ N}) (3.00 \text{ m/s}) = 66 \times 10^3 \text{ W}$

$= \mathbf{66 \text{ kW}}$

3 ENERGY

A body is said to possess energy if it has the capacity to do work. When a body possessing energy does some work, part of its energy is used up. Conversely if some work is done upon an object, the object will be given some energy. Energy and work are mutually convertible.

There are various forms of energy. Heat, electricity, light, sound and chemical energy are all familiar forms. In studying mechanics, we are however concerned chiefly with mechanical energy. This type of energy is a property of movement or position.

3.1 KINETIC ENERGY

Kinetic energy (K.E.), is the capacity of a body to do work by virtue of its motion.

If a body of mass m has a velocity v its kinetic energy is equivalent to the work, which an external force would have to do to bring the body from rest up to its velocity v.

The numerical value of the kinetic energy can be calculated from the formula

$$\text{K.E.} = \frac{1}{2} mv^2 \qquad \ldots \textbf{(8)}$$

This formula can be derived as follows.

Consider a constant force F which, acting on a mass m initially at rest, gives the mass a velocity v. If, in reaching this velocity, the particle has been moving with an acceleration a and has been given a displacement s, then

$F = ma$ (Newton's Law)

$v^2 = 2as$ (Motion of a particle moving with uniform acceleration)

Fs = Work done by the constant force

Combining these relationships, we have

$$\text{Work done} = ma\left(\frac{v^2}{2a}\right) = \frac{1}{2}mv^2$$

But the K.E. of the body is equivalent to the work done in giving the body is velocity.

$$\text{Hence, K.E.} = \frac{1}{2}mv^2$$

☞ ***Since both m and v^2 are always positive, K.E. is always positive and does not depend upon the direction of motion of the body.***

3.2 POTENTIAL ENERGY

Potential energy is energy due to position. If a body is in a position such that if it were released it would begin to move, it has potential energy.

There are two common forms of potential energy, gravitational and elastic.

3.2 (a) Gravitational Potential Energy:-

it is possessed by virtue of height

When an object is allowed to fall from one level to a lower level it gains speed due to gravitational pull, i.e., it gains kinetic energy. Therefore, in possessing height, a body has the ability to convert its gravitational potential energy into kinetic energy.

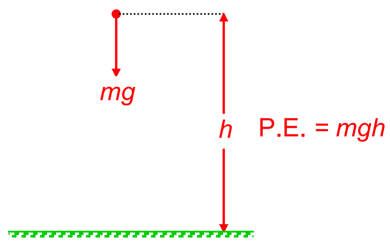


Fig. 4 (a)

The gravitational potential energy is equivalent to the negative of the amount of work done by the weight of the body in causing the descent.

If a mass m is at a height h above a lower level the P.E. possessed by the mass is (mg) (h).

Since h is the height of an object above a specified level, an object below the specified level has negative potential energy.

Therefore $\text{GPE} = \pm\, mgh$ .... **(9)**

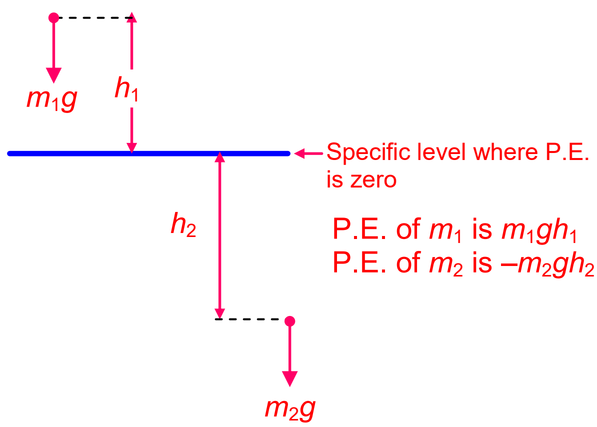


Fig. 4 (b)

☞ *The chosen level from which height is measured has no absolute position. It is important therefore to indicate clearly the zero P.E. level in any problem in which P.E. is to be calculated.*

☞ *GPE = $\pm$ mgh is applicable only when h is very small in comparison to the radius of the earth. We have discussed GPE in detail in 'GRAVITATION'*

3.2 (b) Elastic Potential Energy, It is a property of stretched or compressed springs.

The end of a stretched elastic spring will begin to move if it is released. The spring therefore possesses potential energy due to its elasticity. (i.e., due to change in its configuration)

The amount of elastic potential energy stored in a spring of natural length a and spring constant k when it is extended by a length x is equivalent to the amount of work necessary to produce the extension.

Earlier in the lesson we saw that the work done was $\frac{1}{2}kx^2$ so

$$\text{E.P.E.} = \frac{1}{2}kx^2 \qquad \text{.... (10)}$$

E.P.E. is never negative whether the spring is extended or compressed..

4 WORK-ENERGY THEOREM

Solution using Newton's second law can be difficult if the forces in the problem are complex. An alternative approach that enables us to understand and solve such motion problems is to relate the speed of the particle to its displacement under the influence of some net force. As we shall see in this section, if the work done by the net force on a particle can be calculated for a given displacement, the change in the particle's speed will be easy to evaluate.

As shown in the figure 5 a particle of mass m moving to the right under the action of a constant net force F. Because the force is constant, we know from Newton's second law that the particle will move with a constant acceleration a. If the particle is displaced a distance s, the net work done by the force F is

$$W_{net} = Fs = (ma)s$$

We found that the following relationships are valid when a particle moves at constant acceleration

$$s = \frac{1}{2}(v_i + v_f)t \; ; \quad a = \frac{v_f - v_i}{t}$$

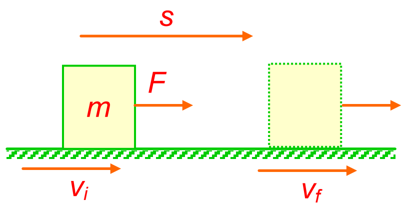


Fig. 5

where v_i is the speed at t = 0 and v_f is the speed at time t. Substituting these expressions

$$W_{net} = m\left(\frac{v_f - v_i}{t}\right)\frac{1}{2}(v_i + v_f)t$$

$$W_{net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$W_{net} = K_f - K_i = \Delta K \qquad \text{... (11)}$$

That is, the work done by the constant net force F_{net} in displacing a particle equals the change in kinetic energy of the particle.

Equation (11) is an important result known as the **work-energy theorem.** For convenience, it was derived under the assumption that the net force acting on the particle was constant.

Now, we shall show that the work-energy theorem is valid even when the force is varying. If the resultant force acting on a body in the x direction is $\sum F_x$, then Newton's second law states that $\sum F_x = ma$. Thus, we express the net work done as

$$W_{net} = \int_{x_i}^{x_f} \left(\sum F_x\right) dx = \int_{x_i}^{x_f} ma\,dx$$

Because the resultant force varies with x, the acceleration and speed also depend on x. We can now use the following chain rule to evaluate W_{net}.

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

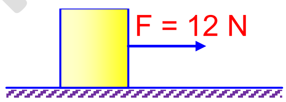
$$W_{net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Illustration 7

Question: **A 2 kg block initially at rest is pulled to the right along a horizontal frictionless surface by a constant force of 12 N, as shown in the figure. Find**

(a) the speed of the block after it has moved 3.0 m.

(b) the acceleration of the block and its final speed using the kinematic equation $v_f^2 = v_i^2 + 2as$.



Solution: The normal force balances the weight of the block, and neither of these forces does work since the displacement is horizontal. Since there is no friction, the resultant external force is the 12 N force. The work done by this force is

$$W = Fs = (12\text{ N})(3.0\text{ m}) = 36\text{ N.m} = 36\text{ J}$$

Using the work-energy theorem and noting that the initial kinetic energy is zero, we get

$$W = K_f - K_i = \frac{1}{2}mv_f^2 - 0$$

$$v_f^2 = \frac{2W}{m} = \frac{2(36\text{J})}{2\text{kg}} = 36\ m^2/s^2$$

v_f **= 6 m/s**

(b) a **= 6.0 m/s²; v_f = 6 m/s.**

Note that result calculated in two ways are same.

4.1 CONSERVATIVE AND NON-CONSERVATIVE FORCE

Suppose a body is taken to a height h from the ground level, work done by the gravity on the body is equal to $-mgh$. When it is allowed to come back to the ground, the work done by it is equal to $+mgh$. So the net work performed $= -mgh + mgh = 0$. Thus, in a gravitational field if a particle describes various displacements and finally comes back to its initial position, the total work performed by the gravity taking into account all the displacements is zero. This is the case with electrostatic fields also. These forces acting in such fields are called **conservative forces**. On the other hand, consider a body moving on a rough surface from A to B and then back from B to A. Work done against frictional forces only add up because in both the displacements work is done against frictional forces only. Hence frictional force cannot be considered as a conservative force. It is a non-conservative force.

4.2. CONSERVATION OF MECHANICAL ENERGY

Kinetic and Potential Energy both are forms of Mechanical Energy. The total mechanical energy of a body or system of bodies will be changed in value if

(a) an external force other than weight causes work to be done (work done by weight is potential energy and is therefore already included in the total mechanical energy),

(b) some mechanical energy is converted into another form of energy (e.g. sound, heat, light etc). Such a conversion of energy usually takes place when a sudden change in the motion of the system occurs. For instance, when two moving objects collide some mechanical energy is converted into sound energy, which is heard as a bang at the impact. Another common example is the conversion of mechanical energy into heat energy when two rough objects rub against each other.

If neither (a) nor (b) occurs then the total mechanical energy of a system remains constant.

This is the Principle of **Conservation of Mechanical Energy** and can be expressed, as,

The total mechanical energy of a system remains constant provided that no external work is done and no mechanical energy is converted into another form of energy. "

When this principle is used in solving problems, a careful appraisal must be made of any external forces, which are acting. Some external forces do work and hence cause a change in the total energy of the system. Other, however, can be present without doing any work and these will not cause any change in energy.

For example, consider a mass m moving along a rough horizontal surface.

The normal reaction N is perpendicular to the direction of motion and does not do any work.

The frictional force μN, acting in the line of motion, does cause the velocity of the mass to change. The frictional force therefore does do work and the total mechanical energy will change.

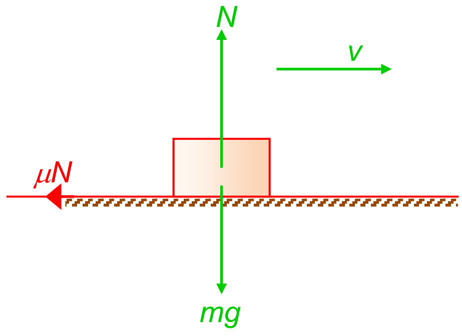


Fig. 6

The principle of conservation of mechanical energy principle is a very useful concept to use in problem solving. It is applicable to any problem where the necessary conditions are satisfied and which is concerned with position and velocity.

Illustration 8

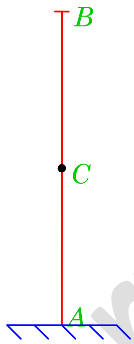
Question: **A body of mass 4 kg is at rest at a height of 10 m above the ground. Calculate its potential energy and kinetic energy after it has fallen through half the height. Also find the velocity at this instant.**

Solution: Total energy at B = K.E. + P.E.

$= 0 + mgh$

$= 4 \times 10 \times 10$

$= \mathbf{400\ J}$



As it descends half the height it loses

Potential energy $= mg\,\dfrac{h}{2}$

$= \dfrac{1}{2}\,mgh$

$= 200$ J

$\therefore$ its P.E. at $C = 400 - 200 = \mathbf{200\ J}$

The loss of potential energy = gain in kinetic energy

$= 400 - 200$

$= \mathbf{200\ J}$

But K.E. $= \dfrac{1}{2}\,mv^2$

$\therefore \quad \dfrac{1}{2} \times 4 \times v^2 = 200$

$\Rightarrow \quad v^2 = 100$

or $\quad v = \mathbf{10\ m/s}$

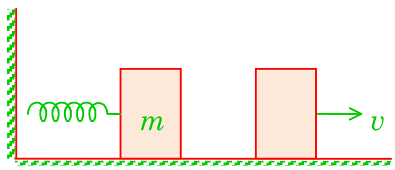
Illustration 9

Question: **A block of mass m = 1kg is pushed against a spring of spring constant k =100 N/m fixed at one end to a wall. The block can slide on a frictionless table as shown in Figure. The natural length of the spring is L_0 = 1m and it is compressed to half its natural length when the block is released. Find the velocity of the block.**

Solution: When the block is released the spring pushes it towards right. The velocity of the block increases till the spring acquires its natural length. Thereafter the block loses contact with the spring and moves with constant velocity. Initially the compression in the spring $= \dfrac{L_o}{2}$.

When the distance of block from the wall becomes x where $x < L_0$ the compression is $(L_0 - x)$. Using the principle of conservation of energy

$$\frac{1}{2}k\left(\frac{L_0}{2}\right)^2 = \frac{1}{2}k(L_o - x)^2 + \frac{1}{2}mv^2$$



Solving this, $v = \sqrt{k/m}\left[\dfrac{L_o^2}{4} - (L_o - x)^2\right]^{\frac{1}{2}}$

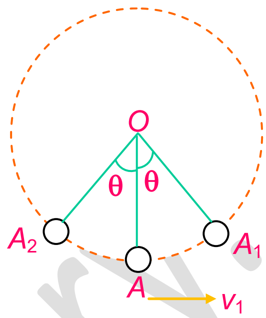
When the spring acquires its natural length $x = L_0$, we have, then $v = \sqrt{\dfrac{k}{m}}\,\dfrac{L_o}{2} = \mathbf{5\ m/s}$

5 MOTION IN A VERTICAL CIRCLE

5.1 BODY SUSPENDED WITH THE HELP OF A STRING

Imagine an arrangement like a simple pendulum where a mass m is tied to a string of length r, the other end of the string being attached to a fixed point O.

Let the body initially lie at the equilibrium position A. Let it be given a velocity v_1 horizontally. If the velocity is small then the body would make oscillations in the vertical plane like a simple pendulum.



Let us now find the velocity of projection v_1 at the lowest point A required to make the body move in a vertical circle.

The following points can be noted while considering the motion:

(i) The motion along the vertical circle is non-uniform since the velocity of body changes along the curve. Hence the centripetal acceleration v^2/r must also change, where v is the instantaneous velocity of the body at any point on the circle.

(ii) The resultant force acting on the body (in the radial direction) provides the necessary centripetal force.

(iii) The velocity decreases as the body moves up form A to B, the topmost point.

When the body is at A, the resultant force acting on it is $T_1 - mg$, where T_1 is the tension in the string.

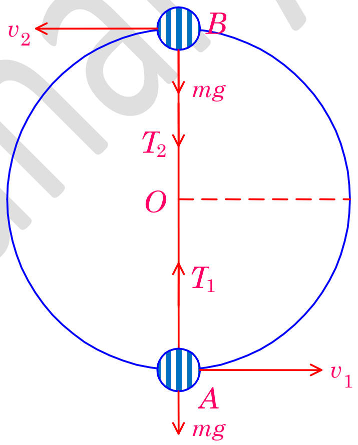


Fig. 7

The centripetal force $T_1 - mg = \dfrac{mv_1^2}{r}$

$$\therefore \quad T_1 = mg + \frac{mv_1^2}{r} \qquad \dots \text{(i)}$$

The tension is always positive (even if v_1 is zero). Therefore, the string will be taut when it is at A. The condition for the body to complete the vertical circle is that the string should be taut all the time i.e., the tension is greater than zero.

The region where the string is most likely to become slack is above the horizontal radius OC. Also the tension would become the least when it is at the topmost point B. Let it be T_2 at B and the velocity of the body be v_2. The resultant force on the body at B is

$$T_2 + mg = \frac{mv_2^2}{r}$$

$$T_2 = \frac{mv_2^2}{r} - mg \qquad \dots \text{(ii)}$$

If the string is to be taut at B, $T_2 \geq 0$.

$$\frac{mv_2^2}{r} - mg \geq 0$$

$$v_2^2 \geq rg$$

$v_2 \geq \sqrt{rg}$ … (iii)

At A, the K.E. of the body = $\frac{1}{2}mv_1^2$

P.E. of the body = 0

Total energy at A = $\frac{1}{2}mv_1^2$

At B, the K.E. of the body = $\frac{1}{2}mv_2^2$

P.E. of the body = $mg(2r)$

Total energy at $B = 2mgr + \frac{1}{2}mv_2^2$

Using the principle of conservation of energy, we have

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 + 2mgr$$

$$v_1^2 = v_2^2 + 4gr \quad \text{… (iv)}$$

Combining equations (iii) and (iv)

$$v_1^2 \geq rg + 4gr$$

$$v_1 \geq \sqrt{5gr}$$

Hence, if the body has minimum velocity of $\sqrt{5gr}$ at the lowest point of vertical circle, it will complete the circle.

The particle will describe complete circle if both v_2 and T_2 do not vanish till the particle reaches the highest point.

The following points are important:

(i) If the velocity of projection at the lowest point A is less than $\sqrt{2gr}$, the particle will come to instantaneous rest at a point on the circle which lies lower than the horizontal diameter. It will then move down to reach A and move on to an equal height on the other side of A. Thus the particle executes oscillations. In this case v vanishes before T does.

We may find an expression for the tension in the string when it makes an angle θ with the vertical. At C, the weight of the body acts vertically downwards, and the tension in the string is towards the centre O.

The weight mg is resolved radially and tangentially.

The radial component is mg cos θ and the tangential component is $mg \sin\theta$.

The centripetal force is $T - mg\cos\theta$

$T - mg\cos\theta = \frac{mv^2}{r}$, where v is the velocity at C,

i.e., $T = m\left(\frac{v^2}{r} + g\cos\theta\right)$ … (v)

The velocity v can be expressed in terms of velocity v_1 at A.

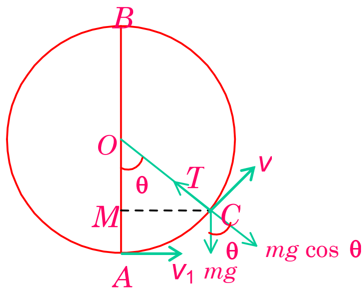


Fig. 8

The total energy at $A = \frac{1}{2}mv_1^2$

The kinetic energy at $C = \frac{1}{2}mv^2$

The potential energy at $C = mg(AM)$

$= mg\,(AO - MO)$

$= mg\,(r - r\cos\theta)$

$= mgr\,(1 - \cos\theta)$

The total energy at $C = \frac{1}{2}mv^2 + mgr\,(1 - \cos\theta)$

$\therefore$ From conservation of energy $\frac{1}{2}mv_1^2 = \frac{1}{2}mv^2 + mgr(1 - \cos\theta)$

$v_1^2 = v^2 + 2gr\,(1 - \cos\theta)$

or $v^2 = v_1^2 - 2gr\,(1 - \cos\theta)$

Substituting in equation (v),

$$T = m\left[g\cos\theta + \frac{v_1^2}{r} - 2g(1 - \cos\theta)\right] = \frac{mv_1^2}{r} + 3mg\left(\cos\theta - \frac{2}{3}\right) \qquad \ldots \text{(vi)}$$

This expression gives the value of the tension in the string in terms of the velocity at the lowest point and the angle θ.

Equation (v) shows that tension in the string decreases as θ increases, since the term '$g\cos\theta$' decreases as θ increases.

When θ is 90°, cos θ = 0, and

$$T_H = \frac{mv^2}{r} \qquad \ldots \text{(vii)}$$

This is obvious because, the weight is vertically downwards whereas the tension is horizontal. Hence the tension alone is the centripetal force.

(ii) If the velocity of projection is greater than $\sqrt{2gr}$ but less than $\sqrt{5gr}$, the particle rises above the horizontal diameter and the tension vanishes before reaching the highest point.

(iii) We have seen that the tension in the string at the highest point is lower than the tension at the lowest point. When the string is above the horizontal, tension may be deduced from equation (vi), if we make θ an obtuse angle. However, in order to have a physical picture of the situation let us study this separately.

At the point D, the string OD makes an angle ϕ with the vertical. The radial component of the weight is mg cos ϕ **towards** the centre O.

$$T + mg\cos\phi = \frac{mv^2}{r}$$

$$T = m\left(\frac{v^2}{r} - g\cos\phi\right) \qquad \dots \text{(viii)}$$

The kinetic energy at $D = \frac{1}{2}mv^2$

Potential energy at $D = mg(AN)$

$$= mg\,(AO + ON)$$

$$= mg(r + r\cos\phi)$$

$$= mgr(1 + \cos\phi)$$

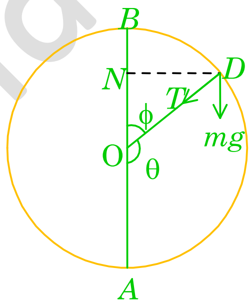


Fig. 9

From conservation of energy $\frac{1}{2}mv_1^2 = \frac{1}{2}mv^2 + mgr(1+\cos\phi)$

$$v^2 = v_1^2 - 2gr(1 + \cos\phi)$$

Substituting in equation (viii),

$$T = m\left[\frac{v_1^2}{r} - 2g(1+\cos\phi) - g\cos\phi\right]$$

$$T = m\left[\frac{v_1^2}{r} - 3g\left(\cos\phi + \frac{2}{3}\right)\right] \qquad \dots \text{(ix)}$$

This equation shows that the tension becomes zero, if

$$\frac{v_1^2}{r} = 3g\left(\cos\phi + \frac{2}{3}\right) \qquad \dots \text{(x)}$$

If the tension is not to become zero,

$$v_1^2 > 3rg\left(\cos\phi + \frac{2}{3}\right)$$

Equation (x) gives the values of ϕ at which the string becomes slack.

$$\cos\phi + \frac{2}{3} = \frac{v_1^2}{3rg}$$

$$\cos\phi = \frac{v_1^2}{3rg} - \frac{2}{3} \qquad \dots \text{(xi)}$$

5.2 A BODY MOVING INSIDE A HOLLOW TUBE OR SPHERE

The same discussion holds good for this case, but instead of tension in the string we have the normal reaction of the surface. If N is the normal reaction at the lowest point, then

$$N - mg = \frac{mv_1^2}{r};$$

$$N = m\left(\frac{v_1^2}{r} + g\right)$$

At the highest point of the circle,

$$N + \text{mg} = \frac{mv_2^2}{r}$$

$$N = m\left(\frac{v_2^2}{r} - g\right)$$

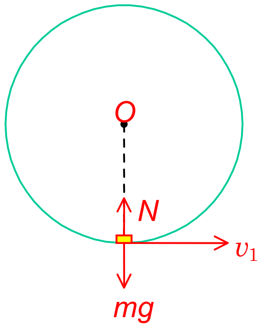


Fig. 10

The condition $v_1 \geq \sqrt{5rg}$ for the body to complete the circle holds for this also.

All other equations (can be) similarly obtained by replacing tension T by reaction R.

5.3 BODY MOVING ON A SPHERICAL SURFACE

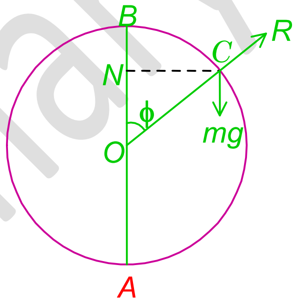


Fig. 11

The small body of mass m is placed on the top of a smooth sphere of radius r.

If the body slides down the surface, at what point does if fly off the surface?

Consider the point C where the mass is, at a certain instant. The forces are the normal reaction R and the weight mg. The radial component of the weight is $mg\cos\phi$ acting towards the centre. The centripetal force is

$$mg\cos\phi - R = \frac{mv^2}{r},$$

where v is the velocity of the body at O.

$$R = m\left(g\cos\phi - \frac{v^2}{r}\right) \qquad \ldots \text{(i)}$$

The body files off the surface at the point where R becomes zero.

$$\text{i.e., } g\cos\phi = \frac{v^2}{r};\ \cos\phi = \frac{v^2}{rg} \qquad \ldots \text{(ii)}$$

To find v, we use conservation of energy

$$\text{i.e., } \frac{1}{2}mv^2 = mg(BN)$$

$$= mg\,(OB - ON) = mgr\,(1 - \cos\phi)$$

$$v^2 = 2rg\,(1 - \cos\phi)$$

$$2(1 - \cos\phi) = \frac{v^2}{rg} \qquad \ldots \text{(iii)}$$

From equations (ii) and (iii) we get

$$\cos\phi = 2 - 2\cos\phi;\ 3\cos\phi = 2$$

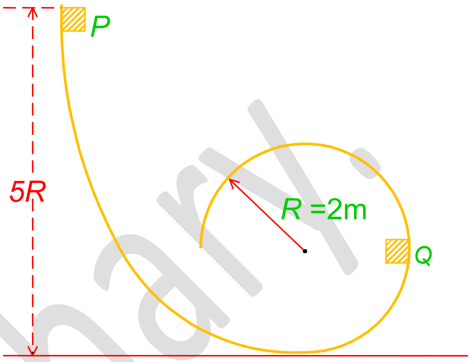
$$\cos\phi = \frac{2}{3};\ \phi = \cos^{-1}\left(\frac{2}{3}\right) \qquad \ldots \text{(iv)}$$

This gives the angle at which the body goes off the surface. The height from the ground of that point = $AN = r(1 + \cos\phi)$

$$= r\left(1+\frac{2}{3}\right)=\frac{5}{3}\, r$$

Illustration 10

Question: **A small block of mass m =1kg slides along the frictionless loop-to-loop track shown in the Figure. (a) If it starts from rest at *P* what is the resultant force acting on it at Q? (b) At what height above the bottom of loop should the block be released so that the force it exerts against the track at the top of the loop equals its weight?**



Solution:

(a) Point Q is at a height R above the ground. Thus, the difference in height between points P and Q is $4R$. Hence, the difference in gravitational potential energy of the block between these points = $4mgR$.

Since the block starts from rest at P its kinetic energy at Q is equal to its change in potential energy. By the conservation of energy.

$$\therefore \quad \frac{1}{2}mv^2=4mgR\,; \quad v^2 = 8gR$$

At Q, the only forces acting on the block are its weight mg acting downward and the force N of the track on block acting in radial direction. Since the block is moving in a circular path, the normal reaction provides the centripetal force for circular motion.

$$N=\frac{mv^2}{R}=\frac{m\times 8gR}{R} = 8\,mg = \mathbf{80\ N}$$

The loop must exert a force on the block equal to eight times the block's weight.

(b) For the block to exert a force equal to its weight against the track at the top of the loop, at the top let the velocity v and normal reaction is N. so that

$$N'+mg = \frac{mv'^2}{R}$$

if $N' = mg$ (force exerts against the track at the top of the loop equal to its weight)

$$\text{then } \frac{mv^2}{R} = mg+mg\,; \quad \frac{mv'^2}{R} = 2mg$$

$$\text{or} \quad v'^2 = 2gR \;\therefore\; mgh=\frac{1}{2}mv'^2\,; \quad h=\frac{v'^2}{2g}=\frac{2gR}{2g}=R$$

The block must be released at a **height of 6m above the bottom of the loop.**

PROFICIENCY TEST

The following questions deal with the basic concepts of this section. Answer the following briefly. Go to the next section only if your score is at least 80%. Do not consult the Study Material while attempting these questions.

1. When a particle rotates in a circle, a central force acts on it directed towards the centre of rotation. Why does this force does no work on the particle?

2. Describe a situation in which friction causes an increase in kinetic energy.

3. A car travelling at a speed v skids a distance d after its brakes lock. Estimate how far it will skid if its brakes lock when its initial speed is $2v$. What happens to the car's kinetic energy as it stops?

4. If the net work done on a particle is zero, what can you conclude about the following:

(a) its acceleration;

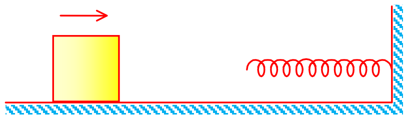
(b) its velocity.

5. A spring of force constant k is cut into three equal parts. What is the force constant of each part?

6. A body of mass 1 kg is dropped from a height of 10m reaches the ground with a speed of 1.2 $\sqrt{gH}$. Calculate the loss of energy due to air friction.

7. Water drawn from a well of (water surface) depth 10 m below the open end of a tube of cross-section 0.245 cm^2 comes out of the tube with a speed of 40 m/s. Calculate the minimum horsepower of motor to be used for this purpose.

8. A 2kg block collides with a horizontal weightless spring of force constant 2N/m. The block compresses the spring 4 metre from rest position. Calculate the speed of the block at the instant of collision



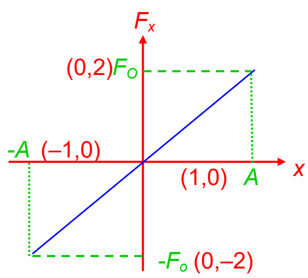
(i) if the surface on which the block slides is frictionless

(ii) if the coefficient of friction between block and horizontal surface is 0.25.

9. The variation of force with position is depicted in figure. Find the work done from

(i) $x = 0$ to $x = -A$.

(ii) $x = +A$ to $x = 0$.



10. A motor car, of mass 1000 kg, runs under a bridge at 72 km/h, the roadway being in the form of an arc of a circle of radius 20 m. Find the reaction between the car and the road at the lowest point of the arc.

ANSWERS TO PROFICIENCY TEST

6. 28 J

7. 98 W

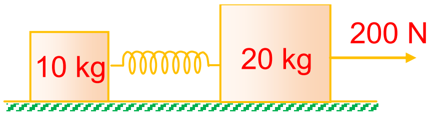
8. (i) 4 m/s;
(ii) 6 m/s

9. (i) 1 J
(ii) −1 J

10. 12 kN

SOLVED OBJECTIVE EXAMPLES

***Example* 1:**



Two masses 10 kg and 20 kg are connected by a massless spring. A force of 200 N acts on 20 kg mass. At the instant when the 10 kg mass has an acceleration 12 m/s² the energy stored in the spring (k = 2400 N/m) will be

(a) 30 J **(b) 3 J**

(c) $\sqrt{3}$ J **(d) 80 J**

Solution:

$F = 10 \times 12 = 120$ N

$F = kx = 2400\,x$

$\therefore\ x = \frac{1}{20}$

Energy stored in the spring $E = \frac{1}{2}kx^2$

$= \frac{1}{2} \times 2400 \times \frac{1}{400} = \mathbf{3\ J}$

$\therefore$ **(b)**

***Example* 2:**

A body is projected at an angle of 30° to the horizontal with kinetic energy 40 J. What will be the kinetic energy at the top most point?

(a) 25 J (b) 40 J (c) 30 J (d) 20 J

Solution:

At the topmost point, the horizontal component of velocity = $u \cos \alpha$

$\therefore$ initial kinetic energy = $\frac{1}{2}mu^2 = 40$ J

Kinetic energy at the topmost point = $\frac{1}{2}mu^2\cos^2\alpha$

$= 40 \cos^2 \alpha$

$= \frac{40 \times 3}{4} = \mathbf{30\ J}$

$\therefore$ **(c)**

***Example* 3:**

Two springs of spring constants 1000 N/m and 2000 N/m are stretched with same source. They will have potential energy in the ratio of

(a) 2 : 1 **(b) $2^2 : 1^2$** **(c) 1 : 2** **(d) $1^2 : 2^2$**

Solution:

Potential energy $= \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{T}{k}\right)^2 = \frac{1}{2}\frac{T^2}{k}$

$E_1 \times k_1 = E_2 \times k_2$

$\frac{E_1}{E_2} = \frac{k_2}{k_1} = \frac{2000}{1000} = \frac{2}{1}$

Ratio is **2 : 1**

∴ **(a)**

***Example* 4:**

A body is slowly lowered on to a massive platform moving horizontally at a speed of 4 m/s. through what distance will the body slide relative to the platform? (The coefficient of friction is 0.2; g = 10 m/s²)

(a) 4 m **(b) 2 m** **(c) 16 m** **(d) 8 m**

Solution:

The frictional force between the body and the platform = μmg, where m is the mass of the body.

Initially the relative velocity = 4 m/s

The relative retardation $= \mu g$

$= 0.2 \times 10$

$= 2 \text{ m/s}^2$

If S is the relative displacement before the relative velocity becomes zero, we have

$0 = 4^2 - 2 \times 2 \times S$

$S = \frac{16}{4} = \mathbf{4\ m}$

∴ **(a)**

***Example* 5:**

A ball falls under gravity from a height 10 m with an initial velocity v_0. It hits the ground, loses 50% of its energy in collision and it rises to the same height. What is the value of v_0?

(a) 14 m/s **(b) 7 m/s** **(c) 28 m/s** **(d) 9.8 m/s**

Solution:

Let v be the velocity when it hits the ground.

Then $v^2 - v_0^2 = 2g \times 10 = 2 \times 9.8 \times 10$

i.e., $v^2 = v_0^2 + 196$

Let v' be the velocity after impact and it reaches the same height 10 m.

$v'^2 - 0 = 2 \times 9.8 \times 10$

$v'^2 = 196$

$v' = 14$ m/s

Ratio of kinetic energy before impact and after impact $= \dfrac{\frac{1}{2}mv^2}{\frac{1}{2}mv'^2} = \dfrac{v^2}{v'^2} = \dfrac{v_0^2+196}{196} = 2.$

$\therefore \quad v_0^2 = 2 \times 196 - 196 = 196$

$v_0 =$ **14 m/s.**

$\therefore$ **(a)**

***Example* 6:**

A block of mass m moving with speed v compresses a spring through a distance x before its speed is halved. What is the value of spring constant?

(a) $\dfrac{3mv^2}{4x^2}$ (b) $\dfrac{mv^2}{4x^2}$ (c) $\dfrac{mv^2}{2x^2}$ (d) $\dfrac{3mv^2}{x^2}$

Solution:

Initial kinetic energy $= \dfrac{1}{2}mv^2$

Final energy $= \dfrac{1}{2}m\left(\dfrac{v}{2}\right)^2 + \dfrac{1}{2}kx^2$

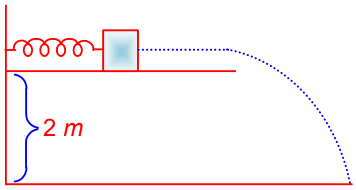
By principle of conservation of energy,

$\dfrac{1}{2}mv^2 = \dfrac{1}{2}m\dfrac{v^2}{4} + \dfrac{1}{2}kx^2$

$\therefore k = \dfrac{3mv^2}{4x^2}$

$\therefore$ **(a)**

Example 7:



A small block of mass 100 g is pressed against a horizontal spring fixed at one end and compression is 5 cm. The spring constant is 100 N/m. When the block moves horizontally it leaves the spring. Where will it hit the ground 2 m below the spring?

(a) Horizontal distance of 1 m from end of spring.

(b) Horizontal distance of 2 m from end of spring.

(c) 0.5 m from free end of spring.

(d) 1.5 m from free end of spring.

***Solution*:**

Let v be the velocity when it leaves the spring.

Then $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$.

$$v^2 = \frac{kx^2}{m} = \frac{100\times(0.05)^2}{100\times10^{-3}}$$

$$v = 10 \times 0.05\sqrt{10}$$

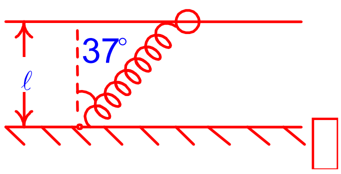
Time to fall vertical distance of 2 metres from the spring

$$= \sqrt{\frac{2s}{g}} = \sqrt{\frac{2\times2}{10}} = \frac{2}{\sqrt{10}}$$

Horizontal distance $= \frac{2}{\sqrt{10}} \times v = \frac{2}{\sqrt{10}} \times 10 \times 0.05 \times \sqrt{10} = 1$ m.

$\therefore$ **(a)**

Example 8:



A spring of natural length ℓ and spring constant k is fixed on the ground and the other is fitted with a smooth ring of mass m which slides on a horizontal rod fixed at a height also equal to ℓ (see Figure).

Initially the spring makes an angle of 37° with the vertical when the system is released from rest. What is the speed of the ring when the spring becomes vertical?

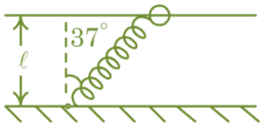
(a) $\frac{\ell}{4}\sqrt{\frac{k}{m}}$

(b) $\ell\sqrt{\frac{k}{m}}$

(c) $2\ell\sqrt{\frac{k}{m}}$

(d) $4\ell\sqrt{\frac{k}{m}}$

***Solution*:**



In the initial position of the ring as shown in the Figure,

The length of spring = $\dfrac{\ell}{\cos 37^\circ} = \dfrac{\ell}{\frac{4}{5}} = \dfrac{5\ell}{4}$

$\therefore$ Extension $= \dfrac{5\ell}{4} - \ell$

$= \dfrac{\ell}{4}$

Energy stored in spring = $\dfrac{1}{2}k\cdot\dfrac{\ell^2}{16}$

This stored energy when released becomes kinetic energy of the ring. If v is the velocity of the ring, kinetic energy when it is vertical = $\dfrac{1}{2}mv^2$

By principle of conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}k\cdot\frac{\ell^2}{16}$$

$$v^2 = \frac{k\ell^2}{16m}$$

$$v = \frac{\ell}{4}\sqrt{\frac{k}{m}}$$

$\therefore$ **(a)**

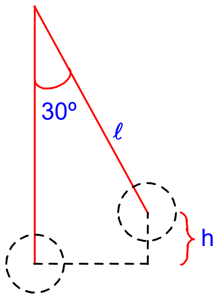
Example 9:

A mass of 1 kg suspended on a thread deviates through an angle of 30°. Find the tension of the thread at the moment the weight passes through the position of equilibrium.

(a) 12.4 N **(b) 15 N** **(c) 24.8 N** **(d) 6.2 N**

Solution:

At the moment the weight passes through the position of equilibrium the tension of the thread $T = mg + \dfrac{mv^2}{\ell}$



By the conservation of energy

$$mgh = \frac{mv^2}{2}$$

$$\therefore \quad v = \sqrt{2gh}$$

But $h = \ell - \ell\cos 30° = \ell(1 - \cos 30°)$

$$\therefore \quad \frac{mv^2}{\ell} = \frac{m}{\ell}\cdot 2gh$$

$$= \frac{m}{\ell}\cdot 2g\ell(1 - \cos 30°)$$

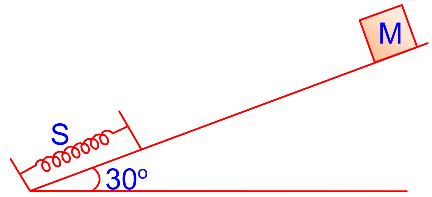
and $T = mg[1 + 2(1 - \cos 30°)]$

Given $m = 1$ kg; g = 9.8 m/s^2; $\cos 30° = \dfrac{\sqrt{3}}{2}$

$\therefore$ $T = 12.4$ N

$\therefore$ **(a)**

***Example* 10:**



An ideal massless spring *S* can be compressed one metre by a force of 100 newton. The same spring is placed at the bottom of a frictionless inclined plane inclined at 30° to horizontal. A block *M* of mass 10Kg is released from rest at the top of the incline and is brought to rest momentarily after compressing the spring 2 metres. What is the speed of mass just before it reaches the spring? (g = 10 m/s^2).

(a) $\sqrt{20}$ m/s **(b)** $\sqrt{30}$ m/s

(c) $\sqrt{10}$ m/s **(d)** $\sqrt{40}$ m/s

***Solution*:**

Applied force on the spring, $F = k.x$,

where x is the distance through which it is compressed.

$$k = \frac{F}{x} = \frac{100N}{1m} = 100 \text{ N/m}$$

Let the mass M slide a distance s metres along the incline before hitting the spring. The spring gets compressed by 2 metres. Hence the mass M slides a total distance $(s + 2)$ metres along the incline. Initially M is placed at a height $(s + 2) \sin 30^\circ$ above the bottom of incline $= \frac{s+2}{2} = h$.

The mass has initially P.E. $= Mgh = \frac{Mg(s+2)}{2}$

When the spring is compressed, the energy has gone entirely into deformation of spring.

$$\frac{1}{2}kx^2 = \frac{Mg(s+2)}{2}$$

$$s + 2 = \frac{kx^2}{Mg} = \frac{100\times 2^2}{10\times g} = \frac{40}{g}$$

$s = \frac{40}{g} - 2 = 2$ m. The mass falls through a height, $s \sin 30^\circ = \frac{s}{2}$.

Gain in K.E. = Loss of P.E.

$$\frac{1}{2}Mv^2 = \text{Mgh} \quad \text{or} \quad v = \sqrt{2gh}$$

$$v = \sqrt{2g\cdot\frac{s}{2}} = \sqrt{2g\times 1} = \sqrt{2g} = \sqrt{20} \text{ m/s.}$$

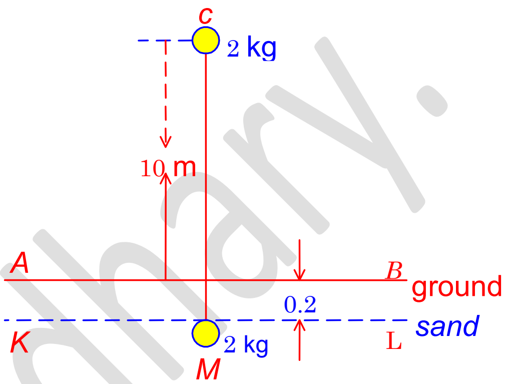
$\therefore$ **(a)**

SOLVED SUBJECTIVE EXAMPLES

***Example* 1:**

A body of mass 2 kg is allowed to fall from the top of a 10 m tall building. It lands on a sandy patch of ground and buries itself 20 cm deep in the sand. Find the average resistance offered to it by sand.

Solution:



AB is the ground level. The body reaches M, 0.2 m below ground level through sand. Taking M as standard (reference) position for calculating potential energy the potential energy at C = mgh = $2 \times 10 \times 10.2 = 204$ J. This is used to do work against the resistance offered by the sand.

If R is the average resistance, then

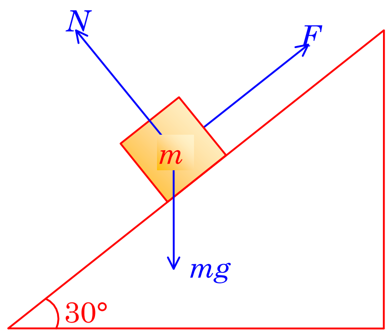
$$R \times 0.2 = 204$$

$$R = \frac{204}{0.2} = \mathbf{1020\ N}$$

***Example* 2:**

A block of mass 2 kg is pulled up on a smooth incline of angle 30° with horizontal. If the block moves with an acceleration of 1 m/s^2, find the power delivered by the pulling force at a time 4 seconds after motion starts. What is the average power delivered during these four seconds after the motion starts?

***Solution*:**



The forces acting on the block are shown in Figure. Resolving forces parallel to incline,

$$F - mg\sin\theta = ma$$

$$\Rightarrow\ F = mg\sin\theta + ma$$

$$= 2 \times 10 \times \sin 30° + 2 \times 1$$

$$= 12\ \text{N}$$

The velocity after 4 seconds = $u + at$

$$= 0 + 1 \times 4$$

$$= 4\ \text{m/s}$$

Power delivered by force at $t = 4$ seconds

$$= \text{Force} \times \text{velocity}$$

$$= 12 \times 4$$

$$= \mathbf{48\ W}$$

The displacement during 4 seconds is given by the formula

$$V^2 = u^2 + 2aS$$

$v^2 = 0 + 2 \times 1 \times S$

$\therefore \quad S = 8$ m

Work done in four seconds = Force × distance = 12 × 8 = 96 J

$\therefore$ average power delivered = $\dfrac{\text{work done}}{\text{time}}$

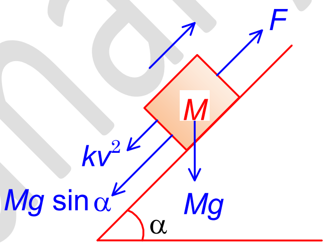
$= \dfrac{96}{4} = \mathbf{24\ W}$

***Example* 3:**

A motorcar of mass 1000 kg attains a speed of 36 km/hr when running down an incline of 1 in 20 with the engine shut off. It can attain a speed of 72 km/hr up the same incline when the engine is switched on. Assuming that the resistance varies as the square of the velocity, find the power (in kW) developed by engine.

Solution:

When the motor car is moving down the plane there is a force Mg sin α down the plane. This is opposed by the resistance, which is proportional to square of the velocity. That is



$Mg \sin\alpha \propto v^2$

$Mg \times \dfrac{1}{20} = kv^2$, where k is a constant.

$\therefore \quad \dfrac{1000 \times g}{20} = k\left(36 \times \dfrac{5}{18}\right)^2$

$k = \dfrac{1000 \times g}{20} \times \left(\dfrac{18}{36 \times 5}\right)^2 \qquad \ldots (1)$

When the engine is on, let the tractive force (force exerted by engine) be F. This is used to overcome the force due to incline and the resistance offered.

$\therefore \quad F = \text{k}\,(72 \times 5/18)^2 + \dfrac{Mg}{20}$

$= k\,(72 \times 5/18)^2 + \dfrac{1000 \times g}{20}$

Substituting the value of k from equation (1)

$F = \dfrac{1000 \times g}{20} \times \left(\dfrac{18}{36 \times 5}\right)^2 \times \left(72 \times \dfrac{5}{18}\right)^2 + \dfrac{1000 \times g}{20}$

$= \dfrac{1000g}{20}\left[\dfrac{18 \times 18}{36 \times 5 \times 36 \times 5} \times \dfrac{72 \times 5 \times 72 \times 5}{18 \times 18} + 1\right]$

$= 2500$ N

Power developed = Force × velocity

$= 2500 \times 72 \times 5/18$

$=$ **50 Kw**

***Example* 4:**

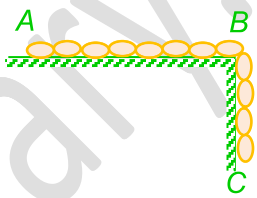
A uniform flexible chain of length l = 60 m is held on a smooth horizontal table so that $\frac{l}{n}$ m overhangs one edge, the chain being perpendicular to the edge. If the chain is released from rest find the velocity with which it leaves the table. ($n = \sqrt{2}$)

Solution:

ABC is the flexible chain and *BC* is the free part which overhangs on one side of the table. The C.G of the part *BC* lies at its middle.

Let the surface of the table be the level of zero potential energy.

Potential energy of the chain at the start = potential energy of the overhang



$= mgh$

$= -\dfrac{mgl}{2n^2}$

The potential energy is negative because the C.G of overhanging part lies below the zero potential energy level.

Kinetic energy of the chain at start = 0

$\therefore$ total energy at the start = kinetic energy + potential energy

$$= -\frac{mgl}{2n^2}$$

Let v be the velocity of chain when its full length gets detached from the table.

Its kinetic energy will be = $\frac{1}{2}mv^2$

potential energy of the chain at that instant = Mass of chain × g × depth of C.G below table level

$$= \frac{mgl}{2}$$

$\therefore$ total energy = $\frac{1}{2}mv^2 + \frac{-mgl}{2}$ joules

By the principle of conservation of energy

$$\frac{1}{2}mv^2 + \frac{-mgl}{2} = -\frac{mgl}{2n^2}$$

$$\frac{1}{2}mv^2 = -\frac{mgl}{2n^2} + \frac{mgl}{2}$$

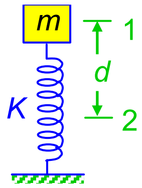
$$= -\frac{mgl}{4} + \frac{mgl}{2} = -\frac{mgl + 2mgl}{4} = \frac{mgl}{4}$$

$$v^2 = \frac{gl}{2} \qquad (\therefore\ n = \sqrt{2})$$

$$v = \sqrt{300} \text{ m/sec}$$

***Example* 5:**

A block of mass *m* released from rest onto an ideal non-deformed spring of spring constant *k* from a negligible height. Neglecting the air resistance, find the compression *d* of the spring. ($mg = k$)



***Solution*:**

According to work energy theorem the increment of the kinetic energy of the block is equal to the algebraic sum of the works performed by all forces acting on it.

i.e., $W = \Delta T$

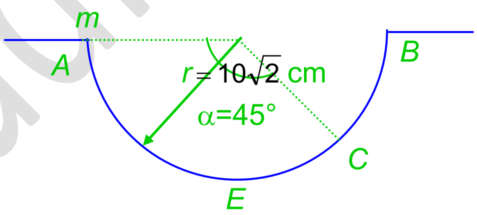
or, $W_{mg} + W_{sp} = 0$

or, $mg\,d + \int_0^d (-kx)\,dx = 0$ i.e., $mgd - \frac{1}{2}kd^2 = 0$

Hence $d = \frac{2mg}{k} = \mathbf{2\ m}$

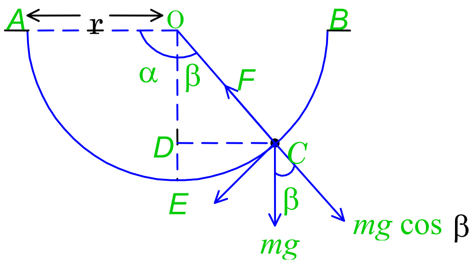
***Example* 6:**

A small particle of mass m = $\sqrt{2}$ kg initially at *A* (see Figure) slides down a frictionless surface *AEB*. When the particle is at the point *C*. Find the angular velocity and the force exerted by the surface.



Solution:

The force diagram of the mass when it is at *C* is shown in Figure.



At the point *C* the total energy = P.E. + K.E.

$$= mg(DE) + \frac{1}{2}mv^2$$

Where v is the linear velocity at *C*.

At the point *A* the total energy = mgr

By the principle of conservation of energy,

$$mgr = mg\,(DE) + \frac{1}{2}mv^2$$

But $v = r\omega$, where ω is the angular velocity.

$$\therefore\quad mgr = mg(OE - OD) + \frac{1}{2}mr^2\omega^2$$

$$\Rightarrow\quad gr = g(OE - OD) + \frac{1}{2}r^2\omega^2$$

$\Rightarrow$ Now, $(OE - OD) = r - r\cos\beta$

$= r\,(1 - \cos\beta)$

$$\therefore\quad gr = gr\,(1 - \cos\beta) + \frac{1}{2}r^2\omega^2$$

$$\omega^2 r^2 = 2gr\cos\beta$$

$$= 2gr\sin\left(\frac{\pi}{2}+\beta\right)$$

$$= 2gr\sin\alpha$$

$$\omega = \sqrt{\frac{2g\sin\alpha}{r}} = \mathbf{10\ rad/s}$$

Now, $m\omega^2 r = 2mg\sin\alpha$

$\therefore$ total force at C, $F = m\omega^2 r + mg\cos\beta$

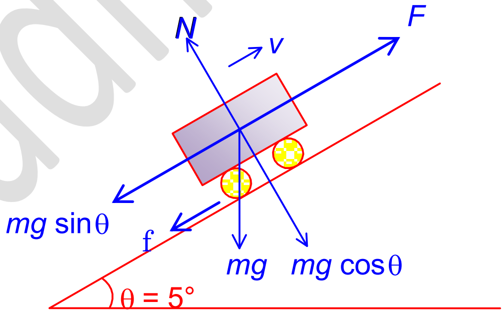
$$= 2mg\sin\alpha + mg\sin\alpha$$

$$= 3\,mg\sin\alpha = \mathbf{30\ N}$$

Example 7:

A 100000 kg engine is moving up a slope of gradient 5° at a speed of 100 m/hr. The coefficient of friction between the engine and the rails is 0.1. If the engine has an efficiency of 4% for converting heat into work, find the amount of coal, the engine has to burn up in one hour. (Burning of 1 kg of coal yields 50000 J.)

Solution:



The forces are shown in Figure.

Net force to move the engine up the slope.

$F = \mu N + mg\sin\theta$

$= mg(\mu\cos\theta + \sin\theta)$

If the engine has to apply an upward force equal to F,

power of engine, $P = Fv$

where v is the velocity equal to 100 m/hr.

Work done by engine, $W = Pt = Fvt$

Efficiency of engine, $\eta = \frac{\text{Output}}{\text{Input}} = \frac{Fvt}{\text{Energy used}}$

Energy used by engine $= \frac{Fvt}{\eta} = \frac{mg(\mu\cos\theta+\sin\theta)vt}{\eta}$

$m = 100000$ kg, $\mu = 0.1$, $\theta = 5°$, $v = 100$ m/hr, $t = 1$ hr

$\eta = \frac{4}{100} = 0.04$

Energy used by engine $= \frac{100000 \times 9.8\ (0.1\cos 5^\circ + \sin 5^\circ) \times 100}{0.04}$

$$= \frac{10^5 \times 9.8(0.1\times 0.9962 + 0.0872)\times 100}{4\times 10^{-2}}$$

$$= \frac{9.8\times 0.1868\times 10^9}{4}$$

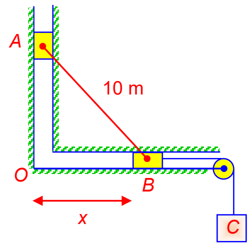
$$= 4.577 \times 10^8\ \text{J}$$

As 1 kg coal yields 50000 J, we have the amount of coal burnt up

$$= \frac{4.577\times10^8}{5\times10^4} = \mathbf{9154\ kg}$$

Example 8:

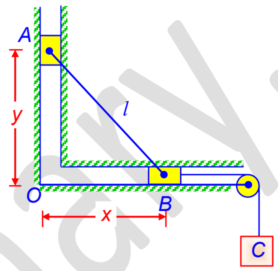
Two bodies A and B connected by a light rigid bar 10 m long move in two frictionless guides as shown in the Figure. If B starts from rest when it is vertically below A, find the velocity (in cm/s) of B when $x = 6$ m. Assume $m_A = m_B = 200$ kg and $m_C = 100$ kg.



Solution:

At the instant, when the bar is as shown in the Figure,

$X^2 + y^2 = L^2$



$$\therefore\quad 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \qquad \text{... (i)}$$

$$\therefore\quad x\frac{dx}{dt} = -y\frac{dy}{dt} \qquad \text{... (ii)}$$

where $\frac{dx}{dt}$ = velocity of B $\frac{dy}{dt}$ = velocity of A.

Applying the law of conservation of energy, loss of potential energy of A, if it is going down when the rod is vertical to the position shown in the Figure = $m_A g$ [10 – 8] = 2 × 200 × 9.8

C moves down 6 m since B moves 6 m along x-axis

Loss of potential energy of C (= mgh) = 100 × 9.8 × 6

Total loss of potential energy = 200 × 9.8 × 2 + 100 × 9.8 × 6 if $x = 6 \Rightarrow y = 8$

= 100 × 9.8 × 10 = 9800 J.

This must be equal to kinetic energy gained

Kinetic energy gained $= \frac{1}{2}m_A(v_A)^2 + \frac{1}{2}m_B(v_B)^2 + \frac{1}{2}m(v_C)^2$

$$= \frac{1}{2}\times200\left(\frac{dy}{dt}\right)^2 + \frac{1}{2}\times200\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}\times100\left(\frac{dx}{dt}\right)^2$$

$$= 100\left(\frac{dy}{dt}\right)^2 + 150\left(\frac{dx}{dt}\right)^2$$

$$= 100\left[\frac{x}{y}\,\frac{dx}{dt}\right]^2 + 150\left(\frac{dx}{dt}\right)^2 \qquad \text{from (ii)}$$

$$= 100\left[\frac{6}{8}\frac{dx}{dt}\right]^2 + 150\left(\frac{dx}{dt}\right)^2$$

$$= \left[100\times\frac{9}{16} + 150\right]\left(\frac{dx}{dt}\right)^2$$

$$= \frac{3300}{16}v_B^2$$

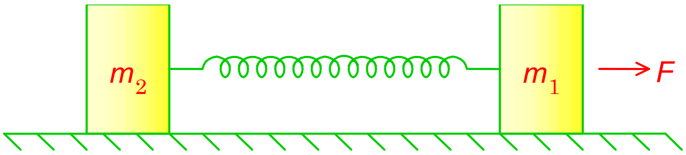
$$\therefore\quad \frac{3300}{16}v_B^2 = 9800$$

$$\therefore\quad v_B = \sqrt{\frac{98\times16}{33}} = 7\times4\sqrt{\frac{2}{33}} = 6.9\ \text{ms}^{-1}.$$

$\therefore$ velocity of B at the required moment = **690 cm/s**

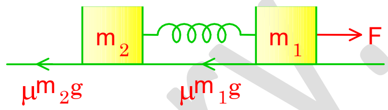
***Example* 9:**

Two bars of masses $m_1 = 4$ kg and $m_2 =$ 8kg connected by a non-deformed light spring rest on a horizontal plane. The coefficient of friction between bars and surface is $\mu = 0.2$. What minimum constant force has to be applied in the horizontal direction to the bar of mass m_1 in order to shift the other bar?



***Solution*:**

Let F be the force applied as shown in the Figure. If F moves through x, the work done will be $F \times x$. This is used to work against friction μm_1 g and store energy in the spring.



Work done against friction = $\mu m_1 g x$;

Energy stored = $\frac{1}{2}kx^2$, where k is the spring constant.

$\therefore \; F \cdot x = \mu m_1 gx + \frac{1}{2}kx^2$ ... (i)

When the mass m_2 moves, the tension in the spring balances the force of friction at m_2.

$\therefore \qquad kx = \mu m_2 g$ ... (ii)

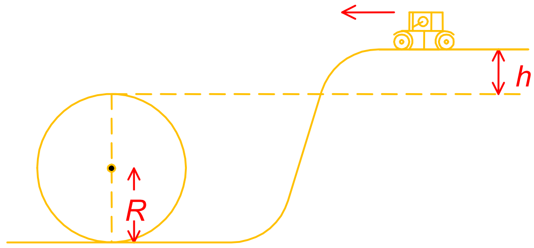
$\therefore$ combining equations (i) and (ii),

$$F \cdot x = \mu m_1 gx + \frac{1}{2}\mu m_2 g \cdot x$$

$$F = \mu g\left(m_1 + \frac{m_2}{2}\right) = \mathbf{16\ N}$$

***Example* 10:**

The Figure shows a loop-to-loop track of radius R = 6m. A car without engine starts from a platform at a distance h above the top of the loop and goes around the loop without falling off the track. Find the minimum value of h for a successful looping. Neglect friction.



Solution:

The speed of the car at the top-most point of the loop is v. The gravitational potential energy is zero at the platform and the car starts with negligible speed. (This is assumed.)

According to the law of conservation of energy,

$0 = -mgh + \frac{1}{2}mv^2 \text{ or } v^2 = 2gh$, where m is mass of the car. … (i)

The car has radial acceleration $\left(\frac{v^2}{R}\right)$. The forces acting on the car are mg due to gravity and reaction N due to contact with the track. These forces are radial at the top of the loop.

$\therefore \qquad mg + N = \frac{mv^2}{R}$

i.e., $\qquad mg + N = \frac{2mgh}{R}$ … (ii)

For h to be minimum, N should be minimum or zero,

i.e., $\qquad 2\,mgh = Rmg$

or, $\quad h_{min} = \frac{R}{2} = 3\text{m}$

MIND MAP

WORK, ENERGY AND POWER

1. Work done by a constant force $W = \vec{F}.\vec{S} = FS\cos\theta$.

2. Work done by a variable force $= \int \vec{F}.d\vec{s}$

3. If force is along the line of motion, work done can be calculated by calculating area enclosed between F-S curve and displacement axis.

4. Work done by conservative force is path independent while that by non-conservative force is path dependent.

5. Power is the rate at which an agent does work.

Average Power, $\bar{P} = \frac{\Delta W}{\Delta t}$

Instantaneous Power

$P = \frac{dw}{dt} = \vec{F}.\vec{v}$

6. Slope of W-t graph gives power of agent.

Area enclosed between P-t curve and time axis gives work done by the agent.

7. Kinetic energy $= \frac{1}{2}mv^2$

8. Gravitational potential energy $= \pm\, mgh$, we take positive GPE when body is above reference level and negative when the body is below the reference level.

9. Spring's elastic energy $= \frac{1}{2}kx^2$

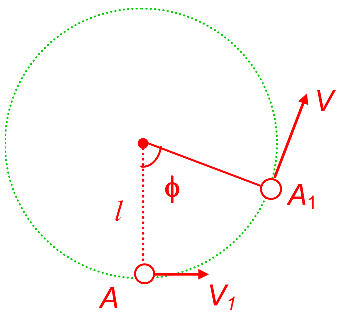
10. Work energy theorem $W_{ext} = K_f - K_i = \Delta K$

MOTION IN A VERTICAL CIRCLE

(*Body suspended with the help of a string*)

If body has velocity v_1 at its lowest position A,

then at any position A_1



Velocity, $V = \sqrt{v_1^2 - 2gl(1-\cos\phi)}$

Tension in string, $T = \frac{Mv_1^2}{l} - 2Mg + 3Mg\cos\phi$

To complete the circle, $v_1 \geq \sqrt{5gl}$.

EXERCISE – I

NEET-SINGLE CHOICE CORRECT

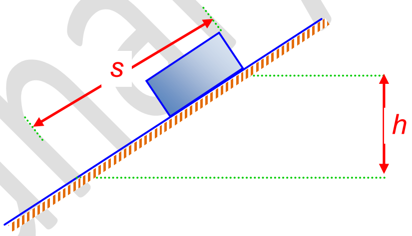
1. The work done by all the forces (external and internal) on a system equals the change in

(a) total energy (b) kinetic energy (c) potential energy (d) none of these

2. The work done by the external forces on a system equals the change in

(a) total energy (b) kinetic energy (c) potential energy (d) none of these

3. The work done against gravity in moving the block of mass *m* a distance *s* up the slope as shown in the figure is



(a) mh (b) mgs

(c) ms (d) mgh

4. An object of mass 10 kg falls from rest through a vertical distance of 10 m and acquires a velocity of 10 m/s. The work done by the push of air on the object is (g = 10 m/s^2)

(a) 500 J (b) –500 J (c) 250 J (d) –250 J

5. A chain of mass *m* and length *l* is placed on a table with one-sixth of it hanging freely from the table edge. The amount of work done to pull the chain on the table is

(a) $mgl/72$ (b) $mgl/36$ (c) $mgl/6$ (d) $mgl/4$

6. A particle moves under the effect of a force $F = cx$ from $x = 0$ to $x = x_1$. The work done is

(a) cx_1^2 (b) $\frac{1}{2}cx_1^2$ (c) cx_1^3 (d) zero

7. Work done in time *t* on a body of mass *m* accelerated from rest to a speed *v* in time t_1 as function of time *t*, is

(a) $\frac{1}{2}\frac{mvt^2}{t_1}$ (b) $\frac{mvt^2}{t_1}$ (c) $\frac{1}{2}\left(\frac{mv}{t_1}\right)^2 t^2$ (d) $\frac{1}{2}m\left(\frac{vt}{t_1}\right)^2$

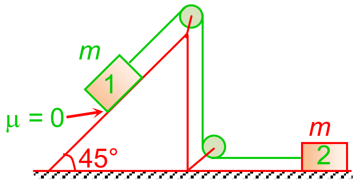
8. How much work must be done by a force on 100 kg body to accelerate it from 0 to 20 m/s in 20 s?

(a) 2×10^3 W (b) 2×10^3 J (c) 2×10^4 J (d) 4×10^4 J

9. A 2 kg body and a 3 kg body have equal momentum. If the kinetic energy of 3 kg body is 10J, the K.E. of 2 kg body will be

(a) 6.66 J (b) 15 J (c) 22.5 J (d) 45 J

10. Coefficient of friction between block 2 and ground is 1.3/√2. Work done by the friction force when blocks are released for 1 second is



(a) $\frac{1.3}{2\sqrt{2}}mg$ (b) $\frac{3mg}{\sqrt{2}}$

(c) 0 (d) $\frac{1.3}{\sqrt{2}}mg$

11. A position dependent force $F = 7 - 2x + 3x^2$ newton acts on a small body of mass 2 kg and displaces it from $x = 0$ to $x = 5$ m. The work done in joules is

(a) 70 (b) 270 (c) 35 (d) 135

12. Two equal masses are attached to the two ends of a spring of spring constant k. The masses are pulled out symmetrically to stretch the spring by a length x over its natural length. The work done by the spring on each mass is

(a) $\frac{1}{2}kx^2$ (b) $-\frac{1}{2}kx^2$ (c) $\frac{1}{4}kx^2$ (d) $-\frac{1}{4}kx^2$

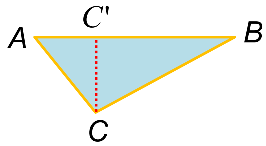
13. An elastic string of unstretched length l and force constant k is stretched by a small amount x. It is further stretched by another small length y. What is the work done in second stretching?

(a) $1/2\,k\,(y^2 - x^2)$ (b) $1/2\,ky\,(2x + y)$ (c) $1/2\,ky^2$ (d) $\frac{1}{2}k(x^2 + y^2)$

14. The natural length of spring is 0.3 m and its spring constant is 30 N/m. How much work is done by the applied external force to stretch the spring from 0.1 to 0.2 m?

(a) 0.68 J (b) 0.45 J (c) 0.55 J (d) 0.70 J

15. A triangle is formed by using three wires AB, BC and CA and is placed in a vertical plane. Coefficient of friction for all the three wires is same. If w_1 and w_2 is the work done by the friction in moving an object from A to B through C' and C respectively, then



(a) $w_1 = w_2$ (b) $w_1 < w_2$

(c) $w_1 > w_2$ (d) the relation depends on the length AC and BC

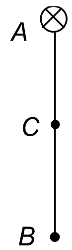
16. The kinetic energy K of a particle moving along a circle of radius R depends on the distance covered as $K = aS^2$. Force acting on the particle is

(a) $2a\frac{S^2}{R}$ (b) $2aS\left(1 + \frac{S^2}{R^2}\right)^{1/2}$ (c) $2aS$ (d) $\frac{2aR^2}{S}$

17. A wind-powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be proportional to

(a) v (b) v^2 (c) v^3 (d) v^4

18. A weightless rigid rod AB of length l carries two equal masses m, one secured at the end and other at the middle of the rod as shown in figure. The rod can rotate in vertical plane around the hinge at A. The minimum horizontal velocity required to be imparted to the end B of rod so as to make the rod go around in a complete circle is



(a) $\sqrt{4gl}$ (b) $\sqrt{5gl}$ (c) $\sqrt{\frac{24gl}{5}}$ (d) $\sqrt{\frac{24gl}{7}}$

19. A uniform chain of length ℓ and mass m overhangs a smooth table with its two third part lying on the table, then the kinetic energy of the chain as it completely slips off the table is

(a) $\frac{4}{9}mgl$ (b) mgl (c) $\frac{2}{9}mgl$ (d) none

20. Under the action of a force, a 2 kg body moves such that its position x as a function of time is given by $x = \frac{t^3}{3}$. where x is in metre and t in seconds. The work done by force in first two seconds is

(a) 1.6 J (b) 16 J (c) 160 J (d) 1600 J

21. A person of mass 60 kg is moving with a velocity of 20 m/s. A boy of mass 40 kg is moving with a speed of 10 m/s. The ratio of kinetic energy of person and boy is

(a) 1 : 2 (b) 6 : 1 (c) 1 : 6 (d) 4 : 3

22. A pump can hoist 9000 kg of coal per hour from a mine of 120 m deep. Then the power in watts, assuming that its efficiency is 75%, is

(a) 4920 watt (b) 5920 watt (c) 3920 watt (d) none of these

23. A running man has half the kinetic energy of that of a boy of half of his mass. The man speeds up by 1 m/s so as to have same kinetic energy as that of the boy. The original speed of the man is

(a) $(\sqrt{2}-1)$ m/s (b) $\sqrt{2}$ m/s (c) $\frac{1}{(\sqrt{2}-1)}$ m/s (d) $\frac{1}{\sqrt{2}}$ m/s

24. A heavy weight is suspended from the spring. A person raises the weight slowly till the spring becomes slack. The work done by him is W. The energy stored in the stretched spring was E. What will be the gain in gravitational potential energy?

(a) $W + E$ (b) $W - E$ (c) W (d) E

25. A block slides down an inclined plane of slope θ with constant velocity. It is then projected up the plane with an initial velocity u. How far up the incline will it move before coming to rest?

(a) $\frac{u^2}{g\sin\theta}$ (b) $\frac{u^2}{2g\sin\theta}$ (c) $\frac{u^2}{4g\sin\theta}$ (d) $\frac{2u^2}{g\sin\theta}$

EXERCISE – II

IIT-JEE- SINGLE CHOICE CORRECT

1. A child builds a tower from three blocks. The blocks are uniform cubes of side 2 cm. The blocks are initially lying on the same horizontal surface and each block has a mass of 0.1 kg. The work done by the child is

(a) 4 J (b) 0.04 J (c) 6 J (d) 0.06 J

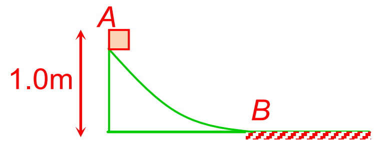
2. A long spring is stretched by 2 cm and its potential energy is *U*. If the spring is stretched by 10 cm, the potential energy stored in it will be

(a) *U*/25 (b) *U*/5 (c) 5*U* (d) 25*U*

3. A particle is acted upon by a conservative force $F = (7\hat{i} - 6\hat{j})$N. The work done by the force when the particle moves from origin (0, 0) to the position (-3m, 4m) is given by

(a) 3 J (b) 10 J (c) –45 J (d) none of these

4. A block weighing 10 N travels down a smooth curved track AB joined to a rough horizontal surface. The rough surface has a friction coefficient of 0.20 with the block. If the block starts slipping on the track from a point 1.0 m above the horizontal surface, then it would move a distance S on the rough surface. The value of S is [g = 10 m s^{-2}]

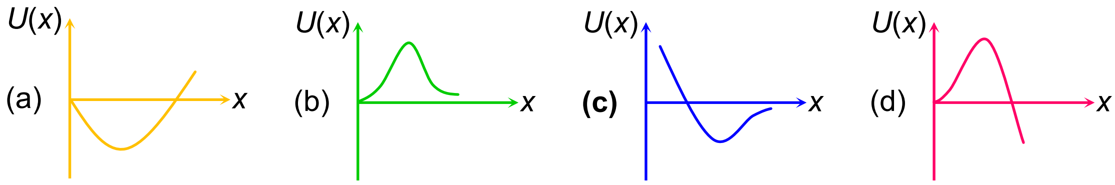


(a) *m* (b) 2*m* (c) 3*m* (d) 5*m*

5. The potential energy between the atoms in a molecule is given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$,where *a* and *b* are positive constants and *x* is the distance between the atoms. The atom is in stable equilibrium when

(a) $x = 0$ (b) $x = \left(\frac{a}{2b}\right)^{1/6}$ (c) $x = \left(\frac{2a}{b}\right)^{1/6}$ (d) $x = \left(\frac{11a}{5b}\right)^{1/6}$

6. A particle, which is constrained to move along the *X*-axis, is subjected to a force in the same direction which varies with the distance *x* of the particle from the origin as $F(x) = -kx + ax^3$. Here *k* and *a* are positive constants. For $x \geq 0$, the graph of the potential energy *U*(*x*) of the particle with its position is



7. A small block of mass m is kept on a rough inclined surface of inclination θ fixed in an elevator. The elevator goes up with a uniform velocity v and the block does not slide on the wedge. The work done by the force of friction on the block in time t will be

(a) zero (b) $mgvt \cos^2\theta$ (c) $mgvt \sin^2\theta$ (d) $mgvt \sin 2\theta$

8. A person wants to drive on the vertical surface of a large cylindrical wooden well commonly known as death well in a circus. The radius of well is R and the coefficient of friction between the tyres of the motorcycle and the wall of the well is μ_s. The minimum speed, the motorcyclist must have in order to prevent slipping should be

(a) $\sqrt{\frac{Rg}{\mu_s}}$ (b) $\sqrt{\frac{\mu_s}{Rg}}$ (c) $\sqrt{\frac{\mu_s g}{R}}$ (d) $\sqrt{\frac{R}{\mu_s g}}$

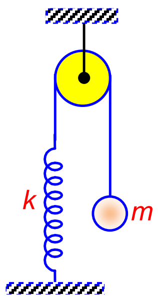
9. A stone of mass 1 kg tied to a light inextensible string of length $L = \frac{10}{3}$ is whirling in a circular path of radius L in vertical plane. If the ratio of the maximum tension to the minimum tension in the string is 4, what is the speed of stone at the highest point of the circle? (Taking g = 10 m/s^2)

(a) 10 m/s (b) $5\sqrt{2}$ m/s (c) $10\sqrt{3}$ m/s (d) 20 m/s

10. A block of mass m moving on a smooth horizontal plane with speed v compresses a spring through a distance x before its speed is halved. The spring constant of the spring is

(a) $mv^2/4x^2$ (b) $3mv^2/4x^2$ (c) $5mv^2/4x^2$ (d) $7mv^2/4x^2$

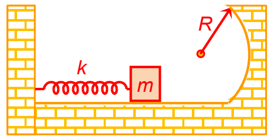
11. Consider the situation shown in the figure. Initially the spring is unstretched when the system is released from rest. Assuming no friction in the pulley, the maximum elongation of the spring is



(a) $\frac{mg}{k}$ (b) $\frac{2mg}{k}$

(c) $\frac{3mg}{k}$ (d) $\frac{4mg}{k}$

12. A smooth track is shown in the figure. A part of track is a circle of radius R. A block of mass m is pushed against a spring of constant k fixed at the left end and is then released. The initial compression of the spring so that the block presses the track with a force *mg* when it reaches the point *P* of the track, where radius of the track is horizontal is

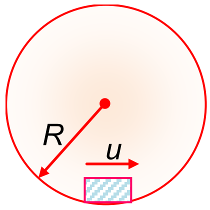


(a) $\sqrt{\frac{mgR}{3k}}$ (b) $\sqrt{\frac{3gR}{mk}}$ (c) $\sqrt{\frac{3mgR}{k}}$ (d) $\sqrt{\frac{3mg}{kR}}$

13. A particle of mass *m* is attached to one end of a string of length *l* while the other end is fixed to a point *h* above the horizontal table. The particle is made to revolve in a circle on the table, so as to make *P* revolutions per second. The maximum value of *P* if the particle is to be in contact with the table will be

(a) $2P\sqrt{gh}$ (b) $\sqrt{\left(\frac{g}{h}\right)}$ (c) $2P\sqrt{\left(\frac{h}{g}\right)}$ (d) $\frac{1}{2\pi}\sqrt{\left(\frac{g}{h}\right)}$

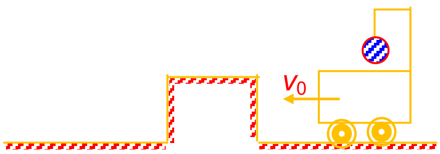
14. A particle is given an initial speed *u* inside a smooth spherical shell of radius *R* = 1 m that it is just able to complete the circle. Acceleration of the particle when its velocity becomes vertical is



(a) $g\sqrt{10}$ (b) *g*

(c) $g\sqrt{2}$ (d) $g\sqrt{6}$

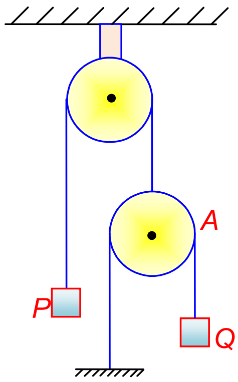
15. A bob is suspended from a crane by a cable of length *l* = 5m. The crane and load are moving at a constant speed v_0. The crane is stopped by a bumper and the bob on the cable swings out an angle of 60°. The initial speed v_0 is (g = 9.8 m/s²)



(a) 10 m/s (b) 7 m/s

(c) 4 m/s (d) 2 m/s

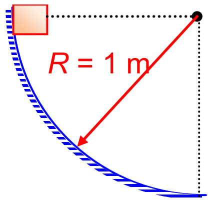
16. If the system in the Figure is released from rest in the configuration shown, find the velocity of the block *Q* after it has fallen through a distance 10 metres, given mass of *P* = mass of *Q* = 10 kg.



(a) 8 m/sec (b) 8.85 m/sec

(c) 9.5 m/sec (d) 10 m/sec

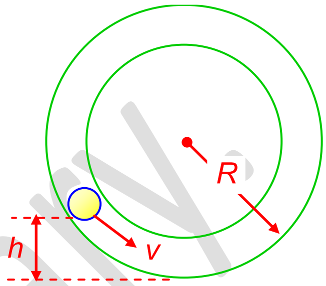
17. A block of mass 1 kg slides down a curved track that is one quadrant of a circle of radius 1m. Its speed at the bottom is 2 m/s. The work done by the frictional force is



(a) –8 J (b) +8 J

(c) 9 J (d) –9J

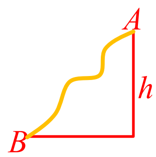
18. With what minimum speed v must a small ball should be pushed inside a smooth vertical tube from a height h so that it may reach the top of the tube? Radius of the tube is R.



(a) $\sqrt{2g(h+2R)}$ (b) $\frac{5}{2}R$

(c) $\sqrt{g(5R-2h)}$ (d) $\sqrt{2g(2R-h)}$

19. When an object is allowed to slide down a hill it stops at the point B because of friction. If friction force depends only on the normal component of the reaction force with coefficient of friction varies along the path, the work done in taking the object slowly from B to A along the hill will be



(a) mgh (b) 2 mgh (c) > 2 mgh (d) < mgh

20. A nail is located at a certain distance vertically below the point of suspension of a simple pendulum. The pendulum bob is released from a position when the string makes 60° with vertical, then the distance of nail from point of suspension such that the bob will just perform revolutions with nail as centre is (The length of pendulum is given as one metre)

(a) 80 cm above the point of suspension

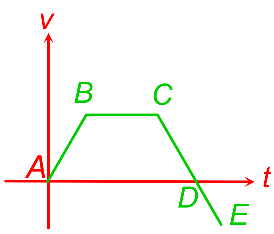
(b) 80 cm below the point of suspension

(c) 60 cm below the point of suspension

(d) 60 cm above the point of suspension

IIT-JEE MORE THAN ONE CHOICE

1. The velocity-time graph of a particle is shown in figure. The work done in the interval



(a) AB is positive (b) BC is positive

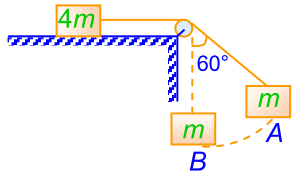
(c) CD is negative (d) DE is zero

2. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that

(a) its velocity is constant (b) its acceleration is constant

(c) its kinetic energy is constant (d) it moves in a circular path.

3. In the system shown in the figure, the mass m moves in a circular arc of angular amplitude 60°. The mass $4m$ remains stationary. Then



(a) the maximum value of coefficient of friction between the mass $4m$ and the surface is 0.5.

(b) the work done by gravitational force on the block m is positive when it moves from A to B.

(c) the power delivered by the tension when m moves from A to B is zero.

(d) the kinetic energy of m in position B equals the work done by gravitational force on the block when it moves from position A to B.

4. A sledge moving over a smooth horizontal surface of ice at velocity v_0 drives out on a horizontal road and comes to halt. The sledge has length l, mass m and friction coefficient between sledge and road is μ. Then

(a) no work is done by the friction to switch the sledge from ice to the road.

(b) a work of $\frac{\mu mgl}{2}$ is done against friction while sledge switches completely on to the road.

(c) the distance covered by the sledge on the road is $\frac{v_0^2}{2\mu g} - \frac{l}{2}$

(d) total distance moved by the sledge before stopping is $\frac{v_0^2}{2\mu g} + \frac{l}{2}$

5. The potential energy U for a force field $\vec{F}$ is such that $U = -Kxy$, where K is a constant. Then

(a) $\vec{F} = Ky\hat{i} + Kx\hat{j}$ (b) $\vec{F} = Kx\hat{i} + Ky\hat{j}$

(c) $\vec{F}$ is a conservative force (d) $\vec{F}$ is a non-conservative force

6. In projectile motion, power of the gravitational force

(a) is constant through out.

(b) is negative for first half, zero at topmost point and positive for rest half.

(c) varies linearly with time.

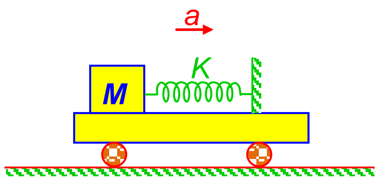
(d) is positive for complete path.

7. Consider two observers moving with respect to each other at a speed v along a straight line. They observe a block of mass m moving a distance l on a rough surface. The following quantities will be different as observed by the two observers

(a) kinetic energy of the block at time t (b) work done by friction

(c) total work done on the block (d) acceleration of the block

8. A block of mass M is attached with a spring of spring constant K. The whole arrangement is placed on a vehicle as shown in the figure. If the vehicle starts moving towards right with an acceleration a (there is no friction anywhere), then



(a) maximum elongation in the spring is $\frac{Ma}{K}$

(b) maximum elongation in the spring is $\frac{2Ma}{K}$

(c) maximum compression in the spring is $\frac{2Ma}{K}$

(d) maximum compression in the spring is zero.

9. A block is suspended by an ideal spring of force constant k. The block is pulled down by applying a constant force F and maximum displacement of block from its initial mean position is x_0. Then

(a) increase in energy stored in spring is kx_0^2

(b) $x_0 = \frac{3F}{2k}$

(c) $x_0 = \frac{2F}{k}$

(d) work done by applied force is Fx_0.

10. A block of mass M is hanging over a smooth and light pulley through a light string. The other end of the string is pulled by a constant force F. The kinetic energy of the block increases by 20 J in 1s.

(a) The total work done on the block in 1s is 20 J.

(b) The tension in the string is F

(c) The work done by the tension on the block is 20 J in 1 s.

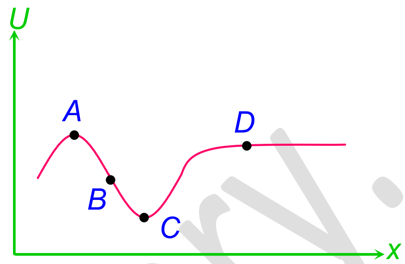
(d) The work done by the force of gravity is –20 J in 1 s.

EXERCISE – III

MATCH THE FOLLOWING

Note: Each statement in column – I has one or more than one match in column –II.

1. Potential energy of conservative field versus x graph is as shown in the figure, where x is the displacement in the direction of force. Four points A, B, C and D are marked in the graph. Match the column I with column II.



Column -I	Column -II
I. At point A	**A.** $\Sigma F_{net} = 0$
II. At point B	**B.** Potential energy is maximum.
III. At point C	**C.** Potential energy is minimum.
IV. At point D	**D.** $\Sigma F_{net} \neq 0$
	E. Potential energy is constant.

REASONING TYPE

Directions: Read the following questions and choose

(A) If both the statements are true and statement-2 is the correct explanation of **statement-1**.

(B) If both the statements are true but statement-2 is not the correct explanation of **statement-1**.

(C) If statement-1 is True and **statement-2** is False.

(D) If statement-1 is False **and statement-2** is True.

1. Statement-1: Non-conservative force always changes the mechanical energy.

Statement-2: Work done by non-conservative force is equal to the change in mechanical energy.

(a) (A) (b) (B) (c) (C) (d) (D)

2. Statement-1: Total mechanical energy is always greater than or equal to the potential energy.

Statement-2: Kinetic energy cannot have negative value.

(a) (A) (b) (B) (c) (C) (d) (D)

3. Statement-1: Work done by non-conservative force over a closed path cannot be zero.

Statement-2: Potential energy is not associated with the non-conservative force.

(a) (A) (b) (B) (c) (C) (d) (D)

4. **Statement-1:** When a body moves vertically upwards, then work done by gravity is negative.

Statement-2: When a body moves vertically upwards, its potential energy increases.

(a) (A) (b) (B) (c) (C) (d) (D)

5. **Statement-1:** In uniform circular motion, work done by tension in a loop is zero.

Statement-2: In uniform motion, tension is always perpendicular to the velocity.

(a) (A) (b) (B) (c) (C) (d) (D)

LINKED COMPREHENSION TYPE

A small sphere of mass m suspended by a thread is first taken aside so that the thread forms the right angle with the vertical and then released.

1. The total acceleration of the sphere as a function of angle θ with the vertical is

(a) $g\sqrt{1+\cos^2\theta}$ (b) $g\sqrt{1+3\cos^2\theta}$

(c) $g\cos\theta$ (d) $g\sin\theta$

2. Then tension in the string as a function of angle θ with the vertical is

(a) $3mg\cos\theta$ (b) $mg\cos\theta$

(c) $mg\sqrt{1+\cos^2\theta}$ (d) $mg\sqrt{1+3\cos^2\theta}$

3. The tension in the thread at the moment the vertical component of the sphere's velocity is maximum is

(a) mg (b) $\frac{mg}{\sqrt{2}}$

(c) $mg\sqrt{3}$ (d) $\frac{mg}{2}$

ANSWERS

EXERCISE – I

NEET-SINGLE CHOICE CORRECT

1. (b)	2. (a)	3. (d)	4. (b)	5. (a)
6. (b)	7. (d)	8. (c)	9. (b)	10. (c)
11. (d)	12. (d)	13. (b)	14. (b)	15. (a)
16. (b)	17. (c)	18. (c)	19. (a)	20. (b)
21. (b)	22. (c)	23. (c)	24. (a)	25. (c)

EXERCISE – II

IIT-JEE-SINGLE CHOICE CORRECT

1. (d)	2. (d)	3. (c)	4. (d)	5. (c)
6. (d)	7. (c)	8. (a)	9. (a)	10. (b)
11. (b)	12. (c)	13. (d)	14. (a)	15. (b)
16. (b)	17. (a)	18. (d)	19. (b)	20. (b)

ONE OR MORE THAN ONE CHOICE CORRECT

1. (a,c)	2. (c,d)	3.(a,b,c,d)	4.(b,c,d)	5. (a,c)
6. (b,c)	7.(a,b,c)	8. (b,d)	9. (c,d)	10. (a,b)

EXERCISE – III

MATCH THE FOLLOWING

1. I – A, B; II – D; III – A, C; IV – A, E

REASONING TYPE

1. (d)	2. (a)	3. (d)	4. (a)	5. (a)

LINKED COMPREHENSION TYPE

1. (b)	2. (a)	3. (c)