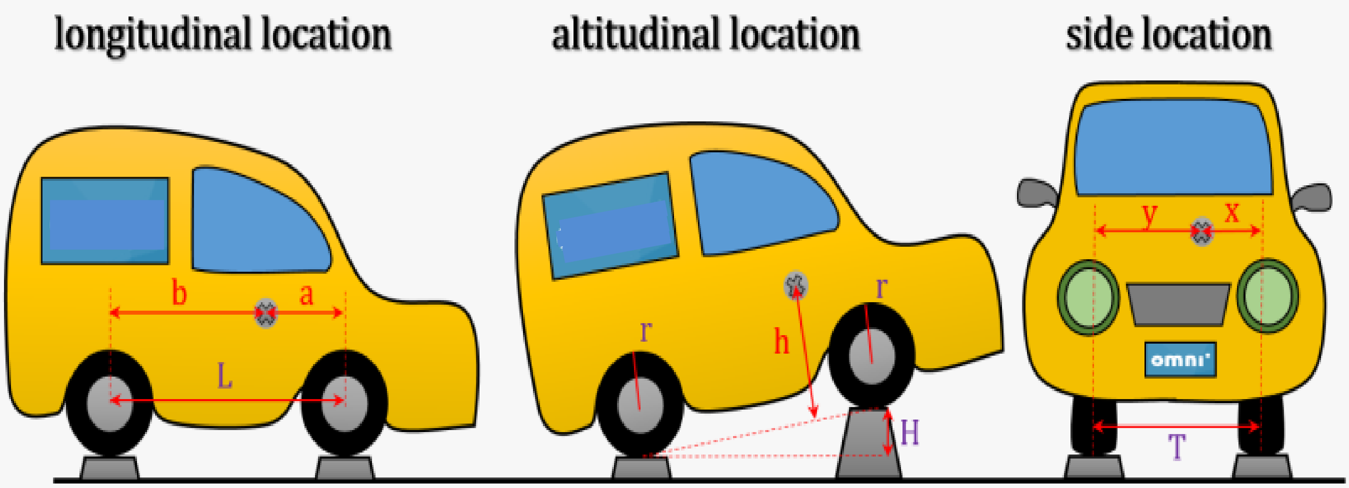
PHYSICS

11th

for JEE, NEET & BOARD

IMPULSE, MOMENTUM & CENTRE OF MASS



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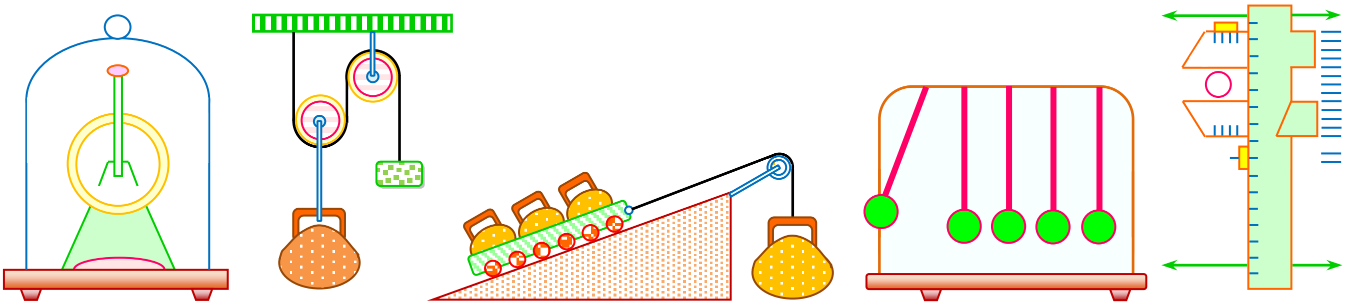

PHYSICS BOOKLET FOR JEE NEET & BOARDS

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IMPULSE, MOMENTUM & CENTER OF MASS

1 IMPULSE AND MOMENTUM

1.1 MOMENTUM

The linear momentum of particle is a vector quantity associated with quantity of motion. It is defined as product of mass of the particle and velocity of particle. i.e, linear momentum $\vec{P}$ of a particle of mass m, moving with velocity $\vec{v}$ is given by

$$\vec{P} = m\vec{v}$$

The direction of linear momentum is in the direction of velocity $\vec{v}$ of the particle. The SI unit for linear momentum is **kg ms^{-1}** and its dimension is **[MLT^{-1}]**.

Using Newton's second law of motion we can relate linear momentum of particle and net force acting on it. The time rate of charge of linear momentum is equal to the resultant force acting on the particle.

This is, $\vec{F} = \dfrac{d\vec{P}}{dt}$

1.2 IMPULSE OF FORCE AND CONSERVATION OF LINEAR MOMENTUM

As we have seen, the force is related to momentum as

$$\vec{F} = \frac{d\vec{P}}{dt} \Rightarrow \vec{F}\,dt = d\vec{P}$$

If momentum of particle changes from $\vec{P}_i$ to $\vec{P}_f$ during a time interval of t_i to t_f, we can write

$$\int_{t_i}^{t_f} \vec{F}\,dt = \int_{\vec{P}_i}^{\vec{P}_f} d\vec{P}$$

$$\Rightarrow \quad \int_{t_i}^{t_f} \vec{F}dt = \vec{P}_f - \vec{P}_i = \Delta\vec{P}$$

The quantity on the left hand side of this equation is called the impulse of force for the time interval $\Delta t = t_f - t_i$. Impulse is represented by $\vec{J}$ and is given by

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}\,dt = \Delta\vec{P}$$

That is, "*The impulse of force equals the change in momentum of the particle*." This statement called '*impulse momentum theorem*,' is equivalent to Newton's second law.

From the equation of impulse, we can see that impulse is a vector quantity having magnitude equal to the area under force-time curve as shown in figure by the shaded area.

Since the force can vary with time, we can define average force $\overline{\vec{F}}$ as

$$\overline{\vec{F}} = \frac{1}{\Delta t}\int_{t_i}^{t_f} \vec{F}\, dt$$

Therefore we can also write,

$$\overline{\vec{F}}\ \Delta t = \Delta\vec{P}$$

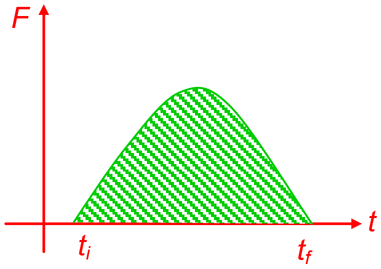


Fig. 1.

From the equation $\vec{F} = \frac{d\vec{P}}{dt}$, we can see that if the resultant force is zero, the time derivative of the momentum is zero and therefore the linear momentum of a particle is constant. This is called 'conservation of linear momentum. This conservation principle, we apply for a particle as well as system of particles also. Hence we can define conservation of linear momentum as

"when the net external force acting on a system is zero, the total linear momentum of the system remains constant".

Illustration 1

Question: **In a particular crash test, an automobile of mass 1500 kg collides with a wall as in figure (a). The initial and final velocities of the automobile are v_i = 15.0 m/s and v_f = 2.6 m/s. If the collision lasts for 0.150 s, find the average force exerted on the automobile.**

Solution:

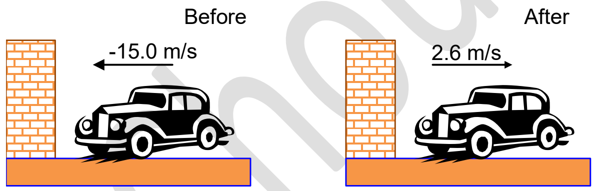


Fig. (a) **Fig. (b)**

The initial and final momenta of the automobile are (taking rightward as positive)

$p_i = mv_i$ = (1500 kg) (–15.0 m/s) = – 2.25 × 10^4 kg.m/s

$p_f = mv_f$ = (1500 kg) (2.6 m/s) = 0.39 × 10^4 kg.m/s

Hence, the impulse is

$J = \Delta p = p_f - p_i$ = 0.39 × 10^4 kg.m/s – (–2.25 × 10^4 kg.m/s)

J = 2.64 × 10^4 kg.m/s

The average force exerted on the automobile is given by

$$\overline{F} = \frac{\Delta p}{\Delta t} = \frac{2.64\times10^4\ kg.m/s}{0.150 s} = 1.76 \times 10^5\ \text{N} = \mathbf{176\ M\ N}$$

Illustration 2

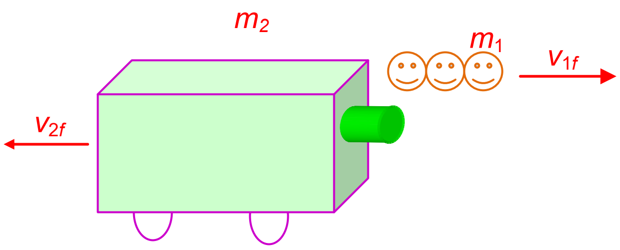
Question: **A baseball player uses a pitching machine to help him improve his batting average. He places the 50-kg machine on a frictionless surface as in figure. The machine fires a 0.15 kg baseball horizontally with a velocity of 36 m/s. What is the recoil speed(in cm/s) of the machine?**

Solution: We take the system which consists of the baseball and the pitching machine. Because of the force of gravity and the normal force, the system is not really isolated. However, both of these forces are directed perpendicularly to the motion of the system. Therefore, momentum is constant in the x-direction because there are no external forces in this direction (as the surface is frictionless).

The total momentum of the system before firing is zero ($m_1v_{1i} + m_2v_2i = 0$). Therefore, the total momentum after firing must be zero; that is,

$$m_1v_{1f} + m_2v_{2f} = 0$$

With m_1 = 0.15 kg, v_{1i} = 36 m/s, and m_2 = 50 kg, solving for v_{2f}, we find the recoil velocity of the pitching machine to be



$$v_{2f} = -\frac{m_1}{m_2}v_{1f} = -\left(\frac{0.15\,\text{kg}}{50\text{ kg}}\right)(36\text{ m/s}) = -0.11\text{ m/s}$$

∴ speed = **11 cm/s**

The negative sign for v_{2f} indicates that the pitching machine is moving to the left after firing, in the direction opposite the direction of motion of the cannon. In the words of Newton's third law, for every force (to the left) on the pitching machine, there is an equal but opposite force (to the right) on the ball. Because the pitching machine is much more massive than the ball, the acceleration and consequent speed of the pitching machine are much smaller than the acceleration and speed of the ball.

2 IMPACT

A collision between two bodies which occurs in a very small interval of time and during which the two bodies exert relatively large forces on each other is called impact. The common normal to the surfaces in contact during the impact is called the line of impact. If centers of mass of the two colliding bodies are located on this line, the impact is a central impact. Otherwise, the impact is said to be eccentric. Our present study will be limited to the central impact of two particles. The analysis of the eccentric impact of two rigid bodies will be considered later.

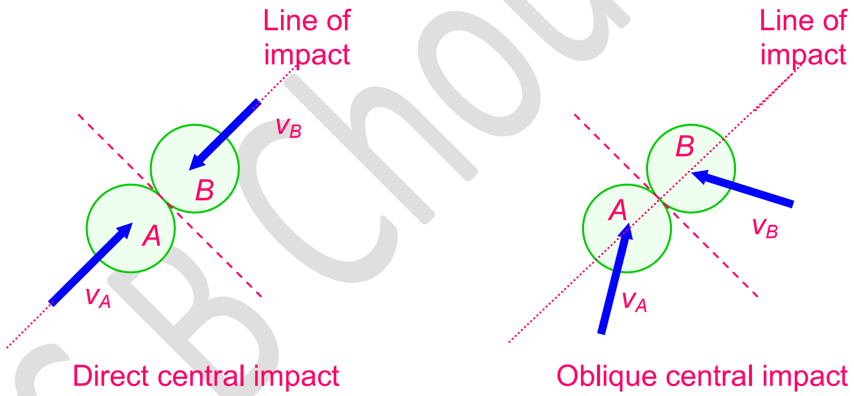


Fig. 2 (a) Fig. 2 (b)

If the velocities of the two bodies are directed along the line of impact, the impact is said to be a direct impact as shown in figure 2(a). If either or both bodies move along a line other than the line of impact, the impact is said to be an oblique impact as in figure 2(b).

2.1 DIRECT CENTRAL IMPACT OR HEAD ON IMPACT

Consider two spheres A and B of mass m_A and m_B, which are moving in the same straight line and to the right with known velocities v_A and v_B as shown in figure. If v_A is larger than v_B, particle A will eventually strike the sphere B. Under the impact, the two spheres will deform and at the end of the period of deformation, they will have the same velocity u as shown in figure. A period of restitution will then take place, at the end of which, depending upon the magnitude of the impact forces and upon the materials involved, the two spheres either will have regained their original shape or will stay permanently deformed. Our purpose here is to determine the velocities v'_A and v'_B of the spheres at the end of the period of restitution as shown in figure.

Considering first the two spheres as a single system, we note that there is no impulsive, external force. Thus, the total momentum of the two particles is conserved, and we write

$$m_Av_A + m_Bv_B = m_Av'_A + m_Bv'_B \qquad \ldots\text{(i)}$$

Since all the velocities considered are directed along the same axis, we had written the relation involving only scalar components.

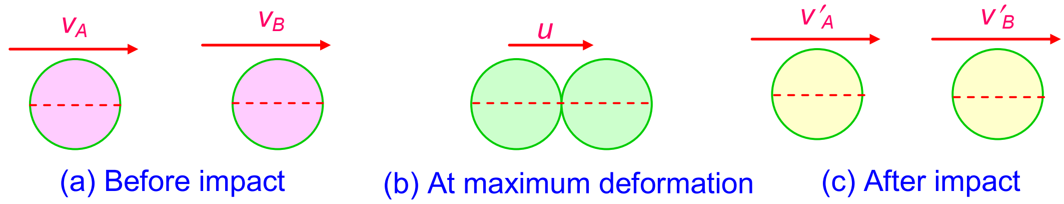


Fig. 3.

A positive value for any of the scalar quantities v_A, v_B, v'_A, or v'_B means that the corresponding vector is directed to the right; a negative value indicates that the corresponding vector is directed to the left.

To obtain the velocities v'_A and v'_B, it is necessary to establish a second relation between the scalars v'_A and v'_B. For this purpose, we use Newton's law of restitution according to which velocity of separation after impact is proportional to the velocity of approach before collisions. In the present situation,

$(v_B' - v'_A) \propto (v_A - v_B)$

or, $(v_B' - v_A') = e\,(v_A - v_B)$ …(ii)

Here e is a constant called as coefficient of restitution. Its value depends on type of collision. The value of the coefficient e is always between 0 and 1. It depends to a large extent on the two materials involved, but it also varies considerably with the impact velocity and the shape and size of the two colliding bodies.

Two particular cases of impact are of special interest.

(i) $e = 0$, Perfectly Plastic Impact. When $e = 0$, equation (ii) yields $v'_B = v'_A$. There is no period of restitution, and both particles stay together after impact. Substituting $v'_B = v'_A = v'$ into equation (i), which expresses that the total momentum of the particles is conserved,

we write, $m_A v_A + m_B v_B = (m_A + m_B)v'$

This equation can be solved for the common velocity v' of the two particles after impact.

(ii) $e = 1$, Perfectly Elastic Impact. When $e = 1$, equation (ii) reduce to

$v'_B - v'_A = v_A - v_B$

Which expresses that the relative velocities before and after impact are equal. The impulses received by each particle during the period of deformation and during the period of restitution are equal. The particles move away from each other after impact with the same velocity with which they approached each other before impact. The velocities v'_A and v'_B can be obtained by solving equation (i) and (ii) simultaneously.

It is worth noting that in the case of a perfectly elastic impact, the total energy of the two particles, as well as their total momentum, is conserved.

It should be noted, however, that in the general case of impact, i.e., when e is not equal to 1, the total energy of the particles is not conserved. This can be shown in any given case by comparing the kinetic energies before and after impact. Some part of the lost kinetic energy transformed into heat and some part spent in generating elastic waves within the two colliding bodies.

2.2 OBLIQUE CENTRAL IMPACT OR INDIRECT IMPACT

Let us now consider the case when the velocities of the two colliding sphere are not directed along the line of impact as shown in figure. As already discussed the impact is said to be oblique. Since velocities v'_A and v'_B of the particles after impact are unknown in direction and magnitude, their determination will require the use of four independent equations.

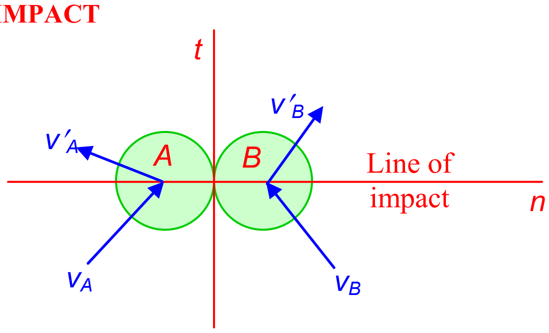


Fig. 4.

We choose as coordinate axes the *n*-axis along the line of impact, i.e., along the common normal to the surfaces in contact, and the *t*-axis along their common tangent. Assuming that the sphere are perfectly smooth and frictionless, we observe that the only impulses exerted on the sphere during the impact are due to internal forces directed along the line of impact i.e., along the *n* axis. It follows that

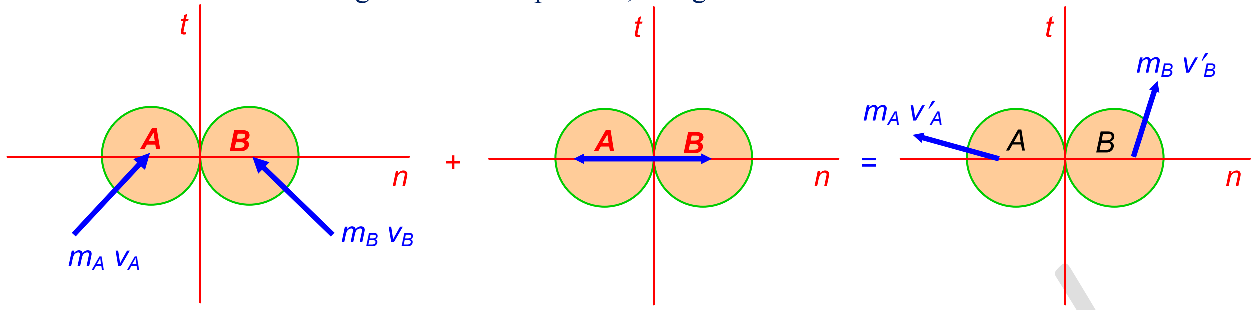


Fig. 5.

(i) The component along the *t* axis of the momentum of each particle, considered separately, is conserved; hence the *t* component of the velocity of each particle remains unchanged. We can write.

$$(v_A)_t = (v'_A)_t\ ;\ (v_B)_t = (v'_B)_t$$

(ii) The component along the *n* axis of the total momentum of the two particles is conserved. We write.

$$m_A(v_A)_n + m_B(v_B)_n = m_A(v'_A)_n + m_B(v'_B)_n$$

(iii) The component along the *n* axis of the relative velocity of the two particles after impact is obtained by multiplying the *n* component of their relative velocity before impact by the coefficient of restitution.

$$(v'_B)_n - (v'_A)_n = e[(v_A)_n - (v_B)_n]$$

We have thus obtained four independent equations, which can be solved for the components of the velocities of *A* and *B* after impact.

Illustration 3

Question: **A block of mass 1.2 kg moving at a speed of 20 cm/s collides head on with a similar block kept at rest. The coefficient of restitution is 0.6. Find the loss of kinetic energy (in μJ) during collision.**

Solution: Suppose the first block moves at a speed v_1 and the second at v_2 after collision. Since the collision is head on, the two blocks move along the original direction of motion of first block. Using the principle of conservation of momentum,

$(1.2 \times 0.2) = 1.2\, v_1 + 1.2\, v_2$

$v_1 + v_2 = 0.2$ … (i)

By Newton's law of restitution,

$v_2 - v_1 = -\,e\,(u_2 - u_1)$

$v_2 - v_1 = -\,0.6\,(0 - 0.2)$

$v_2 - v_1 = 0.12$ … (ii)

Adding equations (i) and (ii),

$2v_2 = 0.32$

$v_2 = 0.16$ m/s or 16 cm/s

$v_1 = 0.2 - 0.16 = 0.04$ m/s $= 4$ cm/s

Loss of K.E. $= \frac{1}{2} \times 1.2 \times (0.2)^2 - \frac{1}{2} \times 1.2 \times (0.16)^2 - \frac{1}{2} \times 1.2 \times (0.04)^2$

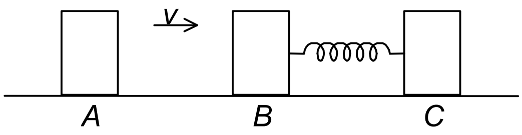
$= 0.6\,[0.04 - 0.0256 - 0.0016]$

$= 0.6 \times 0.0128$

$= 7.7 \times 10^{-3}$ J $=$ **7700 μ J**

Illustration 4

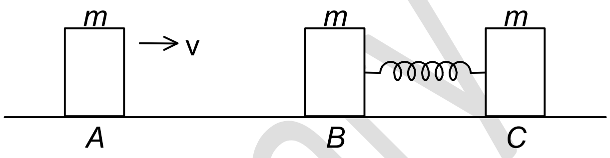
Two blocks B and C of mass m = 10kg each connected by a spring of natural length □ and spring constant k = 5 N/m rest on an absolutely smooth horizontal surface as shown in Figure. A third block A of same mass collides elastically block B velocity v = 1m/s. Calculate the velocities of blocks, when the spring is compressed as much as possible and also the maximum compression.



Solution:

Let A be the moving block and B and C the stationary blocks.

Since A and B are of equal mass, A is stopped dead and B takes off with its velocity. Now B and C move under their mutual action and reaction and so their momentum is conserved.



Let v_1 and v_2 be their instantaneous velocities when the compression of spring is x.

By the principle of conservation of momentum,

$mv = m(v_1 + v_2)$

$v_1 + v_2 = v$ (a constant)

By the principle of conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{2}kx^2$$

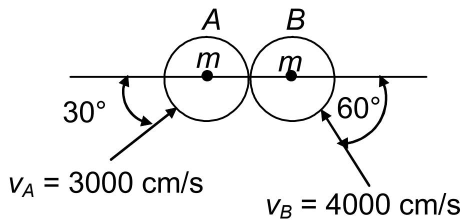
For maximum compression $v_1 = v_2 = \frac{v}{2}$

$$\Rightarrow \quad \frac{k}{m}x_{max}^2 = v^2 - \frac{v^2}{2} = \frac{v^2}{2},$$

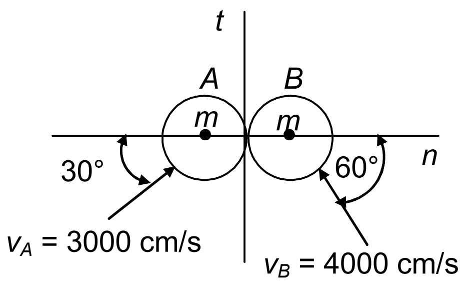
$$x_{max} = \sqrt{\frac{m}{2k}}\cdot v = \mathbf{1m}$$

Illustration 5

Question: **The magnitude and direction of the velocities of two identical frictionless balls before they strike each other are as shown. Assuming e = 0.90, determine the magnitude of the velocity of the each ball after the impact.**



Solution: The impulsive force that the balls exert on each other during the impact are directed along a line joining the centers of the balls called the line of impact. Resolving the velocities into components directed, respectively, along the line of impact and along the common tangent to the surfaces in contact, we write



$(v_A)_n = v_A \cos 30° = +2600$ cm/s

$(v_A)_t = v_A \sin 30° = +1500$ cm/s

$(v_B)_n = -v_B \cos 60° = -2000$ cm/s

$(v_B)_t = v_B \sin 60° = +3460$ cm/s

Since the impulsive forces are directed along the line of impact, the t component of the momentum, and hence the t component of the velocity of each ball, is unchanged. We have,

$(v'_A)_t = 1500$ cm/s $\uparrow$, $(v'_B)_t = 3460$ cm/s $\uparrow$

In the *n* direction, we consider the two balls as a single system and not that by Newton's third law, the internal impulses are, respectively, $F\Delta t$ and $-F\Delta t$ and cancel. We thus write that the total momentum of the balls is conserved.

$$m_A(v_A)_n + m_B(v_B)_n = m_A(v'_A)_n + m_B(v'_B)_n$$

$$m(2600) + m(-2000) = m(v'_A)_n + m(v'_B)_n$$

$$(v'_A)_n + (v'_B)_n = 600 \qquad \text{... (i)}$$

Using law of restitution,

$$(v'_B)_n - (v'_A)_n = e[(v_A)_n - (v_B)_n]$$

$$(v'_B)_n - (v'_A)_n = (0.90)[2600 - (-2000)]$$

$$(v'_A)_n + (v'_A)_n = 4140 \qquad \text{... (ii)}$$

Solving equations (i) and (ii) simultaneously, we obtain

$$(v'_A)_n = -1770\ \text{cm/s} \qquad (v'_B)_n = +2370\ \text{cm/s}$$

$$(v'_A)_n = 1770\ \text{cm/s} \leftarrow \qquad (v'_B)_n = 2370\ \text{cm/s} \rightarrow$$

Resultant Motion: Adding vectorially the velocity components of each ball, we obtain

$v_A' =$ **2320 cm/s** $v_B' =$ **4190 cm/s**

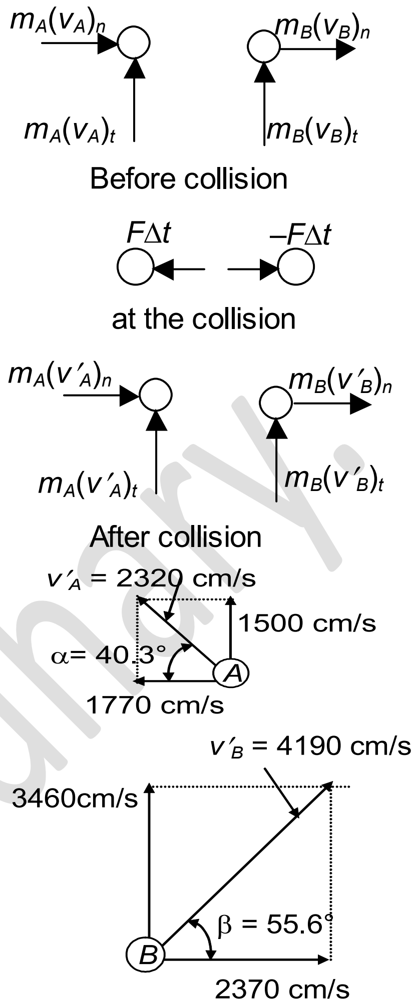
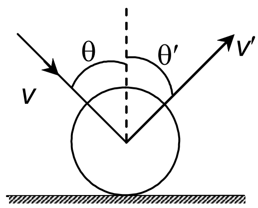


Illustration 6

Question: **A ball of mass *m* hits a floor with a speed *v* making an angle of incidence θ = 45° with normal. The coefficient of restitution is *e* = 3/4. Find the speed of reflected ball and the angle of reflection.**

Solution: Suppose the angle of reflection is θ′ and the speed after collision is *v*′. It is an oblique impact. Resolving the velocity *v* along the normal and tangent, the components are *v* cosθ and *v* sinθ. Similarly, resolving the velocity after reflection along the normal and along the tangent the components are – *v*′ cosθ′ and *v*′ sinθ′.



Since there is no tangential action,

$v \sin\theta = v'\sin\theta'$ ... (i)

Applying Newton's law for collision,

$(-v'\cos\theta' - 0) = -e(v\cos\theta - 0)$

$\therefore\ v'\cos\theta' = ev\cos\theta$ ... (ii)

From equations (i) and (ii),

$v'^2 = v^2\sin^2\theta + e^2 v^2\cos^2\theta$

$$v' = \sqrt{v^2\sin^2\theta + e^2v^2\cos^2\theta}$$

$$v' = \left(v\sqrt{\sin^2\theta + e^2\cos^2\theta}\right) = \frac{5v}{4\sqrt{2}}$$

and $\tan\theta' = \dfrac{\tan\theta}{e}$

$$\theta' = \tan^{-1}\left(\frac{\tan\theta}{e}\right) = \mathbf{53°}$$

3 SYSTEMS OF VARIABLE MASS

We recall that all the principles established so for were derived for the systems which neither gain nor lose mass. But there are various situations in which system loses or gains mass during its motion. e.g. in case of Rocket propulsion, its motion depends upon the continued ejection of fuel from it.

Let us analyse a system of variable mass. Consider the system S shown in figure. Its mass, equal to m at the instant t increases by Δm in the internal of the Δt. The velocity of S at time t is $\vec{v}$ and the velocity of S at time $t + \Delta t$ becomes $\vec{v} + \Delta\vec{v}$, and the absolute velocity of mass absorbed is $\vec{v}_a$ with respect to stationary frame. $\sum \vec{F}_{ext}$ is net external force acting on it during internal Δt

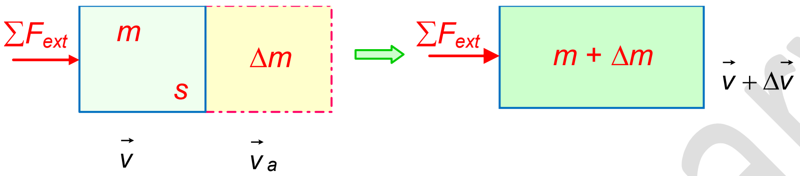


Fig: 6

Applying the Impulse-Momentum theorem,

$$m\vec{v} + \Delta m \vec{v}_a + \sum \vec{F}_{ext}\,\Delta t = (m+\Delta m)\,(\vec{v} + \Delta\vec{v})$$

$$\Rightarrow \qquad \sum \vec{F}_{ext}\,\Delta t = m\Delta\vec{v} + \Delta m\,(\vec{v} - \vec{v}_a) + (\Delta m)\,(\Delta v)$$

Here $\vec{v}_a - \vec{v}$ is relative velocity of mass absorbed with respect to system S, let us write it as $\vec{v}_{rel}$. also last term $\Delta m\ \Delta v$ can be neglected.

We can write, $\sum \vec{F}\,\Delta t = m\Delta\vec{v}$ - (Δm) $\vec{v}_{rel}$

Dividing both sides by Δt and letting Δt approaches zero, we have $\sum \vec{F} = m\dfrac{d\vec{v}}{dt} - \dfrac{dm}{dt}\vec{v}_{rel}$

Rearranging the terms and recalling $\dfrac{d\vec{v}}{dt} = \vec{a}$ where $\vec{a}$ is acceleration of system,

we can write

$$\sum \vec{F}_{ext} + \frac{dm}{dt}\vec{v}_{rel} = m\vec{a}$$

Which shows that the action on S of the mass being absorbed is equivalent to a thrust force $\vec{F}_{th}$ given by,

$$\vec{F}_{th} = \frac{dm}{dt}\vec{v}_{rel}$$

Therefore while analyzing systems of variable mass, we need to consider external forces acting on it as well as a thrust force having magnitude equal to the product of rate at which mass of system changes and the relative velocity of mass coming into the system or going out of the system with respect to the system. If mass of system is increasing, then the direction of thrust is same as that of relative velocity $\vec{v}_{rel}$ and vice versa.

Once we consider the thrust force with the net external force, a system of variable mass can be analyzed in the same way as we analyse systems of constant mass by considering external forces only.

Illustration 7

Question: **The mass of a rocket is 2.8×10^6 kg at launch time of this 2×10^6 kg is fuel. The exhaust speed is 2500 m/s and the fuel is ejected at the rate of 1.4×10^4 kg/s.**

(a) Find thrust on the rocket in mega newton.

(b) What is initial acceleration (in cm/s^2) at launch time? Ignore air resistance.

Solution:

(a) The magnitude of thrust is given by

$= (1.4 \times 10^4 \text{ kg/s}) \times (2500 \text{ ms}^{-1})$

$= 3.5 \times 10^7 \text{ N} = \mathbf{35\ M\ N}$

$$F_{th} = \frac{dM}{dt} v_{rel}.$$

The direction of thrust will be opposite to the direction of relative velocity as mass is decreasing, i.e., upward

(b) to find acceleration, we can use

$$\sum \vec{F}_{ext} + \vec{F}_{th} = M\vec{a}$$

Here external force $\vec{F}$ is weight acting downward and thrust force F_{th} is upward

$\therefore \quad -mg + F_{th} = Ma$ (Taking upward as positive)

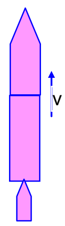
$\Rightarrow \quad a = g - \dfrac{F_{th}}{M}$

$$= \left(-9.8 + \frac{3.5 \times 10^7}{2.8 \times 10^6}\right) \text{ms}^{-2}$$

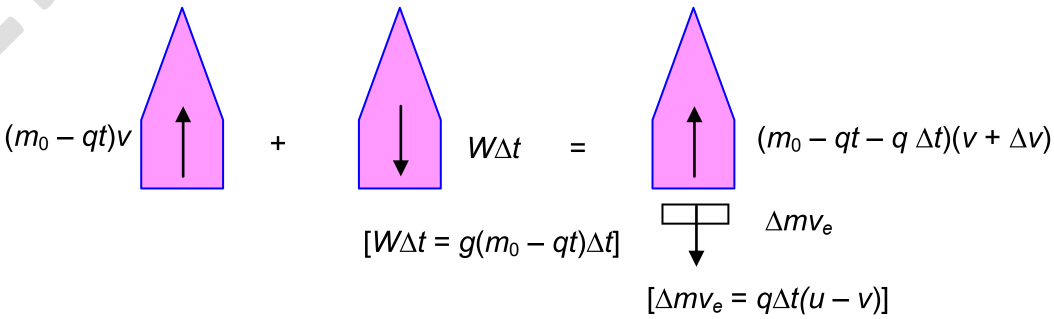
$= \mathbf{270\ cms^{-2}}$

Illustration 8

Question: **A rocket of initial mass m_0 = 2000 kg (including shell and fuel) is fired vertically at time t = 0. The fuel is consumed at a constant rate $q = dm/dt$ = 1000 kg/s and is expelled at a constant speed u = 100m/s relative to the rocket. Find the magnitude of the velocity of the rocket at time t = 1s, neglecting the resistance of the air and variation of acceleration due to gravity. (ln 2 = 0.7)**



Solution: At time t, the mass of the rocket shell and remaining fuel is $m = m_0 - qt$, and the velocity is v. During the time interval Δt, a mass of fuel $\Delta m = q\,\Delta t$ is expelled with a speed u relative to the rocket. Denoting by v_e the absolute velocity of expelled fuel, we apply the principle of impulse and momentum between time t and time $t + \Delta t$.



We write

$$(m_0 - qt)v - g(m_0 - qt)\,\Delta t = (m_0 - qt - q\,\Delta t)\,(v + \Delta v) - q\Delta t\,(u - v)$$

Dividing throughout by Δt and letting Δt approach zero, we obtain

$$-g(m_0 - qt) = (m_0 - qt)\frac{dv}{dt} - qu$$

Separating variables and integrating from $t = 0$, $v = 0$ to $t = t$, $v = v$

$$dv = \left(\frac{qu}{m_0 - qt} - g\right)dt$$

$$\int_0^v dv = \int_0^t \left(\frac{qu}{m_0 - qt} - g\right) dt$$

$\Rightarrow$ $v = [u \ln (m_0 - qt) - gt]_0^t$

$\therefore$ $v = u \ln\left(\frac{m_0}{m_0 - qt}\right) - gt$ **= 60m/s**

PROFICIENCY TEST - I

The following questions deal with the basic concepts of this section. Answer the following briefly. Go to the next section only if your score is at least 80%. Do not consult the Study Material while attempting the questions.

1. A force acts on a mass of 40 kg and changes its velocity from 3 m/s to 12 m/s. Find the impulse of the force.

2. A cricket ball of mass 150 g moving at 30 m/s strikes a bat and returns back along the same line at 20 m/s. If the ball is in contact with the bat for 0.02 second, find the force exerted by the bat on the ball.

3. A 5 kg body has an initial velocity of 10 m/s to the right and a 10 kg body has a velocity 2 m/s towards the left. Both of them collide and stick together after the collision. With what velocity would they move after the collision?

4. An explosive shell of mass 10 kg at rest suddenly explodes into two pieces. If one piece with mass 4 kg is found to move with a velocity 6 m/s towards east, find the velocity of the other piece.

5. A ball moving with a speed of 9 m/s strikes an identical stationary ball such that after collision, the direction of each ball makes an angle of 60° with the original line of motion. Find the speeds of the two balls after collision. Is the kinetic energy conserved in the collision process?

6. If the kinetic energy of a particle is zero, what is its linear momentum? If the total energy of a particle is zero, is its linear momentum necessarily zero? Explain.

7. If two particles have equal kinetic energies, are their momenta necessarily equal? Explain.

8. If two objects collide and one is initially at rest, is it possible for both to be at rest after the collision? Is it possible for one to be at rest after the collision? Explain.

9. Is it possible to have a collision in which all of the kinetic energy is lost? If so, give an example.

10. Explain how linear momentum is conserved when a ball bounces from a floor.

ANSWERS TO PROFICIENCY TEST - I

1. 360 N-s

2. 375 N

3. 2 m/s towards the right

4. 4 m/s towards west

5. $v_1 = v_2 = 9$ m/s; Not conserved

4 SYSTEMS OF PARTICLES: CENTRE OF MASS

Until now we have dealt mainly with single particle. Bodies like block, man, car etc. are also treated as particles while describing its motion. The particle model was adequate since we were concerned only with translational motion. When the motion of a body involves rotation and vibration, we must treat it as a system of particles. In spite of complex motion of which a system is capable, there is a single point, the centre of mass (CM), whose translational motion is characteristic of the system as a whole. Here we shall discuss about location of centre of mass of a system of particles and its motion.

4.1 LOCATION OF CENTRE OF MASS

Consider a set of n particles whose masses are m_1, m_2, m_3 …m_i … m_n and whose position vectors relative to an origin O are $\vec{r}_1, \vec{r}_2, \vec{r}_3 \quad ...\vec{r}_i \quad ...\vec{r}_n$ respectively.

The centre of mass of this set of particles is defined as the point with position vector $\vec{r}_{CM}$

where, $$\vec{r}_{CM} = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{\sum_{i=1}^{n} m_i}$$

In component form above equation can be written as

$$X_{CM} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{i=n} m_i} = \frac{m_1x_1 + m_2x_2 + ...m_nx_n}{m_1 + m_2 + ... + m_n}$$

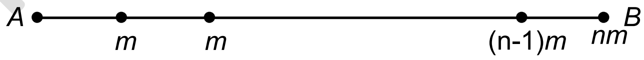
$$Y_{CM} = \frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{i=n} m_i} = \frac{m_1y_1 + m_2y_2 + ...m_ny_n}{m_1 + m_2 + ... + m_n}$$

$$Z_{CM} = \frac{\sum_{i=1}^{i=n} m_i z_i}{\sum_{i=1}^{i=n} m_i} = \frac{m_1z_1 + m_2z_2 + ...m_nz_n}{m_1 + m_2 + ... + m_n}$$

Illustration 9

Question: ***AB* is a light rod of length *n* = 4cm. To the rod masses *m*, 2*m*, 3*m*,… *nm* are attached at distances 1, 2, 3, …….. *n* cm respectively from *A*. Find the distance from *A* of the centre of mass of rod.**

Solution:



Let us take origin at A, then distance of CM from A (origin) X_{CM} can be written as

$$X_{CM} = \frac{m.1 + 2m.2 + 3m.3 + ... + nm.n}{m + 2m + 3m + ... + nm} cm = \frac{m[1^2 + 2^2 + 3^2 + ... + n^2]}{m[1 + 2 + 3 + ... + n]} \text{cm}$$

$$= \frac{n(n+1)(2n+1)/6}{n(n+1)/2} = \frac{2n+1}{3} \text{cm} = \mathbf{3\ cm}$$

4.2 VELOCITY AND ACCELERATION OF THE CENTRE OF MASS

By definition position vector of centre of mass,

$$\vec{r}_{CM} = \frac{\Sigma m_i \vec{r}_i}{\Sigma m_i}$$

Differentiating once w.r.t. time, we will get velocity the centre of mass

$$\vec{V}_{CM} = \frac{d\vec{r}_{CM}}{dt} = \frac{\Sigma\left(m_i \frac{d\vec{r}_i}{dt}\right)}{\Sigma m_i} = \frac{\Sigma(m_i \vec{v}_i)}{\Sigma m_i}$$

Differentiating once more w.r.t. time, we will get acceleration of the centre of mass

$$\vec{a}_{CM} = \frac{d\vec{v}_{CM}}{dt} = \frac{\Sigma\left(m_i \frac{d\vec{v}_i}{dt}\right)}{\Sigma m_i} = \frac{\Sigma(m_i \vec{a}_i)}{\Sigma m_i}$$

4.3 EQUATION OF MOTION FOR A SYSTEM OF PARTICLES

Acceleration of centre of mass $\vec{a}_{CM}$ is given by $\vec{a}_{CM} = \frac{\Sigma m_i \vec{a}_i}{\Sigma m_i} = \frac{1}{M}\Sigma m_i.\vec{a}_i$

Rearranging the expression and using Newton's second law, we get

$$M\vec{a}_{CM} = \Sigma m_i \vec{a}_i = \Sigma \vec{F}_i$$

where $\vec{F}_i$ is the force on *i* th particle.

The force on any particle in the system may include both external forces (from outside the system) and internal forces (from within the system). However by Newton's third law, the force exerted by particle 1 on particle 2, is equal to and opposite the force exerted by particle 2 on particle 1. Thus, when we sum over all internal forces in above equation they cancel in pairs and the net force is only due to external forces. Thus we can write equation of motion of centre of mass in the form.

$$\Sigma \vec{F}_{ext} = M\vec{a}_{CM}$$

Thus the acceleration of the centre of mass of a system is the same as that of a particle whose mass is total mass of the system, acted upon by the resultant external forces acting on the system.

If $\Sigma \vec{F}_{ext} = 0$, then centre of mass of system will move with uniform speed and if initially it were at rest it will remains at rest.

Illustration 10

Question: **A man weighing 70 kg is standing at the centre of a flat boat of mass 350 kg. The man who is at a distance of 10 m from the shore walks 2 m towards it and stops. How far will he be from the shore? Assume the boat to be of uniform thickness and neglect friction between boat and water.**

Solution: Consider that the boat and the man on it constitute a system. Initially, before the man started walking, the centre of mass of the system is at 10 m away from the shore and is at the centre of the boat itself. The centre of mass is also initially at rest.

As no external force acts on this system, the centre of mass will remain stationary at this position.

Let us take this point as the origin and the direction towards the shore as x-axis.

If x_1 and x_2 be the position coordinates of man and centre of boat respectively, at any instant, position coordinate of the centre of mass

$$x_c = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

i.e., $$0 = \frac{70x_1 + 350x_2}{70 + 350}$$

$$x_1 + 5x_2 = 0 \qquad \text{.... (i)}$$

Also, $$x_1 - x_2 = 2 \qquad \text{.... (ii)}$$

Solving equations (i) and (ii),

$$x_1 = \frac{5}{3}m$$

Since the centre of mass of the system remains stationary the man will be at a distance

$10 - \frac{5}{3} =$ **830 cm** from the shore.

4.4 LINEAR MOMENTUM OF A SYSTEM OF PARTICLES

Velocity of centre of mass of a system of particles $\vec{V}_{CM}$ is given by

$$\vec{V}_{CM} = \frac{\Sigma m_i \vec{v}_i}{\Sigma m_i} = \frac{\Sigma m_i \vec{v}_i}{M}$$

Rearranging equation we have,

$$M\vec{V}_{CM} = \Sigma m_i v_i = \Sigma \vec{P}_i = \vec{P}$$

where $\vec{P}$ is total momentum of system.

Thus we conclude that the total linear momentum of the system equals the total mass multiplied by the velocity of centre of mass. In other words, the total linear momentum of the system is equal to that of a single particle of mass M moving with a velocity $\vec{V}_{CM}$.

Also we get, $\Sigma \vec{F}_{ext} = M\vec{a}_{CM} = M\frac{d}{dt}\vec{V}_{CM} = \frac{d}{dt}(M\vec{V}_{CM}) = \frac{d\vec{P}}{dt}$

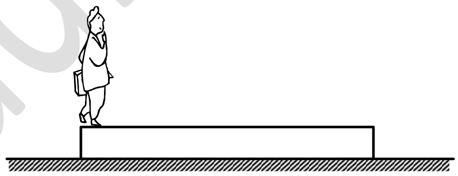
Also, $(\Sigma \vec{F}_{ext})\,dt = d\vec{P}$

The above equation shows that the resultant impulse acting on the system is equal to the change in the resultant momentum of the set of particles.

Also, in the absence of external force, linear momentum of system of particle will remains conserved.

Illustration 11

Question: **A man of mass m = 60 kg is standing over a plank of mass M = 40 kg. The plank is resting on a frictionless surface as shown in figure. If the man starts moving with a velocity v = 10 m/s with respect to plank towards right. Find the velocity with which plank will start moving.**



Solution: Consider man and plank as a system. There is no net external force acting on the system so linear momentum of system will remain conserved.

If plank starts moving with velocity V towards left, then the velocity of man will be $(v - V)$ with respect to surface towards right.

Initial linear momentum of system = 0

Final linear momentum of system = $m(v - V) - MV$.

From conservation of momentum for the system

$m(v - V) - MV = 0$

$$\Rightarrow \quad V = \frac{mv}{m+M} = 6 \text{ m/s}$$

4.5 CENTRE OF MASS OF CONTINUOUS BODIES

For calculating centre of mass of a continuous body, we first divide the body into suitably chosen infinitesimal elements. The choice is usually determined by the symmetry of body.

Consider element dm of the body having position vector $\vec{r}$, the quantity $m_i\vec{r}_i$ in equation of CM is replaced by $\vec{r}\ dm$ and the discrete sum over particles $\frac{\Sigma m_i r_i}{M}$, becomes integral over the body:

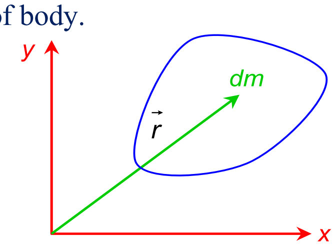


Fig. (7)

$$\vec{r}_{CM} = \frac{1}{M}\int \vec{r}\,dm$$

In component form this equation can be written as

$$X_{CM} = \frac{1}{M}\int x\,dm \;;\; Y_{CM} = \frac{1}{M}\int y\,dm \text{ and } Z_{CM} = \frac{1}{M}\int Z\,dm$$

To evaluate the integral we must express the variable m in terms of spatial coordinates x, y, z or $\vec{r}$.

Illustration 12

Question:

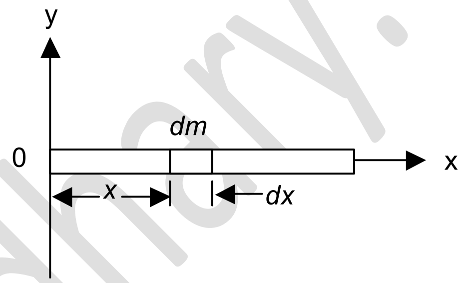
(a) **Show that the centre of mass of a rod of mass *M* and length *L*= 6m lies midway between its ends, assuming the rod has a uniform mass per unit length.**

(b) **Suppose a rod is non-uniform such that its mass per unit length varies linearly with *x* according to the expression $\lambda = \alpha x$, where α is a constant. Find the *x* coordinate of the centre of mass as a fraction of *L*.**

Solution:

(a) By symmetry, we see that $y_{CM} = z_{CM} = 0$ if the rod is placed along the x axis. Furthermore, if we call the mass per unit length λ (the linear mass density), then $\lambda = M/L$ for a uniform rod. If we divide the rod into elements of length dx, then the mass of each element is $dm = \lambda\, dx$. Since an arbitrary element of each element is at a distance x from the origin, equation gives

$$x_{CM} = \frac{1}{M}\int_0^L x\,dm = \frac{1}{M}\int_0^L x\,\lambda\,dx = \frac{\lambda L^2}{2M}$$



Because λ = M/L, this reduces to

$$x_{CM} = \frac{L^2}{2M}\left(\frac{M}{L}\right) = \frac{L}{2} = \mathbf{3m}$$

One can also argue that by symmetry, $x_{CM} = L/2$.

(b) In this case, we replace dm by $\lambda\, dx$, where λ is not constant. Therefore, x_{CM} is

$$x_{CM} = \frac{1}{M}\int_0^L x\,dm = \frac{1}{M}\int_0^L x\,\lambda\,dx = \frac{\alpha}{M}\int_0^L x^2\,dx = \frac{\lambda L^3}{3M}$$

We can eliminate α by noting that the total mass of the rod is elated to α through the relationship

$$M = \int dm = \int_0^L \lambda\,dx = \int_0^L \alpha x\,dx = \frac{\alpha L^2}{2}$$

Substituting this into the expression for x_{CM} gives

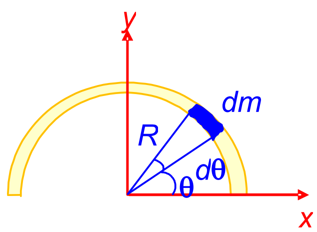
$$x_{CM} = \frac{\alpha L^3}{3\alpha L^2/2} = \frac{2}{3}L = \mathbf{4m}$$

Illustration 13

Question: **Locate the centre of mass of a uniform semicircular rod of radius $R = \pi$ m and linear density λ kg/m.**

Solution:

From the symmetry of the body we see at once that the CM must lie along the y axis, so $x_{CM} = 0$. In this case it is convenient to express the mass element in terms of the angle θ, measured in radians. The element, which subtends an angle $d\theta$ at the origin, has a length $R\,d\theta$ and a mass dm = λ R$d\theta$. Its y coordinate is $y = R \sin\theta$.



Therefore, $y_{CM} = \int \frac{y\,dm}{M}$ takes the

$$y_{CM} = \frac{1}{M}\int_0^{\pi} \lambda R^2 \sin\theta\,d\theta = \frac{\lambda R^2}{M}\left[-\cos\theta\right]_0^{\pi} = \frac{2\lambda R^2}{M}$$

The total mass of the ring is $M = \pi R\lambda$; therefore, $y_{CM} = \frac{2R}{\pi}$ = **2 m**.

4.6 DISTINCTION BETWEEN CENTRE OF MASS AND CENTRE OF GRAVITY

The position of the centre of mass of a system depends only upon the mass and position of each constituent particles,

i.e., $\vec{r}_{CM} = \dfrac{\Sigma m_i \vec{r}_i}{\Sigma m_i}$ … (i)

The location of *G*, the centre of gravity of the system, depends however upon the moment of the gravitational force acting on each particle in the system (about any point, the sum of the moments for all the constituent particles is equal to the moment for the whole system concentrated at *G*).

Hence, if g_i is the acceleration vector due to gravity of a particle *P*, the position vector r_G of the centre of gravity of the system is given by

$\vec{r}_G \times \Sigma m_i g_i = \Sigma(\vec{r}_i \times m_i g_i)$ … (ii)

It is only when the system is in a uniform gravitational field, where the acceleration due to gravity (g) is the same for all particles, that equation (ii)

Becomes $\vec{r}_G = \dfrac{\Sigma m_i \vec{r}_i}{\Sigma m_i} = \vec{r}_{CM}$

In this case, therefore the centre of gravity and the centre of mass coincide.

If, however the gravitational field is not uniform and g_i is not constant then, in general equation (ii) cannot be simplified and $r_G \neq r_{CM}$.

Thus, for a system of particles in a uniform gravitational field, the centre of mass and the centre of gravity are identical points but in a variable gravitational field, the centre of mass and the centre of gravity are in general, two distinct points.

PROFICIENCY TEST– II

The following questions deal with the basic concepts of this section. Answer the following briefly. Go to the next section only if your score is at least 80%. Do not consult the Study Material while attempting the questions.

1. Can the centre of mass of a body lie outside the body? If so give examples.

2. Three balls are thrown into air simultaneously. What is the acceleration of their centre of mass while they are in motion?

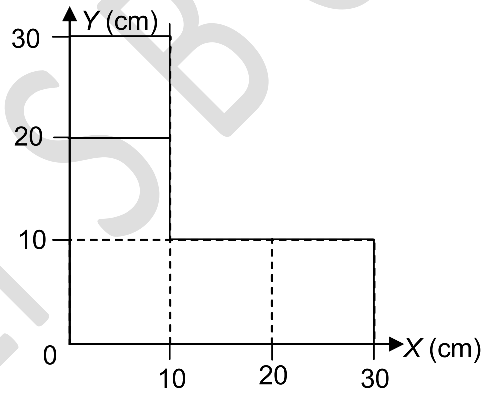
3. As a ball falls towards the earth, the momentum of the ball increases. Reconcile this fact with the law of conservation of momentum.

4. A bomb, initially at rest, explodes into several pieces.
(a) Is linear momentum constant?
(b) Is kinetic energy constant? Explain.

5. The mass of the moon is about 0.013 times the mass of earth and the distance from the centre of the moon to the centre of earth is about 60 times the radius of earth. How far is the centre of mass of earth-moon system from the centre of earth?

6. A 2.0 kg particle has a velocity $(2.0\vec{i} - 4.0\vec{j})$ m/s, and a 3.0 kg particle has a velocity $(2.0\vec{i} + 6.0\vec{j})$ m/s. Find
(a) the velocity of the centre of mass and
(b) the total momentum of the system.

7. A uniform piece of sheet is shaped as shown in the figure. Compute x and y co-ordinates of centre of mass of the piece.



8. A 2.0 kg particle has a velocity of $\vec{v}_1 = (2.0\vec{i} - 10t\vec{j})$ m/s, where t is in seconds. A 3.0 kg particle moves with a constant velocity of $\vec{v}_2 = 7.0\vec{i}$ m/s. At $t = 0.50$ s, find (a) the velocity of the centre of mass, (b) the acceleration of the centre of mass, and (c) the total momentum of the system.

ANSWERS TO PROFICIENCY TEST– II

5. 4930 km, since the radius of earth is 6400 km.

6. (a) $2\hat{i}+2\hat{j}$ m/s

(b) $10\hat{i}+10\hat{j}$ kg-m/s

7. 11cm; 11cm

8. (a) $5\hat{i}-2\hat{j}$ m/s

(b) $-4\hat{j}$ m/s^2

(c) $25\hat{i}-10\hat{j}$ kg-m/s

SOLVED OBJECTIVE EXAMPLES

***Example* 1:**

A bomb of 12 kg explodes into two pieces of masses 4 kg and 8 kg. The velocity of 8 kg mass is 6 m/s. The kinetic energy of the other is

(a) 48 J **(b) 32 J** **(c) 24 J** **(d) 288 J**

Solution:

Momentum of 8 kg mass = 8 × 6 = 48 kg m/s

Hence momentum of 4 kg mass will be the same as this since the bomb was originally at rest. Hence the speed of 4 kg mass $= \dfrac{48}{4} = 12$ m/s

Hence its kinetic energy $= \dfrac{1}{2}mv^2 = \dfrac{1}{2} \times 4 \times 144 = \mathbf{288\ J}$

∴ **(d)**

***Example* 2:**

Water flows through a pipe bent at an angle α to the horizontal with a velocity *v*. What is the force exerted by water on the bend of the pipe of area of cross section *S*?

(a) $2\rho v^2 S \sin\dfrac{\alpha}{2}$ **(b) $2\rho v^2 S \cos\alpha$** **(c) $2\rho v^2 S \sin\alpha$** **(d) $2\rho v^2 S \sin\alpha\cos\alpha$**

Solution:

Let us take horizontal direction as *X*-axis and perpendicular to it as *Y*-axis

$\vec{p_i}$ = (Initial momentum of water flowing per sec)

$= (Sv\rho)\, v\, \hat{i} = S\rho v^2\, \hat{i}$

$\vec{p_f}$ = (Final momentum of water flowing per second)$= S\rho v^2\, (\cos\alpha\,\hat{i} + \sin\alpha\,\hat{j})$

Rate of change of momentum = Force exerted by water on the bend of the pipe

$= S\rho v^2\,(\cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}) - S\rho v^2\,\hat{i}$

$= S\rho v^2\,[(\cos\alpha - 1)\hat{i} + (\sin\alpha\,\hat{j})]$

$= S\rho v^2\,\sqrt{(\cos\alpha - 1)^2 + \sin^2\alpha}$

$= \mathbf{2S\rho v^2 \sin\dfrac{\alpha}{2}}$

∴ **(a)**

***Example* 3:**

A block of mass *m* moving with speed *v* collides with another block of mass 2*m* at rest. The lighter block comes to rest after collision. What is the value of coefficient of restitution?

(a) $\dfrac{1}{2}$ **(b) $\dfrac{1}{3}$** **(c) $\dfrac{3}{4}$** **(d) $\dfrac{1}{4}$**

Solution:

Suppose the second block moves at a speed v' after collision.

By conservation of momentum, $mv = 2m\,v'$ or $v' = \dfrac{v}{2}$

Velocity of separation $= \dfrac{v}{2}$

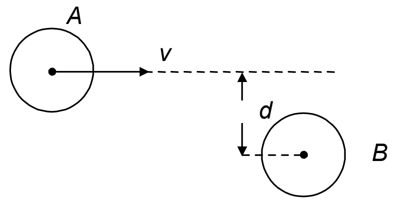
Velocity of approach $= v$

By definition, $e = \dfrac{\text{Velocity of separation}}{\text{Velocity of approach}} = \dfrac{1}{2}$.

∴ (a)

***Example* 4:**

A disk A of radius r moving on perfectly smooth surface at a speed v undergoes an elastic collision with an identical stationary disk B. Find the velocity of the disk B after collision if the impact parameter is d as shown in Figure.

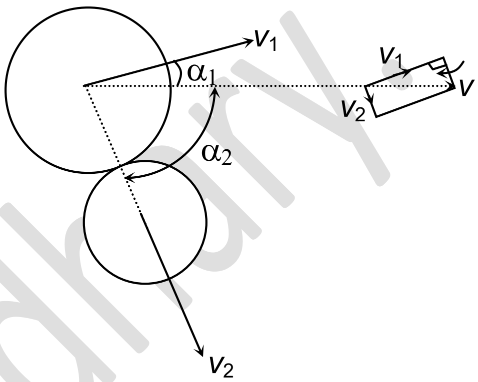


(a) $v=\sqrt{1-\frac{d^2}{4r^2}}$ (b) $\frac{vd}{2r}$

(c) $\frac{vr}{2d}$ (d) $v\sqrt{4r^2-d^2}$

Solution:

One of the disks is at rest before impact. After the impact its velocity will be in the direction of the centre line at the moment of contact because this is the direction in which the force acted on it.



Thus, $\sin\alpha_2 = \frac{d}{2r}$

$\alpha_1 + \alpha_2 = \frac{\pi}{2}$

Since the masses of both disks are equal, the triangle of momenta turns into triangle of velocities. We have

$$v_1 = v\cos\alpha_1 = v\sin\alpha_2 = \frac{vd}{2r},\quad v_2 = v\cos\alpha_2 = v\sqrt{1-\frac{d^2}{4r^2}}$$

∴ (a)

***Example* 5:**

A gun is mounted on a gun carriage movable on a smooth horizontal plane and the gun is elevated at an angle 45° to the horizon. A shot is fired and leaves the gun inclined at an angle θ to the horizon. If the mass of gun and carriage is n times that of the shot, find the value of θ.

(a) $\theta=\tan^{-1}\left(\frac{n}{n+1}\right)$ (b) $\theta=\tan^{-1}\left(\frac{n+1}{n}\right)$ (c) $\theta=\tan^{-1}\left(\frac{2n}{n+1}\right)$ (d) $\theta = \tan^{-1}(2)$

Solution:

Let m be the mass of shot.

mn = mass of gun, w = velocity of shot relative to gun, v = velocity of recoil of gun

Since the gun is inclined at an angle α to horizontal, the direction of w makes an angle α with horizontal. The horizontal and vertical components are $w\cos\alpha$ and $w\sin\alpha$. When the shot leaves the muzzle the horizontal velocity relative to ground = $w\cos\alpha - v$.

The vertical component of shot relative to ground is the same as relative to gun since the gun moves horizontally. If the shot leaves at an angle θ to horizontal,

$$\tan\theta = \frac{\text{Vertical component of velocity of shot}}{\text{Horizontal component of velocity of shot}}$$

$$= \frac{w\sin\alpha}{w\cos\alpha - v} \qquad \text{... (i)}$$

By conservation of momentum in horizontal direction,

$$mnv = m(w\cos\alpha - v),\qquad v = \frac{w\cos\alpha}{(n+1)}$$

Substituting in (i), $\tan\theta = \dfrac{w\sin\alpha}{w\cos\alpha - \dfrac{w\cos\alpha}{n+1}}$

$$\tan\theta = \frac{(n+1)\sin\alpha}{n\cos\alpha} = \left(1+\frac{1}{n}\right)\tan\alpha,$$

$$\theta = \tan^{-1}\left(\frac{n+1}{n}\right) \quad (\because \tan 45° = 1)$$

$\therefore$ **(b)**

***Example* 6:**

A neutron of mass *m* collides elastically with a nucleus of mass *M* which is at rest. If the initial kinetic energy of neutron is K_0, calculate the kinetic energy that it can lose during the collision.

(a) $\frac{MmK_0}{(M+m)^2}$ (b) $\frac{4MmK_0}{(M+m)^2}$ (c) $\frac{2MmK_0}{(M+m)^2}$ (d) $\frac{MmK_0}{(M+m)}$

Solution:

The maximum energy loss occurs in a head on collision. Let v is the velocity of neutron before collision and v_2 its velocity after collision and v_1 the velocity of nucleus after collision.

M = Mass of nucleus, m = Mass of neutrons

By principle of conservation of momentum,

$Mv_1 + mv_2 = mv$

$v_1 - v_2 = v$

Solving, $v_2 = \frac{v(m-M)}{(m+M)}$

Loss of kinetic energy

$= \frac{1}{2}mv^2 - \frac{1}{2}mv_2^2$

$= \frac{1}{2}mv^2\left[1-\frac{v_2^2}{v^2}\right]$

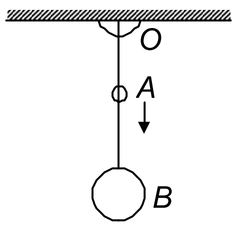
$= \frac{1}{2}mv^2\left[1-\frac{(m-M)^2}{(m+M)^2}\right]$

$= K_0\frac{4mM}{(M+m)^2}$

$\therefore$ **(b)**

***Example* 7:**

A smooth rubber cord of length ℓ with spring constant *k* is suspended from *O*. The other end is fitted with a bob *B*. A small sleeve of mass *m* starts falling from *O*. Neglecting the masses of the cord and bob, find the maximum elongation of the cord.



(a) $\frac{mg}{k}\left[1+\sqrt{1+\frac{2k\ell}{mg}}\right]$ (b) $\frac{mg}{k}\left[\sqrt{1+\frac{2k\ell}{mg}}\right]$

(c) $\frac{mg}{k}$ (d) $\frac{mg}{k}\left[1+\sqrt{\frac{k}{mg}}\right]$

Solution:

Let the cord extend by e. Then by conservation of energy, $mg(\ell + e) = \frac{1}{2}ke^2$

$ke^2 - 2mge - 2mg\ell = 0$

$$e = \frac{2mg \pm \sqrt{4m^2g^2 + 4k \cdot 2mg\ell}}{2k}$$

$$e = \frac{mg}{k} \pm \frac{mg}{k}\sqrt{1 + \frac{2k\ell}{mg}}$$

Discarding the negative sign,

$$e = \frac{mg}{k} + \frac{mg}{k}\sqrt{1 + \frac{2k\ell}{mg}} = \frac{mg}{k}\left[1 + \sqrt{1 + \frac{2k\ell}{mg}}\right]$$

∴ **(a)**

***Example* 8:**

Sand drops from a stationary hopper at the rate 5 kg/s on to a conveyor belt moving with constant speed of 2 m/s. What is the power delivered by the motor drawing the belt?

(a) 10 watt **(b) 20 watt** **(c) 30 watt** **(d) 40 watt**

Solution:

This problem illustrates exertion of tangential force on a body due to gain of mass.

Tangential force = Rate of gain of tangential momentum.

$$= v \cdot \frac{dm}{dt}$$

This is the force needed to keep the belt moving with uniform velocity. The motor must exert this moment of force.

Force needed $= v \cdot \frac{dm}{dt} = 2 \times 5 = 10$ newton.

Power = Force × velocity = 10 × 2 = **20 watts**

∴ **(b)**

***Example* 9:**

Two particles of equal mass have velocities $\vec{v}_1 = 2\hat{i}$ m/s and $\vec{v}_2 = 2\hat{j}$ m/s. First particle has an acceleration $\vec{a}_1 = (3\hat{i} + 3\hat{j})$ m/s² while the acceleration of the other particle is zero. The center of mass of the two particles moves on a

(a) circle **(b) parabola** **(c) straight line** **(d) ellipse**

Solution:

$$\vec{v}_{COM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

$$= \frac{\vec{v}_1 + \vec{v}_2}{2} \quad (m_1 = m_2)$$

$$= (\hat{i} + \hat{j}) \text{ m/s}$$

Similarly, $\vec{a}_{COM} = \frac{\vec{a}_1 + \vec{a}_2}{2} = \frac{3}{2}(\hat{i} + \hat{j})$ m/s²

Since $\vec{v}_{COM}$ is parallel to $\vec{a}_{COM}$ the path will be a **straight line**.

∴ **(c)**

Example **10:**

A rope thrown over a pulley has a ladder with a man of mass *m* on one of its ends and a counterbalancing mass *M* on its other end. The man climbs with a velocity v_r relative to ladder. Ignoring the masses of the pulley and the rope as well as the friction of the pulley axis, the velocity of the centre of mass of this system is

(a) $\frac{m}{M}v_r$ (b) $\frac{m}{2M}v_r$ (c) $\frac{M}{m}v_r$ (d) $\frac{2M}{m}v_r$

Solution:

The rope tension is the same on the left and right hand side at every instant, and, consequently, momentum of both sides are equal

$\therefore$ $Mv = (M - m)(-v) + m(v_r - v)$

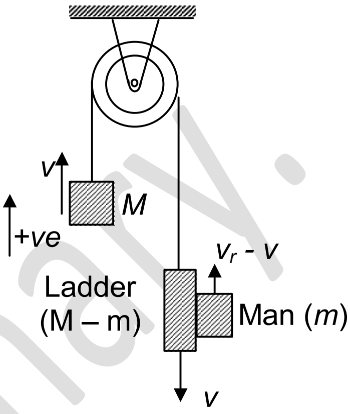
or $v = \frac{m}{2M}v_r$

Momentum of the centre of mass is

$P = P_1 + P_2$

or $v_{COM} = v = \frac{m}{2M}v_r$

$\therefore$ **(b)**



SOLVED SUBJECTIVE EXAMPLES

***Example* 1:**

Two men each of mass *m* = 50 kg, stand on the edge of a stationary buggy of mass *M* = 100kg. Assuming friction to be negligible, find the speed of the buggy after both men jump off with the same horizontal velocity *u* =24 m/s relative to buggy one after the other.

Solution:

Let velocity of buggy just after jumping of first man is $\vec{u}_1$.

The real velocity of first man is $\vec{u}+\vec{u}_1=\vec{u}_m$

Applying conservation of linear momentum,

$$0=(M+m)\vec{u}_1+m\vec{u}_m$$

$$\vec{u}_1=\frac{-m(\vec{u}+\vec{u}_1)}{M+m}$$

$$\vec{u}_1=\frac{-m\vec{u}}{M+2m} \qquad \text{...(i)}$$

Let the velocity of buggy after jump of second men is $\vec{u}_2$

The real velocity of second men is $\vec{u}+\vec{u}_2$

Again applying law of conservation of momentum, $(M+m)\vec{u}_1=M\vec{u}_2+m(\vec{u}+\vec{u}_2)$

From (i), putting the value of $\vec{u}_1$, we get $\quad \vec{u}_2=-\frac{m(2M+3m)}{(M+m)(M+2m)}\vec{u}$

Speed of buggy after both men will jump is u_2 = **14m/s**

***Example* 2:**

An object of mass 5 kg is projected with a velocity of 20 m/s at an angle of 60° with the horizontal. At the highest point of its path, the projectile explodes and breaks up into two fragments of masses 1 kg and 4 kg. The fragments separate horizontally after the explosion. Due to explosion, the kinetic energy of the system at the highest point gets doubled. Find the separation (in cm) between the two fragments when they hit the ground. ($\sqrt{3}=1.7$)

Solution:

Let the velocities of 1 kg fragment be u_1 and 4 kg fragment be u_2.
Then by conservation of linear momentum

$$5(20\cos 60°)=4u_2+u_1 \qquad \text{...(i)}$$

and $$\frac{1}{2}4u_2^2+\frac{1}{2}(1)(u_1)^2=2\left(\frac{1}{2}5(20\cos 60°)^2\right)$$

$$4u_2^2+u_1^2=1000 \qquad \text{...(ii)}$$

From (i) and (ii)

u_1 = 30m/s

u_2 = 5m/s

Relative velocity along x-axis = u_x = 25 m/s.

∴ Separation = x = $u_x t$ = $u_x\left(\frac{u\sin\theta}{g}\right)$

$$x=\frac{25\times 20\times \sin 60}{10}$$

$$=25\times\sqrt{3}=\mathbf{4250\ cm}$$

***Example* 3:**

A body *A* moving with velocity 10 m/s make a head on collision with a stationary body *B* of same mass. As a result of collision the kinetic energy of system decreases by one percent. Find the magnitude and direction of the velocity of particle *A* after collision.

Solution:

Let m be the mass of A and m the mass of B.
Let v_1 be the velocity of A and v_2 the velocity of B after collision.
By the principle of conservation of momentum,

$$mv_1 + mv_2 = mv + 0$$

$$\therefore \quad v_1 + v_2 = v \qquad \text{... (i)}$$

Given, $\dfrac{K_i - K_f}{K_i} = \dfrac{1}{100}$

$$\Rightarrow \quad 1 - \frac{K_f}{K_i} = \frac{1}{100} \quad \frac{K_f}{K_i} = 1 - \frac{1}{100} = \frac{99}{100}$$

$$\Rightarrow \quad \frac{\frac{1}{2}mv_2^2 + \frac{1}{2}mv_1^2}{\frac{1}{2}mv^2} = \frac{99}{100}$$

$$\Rightarrow \quad \frac{v_2^2 + v_1^2}{v^2} = \frac{99}{100}$$

$$\Rightarrow \quad v_2^2 + v_1^2 = \frac{99}{100}v^2$$

$$(v_1 + v_2)^2 = v^2 \qquad \text{[from (i)]}$$

$$\therefore \quad v_1^2 + v_2^2 + 2v_1v_2 = v^2$$

$$\frac{99}{100}v^2 + 2v_1v_2 = v^2$$

$$\Rightarrow \quad 2v_1v_2 = \frac{v^2}{100}$$

or $\quad v_1v_2 = \dfrac{v^2}{200}, \qquad v_1 + v_2 = 10$

$$v_1v_2 = \frac{10\times10}{200} = \frac{1}{2}$$

$$v_1(10 - v_1) = \frac{1}{2}$$

$$10v_1 - v_1^2 = \frac{1}{2} \text{ or } v_1^2 - 10v_1 + \frac{1}{2} = 0$$

$$2v_1^2 - 20v_1 + 1 = 0$$

$$v_1 = \frac{20 \pm \sqrt{400-8}}{4}$$

= 5 cm/s in the same direction.

***Example* 4:**

A block of mass 37.5 kg is placed on a table of mass 12.25 kg, which can move without friction on a level floor. A particle of mass 0.25 kg moving horizontally with velocity 302 m/s strikes the block inelastically (a) Find the distance through which the block moves relative to the table before they acquire a common velocity (b) also compute the common velocity, if the coefficient of friction between block and table is 0.25.

Solution:

(a) Applying the principle of conservation of momentum to the inelastic impact, we have

$0.25 \times 302 = (0.25 + 37.5 + 12.25)v$, where v is the common velocity of the system.

$V_{common} = \frac{0.2\times302}{50}$ = **151 cm/s**

(b) Let u be the velocity of block immediately after impact. Then, $0.25 \times 302 = (0.25 + 37.5)\, u$

$$u = \frac{0.25\times302}{37.75} = 2 \text{ m/s}$$

Let a_1 and a_2 be the retardation of the block and acceleration of the table respectively.

Then $(0.25 + 37.5)\, a_1 = 12.25\, a_2$ = kinetic frictional force

Because $F_k = \mu_k mg = 0.25 \times [0.25 + 37.5]\, g = 0.25 \times 37.75\, g$

$a_1 = 2.45$ m/s^2, $a_2 = 7.55$ m/s^2

Relative retardation of block $= a_1 + a_2 = 2.45 + 7.55 = 10$ m/s^2

$\therefore$ $v^2 = 2 \times 10 \times s,$

$v = 2$ m/s,

$4 = 20$ s

$s = \frac{4}{20} = \frac{1}{5} m$ = **20 cm**

***Example* 5:**

A ball of mass *m* is projected with speed *u* into the barrel of spring gun of mass *M* initially at rest on a frictionless surface. The mass *m* sticks in the barrel at the point of maximum compression of the spring. What percentage fraction of the initial kinetic energy of the ball is stored in the spring? Neglect the friction. ($m = 3M$)

Solution:

Let v be the velocity of system after the ball of mass m sticks in the barrel. Applying law of conservation of linear momentum, we have

$mu = (m + M)v$ … (i)

The initial K.E. $\frac{1}{2}mu^2$ of the ball is converted into elastic potential energy $\frac{1}{2}kx^2$ of the spring and kinetic energy $\frac{1}{2}(m + M)v^2$ of the whole system. That is

$\frac{1}{2}mu^2 = \frac{1}{2}kx^2 + \frac{1}{2}(m + M)v^2$ … (ii)

where k is the spring constant and x is its maximum compression.

Dividing equation (ii) by $\frac{1}{2}mu^2$,

$$1 = \frac{\frac{1}{2}kx^2}{\frac{1}{2}mu^2} + \frac{\frac{1}{2}(m+M)v^2}{\frac{1}{2}mu^2} \quad \text{…(iii)}$$

$$1 = \frac{kx^2}{mu^2} + \frac{(m+M)v^2}{mu^2} \quad \text{…(iv)}$$

From equation (i), $\frac{v}{u} = \frac{m}{(M+m)}$

Substituting this value in equation (iv),

$$1 = \frac{kx^2}{mu^2} + \frac{(m+M)}{m}\cdot\frac{m^2}{(m+M)^2} = \frac{kx^2}{mu^2} + \frac{m}{m+M} \quad \frac{kx^2}{mu^2} = 1 - \frac{m}{m+M} = \frac{M}{(m+M)}$$

The energy stored in spring $= \frac{1}{2}kx^2$

Initial K.E. of the ball $= \frac{1}{2}mu^2$.

Hence, $\frac{kx^2}{mu^2}$ represents the fraction of initial energy, which is stored in the spring.

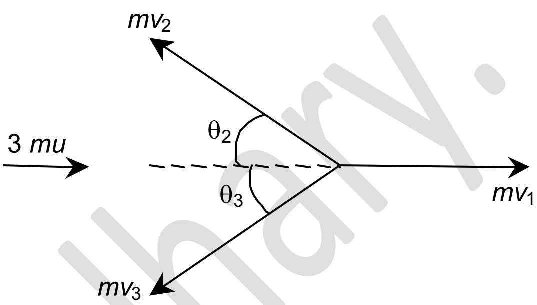
$\therefore$ % fraction = $\frac{M}{m+M}\times 100$ = **25%**

***Example* 6:**

A shell flying with a velocity *u* = 500 m/s bursts into three identical fragments so that the kinetic energy of the system increases *k* times. What maximum velocity can one of the fragments obtain if *k* = 1.5?

Solution:

Let the mass of the shell be 3m.
The mass of each fragment is *m*.
The particle with maximum velocity must be in the forward direction.
By law of conservation of momentum,
$3mu = mv_1 - mv_2 \cos\theta_2 - mv_3 \cos\theta_3$
$3u = v_1 - v_2 \cos\theta_2 - v_3 \cos\theta_3$



$v_1 = 3u + v_2 \cos\theta_2 + v_3 \cos\theta_3$ … (i)
Also $mv_2 \sin\theta_2 = mv_3 \sin\theta_3$ … (ii)
If v_1 is to be maximum $\theta_2 = \theta_3 = 0$
From (2), if $\theta_2 = \theta_3$, $v_2 = v_3 = v$ (say)
Equation (i) becomes $v_1 = 3u + 2v$

$v = \frac{v_1 - 3u}{2}$ … (iii)

Using the principle of conservation of energy

$\frac{1}{2}(3m)u^2 = \frac{1}{k}\left(\frac{1}{2}mv_1^2 + 2\times\frac{1}{2}mv^2\right)$

$3ku^2 = v_1^2 + 2v^2$ … (iv)

Substituting for *v* from (iii) $3ku^2 = v_1^2 + \frac{1}{2}(v_1^2 + 9u^2 - 6v_1u)$

Solving for v_1

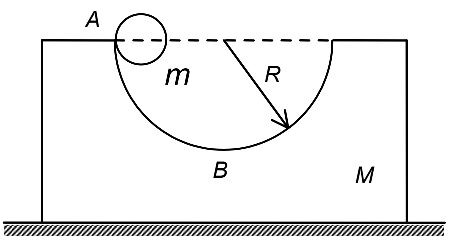
$v_1 = u\left[1+\sqrt{2(k-1)}\right]$

For u = 500 m/s and k = 1.5

$v_1 = 500\left[1+\sqrt{2(1.5-1)}\right]$ = **1000 m/s**

***Example* 7:**

A block of mass *M* = 4 kg with a semicircular track of radius *R* = 5 m rests on a horizontal frictionless surface. A uniform cylinder of radius *r* = 1m and mass *m* = 6kg is released from rest at the top point *A* (see Figure). The cylinder slips on the semicircular frictionless track. How far has the block moved when the cylinder reached the bottom (point *B*) of the track? How fast is the block moving when the cylinder reaches the bottom of the track? ($\sqrt{2} = 1.4$)



Solution:

The horizontal component of forces acting on *M-m* system is zero and the centre of mass of the system cannot have any horizontal displacement.

When the cylinder is at B its displacement relative to the block in the horizontal direction is $(R - r)$. Let the consequent displacement of the block to the left be x. The displacement of the cylinder relative to the ground is $(R - r - x)$.

Since the centre of mass has no horizontal displacement

$$Mx = m(R - r - x)$$

$$x(M + m) = (R - r)m$$

$$x = \frac{(R-r)m}{(M+m)}$$

When the cylinder is at A, the total momentum of the system in the horizontal direction is zero. If v is the velocity of the cylinder at B and V, the velocity of the block at the same instant, then

$mv + MV = 0$, by principle of conservation of momentum.

Potential energy of the system at A $= mg(R - r)$

Kinetic energy of the cylinder at B $= \frac{1}{2}mv^2$

The kinetic energy of the block at that instant $= \frac{1}{2}MV^2$

By principle of conservation of energy,

$$mg(R-r) = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$$

since $v = -\frac{MV}{m}$

$$mg(R-r) = \frac{1}{2}m\left(-\frac{MV}{m}\right)^2 + \frac{1}{2}MV^2 = \frac{V^2}{2}\left(\frac{M^2}{m}+M\right)$$

$$mg(R-r) = \frac{V^2}{2m}\left(M^2+Mm\right)$$

$$V^2 = \frac{2m^2g(R-r)}{(M^2+Mm)}$$

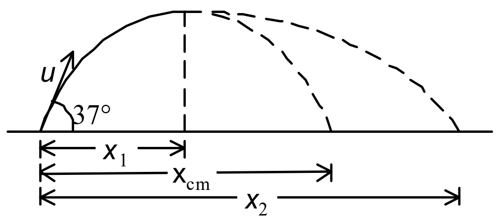
$V =$ **840 cm/s**

***Example* 8:**

A projectile is fired at a speed of 100 m/s at an angle of 37° above horizontal. At the highest point the projectile breaks into two parts of mass ratio 1 : 3, the lighter coming to rest. Find the distance from the launching point to the point where the heavier piece lands.

Solution:

Refer the Figure. At the highest point, the projectile has horizontal velocity. The lighter part comes to rest. Hence the heavier part will move with increased velocity in the horizontal direction. In the vertical direction both parts have zero velocity and undergo same acceleration. Hence they will cover equal vertical displacements in a given time. Thus both will hit the ground together. As internal forces do not affect the motion of the centre of mass, the centre of mass hits the ground at the position where the original projectile would have landed. The range of the original projectile is



$$X_m = \frac{2u^2\sin\theta\cos\theta}{g}$$

$$= \frac{2\times(100)^2\times\frac{3}{5}\times\frac{4}{5}}{10} = 960 \text{ m}$$

where $\sin\theta = \frac{3}{5}, \cos\theta = \frac{4}{5}$ and $g = 10$ m/s^2.

The centre of mass will hit the ground at this position. As the lighter mass comes to rest after breaking it falls down vertically and hits the ground at half the range = 480 m. If the heavier block hits the ground at x_2,

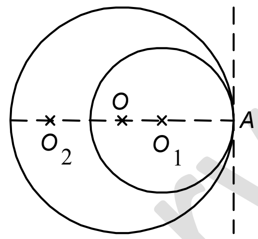
$$x_{cm} = \frac{m_1x_1+m_2x_2}{m_1+m_2}$$

$$960 = \frac{\frac{M}{4}\times480+\frac{3M}{4}\times x_2}{M}$$

Solving, $x_2 = \mathbf{1120\ m}$

Example 9:

A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of diameter 42 cm is removed from one edge of the plate as shown in Figure. Find the distance of centre of mass of the remaining portion.



Solution:

Let O be the centre of circular plate and O_1, the centre of circular portion removed from the plate. Let O_2 be the centre of mass of the remaining part.

Area of original plate $= \pi R^2 = \pi\left(\frac{56}{2}\right)^2 = 28^2\ \pi\ \text{cm}^2$

Area removed from circular part $= \pi r^2$

$$= \pi\left(\frac{42}{2}\right)^2 = (21)^2\pi\ \text{cm}^2$$

Let σ be the mass per cm^2. Then

mass of original plate, $m = (28)^2\ \pi\sigma$

mass of the removed part, $m_1 = (21)^2\ \pi\sigma$

mass of remaining part, $m_2 = (28)^2\ \pi\sigma - (21)^1\ \pi\sigma = 343\ \pi\sigma$

Now the masses m_1 and m_2 may be supposed to be concentrated at O_1 and O_2 respectively. Their combined centre of mass is at O. Taking O as origin we have from definition of centre of mass,

$$x_{cm} = \frac{m_1x_1+m_2x_2}{m_1+m_2}$$

$x_1 = OO_1 = OA - O_1A = 28 - 21 = 7$ cm

$x_2 = OO_2 = ?,\ x_{cm} = 0.$

$$\therefore \quad 0 = \frac{(21)^2\pi\sigma\times7+343\pi\sigma\times x_2}{(m_1+m_2)}$$

$$x_2 = -\frac{(21)^2\pi\sigma\times7}{343\pi\sigma} = -\frac{441\times7}{343} = -9\ \text{cm}.$$

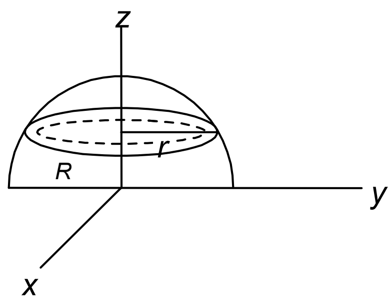
This means that centre of mass of the remaining plate is at a distance **9 cm** from the centre of given circular plate opposite to the removed portion.

Example 10:

Find the z-coordinate of centre of mass of a uniform solid hemisphere of radius R = 8m and mass M with centre of sphere at origin and the flat of the hemisphere in the x, y plane.

Solution:

Let the centre of the sphere be the origin and let the flat of the hemisphere lie in the x-y plane as shown. By symmetry, x and y coordinates of centre of mass $\bar{x}=\bar{y}=0$. Consider the hemisphere divided into a series of slices parallel to x, y plane. Each slice is of thickness dz.



The slice between z and $(z + dz)$ is a disk of radius, $r = \sqrt{R^2-z^2}$.

Let ρ be the constant density of the uniform hemisphere.

Mass of the slice, $dm = (\rho\pi r^2)dz = \rho\pi\,(R^2 - z^2)\,dz$

The $\bar{z}$ value is obtained by $\bar{z} = \dfrac{\int_0^R z\,dm}{M}$

$$= \frac{\int_0^R \pi\rho(R^2 z - z^3)dz}{M}$$

$$= \frac{\pi\rho}{M}\left[\left(\frac{R^2 z^2}{2} - \frac{z^4}{4}\right)\right]_{z=0}^{z=R}$$

$$\Rightarrow \quad \bar{z} = \frac{\pi\rho\left(\frac{R^4}{2} - \frac{R^4}{4}\right)}{M}$$

$$\Rightarrow \quad \bar{z} = \frac{\rho\pi R^4}{4M}$$

Since $2M = \rho\left(\frac{4}{3}\pi R^3\right)$,

we have $\bar{z} = \dfrac{(\rho\pi R^4/4)}{(\rho 2\pi R^3/3)} = \dfrac{3}{8}R$ **= 3 m**

MIND MAP

IMPULSE AND MOMENTUM

1. Important formulae

- $\vec{P} = M\vec{v}$
- $\vec{F} = \frac{\Delta \vec{P}}{\Delta t}, \vec{F} = \frac{d\vec{p}}{dt}$
- $\vec{J} = \int \vec{F}.dt = \Delta \vec{p}$

2. Conservation of linear momentum

- If net force acting on a body or system of bodies is zero, the momentum of body or system of body remains conserved.

3. Classification of impact on the basis of direction of force

- Central
  - Direct or head-on
  - Indirect or oblique
- Eccentric

4. Classification of impact on the basis of nature of colliding bodies

- Elastic
- Inelastic
- Perfectly inelastic

5. Analysis of collision

- Apply conservation of momentum along the line of collision.
- Apply law of restitution along the line of collision
  i.e., $v_2 - v_1 = e\,(u_1 - u_2)$
- $e = 1$ for perfectly elastic collision.
- $e = 0$ for perfectly inelastic collision
- $0 < e < 1$ for other collisions.

6. Equation of motion for variable mass system

- $\vec{F}_{ext} + \vec{F}_{th} = M\vec{a}$
- where, $\vec{F}_{th} = \frac{-dM}{dt}\vec{v}_{rel}$

MIND MAP

SYSTEM OF PARTICLES

1. LOCATION OF CM

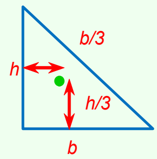
- For a system of particles

$$X_{CM}=\frac{\sum M_iX_i}{\sum M_i},\quad Y_{CM}=\frac{\sum M_iY_i}{\sum M_i},\quad Z_{CM}=\frac{\sum M_iZ_i}{\sum M_i}$$

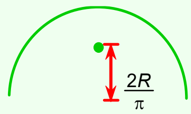
- For a continuous mass system

$$X_{CM}=\frac{\int xdm}{\int dm},\quad Y_{CM}=\frac{\int ydm}{\int dm},\quad Z_{CM}=\frac{\int zdm}{\int dm}$$

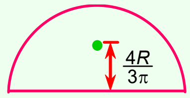
2. LOCATION OF CM FOR UNSYMMETRICAL BODIES



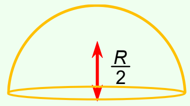
Right angled triangular lamina



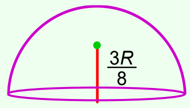
Semicircular ring



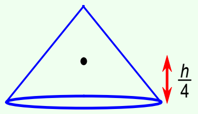
Semicircular disc



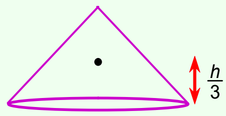
Hemispherical shell



Hemisphere



Right circular cone (solid)



Right circular cone (hollow)

3. MOTION OF CM

- **Velocity of CM,**

$$\vec{V}_{cm}=\frac{\sum M_i\vec{V_i}}{\sum M_i}$$

- **Acceleration of CM,**

$$\vec{a}_{cm}=\frac{\sum M_i\vec{a_i}}{\sum M_i}$$

4. NEWTON'S LAW

- Newton's second law of motion applicable for the system of particles:

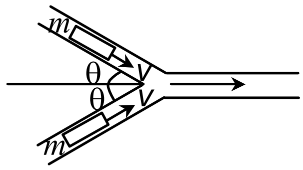
$$\sum\vec{F}_{ext}=\left(\sum M_i\right)\vec{a}_{CM}$$

- If net external force on a system of body is zero and initially its center of mass were at rest then center of mass will remain at rest.

EXERCISE – I

NEET-SINGLE CHOICE CORRECT

1. Two vehicles of equal masses are moving with same speed v on two roads making an angle θ. They collide inelastically at the junction and then move together. The speed of the combination is

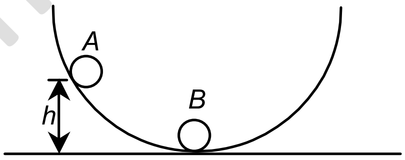


(a) $v\cos\theta$ (b) $2v\cos\theta$ (c) $\frac{v}{2}\cos\theta$ (d) $\frac{v}{2}\cos\frac{\theta}{2}$

2. Two particles having position vectors $\vec{r}_1 = (3\hat{i}+5\hat{j})$ metres and $\vec{r}_2 = (-5\hat{i}-3\hat{j})$ metres are moving with velocities $\vec{v}_1 = (4\hat{i}+3\hat{j})$ and $\vec{v}_2 = (a\hat{i}+7\hat{j})$ m/s. If they collide after 2 seconds, the value of a is

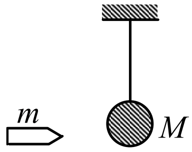
(a) 2 (b) 4 (c) 6 (d) 8

3. A sphere A of mass 4 kg is released from rest on a smooth hemispherical shell of radius 0.2 m. The sphere A slides down and collides elastically with another sphere B of mass 1 kg placed on the bottom of the shell. If the sphere B has to just reach the top, the height h from where the sphere A should be released is



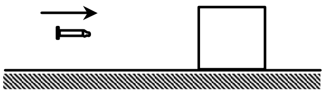
(a) 0.08 m (b) 0.02 m
(c) 0.18 m (d) 0.10 m

4. A bullet of mass m is fired along the bob of a pendulum hanging by a string. If α is angle of deflection of the bob after the bullet hits the bob, the angle α is maximum when



(a) bullet passes through the bob
(b) bullet gets stuck inside the bob
(c) bullet is reflected back
(d) in all circumstances

5. A bullet of mass 20 g travelling horizontally with a speed of 500 m/s passes through a wooden block of mass 10.0 kg initially at rest on a surface. The bullet emerges with a speed of 100 m/s and the block slides 20 cm on the surface before coming to rest, the coefficient of friction between the block and the surface. (g = 10 m/s^2)



(a) 0.16 (b) 0.6 (c) 0.5 (d) 0.25

6. When two bodies stick together after collision, the collision is said to be

(a) partially elastic (b) elastic
(c) perfectly inelastic (d) none of the above

7. A sphere of mass m moving with a constant velocity u hits another stationary sphere of same mass. If e is the coefficient of restitution, the ratio of velocities of two spheres after collision is

(a) $\frac{1-e}{1+e}$ (b) $\frac{1+e}{e}$ (c) $\frac{e+1}{e-1}$ (d) $\frac{e-1}{e+1}$

8. A body of mass m_1 strikes a stationary body of mass m_2. If the collision is elastic, the fraction of kinetic energy transferred by the first body to the second is

(a) $\dfrac{m_1 m_2}{(m_1 + m_2)}$ (b) $\dfrac{2m_1 m_2}{(m_1 + m_2)}$ (c) $\dfrac{4m_1 m_2}{(m_1 + m_2)^2}$ (d) $\dfrac{2m_1 m_2}{(m_1 + m_2)^2}$

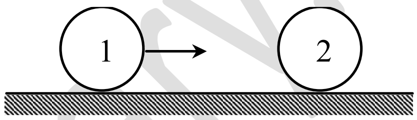
9. In the elastic collision of a heavy vehicle moving with a velocity of 10 ms^{-1} and a small stone at rest, the stone will fly away with a velocity equal to

(a) 5 ms^{-1} (b) 10 ms^{-1} (c) 20 ms^{-1} (d) 40 ms^{-1}

10. A body of mass 2 kg moving with a velocity of 6 m/s strikes inelastically to another body of same mass at rest. The amount of heat evolved during collision is

(a) 36 J (b) 18 J (c) 9 J (d) 3 J

11. Ball 1 collides with an another identical ball 2 at rest as shown in figure. For what value of coefficient of restitution *e*, the velocity of second ball becomes two times that of 1 after collision



(a) 1/3 (b) 1/2 (c) 1/4 (d) 1/6

12. A ball *P* of mass 2 kg undergoes an elastic collision with another ball Q at rest. After collision, ball *P* continues to move in its original direction with a speed one-fourth of its original speed. What is the mass of ball Q?

(a) 0.9 kg (b) 1.2 kg (c) 1.5 kg (d) 1.8 kg

13. Two masses of 1 g and 9 g are moving with equal kinetic energies. The ratio of the magnitudes of their respective linear momentum is

(a) 1 : 9 (b) 9 : 1 (c) 1 : 3 (d) 3 : 1

14. The bullet of mass *a* and velocity *b* is fired into a large block of mass *c*. If bullet sticks to it then the final velocity of the system is

(a) $\dfrac{a+c}{a}\times b$ (b) $\dfrac{a}{a+c}\times b$ (c) $\dfrac{a+b}{c}\times a$ (d) $\dfrac{c}{a+b}\times b$

15. Two masses m_a and m_b moving with velocities $\vec{v}_a$ and $\vec{v}_b$ collide elastically and after that m_a and m_b move with velocities $\vec{v}_b$ and $\vec{v}_a$ respectively. Then the ratio m_a/m_b is

(a) $\dfrac{v_a - v_b}{v_a + v_b}$ (b) $\dfrac{m_a + m_b}{m_a}$ (c) 1 (d) ½

16. If two balls each of mass 0.06 kg moving in opposite directions with same speed 4 m/s collide and rebound with the same speed, then the impulse imparted to each ball due to other is

(a) 0.48 kg m/s (b) 0.24 kg m/s (c) 0.81 kg m/s (d) zero

17. A ball approaches a moving wall of infinite mass with speed *v* along normal to the wall. The speed of the wall is *u* away from the ball and $u < v$. The speed of ball after an elastic collision is

(a) $u + v$ away from the wall (b) $2u + v$ away from the wall
(c) $v - u$ towards from the wall (d) $v - 2u$ away from the wall

18. Blocks *A* and *B* of equal masses are arranged as shown in figure. The surface of *A* is smooth while *B* is rough and has a coefficient of friction 0.1 with surface. The block *A* moves with speed 10 m/s and collides with *B*. The collision is perfectly elastic. Find the distance moved by *B* before it comes to rest.

(a) 25 m (b) 100 m (c) 50 m (d) 75 m

19. A sphere collides with another sphere of identical mass kept at rest. After collision, the two spheres move. The collision is perfectly inelastic, then the angle between the directions of motion of the two spheres is

(a) 0° (b) 45° (c) different from 90° (d) 90°

20. If momentum is increased by 20%, then K.E. increased by

(a) 44%(b) 55% (c) 66% (d) 77%

21. A bullet is shot from a rifle. As a result the rifle recoils. The kinetic energy of rifle as compared to that of bullet

(a) is less (b) is greater
(c) is equal (d) cannot be concluded

22. A body falling vertically downwards under gravity breaks in two parts of unequal masses. The centre of mass of the two parts taken together shifts horizontally towards

(a) heavier piece
(b) lighter piece
(c) does not shift horizontally
(d) depends on the vertical velocity at the time of breaking

23. Two blocks of masses 10 kg and 4 kg connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 m/s to the heavier block in the direction of the lighter block. The velocity of the centre of mass is

(a) 30 m/s (b) 20 m/s (c) 10 m/s (d) 5 m/s

24. A nucleus moving with a velocity $\vec{v}$ emits an α particle. Let the velocities of the α- particle and the remaining nucleus be $\vec{v}_1$ and $\vec{v}_2$ and their masses be m_1 and m_2. Then

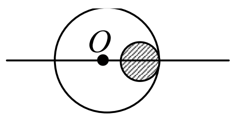
(a) $\vec{v}$, $\vec{v}_1$ and $\vec{v}_2$ must be parallel to each other,

(b) None of the two of $\vec{v}$, $\vec{v}_1$ and $\vec{v}_2$ should be parallel to each other.

(c) $\vec{v}_1 + \vec{v}_2$ must be parallel $\vec{v}$

(d) $m_1 \vec{v}_1 + m_2 \vec{v}_2$ must be parallel to $\vec{v}$

25. From a uniform circular plate of radius R, a small circular plate of radius $R/4$ is cut off as shown. If O is the center of the complete plate, then the distance of the new center of mass of the remaining plate from O will be

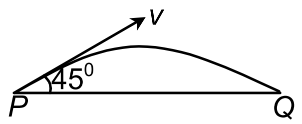


(a) $R/20$ (b) $R/16$ (c) $R/15$ (d) $\frac{3}{4}R$

EXERCISE – II

IIT-JEE- SINGLE CHOICE CORRECT

1. A projectile of mass m is fired with velocity v from a point P as shown. Neglecting air resistance, the magnitude of the change in momentum between the points P and Q is

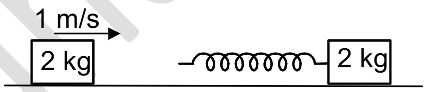


(a) zero (b) $\frac{1}{2}mv$ (c) $mv\sqrt{2}$ (d) 2 mv

2. A body of mass 1 kg initially at rest, explodes and breaks into three fragments of masses in the ratio 1 : 1 : 3. the two pieces of equal mass fly off perpendicular to each other with a speed of 15 m/s each. The speed of the heavier fragment is

(a) $5\sqrt{2}$ m/s (b) 45 m/s (c) 5 m/s (d) 15 m/s

3. A block of mass 2 kg is moving on a frictionless horizontal surface with a velocity of 1 m/s towards another block of equal mass kept at rest. The spring constant of the spring fixed at one end of stationary mass is 100 N/m. Find the maximum compression of the spring.



(a) 5 cm (b) 10 cm (c) 15 cm (d) 20 cm

4. The truck moving on a smooth horizontal surface with a uniform speed u is carrying stone-dust. If a mass Δm of the stone-dust 'leaks' from the truck through a hole in its bottom in a time Δt, the force needed to keep the truck moving at its uniform speed is

(a) $u\,\Delta m/\Delta t$ (b) $\Delta m du / dt$ (c) $u\frac{\Delta m}{\Delta t}+(\Delta m)\frac{du}{dt}$ (d) zero

5. A body of mass 2 kg moving with a velocity $(\hat{i}+2\hat{j}-3\hat{k})ms^{-1}$ collides with another body of mass 3 kg moving with a velocity $(2\hat{i}+\hat{j}+\hat{k})$ in ms^{-1}. If they stick together, the velocity in ms^{-1} of the composite body is

(a) $\frac{1}{5}(8\hat{i}+7\hat{j}-3\hat{k})$ (b) $\frac{1}{5}(-4\hat{i}+\hat{j}-3\hat{k})$

(c) $\frac{1}{5}(8\hat{i}+\hat{j}-\hat{k})$ (d) $\frac{1}{5}(-4\hat{i}+7\hat{j}-3\hat{k})$

6. A big particle of mass (3 + m) kg blasts into 3 pieces, such that a particle of mass 1 kg moves along x-axis, with velocity 2 m/s and a particle of mass 2 kg moves with velocity 1 m/s perpendicular to direction of 1 kg particle. If the third particle moves with velocity $\sqrt{2}$ m/s, then m is

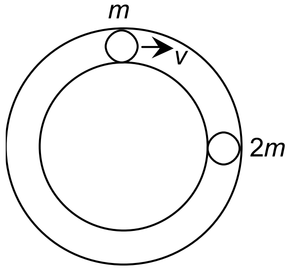
(a) 2 kg (b) 1 kg (c) $2\sqrt{2}$ kg (d) none of these

7. A shell of mass m is fired from a gun of mass M placed on smooth horizontal surface at an angle α with a speed u with respect to gun then, find the range of the shell.

(a) $\frac{v^2\sin 2\alpha}{g}$ (b) $\left(\frac{v^2\sin 2\alpha}{g}\right)\left(\frac{M}{M+m}\right)$

(c) $\frac{(v\cos\alpha - v)^2}{g}$ (d) $\frac{mv^2\sin 2\alpha}{Mg}$

8. A particle of mass m moving with a speed v collides elastically with another particle of mass $2m$ on a horizontal circular tube of radius R, then select the correct alternative(s).



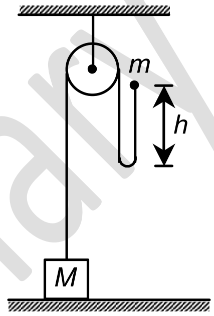
(a) the time after which the next collision will take place is $\frac{2\pi R}{v}$

(b) the time after which the next collision will take place is proportional to m

(c) the time after which the next collision will take place is inversely proportional to m

(d) the time after which the next collision will take place is dependent of the mass of the balls.

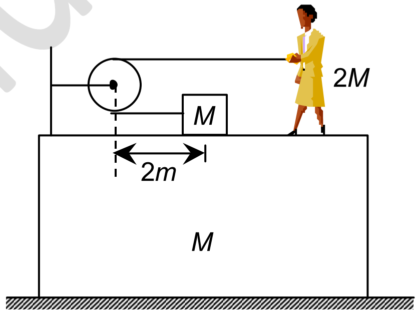
9. Two masses M and m are tied with a string and arranged as shown. The velocity of block M when it loses the contact is



(a) $2\sqrt{gh}$ (b) $\frac{m\sqrt{gh}}{(m+M)}$

(c) $\frac{2m\sqrt{gh}}{(m+M)}$ (d) $\frac{2M\sqrt{gh}}{(m+M)}$

10. A block of mass M is tied to one end of a massless rope. The other end of the rope is in the hands of a girl of mass 2M as shown in the figure. The block and the girl are resting on a rough wedge of mass M as shown in the figure. The whole system is resting on a smooth horizontal surface. The girl pulls the rope. Pulley is massless and frictionless. What is the displacement of the wedge when the block meets the pulley? (girl does not leave her position during the pull)

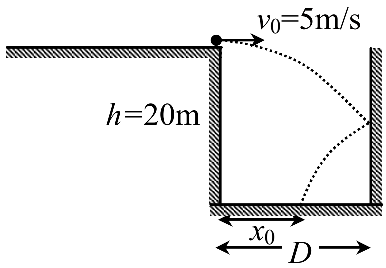


(a) 0.5 m (b) 1 m (c) zero (d) 2/3 m

11. A uniform chain of length l and mass m is hanging vertically from its ends A and B which are close together. At a given instant the end B is released. What is the tension at A when B has fallen a distance x $[x < l]$?

(a) $\frac{mg}{2}\left(1+\frac{3x}{l}\right)$ (b) $mg\left(1+\frac{2x}{l}\right)$ (c) $\frac{mg}{2}\left(1+\frac{x}{l}\right)$ (d) $\frac{mg}{2}\left(1+\frac{4x}{l}\right)$

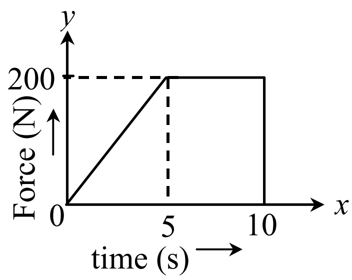
12. A ball rolls off a horizontal table with velocity v_0 = 5 m/s. The ball collides elastically from a vertical wall at a horizontal distance D (= 8 m) from the table, as shown in figure. The ball then strikes the floor a distance x_0 from the table (g = 10 m/s^2). The value of x_0 is



(a) 6 m (b) 4 m

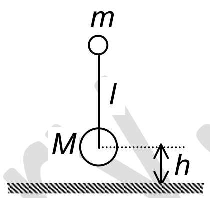
(c) 5 m (d) 7 m

13. A 20 kg block is initially at rest on a horizontal surface for which the coefficient of friction is 0.6. If a horizontal force F is applied such that it varies with time as shown in figure, the speed of the block after 10 s is
($g = 10\ \text{m/s}^2$)



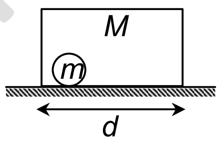
(a) 22 m/s (b) 30 m/s
(c) 24 m/s (d) none of these

14. A small ball of mass m is connected by an inextensible massless string of length l with another ball of mass $M = 4m$. They are released with zero tension in the string from a height h as shown. The time when the string becomes taut for the first time after the mass M collides with the ground is (all collisions are elastic)



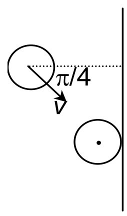
(a) $\dfrac{l}{2\sqrt{2gh}}$ (b) $\dfrac{l}{\sqrt{2gh}}$ (c) $\dfrac{2l}{\sqrt{2gh}}$ (d) none of these

15. A box of mass M is placed on a smooth horizontal surface and ball of mass m is placed on smooth surface of the box. An impulse J is imparted in horizontal direction to the ball. Find the time period of oscillation (assuming all collision are elastic)



(a) $\dfrac{2d(m+M)^2}{mJ}$ (b) $\dfrac{2dm}{J}$ (c) $\dfrac{2dM^2}{(m+M)J}$ (d) $\dfrac{2dM}{(m+M)^2 J}$

16. In a carom-board game the striker and the coins are identical and of mass m. In a particular hit the coin is hit when it is placed close to the edge of the board as shown in figure such that the coin travels parallel to the edge. If the striker is moving with speed v before the strike, then the net impulse on the striker during collision if its moves perpendicular to the edge is (all collisions the elastic)

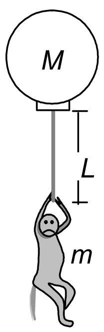


(a) $mv\sqrt{\dfrac{5}{2}}$ (b) $2\,mv$ (c) $\dfrac{mv\sqrt{3}}{2}$ (d) mv

17. Consider a system of two identical particles. One of the particles is at rest and the other has an acceleration $\vec{a}$. The centre of mass has an acceleration

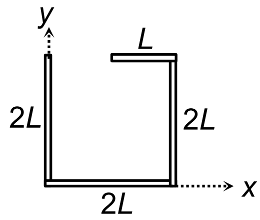
(a) zero (b) $\dfrac{1}{2}\vec{a}$ (c) $\vec{a}$ (d) $2\vec{a}$

18. The balloon, the light rope and the monkey shown in figure are at rest in the air. If the monkey reaches the top of the rope, by what distance does the balloon descend? Mass of the balloon = M, mass of the monkey = m and the length of the rope ascended by the monkey = L



(a) $\dfrac{mL}{m+M}$ (b) $\dfrac{ML}{m+M}$
(c) $\dfrac{mL}{2m+M}$ (d) none

19. A uniform rod of length $7L$ is bent in the shape as shown in the figure. The co-ordinates of the centre of mass of the system are



(a) $\frac{15}{7}L, \frac{6}{7}L$ (b) $\frac{15}{14}L, \frac{6}{7}L$

(c) $\frac{15}{7}L, \frac{6}{14}L$ (d) $\frac{15}{14}L, \frac{6}{14}L$

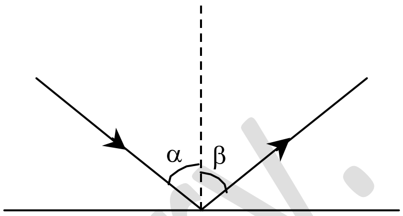
20. A cannon shell is fired to hit a target at a horizontal distance R. However it breaks into two equal parts at its highest point. One part returns to the cannon. The other part

(a) will fall at a distance R beyond target
(b) will fall at a distance $3R$ beyond target
(c) will hit the target
(d) will fall at a distance $2R$ beyond target

ONE OR MORE THAN ONE CHOICE CORRECT

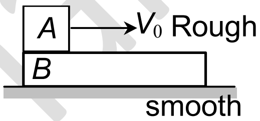
1. In head on elastic collision of two bodies of equal masses
(a) the velocities are interchanged
(b) the momenta are interchanged
(c) the faster body slows down and the slower body speeds up
(d) kinetic energy is conserved

2. A ball strikes the ground at an angle α and rebound at an angle β with the vertical as shown in the figure. Then



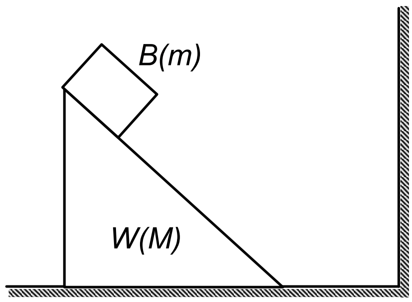
(a) coefficient of restitution is $\dfrac{\tan\alpha}{\tan\beta}$
(b) if $\alpha < \beta$ the collision is inelastic
(c) if $\alpha = \beta$ the collision is elastic
(d) the momentum of the ball is conserved.

3. In a two blocks system an initial velocity V_0 (w.r.t. ground) is given to block A



(a) the momentum of block A is not conserved
(b) the momentum of system of blocks A and B is conserved
(c) the increase in momentum of B is equal to the decrease in momentum of block A
(d) kinetic energy is conserved

4. In figure, the block B of mass m starts from rest at the top of a wedge W of mass M. All surfaces are frictionless. W can slide on the ground. B slides down onto the ground, moves along it with a speed v, has an elastic collision with the wall, and climbs back onto W then which of the following options is correct.



(a) B will reach the top of W again
(b) From the beginning, till the collision with the wall, the centre of mass of B plus W does not move horizontally.
(c) after the collision, the centre of mass of B plus W moves with the velocity $\dfrac{2mv}{m+M}$
(d) when B reaches its highest position on W, the speed of W is $\dfrac{2mv}{m+M}$

5. Which of the following(s) depend(s) on the choice of the inertial reference frame
(a) Momentum
(b) change in momentum
(c) Kinetic energy
(d) change in kinetic energy

6. A woman holding a large ball stands on a frictionless, horizontal sheet of ice. She throws the ball with speed v_0 at an angle α above the horizontal
(a) momentum of the woman will be conserved
(b) momentum of the ball will be conserved
(c) momentum of the ball plus woman will be conserved if $\alpha = 0°$
(d) horizontal component of momentum of the ball plus woman will be conserved anyways

7. A net force with x- component $\sum F_x$ acts on an object from time t_1 to time t_2. The x-component of the momentum of the object is same at t_1 as it is at t_2, then which of the following option(s) is/are possible

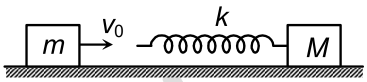
(a) $t_1 = t_2$, $\sum F_x$ is variable

(b) $t_1 \neq t_2$, $\sum F_x = 0$

(c) $t_1 \neq t_2$, $\sum F_x$ is variable

(d) $t_1 = t_2$, $\sum F_x = 0$

8. A block of mass m moving on a smooth horizontal plane with a velocity v_0 collides with a stationary block of mass M at the back of which a spring of spring constant k is attached, as shown in the figure. Select the correct alternative(s)



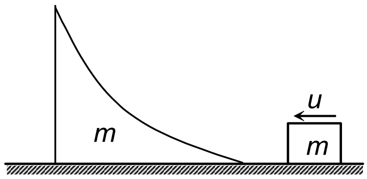
(a) velocity of centre of mass is $\dfrac{m}{m+M}v_0$

(b) initial kinetic energy of the system in centre of mass frame is $\dfrac{1}{4}\left(\dfrac{mM}{m+M}\right)v_0^2$

(c) maximum compression in the spring is $v_0\sqrt{\dfrac{mM}{(m+M)}\dfrac{1}{k}}$

(d) when the spring is in state of maximum compression the kinetic energy in the centre of mass frame is zero

9. A block of mass m is moving with velocity u on a horizontal smooth surface towards a wedge of same mass initially kept at rest. Wedge is free to move in any direction. Initially the block moves up the smooth incline plane of the wedge to a height h and again moves down back to the horizontal plane. After this process, velocity of the



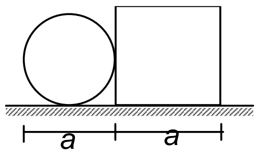
(a) wedge will be $\left(\dfrac{h}{h+1}\right)u$

(b) wedge will be u

(c) block will be $\left(\dfrac{h}{h+1}\right)u$

(d) block will be zero

10. A circular plate of diameter a is kept in contact with a square plate of edge a as shown in figure. The density of the material and the thickness are same everywhere. Then



(a) x-coordinate of the centre of mass will lie inside the square plate.
(b) x-coordinate of the centre of mass will lie inside the circular plate.
(c) y-coordinate of the centre of mass will lie inside the circular plate.
(d) y-coordinate of the centre of mass will lie inside the square plate.

EXERCISE – III

MATCH THE FOLLOWING

Note: Each statement in column – I has one or more than one match in column –II.

1. Consider a head-on collision between two particles of masses m_1 and m_2. The initial speeds of the particles are u_1 and u_2 in the same direction. Coefficient of restitution between two body is e and speeds after collision are v_1 and v_2 in same direction as before collision.

Column-I	Column-II
I. $m_1 = m_2$, $u_2 = 0$ and $e = 1$	**A.** $v_1 = u_1,\ v_2 = u_2$
II. $m_1 >> m_2$, $e = 1$ and $u_2 \neq 0$	**B.** $v_2 = 2u_1 - u_2$ and $v_1 = u_1$ $(u_2 \neq 0)$
III. $m_1 >> m_2$, $e = 1$ and $u_2 = 0$	**C.** $v_1 = 0,\ v_2 = u_1$
IV. $m_2 >> m_1$, $e = 1$	**D.** $v_2 = 2u_1,\ v_1 = u_1$
	E. $v_1 - v_2 = u_2 - u_1$

REASONING TYPE

Directions: Read the following questions and choose

(A) If both the statements are true and statement-2 is the correct explanation of **statement-1**.
(B) If both the statements are true but statement-2 is not the correct explanation of **statement-1**.
(C) If statement-1 is True and **statement-2** is False.
(D) If statement-1 is False **and statement-2** is True.

1. **Statement-1:** In oblique elastic collision of two bodies, momentum is not conserved along a line making non-zero angle with line of impact.
Statement-2: In oblique collision of same masses, one at rest initially, bodies go at right angle to each other after collision.
(a) (A) (b) (B) (c) (C) (d) (D)

2. **Statement-1:** In elastic collision, kinetic energy may not be conserved during the collision time.
Statement-2: In elastic collision potential energy of bodies may change during collision time.
(a) (A) (b) (B) (c) (C) (d) (D)

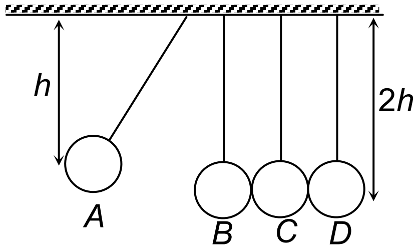
3. **Statement-1:** $\vec{F} = \frac{d\vec{P}}{dt}$ is true for the system of variable mass as treating the mass variable.
Statement-2: If net external forces on a system of variable mass is zero, instantaneous acceleration of centre of mass of system may be non zero.
(a) (A) (b) (B) (c) (C) (d) (D)

4. **Statement-1:** Area per unit mass of force-time graph gives change in velocity.
Statement-2: An impulse $\vec{I}$ changes the momentum of a body by $\vec{P}$ then $\vec{I} = \vec{P}$
(a) (A) (b) (B) (c) (C) (d) (D)

5. **Statement-1:** Two balls are thrown simultaneously in air. The acceleration of centre of mass of the two balls while in air depends on the masses of the two balls.
Statement-2: The acceleration of centre of mass is given by $\vec{a} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$
(a) (A) (b) (B) (c) (C) (d) (D)

LINKED COMPREHENSION TYPE

Figure shows on arrangement of four steel bob of mass m, supported by a vertical string and touches the neighbouring bob. The centres of gravity of the bobs are at a distance of ($2h$) below their points of suspension. Bob A is displaced to the left so that its string is taut with its centre of gravity at a vertical distance h below its point of suspension. The bob A is then released from rest from the position shown in the figure. Assume the collision between the bobs to be perfectly elastic.



1. After impact the velocity of bob A is

(a) $\sqrt{2gh}$ (b) $2\sqrt{2gh}$ (c) zero (d) $4\sqrt{2gh}$

2. The vertical height raised by the sphere D is

(a) $2h$ (b) h (c) zero (d) $h/2$

3. If coefficient of restitution is $e = 0.2$, then velocity of D after impact is

(a) $\frac{27}{125}\sqrt{2gh}$ (b) $\frac{27}{29}\sqrt{2gh}$ (c) $\frac{1}{125}\sqrt{2gh}$ (d) $\sqrt{2gh}$

ANSWERS

EXERCISE – I

NEET-SINGLE CHOICE CORRECT

1. (a)	2. (d)	3. (a)	4. (c)	5. (a)
6. (c)	7. (a)	8. (c)	9. (c)	10. (b)
11. (a)	12. (b)	13. (c)	14. (b)	15. (c)
16. (a)	17. (d)	18. (c)	19. (a)	20. (a)
21. (a)	22. (c)	23. (c)	24. (d)	25. (a)

EXERCISE – II

IIT-JEE-SINGLE CHOICE CORRECT

1. (c)	2. (a)	3. (b)	4. (d)	5. (a)
6. (a)	7. (b)	8. (a)	9. (c)	10. (a)
11. (a)	12. (a)	13. (c)	14. (b)	15. (b)
16. (a)	17. (b)	18. (a)	19. (b)	20. (a)

ONE OR MORE THAN ONE CHOICE CORRECT

1. (a,c,d)	2. (a,b,c)	3. (a,b,c)	4. (b,c,d)	5.(a,c,d)
6. (c,d)	7. (a,b,c,d)	8. (a,c,d)	9. (b,d)	10. (a,d)

EXERCISE – III

MATCH THE FOLLOWING

1. I – C, E ; II – B, E ; III – D, E ; IV – E

REASONING TYPE

1. (d)	2. (a)	3. (c)	4. (a)	5. (d)

LINKED COMPREHENSION TYPE

1. (c)	2. (b)	3. (a)