PHYSICS

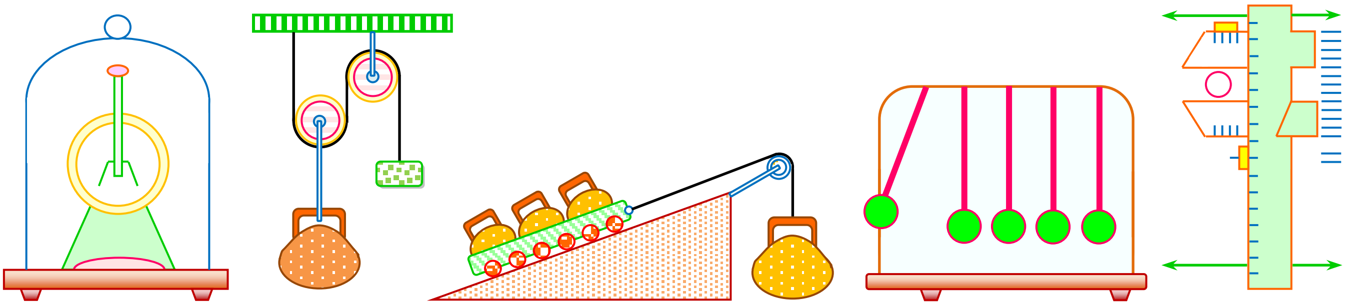
for JEE, NEET & BOARD

11th

GRAVITATION

Key Features

- ✓ All-in one Study Material (for Boards/IIT/Medical/Olympiads)
- ✓ Multiple Choice Solved Questions for Boards & Entrance Examinations
- ✓ Concise, conceptual & trick – based theory
- ✓ Magic trick cards for quick revision & understanding
- ✓ NCERT & Advanced Level Solved Examples



GRAVITATION

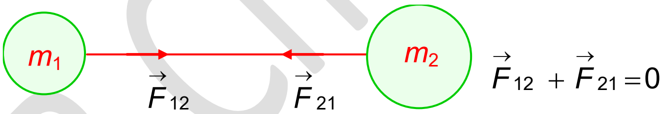
1 NEWTON'S LAW OF GRAVITATION

Newton's law of gravitation states that every particle in the universe attracts every other particle with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between them. The direction of the force is along the line joining the particles.

Therefore from Newton's law of gravitation

$$\vec{F} = \frac{Gm_1 m_2}{r^2}\,\hat{r} \qquad \dots (1)$$

where G is called the gravitational constant and $\hat{r}$ is the unit vector along the line joining the two mass particles.



The gravitational force between two particles form an action reaction pair.

Illustration 1

Question: **A mass M is split into two parts m and $(M - m)$, which are then separated by a certain distance. What ratio (M/m) maximizes the gravitational force between the parts?**

Solution: If 'r' is the distance between m and $(M - m)$, the gravitational force will be

$$F = G\frac{(M-m)m}{r^2} = \frac{G}{r^2}\left[Mm - m^2\right]$$

for F to be maximum, $\dfrac{dF}{dm} = 0$,

i.e., $\dfrac{d}{dm}\left[\dfrac{G}{r^2}(Mm - m^2)\right] = 0$

or, $M - 2m = 0$ $\quad \left[\because \dfrac{G}{r^2} \neq 0\right]$

or, $M/m = \mathbf{2}$,

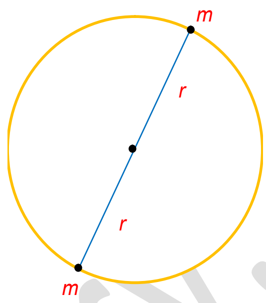
i.e., the force will be maximum when two parts are equal.

Illustration 2

Question: **Two particles of equal mass m = 10 kg are moving in a circle of radius r = 1.65m under the action of their mutual gravitational attraction. Find the speed (in μm/s) of each particle. (Take G = 6.6×10^{-11} Nm²/kg²)**

Solution: The particles will always remain diametrically opposite so that the force on each particle will be directed along the radius.

Considering the circular motion of one particle, we have,

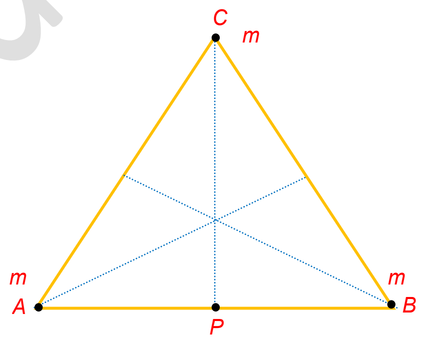


$$\frac{mv^2}{r} = \frac{Gm.m}{(2r)^2} \quad \therefore v = \sqrt{\frac{Gm}{4r}} = \mathbf{10\ \mu m/s}$$

Illustration 3

Question: **Three equal particles each of mass m =10 kg are placed at the three corners of an equilateral triangle of side a = 1mm. Find the force exerted (in CGS units) by this system on another particle of mass m placed at (a) the mid–point of a side (b) at the center of the triangle. (Take $G = 6.6 \times 10^{-11}$ Nm²/kg²)**

Solution: As gravitational force is a two body interaction, the principle of superposition is valid, i.e., resultant force on the particle of mass m at P is



$$\vec{F} = \vec{F}_A + \vec{F}_B + \vec{F}_C$$

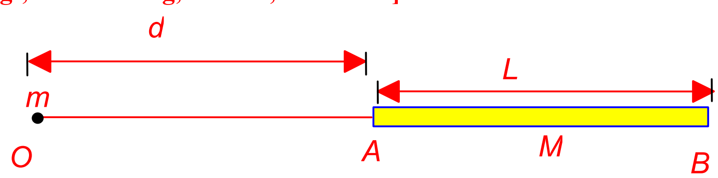
(a) As shown in the above figure, when P is at the mid–point of a side, $\vec{F}_A$ and $\vec{F}_B$ will be equal in magnitude but opposite in direction. So they will cancel each–other. So the point mass m at P will experience a force due to C only, i.e.,

$$F = F_C = \frac{Gmm}{(CP)^2} = \frac{Gm^2}{(a\sin 60^0)^2} = \frac{4Gm^2}{3a^2} = \mathbf{880\ dynes} \text{ along } PC$$

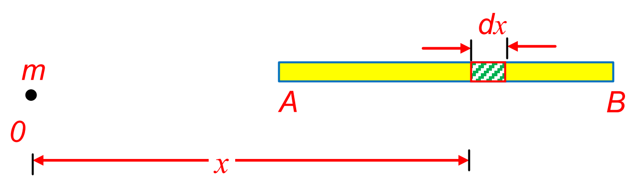
(b) From symmetry, the net force on the particle at the center of triangle = **0**

Illustration 4

Question: **Find the gravitational force (in micronewton) of attraction on the point mass 'm' placed at O by a thin rod of mass M and length L as shown in figure. [take m = 10 kg, G = 6.6×10^{-11} Nm²/kg², M = 1000 kg, L = 1m, d = 10 cm]**



Solution:



First we need to find the force due to an element of length dx. The mass of the element is $dm = \left(\frac{M}{L}\right)dx$.

so, $dF = G\frac{Mm}{L}\frac{dx}{x^2}$

$\therefore$ The net gravitational force is

$$F = \frac{GMm}{L}\int_d^{d+L}\frac{dx}{x^2} = \frac{GMm}{L}\left[\frac{1}{d} - \frac{1}{L+d}\right]$$

$$= \frac{GMm}{d(L+d)} = \mathbf{6\ \mu N}$$

Notice that when $d >> L$, we find $F = \frac{GMm}{d^2}$, the result for two point masses.

2 GRAVITATIONAL FIELD

Gravitational field due to a mass is defined as the region of space in which it interacts with other masses. In order to get the extent of interaction between two masses we defined another quantity called gravitational field intensity. Gravitational field intensity due to a mass m at a distance r is defined as the force acting on unit mass kept at a distance r. The gravitational field intensity is a vector quantity and its direction is the direction along which the unit mass has a tendency to move. The unit of gravitational field intensity is N/kg and its dimensions are $[LT^{-2}]$.

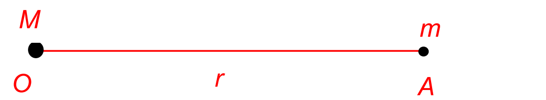
CALCULATION OF GRAVITATIONAL FIELD

(a) Gravitational field intensity due to a point mass

Consider a point mass M at O and let us calculate gravitational intensity at A due to this point mass.

Suppose a test mass is placed at A.

By Newton's law of gravitation, force on the test mass

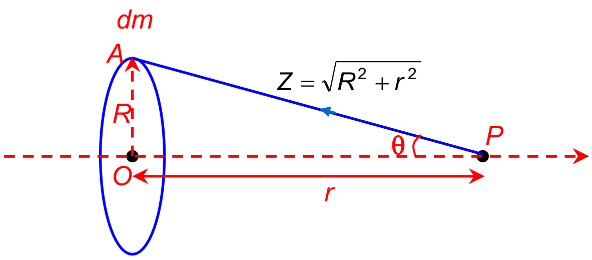


$$F = \frac{GMm}{r^2} \text{ along } \vec{AO}$$

$$\vec{E} = \frac{\vec{F}}{m} = -\frac{GM}{r^2}\hat{e}_r \qquad \ldots (2)$$

(b) Gravitational field intensity due to a uniform circular ring, at a point on its axis.

Figure shows a ring of mass M and radius R. Let P is the point at a distance r from the centre of the ring. By symmetry the field must be towards the centre that is along $\vec{PO}$



Let us assume that a particle of mass dm on the ring say, at point A. Now the distance AP is $\sqrt{R^2 + r^2}$. Again the gravitational field at P due to dm is along $\vec{PA}$ and its magnitude is

$$dE = \frac{Gdm}{Z^2}$$

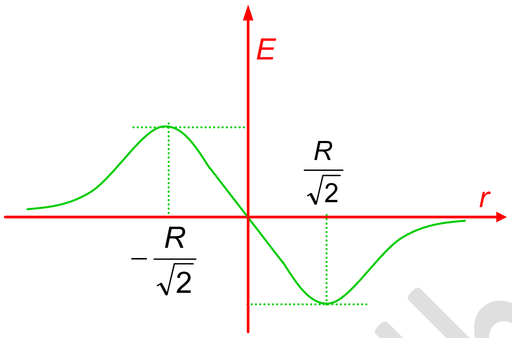
$$\therefore dE\cos\theta = \frac{Gdm}{Z^2}\cos\theta$$

Net gravitational field $E = \dfrac{G\cos\theta}{Z^2}\int dm$

$$= \frac{GM}{Z^2}\frac{r}{Z}$$

$$= \frac{GMr}{(r^2+R^2)^{3/2}} \text{ along } \vec{PO} \qquad \dots (3)$$

Variation of gravitational field due to a ring as a function of its axial distance.



Important points:

(i) If $r >> R$, $r^2 + R^2 \simeq r^2$

$\therefore \quad E = -\dfrac{GMr}{r^3} = -\dfrac{GM}{r^2}$ [where negative sign is because of attraction]

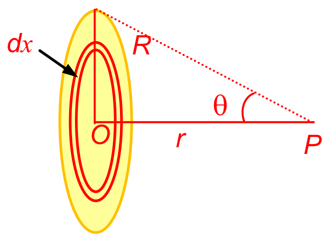
Thus, for a distant point, a ring behaves as a point mass placed at the center of the ring.

(ii) If $r << R$, $r^2 + R^2 \simeq R^2$

$\therefore \quad E = -\dfrac{GMr}{R^3}$

i.e., $E \propto r$

(c) Gravitational field intensity due to a uniform disc at a point an its axis



Let the mass of disc be M and its radius is R and P is the point on its axis where gravitational field is to be calculated.

Let us draw a circle of radius x and centre at O. We draw another concentric circle of radius $x + dx$. The part of disc enclosed between two circle can be treated as a uniform ring of radius x. The area of this ring is $2\pi x dx$

Therefore mass dm of the ring $= \dfrac{M}{\pi R^2} 2\pi x dx$

$$= \frac{2Mxdx}{R^2}$$

gravitational field at P due to the ring is,

$$dE = \frac{G\left(\frac{2Mxdx}{R^2}\right)r}{(r^2+x^2)^{3/2}}$$

$$\int dE = \frac{2GMr}{R^2}\int_O^a \frac{xdx}{(r^2+x^2)^{3/2}}$$

$$= \frac{2GMr}{R^2}\left[-\frac{1}{\sqrt{r^2+x^2}}\right]_0^R$$

$$= \frac{2GMr}{R^2}\left[\frac{1}{r}-\frac{1}{\sqrt{r^2+R^2}}\right]$$

in terms of θ

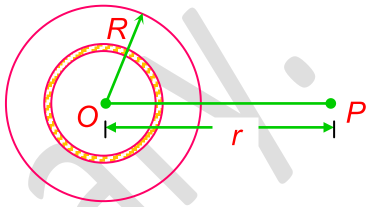
$$E = \frac{2GM}{R^2}(1-\cos\theta) \qquad \ldots (4)$$

(d) Gravitational field due to a uniform solid sphere

Case I:

Field at an external point

Let the mass of sphere is M and its radius is R we have to calculate the gravitational field at P.



$$\int dE = \int \frac{Gdm}{r^2}$$

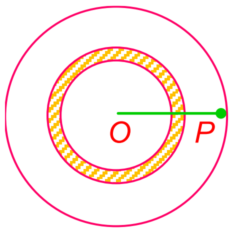
$$= \frac{G}{r^2}\int dm = \frac{GM}{r^2} \qquad \ldots (5)$$

Thus, a uniform sphere may be treated as a single particle of equal mass placed at its centre for calculating the gravitational field at an external point.

Case II

Field at an internal point

Suppose the point P is inside the solid sphere, in this case $r < R$ the sphere may be divided into thin spherical shells all centered at O. Suppose the mass of such a shell is dm. Then



$$dE = \frac{Gdm}{r^2} \text{ along } PO$$

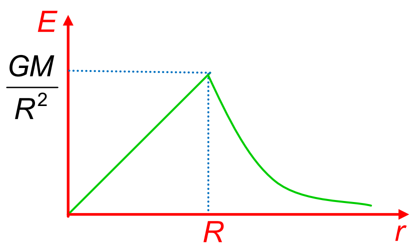
$$= \frac{G}{r^2}\int dm$$

where $\int dm = \frac{M}{\frac{4}{3}\pi R^3}\frac{4}{3}\pi r^3 = \frac{Mr^3}{R^3}$

$$\therefore \quad E = \frac{GM}{R^3}r \qquad \ldots (6)$$

Therefore gravitational field due to a uniform sphere at an internal point is proportional to the distance of the point from the centre of the sphere. At the centre $r = 0$ the field is zero. At the surface of the sphere $r = R$

$$E = \frac{GM}{R^2} \qquad \ldots (7)$$



☞ Gravitational field due to solid sphere is continuous but it is not differentiable function.

(e) Field due to uniform thin spherical shell

Case I

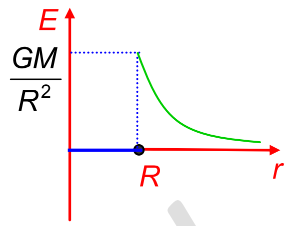
When point lies inside the spherical shell

$$\int dE = \frac{G}{r^2}\int m_{\text{enclosed}} = 0 \qquad \ldots (8)$$

Case II

When point P lies outside the spherical shell

$$\int dE = \frac{G}{r^2}\int dm = \frac{GM}{r^2} \qquad \ldots (9)$$

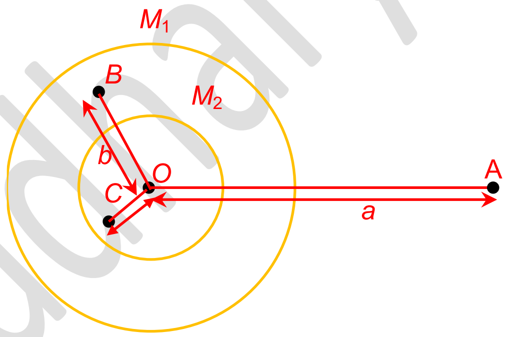


☞ ***Gravitational field due to thin spherical shell is both discontinuous and non-differentiable function.***

Illustration 5

Question: **Two concentric shells of masses M_1 and M_2 are situated as shown in figure. Find the force on a particle of mass $m = 1$ kg when the particle is located at (a) $r = a$ (b) $r = b$ (c) $r = c$. The distance r is measured from the center of the shell.**

[M_1 = 500 kg, M_2 = 1500kg, a = $\sqrt{6.6}$ m, b = $\sqrt{3.3}$, G = 6.6×10^{-11} kgm²/kg²] Take 10^{-8} N as unit]



Solution: We know that attraction at an external point due to spherical shell of mass M is $\frac{GM}{r^2}$ while at an internal point is zero. So

(a) for $r = a$, the point is external for both the shell ; so

$$E_A = \frac{G(M_1 + M_2)}{a^2}$$

$$\therefore\ F_A = mE_A = \frac{G\,[M_1 + M_2]m}{a^2} = \textbf{2 units}$$

(b) For $r = b$, the point is external to the shell of mass M_2 and internal to the shell of mass M_1 ; so

$$E_B = \frac{GM_2}{b^2} + O$$

$$\therefore\ F_B = mE_B = \frac{GM_2 m}{b^2} = \textbf{3 units}$$

(c) For $r = c$, the point is internal to both the shells, so

$$E_C = 0 + 0 = 0$$

$$\therefore\ F_C = m\,E_C = \mathbf{0}$$

3 GRAVITATIONAL POTTENTIAL

At a point in a gravitational field, potential (V) is defined as the work done by the external agent against the gravitational field in bringing unit mass from infinity to that point.

Mathematically ,

$$V = \frac{W}{m}$$

It is a scalar having dimensions [$L^2\,T^{-2}$] and SI unit J/kg.

⇒ By the definition of potential energy, $U = W$

So, $V = \dfrac{U}{m}$

i.e., $U = mV$

Thus, gravitational potential at a point represents potential energy of a unit point mass at that point.

$\Rightarrow$ by definition of work $W = \int \vec{F}_{ext} . d\vec{r}$

$\therefore$ But, $\vec{F}_{ext} = -\vec{F}_{gravitation}$

$\therefore$ W = $-\int \vec{F}_{gravitational} . d\vec{r}$

So, $V = -\dfrac{\int \vec{F}_{gravitational} . d\vec{r}}{m} = -\int \vec{E} . d\vec{r}$ $\left[\because \dfrac{\vec{F}_{gravitational}}{m} = \vec{E} \right]$

i.e., $dV = -Edr$

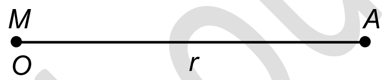
or $E = -\dfrac{dV}{dr}$ … (10)

So the potential can also be defined as a scalar function of position whose negative gradient. i.e., space derivative gives field intensity.

$\Rightarrow$ *Negative of the slope of V vs r graph gives intensity.*

Calculation of Gravitational potential

(a) Gravitational potential at a point due to a point mass



We have, gravitational field due to a point mass

$$E = -\frac{GM}{r^2}$$

[The negative sign is used as gravitational force is attractive]

$\therefore$ $V = -\int Edr = -\dfrac{GM}{r} + C$

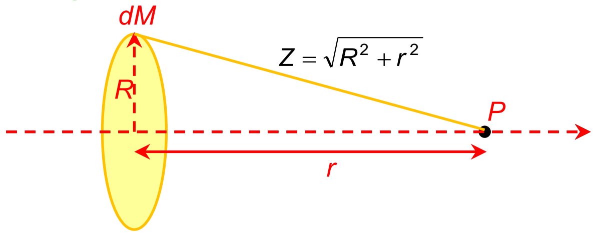
when, $r = \infty$, $V = 0$; so $C = 0$

$\therefore$ $V = -\dfrac{GM}{r}$ … (11)

(b) Gravitational potential at a point due to a ring

Let M be the mass and R be the radius of a thin ring.

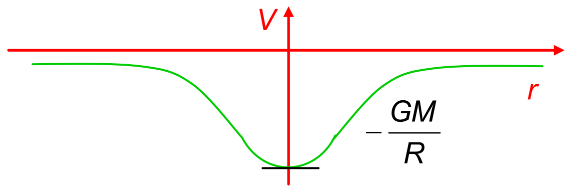
Considering a small element of the ring and treating it as a point mass, the potential at the point P is



$$dV = -\frac{G\,dM}{Z} = -\frac{G\,dM}{\sqrt{R^2 + r^2}}$$

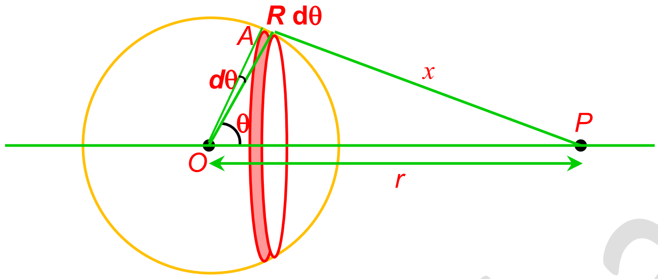
Hence, the total potential at the point P is given by

$$V = -\int \frac{G\,dM}{\sqrt{R^2 + r^2}} = -\frac{GM}{\sqrt{R^2 + r^2}} \quad \text{… (12)}$$



at $r = 0$ $\frac{dV}{dr} = 0$ $\therefore$ gravitational field is zero at the centre.

(c) Gravitational potential at a point due to a spherical shell (hollow sphere)



Consider a spherical shell of mass M and radius R. P is a point at a distance 'r' from the center O of the shell.

Consider a ring at right angles to OP. Let θ be the angular position of the ring from the line OP.

The radius of the ring = $R \sin\theta$

The width of the ring = $R\, d\theta$

The surface area of the ring = $(2\pi R \sin\theta).\, R\, d\theta$

$= 2\pi R^2 \sin\theta\, d\theta$

The mass of the ring $= (2\pi R^2 \sin\theta\, d\theta) \times \frac{M}{4\pi R^2}$

$= \frac{M \sin\theta\, d\theta}{2}$

If 'x' is the distance of the point P from a point of the ring, then the potential at P due to the ring.

$$dV = -\frac{GM \sin\theta\, d\theta}{2x}$$

From the 'cosine-property' of the triangle OAP,

$$x^2 = R^2 + r^2 - 2Rr\cos\theta$$

Differentiating,

$$2x\, dx = 2\, Rr \sin\theta\, d\theta$$

$$\therefore \quad \sin\theta\, d\theta = \frac{x\, dx}{Rr}$$

Substituting the above value of $\sin\theta\, d\theta$ in equation (i), we get

$$dV = -\frac{GM}{2x} \times \frac{x\, dx}{Rr}$$

$$= -\frac{GM}{2Rr} dx$$

Case I:

When the point P lies outside the shell.

$$V = -\frac{GM}{2Rr}\int_{r-R}^{R+r} dx = -\frac{GM}{2Rr}[x]_{r-R}^{r+R}$$

$$V = -\frac{GM}{2Rr}[(R+r)-(r-R)]$$

$$V = -\frac{GM}{r} \quad \ldots (13)$$

This is the potential at P due to a point mass M at O.

For external point, a spherical shell behaves as a point mass supposed to be placed at it center.

Case II:

When the point P lies inside the spherical shell.

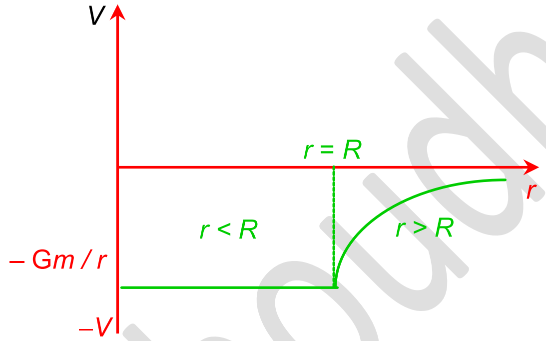
$$V = -\frac{GM}{2Rr}\int_{R-r}^{R+r} dx = -\frac{GM}{2Rr}[x]_{R-r}^{R+r}$$

or $$V = -\frac{GM}{R} \quad \ldots (14)$$

This expression is independent of r. Thus, the potential at every point inside the spherical shell is the same and it is equal to the potential of the surface of the shell.

Thus, the gravitational field inside a spherical shell is zero everywhere.

Graphical representation of the variation of V with r in case of a hollow spherical shell

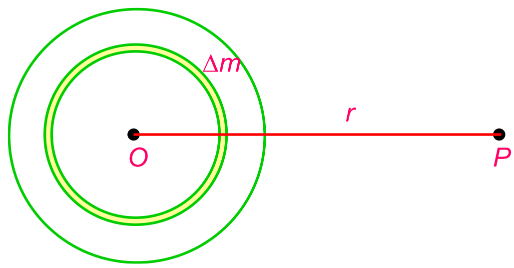


(d) Gravitational potential due to a homogeneous solid sphere

Case I:

When the point P lies outside the sphere

Consider a homogenous solid sphere of mass M and radius 'R'. P is a point at a distance 'r' from the center of the sphere.



The solid sphere may be supposed to be made up of large number of thin concentric spherical shells. Consider one such shell of mass Δm.

The potential at P due to the shell $= -\frac{G\,\Delta m}{r}$

So, the potential at P due to the entire sphere.

$$V = -\sum\frac{G\,\Delta m}{r} = -\frac{G}{r}\sum \Delta m$$

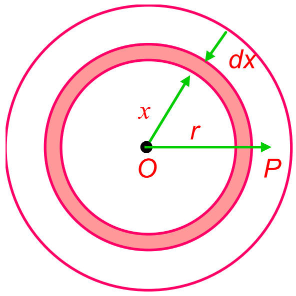
$$V = -\frac{GM}{r} \quad [\because M = \Sigma\,\Delta m] \quad \ldots (15)$$

Hence for an external point, a solid sphere behaves as if the whole of its mass is concentrated at the centre.

Case II:

When the point P lies inside the sphere

Let us consider a concentric spherical surface thorough the point P. The potential at P arises out of the inner sphere and the outer thick spherical shell.



$\therefore$ $V = V_1 + V_2$, where V_1 = potential due to the inner sphere and V_2 = potential due to the outer thick shell.

The mass of the inner sphere = $\frac{4\pi r^3}{3}\rho$, where

ρ = density of the sphere = $\frac{M}{\frac{4}{3}\pi R^3} = \frac{3M}{4\pi R^3}$

The potential at P due to this sphere

$$V_1 = -\frac{G\left[\frac{4\pi r^3}{3}\right]\rho}{r} = -\frac{4\pi G\rho}{3}r^2 \qquad \text{... (i)}$$

To find V_2, consider a thin concentric shell of radius x and thickness dx.

The volume of the shell = $4\pi x^2\, dx$

The mass of the shell = $4\pi x^2\, dx\, \rho$.

The potential at P due to this shell

$$V_2 = -\int_r^R 4\pi G\rho x\, dx$$

$$= -4\pi G\rho\left[\frac{x^2}{2}\right]_r^R = -4\pi G\rho\left[\frac{R^2}{2} - \frac{r^2}{2}\right]$$

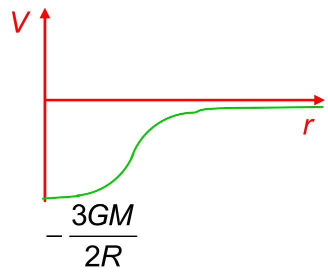
$$-2\pi G\rho\left[R^2 - r^2\right]$$

$$\therefore\ V_1 + V_2 = -\frac{4\pi G\rho r^2}{3} - 2\pi G\rho\left[R^2 - r^2\right]$$

$$= -\frac{4\pi G\rho}{3}\left[r^2 + \frac{3R^2}{2} - \frac{3r^2}{2}\right]$$

$$= -\frac{4\pi G\rho}{3}\left[\frac{3R^2}{2} - \frac{r^2}{2}\right]$$

$$= -\frac{4\pi G}{3}\cdot\frac{3M}{4\pi R^3}\left[\frac{3R^2 - r^2}{2}\right]$$



or, $$V = -\frac{GM}{2R^3}\left[3R^2 - r^2\right] \qquad \text{... (16)}$$

at $r = 0$ $\frac{dV}{dr} = 0$

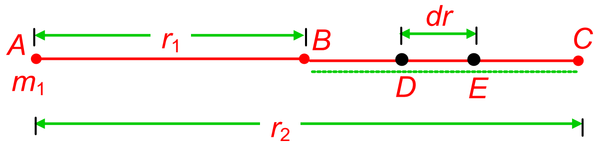
Hence gravitational field is 0 at the centre of a solid sphere.

4 GRAVITATIONAL POTENTIAL ENERGY

The potential energy of a system corresponding to a conservative force is defined as

$$\int_{U_i}^{U_f} dU = -\int \vec{F} \cdot \vec{dr}$$

i.e., the change in potential energy is equal to negative of work done



Let a particle of mass m_1 be kept fixed at point A and another particle of mass m_2 is taken from a point B to C. Initially, the distance between the particle is r_1 & finally it becomes $AC = r_2$. We have to calculate the change in gravitation in potential energy of the system of two particles.

Consider a small displacement when the distance between the particles changes from r to $r + dr$. In the figure this corresponds to the second particle going from D to E.

Force acting on second particle is

$$\vec{F} = \frac{Gm_1m_2}{r_2} \text{ along } \vec{DA}$$

$$\therefore \qquad dW = \vec{F} \cdot \vec{dr} = -\frac{Gm_1m_2}{r^2} dr$$

$$dU = -dW = \int \frac{Gm_1m_2 \; dr}{r^2}$$

$$\int_{r_1}^{r_2} dU = Gm_1m_2 \int_{r_1}^{r_2} \frac{dr}{r^2} = Gm_1m_2 \left(-\frac{1}{r}\right)_{r_1}^{r_2}$$

$$U(r_2) - U(r_1) = Gm_1m_2 \left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad \ldots \text{(i)}$$

We choose the potential energy of the two particles system to be zero when the distance between them is infinity. This means $U(\infty) = 0$

Hence $U(r) = U(r) - U(\infty)$

Now in equation (i)

Take $r_1 = r$ and $r_2 = \infty$

$$U(\infty) - U(r) = Gm_1m_2 \left(\frac{1}{r} - \frac{1}{\infty}\right)$$

$$\therefore \qquad U(r) = -\frac{Gm_1m_2}{r_1} \qquad \ldots \textbf{(17)}$$

Hence when two masses m_1 and m_2 separated by a distance their gravitational potential energy is

$$U(r) = \frac{-Gm_1m_2}{r}$$

Illustration 6

Question: Three particles each of mass m = 100 kg are placed at the corners of an equilateral triangle of side d = 1 cm. Calculate

(a) the potential energy of the system (in μJ)

(b) work done on the system if the side of the triangle is changed from d to $2d$ (in μJ)

Solution:

(a) $$U = -\frac{3Gmm}{d}$$

$$= -\frac{3Gm^2}{d} = \mathbf{-198\ \mu J}$$

(b) $$U_i = -\frac{3Gm^2}{d}$$

$$U_f = -\frac{3Gm^2}{2d}$$

∴ Work done $= W = \Delta U = U_f - U_i = \frac{3Gm^2}{2d} = \mathbf{99\ \mu J}$

5 VARIATION IN ACCELERATION DUE TO GRAVTY

(a) With altitude

At the surface of the earth,

$$g = \frac{GM}{R^2}$$

For a height h above the surface of the earth,

$$g' = \frac{GM}{(R+h)^2}$$

$$\frac{g'}{g} = \frac{R^2}{(h+R)^2} = \frac{1}{\left(1+\frac{h}{R}\right)^2}$$

or, $$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

So, with increase in height, g decreases. If $h << R$.

$$g' = g\left[1+\frac{h}{R}\right]^{-2} \approx g\left[1-\frac{2h}{R}\right]$$

or, $$g' = g\left[1-\frac{2h}{R}\right] \quad \ldots (18)$$

(b) With depth

At the surface of the earth,

$$g = \frac{GM}{R^2}$$

for a point at a depth d below the surface,

$$g' = \frac{GM}{R^3}[R-d]$$

∴ $$\frac{g'}{g} = \left(\frac{R-d}{R}\right)$$

i.e., $g' = g\left[1-\frac{d}{R}\right]$ … (19)

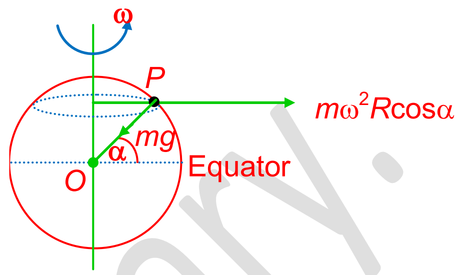
So with increase in depth below the surface of the earth, g decreases and at the center of the earth it becomes zero.

It should be noted that value of g decreases if we move above the surface or below the surface of the earth.

(c) Due to rotation of earth

The earth is rotating about its axis from west to east. So, the earth is a non-inertial frame of reference. Every body on its surface experiences a centrifugal force $m\omega^2 R\cos\alpha$. Where α is latitude of the place.

The net force on a particle on the surface of the earth



$$F = \sqrt{m^2g^2 + m^2\omega^4R^2\cos^2\alpha + 2(mg)(m\omega^2R\cos\alpha)[\cos(180-\alpha)]} = mg' \quad \dots (20)$$

Therefore,

(i) g is maximum (= g) when cos α = minimum = 0, i.e., α = 90°, i.e., at the pole.

(ii) g is minimum ($= g - \omega^2R$) when cos α = maximum = 1, i.e., , α = 0°, i.e., at the equator.

Illustration 7

Question: **Calculate the acceleration due to gravity (in cm/s²) at the surface of Mars if its diameter is 6760 km and mass one tenth that of the earth. (Given that the diameter of earth is 12742 km and acceleration due to gravity on its surface is 9.8 m/s²)**

Solution: We know that

$$g = \frac{GM}{R^2}$$

$$\therefore \quad \frac{g_M}{g_E} = \left(\frac{M_M}{M_E}\right)\left(\frac{R_E}{R_M}\right)^2 = \left[\frac{1}{10}\right]\left[\frac{12742}{6760}\right]^2$$

or, $$\frac{g_M}{g_E} = 0.35$$

$\therefore \quad g_M = 0.35 \times g_E = 9.8 \times 0.35 = 3.48\ \text{m/s}^2 =$ **348 cm/s²**

Illustration 8

Question: **The mass of moon is given as $x \times 10^{20}$ kg and its density is $y \times 10^2$ kg/m³. If acceleration due to gravity on its surface is 1.62 m/s² and its radius is 1.74×10^6 m ($G = 6.67 \times 10^{-11}$ MKS units). Then find x and y.**

Solution: We know that

$$g = \frac{GM}{R^2}$$

$$\therefore \quad M = \frac{gR^2}{G} = \frac{1.62 \times (1.74 \times 10^6)^2}{6.67 \times 10^{-11}}$$

or, $M = 7.35 \times 10^{22}$ kg $\Rightarrow x =$ **735**

Now, $$\rho = \frac{M}{V} = \frac{gR^2}{G\left(\frac{4}{3}\pi R^3\right)}\ \frac{3g}{4\pi GR}$$

$\therefore \qquad \rho = \dfrac{3\times 1.62}{4\times 3.14\times 6.67\times 10^{-11}\times 1.74\times 10^{6}}$

$= 3.3\times 10^{3}\ \text{kg/m}^{3}$

$\Rightarrow \qquad y = \mathbf{33}$

6 ESCAPE SPEED

It is the minimum speed with which a body must be projected from the surface of a planet (usually the earth) so that it permanently overcomes and escapes from the gravitational field of the planet (the earth). We can also say that a body projected with escape speed will be able to go to a point which is at infinite distance from the earth.

If a body of mass m is projected with speed v from the surface of a planet of mass M and radius R, then

K.E. $= \dfrac{1}{2}mv^2$; $G.P.E. = -\dfrac{GMm}{R}$

Total mechanical energy (T.M.E.) of the body $= \dfrac{1}{2}mv^2 - \dfrac{GMm}{R}$

If the v' is the speed of the body at infinity, then

T.M.E. at infinity $= 0 + \dfrac{1}{2}mv'^2 = \dfrac{1}{2}mv'^2$

Applying the principle of conservation of mechanical energy, we have

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{1}{2}mv'^2$$

or, $\quad v^2 = \dfrac{2GM}{R} + v'^2$

v will be minimum when $v' \to 0$, i.e.,

$$V_e = V_{\min} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} \qquad \ldots (21)$$

$$\left[\because g = \frac{GM}{R^2}\right]$$

Important points

(i) Escape speed is independent of the mass and direction of projection of the body.

(ii) For earth as $g = 9.8\ \text{m/s}^2$ and $R = 6400$ km

$$V_e = \sqrt{2\times 9.8\times 6.4\times 10^6} = 11.2\ \text{km/s}$$

Illustration 9

Question: **What will be the acceleration due to gravity on the surface of the moon if its radius were $\frac{1}{4}$th the radius of the earth and its mass $\left(\frac{1}{80}\right)$th the mass of the earth. Given g = 10 m/s².**

Solution: As on the surface of planet,

$$g = \frac{GM}{R^2}$$

we have, $\dfrac{g_M}{g_E} = \dfrac{M_M}{M_E}\times\left(\dfrac{R_E}{R_M}\right)^2 = \dfrac{1}{80}\times(4)^2 = \dfrac{1}{5}$

$\therefore \qquad g_M = \dfrac{g}{5} = \dfrac{10}{5} = \mathbf{2\ m/s^2}$

7 MOTION OF A SATWELLITE

Consider a satellite of mass *m* revolving in a circle around the earth. If the satellite is at a height *h* above the earth's surface, the radius of its orbit is $r = R + h$, where *R* is the radius of the earth. The gravitational force between *m* and *M* provides the necessary centripetal force for circular motion.

(a) ORBITAL VELOCITY (V_0)

The velocity of a satellite in its orbit is called orbital velocity. Let V_0 be the orbital velocity of the satellite, then

$$\frac{GMm}{r^2} = \frac{mv_0^2}{r}$$

$$\Rightarrow \quad v_0 = \sqrt{\frac{GM}{r}} \qquad \dots (22)$$

$$\text{or,} \quad v = \sqrt{\frac{GM}{R+h}} \qquad (\because r = R + h)$$

Important points

- Orbital velocity is independent of the mass of the orbiting body and is always along the tangent to the orbital.
- Close to the surface of the earth, $r \approx R$ as $h << R$

$$\therefore \quad V_0 = \sqrt{\frac{GM}{R}} = \sqrt{gR} = \sqrt{10 \times 6.4 \times 10^6} \approx 8 \text{ km/s}$$

- Close to the surface of the planet

$$V_0 = \sqrt{\frac{GM}{R}} = \frac{V_e}{\sqrt{2}}$$

$$\text{i.e.,} \quad V_e = \sqrt{2}V_0$$

(b) TIME PERIOD OF A SATELLITE

The time taken by a satellite to complete one revolution is called the time period (*T*) of the satellite
It is given by

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{GM}}$$

$$\text{or,} \quad T = \frac{2\pi r\sqrt{r}}{\sqrt{GM}} \quad \text{or,} \quad T^2 = \left(\frac{4\pi^2}{GM}\right)r^3 \qquad \dots (23)$$

$$\Rightarrow \quad T^2 \propto r^3$$

(c) ANGULAR MOMENTUM OF A SATELLITE (*L*)

In case of satellite motion, angular momentum will be given by

$$L = mvr = mr\sqrt{\frac{GM}{r}}$$

$$L = \left(m^2GMr\right)^{1/2} \qquad \dots (24)$$

- *In case of satellite motion, the net force on the satellite is centripetal force. The torque of this force about the center of the orbit is zero. Hence, angular momentum of the satellite is conserved, i.e., L = constant.*

(d) ENERGY OF A SATELLITE

The P.E. of a satellite is

$$U = mV = -\frac{GMm}{r} \quad \left[\because V = -\frac{GM}{r}\right]$$

The kinetic energy of the satellite is

$$K = \frac{1}{2}mv_0^2 \quad = \frac{GMm}{2r} \quad \left[\because v_0 = \sqrt{\frac{GM}{r}}\right]$$

Total mechanical energy of the satellite $= -\frac{GMm}{r} + \frac{GMm}{2r} = -\frac{GMm}{2r}$ … **(25)**

Important points

☞ We have,

$$\frac{K}{E} = -1 \text{ i.e., } K = -E$$

Also, $\frac{U}{E} = 2$

⇒ $U = 2E$

☞ ***Total energy of a satellite in its orbit is negative. Negative energy means that the satellite is bound to the central body by an attractive force and energy must be supplied to remove it from the orbit to infinity.***

Binding energy of the satellite

The energy required to remove the satellite from its orbit to infinity is called binding energy of the satellite, i.e.

$$\text{Binding energy} = -E = \frac{GMm}{2r}$$

7.1. GEO-STATIONARY SATELLITE

If there is a satellite rotating in the direction of earth's rotation, i.e., from west to east, then for an observer on the earth the angular velocity of the satellite will be $(\omega_S - \omega_E)$.

However, if $\omega_S - \omega_E = 0$, satellite will appear stationary relative to the earth. Such a satellite is called 'Geo-stationary satellite' and is used for communication purposes.

The orbit of a geostationary satellite is called '**Parking orbit**'.

We know that,

$$T^2 = \frac{4\pi^2}{GM}r^3 \quad \text{… (26)}$$

For geostationary satellite, $T = 24$ hours.

Putting this value of T in the above equation, we get

$r \approx 42000$ km

or, $h \approx 36000$ km.

where h is the height of the satellite from the surface of the earth.

7.2. WEIGHTLESSNESS IN A SATELLITE

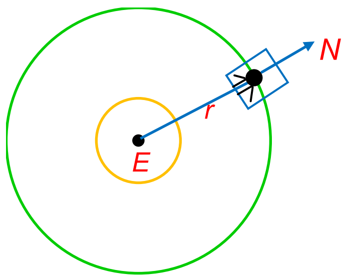
The radial acceleration of the satellite is given by

$$a_r = \frac{F_r}{m} = \frac{GMm}{r^2 \times m} = \frac{GM}{r^2}$$

For a astronaut inside the satellite, we have

$$\frac{GMm_a}{r^2} - N - m_a a_r = 0$$

where m_a is mass of astronaut a_r is radial acceleration of satellite and N is normal reaction on the astronaut



or $$\frac{GMm_a}{r^2} - N - \frac{GMm_a}{r^2} = 0$$

⇒ $N = 0$

Hence, the astronaut feels weightlessness.

Illustration 10

Question: **Calculate the orbital velocity (in m/s) of a satellite revolving at a height h above the earth's surface if $h = R$. ($g = 10$ m/s^2, $R = 6400$ km)**

Solution: For the orbital velocity in a circular orbit, we have

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+h}} \qquad (\because r = R + h)$$

$$\Rightarrow v = \sqrt{\frac{gR^2}{2R}} = \sqrt{\frac{gR}{2}} \qquad (\because GM = gR^2 \text{ and } h = R)$$

$$\Rightarrow v = \sqrt{\frac{9.8 \times 6400 \times 10^3}{2}} = 5.6 \text{ km/s} = \mathbf{5600 \text{ m/s}}$$

Illustration 11

Question: **Two planets have masses in the ratio 1 : 10 and radii in the ratio 2: 5.**

(a) If density of first is 25 kg/m^3 find the density of 2nd planet

(b) the acceleration due to gravity of 1st is 10 m/s^2, find the acceleration due to gravity of the other

(c) escape velocity of one is 4 km/s, then find escape velocity of the 2nd.

Solution: Let M_1, M_2 by the masses and R_1, R_2 be the radii of the planets.

$$\Rightarrow \frac{M_1}{M_2} = \frac{1}{10} \text{ and } \frac{R_1}{R_2} = \frac{2}{5}$$

(a) Ratio of densities $= \frac{d_1}{d_2}$ or, $\frac{d_1}{d_2} = \left[\frac{M_1}{\frac{4}{3}\pi R_1^3}\right]\left[\frac{\frac{4}{3}\pi R_2^3}{M_2}\right]$

or, $\frac{d_1}{d_2} = \frac{M_1}{M_2}\left[\frac{R_2}{R_1}\right]^3 \Rightarrow \frac{d_1}{d_2} = \left[\frac{1}{10}\right]\left[\frac{5}{2}\right]^3 = \frac{25}{16} \Rightarrow d_2 = $ **16 kg/m^3**

(b) Acceleration due to gravity at the surface $= g = \frac{GM}{R^2}$

$$\therefore \frac{g_1}{g_2} = \frac{M_1}{M_2}\left[\frac{R_2}{R_1}\right]^2$$

$$= \frac{1}{10}\left[\frac{5}{2}\right]^2 = \frac{5}{8} = \Rightarrow g_2 = \frac{8}{5} \times 10 \text{ m/s}^2 = \mathbf{16 \text{ m/s}^2}$$

(c) Escape velocity $= \sqrt{\frac{2GM}{R}}$

$$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{M_1}{M_2}}\sqrt{\frac{R_2}{R_1}} = \sqrt{\frac{1}{10} \times \frac{5}{2}} = \frac{1}{2} \Rightarrow v_2 = 2v_1 = \mathbf{8 \text{ km/s}}$$

Illustration 12

Question: **The mean distance of a plant from the sun is 4 times that of the earth from the sun. Find the number of years required for the planet to make one revolution about the sun.**

Solution: For planets revolving around the sun, $T^2 \propto r^3$

$$\Rightarrow \frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3} \qquad [T_1\text{: time-period of mars and } T_2\text{: time-period of earth}]$$

$$\Rightarrow \frac{T_1}{T_2} = \left[\frac{r_1}{r_2}\right]^{3/2}$$

$$\Rightarrow T_1 = T_2\left[\frac{r_1}{r_2}\right]^{3/2} = (1 \text{ year})\, 4 = \mathbf{8\ years}$$

PROFICIENCY TEST

The following questions deal with the basic concepts of this section. Answer the following briefly. Go to the next section only if your score is at least 80%. Do not consult the Study Material while attempting these questions.

1. Spheres of the same material and same radius r are touching each other; Show that gravitational force between them is directly proportional to r^4.

2. Assuming earth to be a sphere of uniform mass density, how much would *a* body weigh half way down the centre of the earth if it weighed 100 N on the surface?

3. Suppose the earth increases its speed of rotation. The new time period at which the weight of a body on the equator becomes zero. Take $g = 10$ m/s^2 and radius of earth $R = 6400$ km. T can be written as $n \times \pi$ sec, find the value n?

4. Gravitational potential at $x = 2$ m is decreasing at a rate of 10 J/Kg-m along the positive x-direction. It implies that the magnitude of gravitational field at $x = 2$ m is also 10 N/kg. Is this statement true of false?

5. The gravitational potential in a region is given by, $V = 10\sqrt{2}\ (x + y)$ m N/kg. Find the magnitude of the gravitational force on a particle of mass 0.5 kg placed at the origin.

6. The mass of moon is one percent the mass of earth. What is the ratio of gravitational pull of the earth on moon and that of the moon on earth?

7. If the earth is at 4 times of its present distance from the sun, what would be the duration of the year?

8. Three masses of 1 kg, 2 kg and 3 kg are placed at the vertices of an equilateral triangle of side 1 m. Find the gravitational potential energy of this system.
Take $G = 6.6 \times 10^{-11}$ N-m^2/kg^2, 10^{-12} J as unit.

9. What depth below the surface of earth the weight of body decreases by $\frac{1}{4}$ times of the value on the surface.

10. A space ship is launched into a circular orbit close to the earth's surface. What additional velocity (in m/s) has now to be imparted to the space ship in the orbit to overcome the gravitational pull. Radius of earth = 6400 km, $g = 10$ m/s^2.

ANSWERS TO PROFICIENCY TEST

2. 50 N

3. $n = 1600$

4. False

5. 10 N

6. Unity

7. 8 year

8. 726 unit

9. 1600 km

10. $v_e - v_0 \approx 3200$ m/s

SOLVED OBJECTIVE EXAMPLES

Example 1:

Two particles of masses *M* and *m* are initially at rest and infinitely separated. When they move towards each other due to gravitational attraction, their relative velocity at any instant in terms of distance '*d*' between them at that instant is

(a) $\left(\frac{2Gd}{M+m}\right)^{1/2}$ (b) $\left[\frac{2G(M+m)}{d}\right]^{1/2}$ (c) $\frac{2G(M+m)}{d}$ (d) $\frac{2Gd}{M+m}$

Solution:

As the two particles moves in the influence of gravitational (conservative force) mechanical energy will be conserved.

Let p = momentum of mass m

So $U_i + K_i = U_f + K_f$

$$0+0=-\frac{GMm}{d}+\frac{p^2}{2m}+\frac{(-p)^2}{2M}$$

Solving, $p = Mm\sqrt{\frac{2G}{d(m+M)}}$

Velocity of m, $v_1 = M\sqrt{\frac{2G}{d(m+M)}}$

Velocity of M, $v_2 = -m\sqrt{\frac{2G}{d(m+M)}}$

$\Rightarrow$ relative velocity $= v_1 - v_2 = (M+m)\sqrt{\frac{2G}{d(m+M)}} = \sqrt{\frac{2G(m+M)}{d}}$

$\therefore$ **(b)**

Example 2:

The weight of a body on earth is denoted by *W* and the acceleration due to gravity is *g*. Newton's second law can be written as $F=\frac{W}{g}a$. The acceleration due to gravity on the moon is g_1. What is the expression for Newton's law on the moon?

(a) $F=\frac{W}{g_1}a$ (b) $F=\frac{W}{g}g_1a$ (c) $F=\frac{W}{g}a$ (d) $F=\frac{W}{g_1}ga$

Solution:

The mass of the body on earth is $\frac{W}{g}$. The mass of the body on the moon remains the same namely $\frac{W}{g}$.

Hence Newton's law on the moon is $F = \frac{W}{g}$ **a**.

$\therefore$ **(c)**

Example 3:

The orbital velocity of an artificial satellite in a circular orbit just above earth's surface is v_0. For a satellite orbiting in a circular orbit at an altitude of half of earth's radius is

(a) $\sqrt{\frac{3}{2}}v_0$ (b) $\sqrt{\frac{2}{3}}v_0$ (c) $\frac{3}{2}v_0$ (d) $\frac{2}{3}v_0$

Solution:

Orbital velocity = $\sqrt{\frac{g_0R^2}{R+h}}$ where R is radius of earth.

If $h = 0$, $v_0=\sqrt{\frac{g_0R^2}{R}}=\sqrt{g_0R}$

If $h = \frac{R}{2}$, $v=\sqrt{\frac{g_0R^2}{R+\frac{R}{2}}}=\sqrt{\frac{2g_0R}{3}} = \sqrt{\frac{2}{3}}v_0$.

∴ **(b)**

Example 4:

The dimensional formula for gravitational constant is

(a) $[M^{-1} L^3 T^{-2}]$ (b) $[M^3 L^{-1}T^{-2}]$ (c) $[M^{-1}L^2T^3]$ (d) $[M^2L^3T^{-1}]$

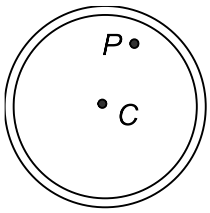
Solution:

$$G = \frac{Fr^2}{m_1m_2} = \frac{MLT^{-2}L^2}{M^2} = M^{-1}L^3T^{-2}$$

∴ **(a)**

Example 5:

The force between a hollow sphere and a point mass at P inside it as shown in the figure



(a) is attractive and constant

(b) is attractive and depends on the position of the point with respect to centre C

(c) is zero

(d) is repulsive and constant.

Solution:

Since gravitational field inside hollow sphere is zero. Therefore force acting on the particle P is zero.

∴ **(c)**

Example 6:

A body of mass *m* is moved to a height equal to the radius of the earth *R*. The increase in its potential energy is

(a) *mgR* **(b) 2*mgR*** **(c) (1/2) *mgR*** **(d) (1/4) *mgR***

Solution:

$$\Delta U = \frac{-GMm}{2R} + \frac{GMm}{R} = \frac{GMm}{2R} = \frac{mgR}{2}$$

$\therefore$ **(c)**

Example 7:

A binary star system consists of two stars. One star has twice the mass of the other. The star rotates about their common centre of mass

(a) Star having the smaller mass has twice angular momentum compared to heavier star

(b) Both stars have same angular momentum about centre of mass

(c) Both stars have same linear speed

(d) Both the stars have same kinetic energy

Solution:

If distance between the two stars is 3 l then angular momentum of mass m about common centre of mass = $m (2l)^2 \omega$ and angular momentum of mass 2m about common centre of mass = $(2m)\, l^2\omega$.

$\therefore$ **(a)**

Example 8:

The escape velocity of a particle of mass *m* varies as

(a) m^2 **(b) m** **(c) m^0** **(d) m^{-1}**

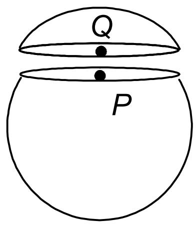
Solution:

$v_{escape} = \sqrt{\frac{2GM}{R}}$ Where M is mass of planet (it is independent of mass of particle m)

$\therefore$ **(c)**

Example 9:

A spherical shell is cut into two pieces along a chord as shown in figure. For points *P* and *Q*



(a) $E_P > E_Q$ **(b) $E_P < E_Q$**

(c) $E_P = E_Q = 0$ **(d) $E_P = E_Q \neq 0$**

Solution:

$E_P + E_Q = 0$ but $E_P = E_Q \neq 0$

$\therefore$ **(d)**

SOLVED SUBJECTIVE EXAMPLES

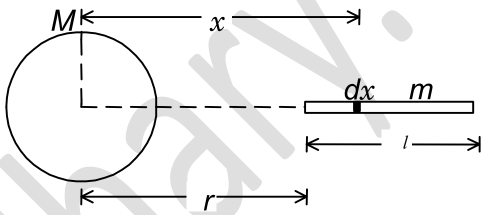
***Example* 1:**

A uniform sphere of mass M = 100 kg and a thin uniform rod of length l = 30cm and mass m = 300 kg oriented as shown in the figure. The distance of centre of sphere to the nearest end of the rod is r = 3 m. The gravitation force between them is given as x × 10^{-8} N. Find x

($G = 6.6 \times 10^{-11}$ Nm²/kg²)

Solution:

Since the sphere is uniform its entire mass may be considered to be concentrated at its centre. The force on the elementary mass dm is



$$dF = \frac{GMdm}{x^2}$$

But $dm = \frac{m}{l}dx$

$$F = \int_r^{r+\ell} \frac{GMm}{lx^2}dx = -\frac{GMm}{l}\left[\frac{1}{x}\right]_r^{r+\ell} = -\frac{GMm}{l}\left[\frac{1}{r+l} - \frac{1}{r}\right]$$

$$F = \frac{GmM}{l}\left(\frac{l}{r(r+l)}\right) = \frac{GMm}{r(r+l)} = 2 \times 10^{-7}\ \text{N}$$

x = **20**

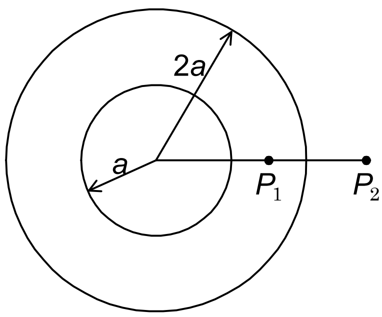
***Example* 2:**

A uniform solid sphere of mass M = 225 kg and radius a = 1 m is surrounded symmetrically by a thin spherical shell of equal mass and radius 2a. If the gravitational field is given as $x \times G$ at a distance. Find x (where G universal gravitational constant)

(a) $\frac{3}{2}a$ from centre **(b) $\frac{5}{2}a$ from centre.**

Solution:

(a) The situation is indicated in the figure in the two cases.



The point P_1 is at a distance $\frac{3}{2}a$ from centre and P_2 is at a distance $\frac{5}{2}a$ from centre. As P_1 is inside the cavity of the thin spherical shell the field here due to the shell is zero. The field due to the solid sphere is

$$E = \frac{GM}{\left(\frac{3}{2}a\right)^2} = \frac{4GM}{9a^2}$$

This also represents the resultant field at P_1.

Resultant field $=\dfrac{4GM}{9a^2}$. The direction is towards centre. = 100 G

$x = 100$

(b) In this case P_2 is outside the sphere as well as the shell. Both may be replaced by single particles of same mass at the centre.

The field due to each of them at $P_2 = \dfrac{GM}{\left(\frac{5}{2}a\right)^2} = \dfrac{4GM}{25a^2}$

Resultant field $=\dfrac{4GM}{25a^2}+\dfrac{4GM}{25a^2} = \dfrac{8GM}{25a^2}$ = 72 G

x = **72**

This is also acting towards the common centre.

***Example* 3:**

An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from earth. Determine

(a) the height of satellite above earth's surface (in km)

(b) if the satellite is suddenly stopped, find the speed (in m/s) with which the satellite will hit the earth's surface after falling down. (R = 6400 km, g = 10 m/s^2)

Solution:

Escape velocity $=\sqrt{2gR}$, where g is acceleration due to gravity on surface of earth and R the radius of earth.

Orbital velocity $=\dfrac{1}{2}v_e = \dfrac{1}{2}\sqrt{2gR} = \sqrt{\dfrac{gR}{2}}$

(a) If h is the height of satellite above earth

$$\frac{mv_0^2}{R+h} = \frac{GMm}{(R+h)^2}$$

$$v_0^2 = \frac{GM}{R+h} = \frac{gR^2}{(R+h)} \quad \therefore \left(\frac{1}{2}v_e\right)^2 = \frac{gR^2}{R+h}$$

$$\frac{gR}{2} = \frac{gR^2}{R+h} \text{ from equation (1)}$$

$R + h = 2R$

$h = R =$ **6400 km**

(b) If the satellite is stopped in orbit, the kinetic energy is zero and its potential energy is $\frac{-GMm}{2R}$,

Total energy $= \frac{-GMm}{2R}$

When it reaches the earth let v be its velocity.

Hence the kinetic energy $= \frac{1}{2}mv^2$

Potential energy $= -\frac{GMm}{R}$

$$\therefore \frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{2R}$$

$$v^2 = 2GM\left(\frac{1}{R} - \frac{1}{2R}\right) = \frac{2gR^2}{2R}$$

$$= gR$$

$v = \sqrt{gR}$ = **8000 m/s**

***Example* 4:**

Two satellites S_1 and S_2 revolve round a planet in coplanar circular orbits in the same sense. Their periods of revolutions are 1 hour and 8 hours respectively. The radius of orbit of S_1 is 10^3 km. When S_2 is closest to S_1. ($\pi = 3.141$) Find

(i) speed of S_2 relative to S_1 (in km/hr)

(ii) the angular speed of S_2 actually observed by an astronaut in S_1. (in milli rad/hr)

Solution:

If r_1 and r_2 are radii of orbits of S_1 and S_2, T_1 and T_2 their respective periods, we have by Kepler's third law

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

$$r_2^3 = r_1^3\left(\frac{T_2}{T_1}\right)^2 = (10^3)^3\left(\frac{8}{1}\right)^2 = (10^3 \times 4)^3$$

$\therefore$ $= 4 \times 10^3$ km

(i) If the orbital speeds of satellites S_1 and S_2 are v_1 and v_2.

$$v_1 = \frac{2\pi r_1}{T_1}$$

$$v_2 = \frac{2\pi r_2}{T_2}$$

Speed of S_2 relative to S_1 $= |v_2 - v_1|$

$$= \left|\frac{2\pi r_2}{T_2} - \frac{2\pi r_1}{T_1}\right| = 2\pi\left|\frac{r_2}{T_2} - \frac{r_1}{T_1}\right| = 2\pi\left|\frac{4\times10^3}{8} - \frac{10^3}{1}\right|$$

$$= 2\pi \times 10^4 \times \frac{1}{2} = 3.141 \times 10^3 \text{ km/hr} = \mathbf{3141\ km/hr}$$

(ii) Angular speed of S_2 relative to S_1 $= \frac{v_2 - v_1}{r_2 - r_1} = -\frac{3.141\times10^3}{4\times10^3 - 10^3}$

$= -1.047$ rad/hour = **1047 milli rad/hour**

***Example* 5:**

Two bodies of masses $M_1 = 4$ kg and $M_2 = 9$ kg are placed at a distance $d = 12.5$ m apart. The potential at the position where the gravitational field due to them is zero is written as xG. Find x.

Solution:

Let the field be zero at a point distant x from M_1.

$$\therefore \quad \frac{GM_1}{x^2} = \frac{GM_2}{(d-x)^2} \quad \therefore \quad \frac{x}{d-x} = \sqrt{\frac{M_1}{M_2}}$$

$$x\sqrt{M_2} = \sqrt{M_1}\cdot d - x\sqrt{M_1}$$

$$x\left[\sqrt{M_1} + \sqrt{M_2}\right] = \sqrt{M_1}\cdot d$$

$$x = \frac{d\sqrt{M_1}}{\sqrt{M_1} + \sqrt{M_2}}$$

$$d - x = \frac{d\sqrt{M_2}}{\sqrt{M_1} + \sqrt{M_2}}$$

Potential at this point due to both the masses will be

$$= -\frac{GM_1}{x} - \frac{GM_2}{(d-x)} = -G\left[\frac{M_1\left(\sqrt{M_1} + \sqrt{M_2}\right)}{d\sqrt{M_1}} + \frac{M_2\left(\sqrt{M_1} + \sqrt{M_2}\right)}{d\sqrt{M_2}}\right]$$

$$= -\frac{G}{d}\left(\sqrt{M_1} + \sqrt{M_2}\right)^2 = -\frac{G}{d}\left(M_1 + M_2 + 2\sqrt{M_1 M_2}\right) = -2G$$

$x = -2$

***Example* 6:**

The distance between earth and moon is 4×10^5 km and the mass of earth is 81 times the mass of moon. Find the position (take 10^4 km as unit) of a point on the line joining the centres of earth and moon, where the gravitational field is zero.

Solution:

Let x be the distance of the point of no net field from earth. The distance of this point from moon is $(r - x)$, where $r = 3.8\times10^5$ km.

The gravitational field due to earth $=\frac{GM_e}{x^2}$ and that due to moon $=\frac{GM_m}{(r-x)^2}$. For the net field to be zero these are equal and opposite.

$$\frac{GM_e}{x^2}=\frac{GM_m}{(r-x)^2}$$

$$\frac{M_e}{M_m}=\frac{x^2}{(r-x)^2}.\text{ But }\frac{M_e}{M_m}=81$$

$$\therefore\quad 81=\frac{x^2}{(r-x)^2}$$

$$\frac{x}{r-x}=9$$

$$9r-9x=x$$

$$10x=9r$$

$$x=\frac{9}{10}r=\frac{9}{10}\times4\times10^5=3.6\times10^5\text{ km}=36\times10^4\text{ km}\Rightarrow\quad x=\mathbf{36\text{ unit}}$$

Example 7:

A body is projected vertically upwards with speed v_0. Show that its speed v at a height is given by $v_0^2-v^2=\frac{2gh}{1+\frac{h}{R}}$, where R is the radius of earth. Hence deduce the maximum height (in km) reached by the body fired with a speed of 60% of escape speed.

Solution:

Kinetic energy on the surface of earth $=\frac{1}{2}mv_0^2$

Potential energy on the surface of earth $=\frac{-GMm}{R}$

Total energy $=\frac{1}{2}mv_0^2-\frac{GMm}{R}$

Kinetic energy at a height $h=\frac{1}{2}mv^2$

Potential energy at this height $=\frac{-GMm}{(R+h)}$

Total energy $=\frac{1}{2}mv^2-\frac{GMm}{R+h}$

By the principle of conservation of energy,

$$\frac{1}{2}mv^2-\frac{GMm}{R+h}=\frac{1}{2}mv_0^2-\frac{GMm}{R}$$

$$\frac{1}{2}\left(v_0^2 - v^2\right) = \frac{GM}{R} - \frac{GM}{(R+h)}$$

But $GM = gR^2$

$$\frac{1}{2}\left(v_0^2 - v^2\right) = \frac{gR^2h}{R(R+h)}$$

$$v_0^2 - v^2 = \frac{2gRh}{R+h} = \frac{2gh}{1+\frac{h}{R}}$$

For maximum height $v = 0$,

v_0 = 60% of escape speed

$$= 0.6\sqrt{2gR}$$

$$\therefore \qquad \left(0.6\sqrt{2gR}\right)^2 - 0 = \frac{2gh_{max}}{1+\frac{h_{max}}{R}}$$

$$0.36R = \frac{h_{max}}{1+\frac{h_{max}}{R}}$$

$$0.36R + 0.36h_{max} = h_{max}$$

$$0.36R = 0.64h_{max}$$

h_{max} = **3600 km**

***Example* 8:**

A person brings a mass of 1 kg from infinity to a point *A*. Initially the mass was at rest but it moves at a speed of 2 m/s as it reaches *A*. The work done by the person on the mass is –3J. The potential at infinity is –10 J. Then find the potential at *A*.

Solution:

Work done by external agent

$W_{ent}, = \Delta U + \Delta K$

$= U_A - U_\infty + K_A - K_\infty$

$= U_A - 10 + \frac{1}{2} \times mv^2 - 0$

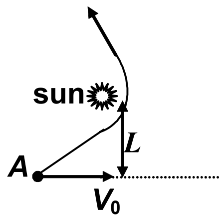
$-3 = U_A - 10 + \frac{1}{2} \times 1 \times 4$

$+ 5\text{ J} = U_A$

So V_A = **5 J/kg**

***Example* 9:**

A cosmic body *A* moves towards star with velocity v_0 (when far from the star) and aiming parameter *L* and arm of velocity vector v_0 relative to the centre of the star as shown in figure. Find the minimum distance (take 10^8 m as unit) by which this body will get to the star. Mass of the star is *M*.



[$M = 10^{20}$ kg, $G = \frac{2}{3}\times 10^{-10} Nm^2/kg^2$, $v_0 = \sqrt{\frac{200}{3}}$ m/s, $L = \sqrt{15}\times 10^8$ m]

Solution:

$r =$ minimum distance

conservation of angular momentum about star $mv_0L = mrv_1$

$$\frac{1}{2}mv_0{}^2 - 0 = \frac{1}{2}mv_1^2 - \frac{GMm}{r} \quad ; \quad \frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{v_0L}{r}\right)^2 - \frac{GMm}{r} \Rightarrow r^2 + \frac{3GM}{v_0^2}r - L^2 = 0$$

Solving $r = \frac{GM}{v_0^2}\left[\sqrt{1+\left(\frac{rv_0^2}{GM}\right)} - 1\right]$

Putting the values $r = 3\times 10^8$ m = **3 unit**

***Example* 10:**

The density inside an isolated large solid sphere of radius $a = 4$ km is given by $\rho = \rho_0\, a/r$ where ρ_0 is the density at the surface and equals to 10^9 kg/m^3 and *r* denotes the distance from the centre. Find the gravitational field (in m/s^2) due to this sphere at a distance 2*a* from its centre. Take $G = 6.65\times 10^{-11}$ Nm^2/kg^2.

Solution:

The gravitational field at the given point is $E = \frac{GM}{(2a)^2} = \frac{GM}{4a^2}$ … (i)

The mass *M* may be calculated as follows. Consider a concentric shell of radius *r* and thickness *dr*. Its volume is

$$dV = (4\pi r^2)dr$$

and its mass is $dM = \rho dV = \left(\rho_0 \frac{a}{r}\right)(4\pi r^2 dr) = 4\pi\rho_0 ar\ dr$.

The mass of the whose sphere is $M = \int_0^a 4\pi\rho_0 ar\ dr = 2\pi\rho_0 a^3$

Thus, by (i) the gravitational field is $E = \frac{2\pi G\rho_0 a^3}{4a^2} = \frac{1}{2}\pi G\rho_0 a$

$\therefore \quad E = \frac{1}{2}\times\frac{22}{7}\times 6.65\times 10^{-11}\times 10^9\times 4\times 10^3$ m/s^2 $\Rightarrow$ $E =$ **418 m/s^2**

MIND MAP

GRAVITATION

1. Newton's law of gravitation $\vec{F} = \dfrac{Gm_1m_2}{r^2}\hat{r}$

2. Gravitational force when a particle of mass m is placed in gravitational field $\vec{F} = m\vec{E}$

3. Field due to a point mass $= \dfrac{Gm}{r^2}\hat{r}$

4. Field due to ring of radius R at an axial distance $= \dfrac{Gmr}{(R^2+r^2)^{3/2}}$

5. Field due to disc of radius $R = \dfrac{2Gmr}{R^2}\left(\dfrac{1}{r} - \dfrac{1}{\sqrt{r^2+R^2}}\right)$

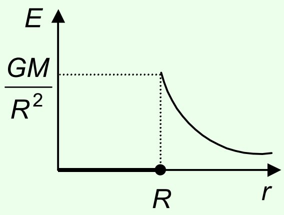
6. Field due to thin spherical shell: inside = 0

outside = $\dfrac{GM}{r^2}$

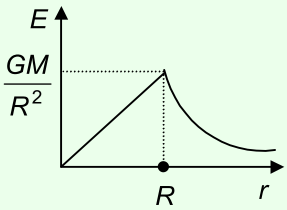
7. Field due to solid sphere

Inside = $\dfrac{GMr}{R^3}$ Outside = $\dfrac{GM}{r^2}$

Graph
Spherical shell



Graph
Solid sphere



8. Gravitational potential due to a point mass $= -\dfrac{Gm}{r}$

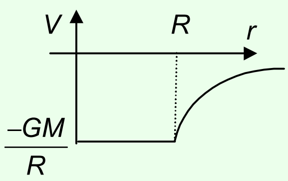
9. Gravitational potential due to a ring of radius R

$= \dfrac{-GM}{\sqrt{R^2+r^2}}$

10. Gravitational potential due to uniform spherical shell

Inside = $\dfrac{-GM}{R}$; constant

outside = $\dfrac{-GM}{r}$



11. Escape velocity

$= \sqrt{\dfrac{2GM}{R}}$

12. orbital speed $= \sqrt{\dfrac{GM}{R}}$

13. Time period(T) $= \dfrac{2\pi r\sqrt{r}}{\sqrt{GM}}$

14. Gravitational potential energy when two masses m_1 & m_2 are at a distance r apart

$= \dfrac{-Gm_1m_2}{r}$

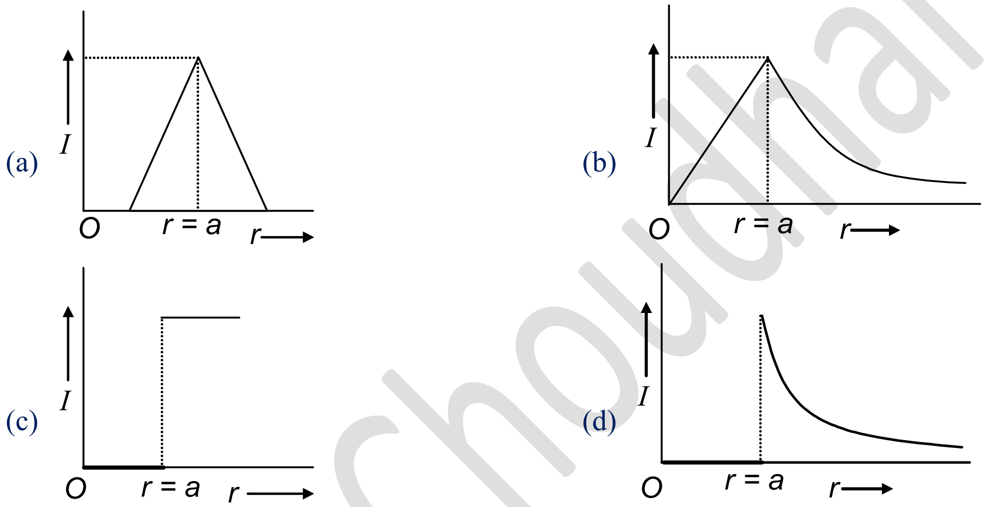
EXERCISE – I

NEET-SINGLE CHOICE CORRECT

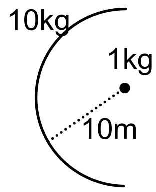
1. The ratio of the radii of the planets P_1 and P_2 is k_1. The ratio of the acceleration due to the gravity on their surface is k_2. The ratio of the escape velocities from them will be

(a) $k_1 k_2$ (b) $\sqrt{k_1 k_2}$ (c) $\sqrt{(k_1/k_2)}$ (d) $\sqrt{(k_2/k_1)}$

2. Which of the following graphs represents correctly the variation of intensity of gravitational field I with the distance r from the centre of a spherical shell of mass M and radius a?



3. A particle of mass 1 kg is placed on centre of curvature of a fixed uniform semicircular ring of radius 10 m and mass 10 kg as shown in the figure. The work done to displace the particle from centre of curvature to infinity (in Joules) (G = universal gravitational constant)



(a) G (b) 10 G (c) G/10 (d) $G/5$

4. The period of a satellite in a circular orbit of radius R is T. The period of another satellite in a circular orbit of radius $4R$ is

(a) $4T$ (b) $T/4$ (c) $8T$ (d) $T/8$

5. The period of a satellite in a circular orbit around a planet is independent of

(a) the mass of the planet (b) the radius of the orbit

(c) the mass of the satellite (d) all of three parameters a, b and c

6. At a height above the surface of the earth equal to the radius of the earth the value of acceleration due to gravity. (Where g is acceleration due to gravity on the surface of the earth) will be nearly

(a) zero (b) $\sqrt{g}$ (c) $\frac{g}{4}$ (d) $\frac{g}{2}$

7. A simple pendulum has a time period T_1 when on the earth's surface, and T_2 when taken to a height R above the earth's surface, where R is radius of earth. The value of T_2/T_1 is

(a) 1 (b) $\sqrt{2}$ (c) 4 (d) 2

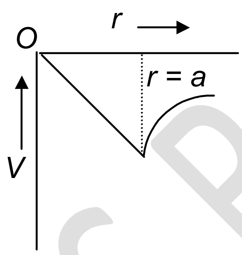
8. The rotation of the earth about its axis speeds up such that a man on the equator becomes weightless. In such a situation, what would be the duration of one day? (Radius of earth = R, acceleration due gravity at earth surface = g)

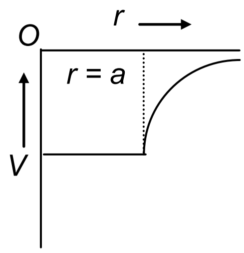
(a) $2\pi\sqrt{\frac{R}{g}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{R}{g}}$ (c) $2\pi\sqrt{Rg}$ (d) $\frac{1}{2\pi}\sqrt{Rg}$

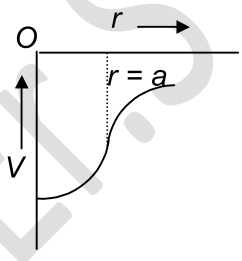
9. If the value of escape velocity on a planet is $2\sqrt{\frac{GM}{K}}$ the radius of the planet will be

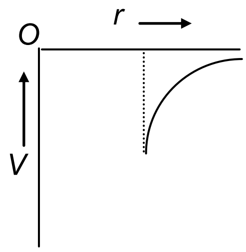
(a) 0.5 K (b) K (c) 2K (d) 5K

10. P is a point at a distance r from the centre of a uniform solid sphere of radius a. The gravitational potential at P is V. If V is plotted as a function of r, which is the correct curve?

(a) 

(b) 

(c) 

(d) 

11. A satellite orbiting the earth in a circular orbit of radius R completes one revolution in 3 hours. Orbital radius of geostationary satellite is 42000 km. The orbital radius R is

(a) 7000 km (b) 10500 km (c) 14000 km (d) 15000 km

12. If mass M is splited into two parts m and $(M–m)$ which are then separated by a distance, the ratio of m/M that maximises the gravitational force between the two parts is

(a) 1 : 2 (b) 1 : 1 (c) 1 : 3 (d) 1 : 4

13. The distance of the centres of moon and earth is D. The mass of the earth is 81 times the mass of the moon. At what distance from the centre of the earth, the gravitation force will be zero?

(a) $D/2$ (b) $2D/3$ (c) $4D/3$ (d) $9D/10$

14. If a particle is fired vertically upwards from the surface of earth and reaches a height of 6400 km, the initial velocity of the particle is (assume $R = 6400$ km and $g = 10$ ms^{-2})

(a) 4 km/sec (b) 2 km/sec (c) 8 km/sec (d) 16 km/sec

15. A space-ship is launched into a circular orbit close to the surface of the earth. The additional velocity now imparted to the space-ship in the orbit to overcome the gravitational pull is nearly

(a) 11.2 km/sec (b) 5 km/sec (c) 3.2 km/sec (d) none of these

16. If V is the gravitational potential on the surface of the earth, then what is its value at the centre of the earth?

(a) $2V$ (b) $3V$ (c) $\frac{2}{3}V$ (d) $\frac{3}{2}V$

17. A body is projected upwards with a velocity of 4×11.2 km/s from the surface of earth. What will be the velocity of the body when it escapes the gravitational pull of earth?

(a) 11.2 km/s (b) 2×11.2 km/s (c) 3×11.2 km/s (d) $\sqrt{15} \times 11.2$ km/s

18. Which of the following is not true for a geostationary satellite?

(a) Its time period is 24 hr

(b) Its angular speed is equal to that of earth about its own axis

(c) It is stationary in space

(d) It revolves from west to east

19. A particle is projected from the surface of earth with an initial speed of 4.0 km/s. Find the maximum height from the surface of earth attained by the particle. Radius of earth = 6400 km, and acceleration due to gravity at the earth surface $g = 9.8$ m/s^2.

(a) 350 km (b) 935 km (c) 90 km (d) none of these

20. A body of mass m is moved to a height equal to the radius of the earth R. The increase in its potential energy is

(a) mgR (b) $2\,mgR$ (c) $(1/2)mgR$ (d) $(1/4)\,mgR$

21. The value of g will be 1% of its value at the surface of earth at a height of ($R_e = 6400$ km)

(a) 6400 km (b) 57600 km (c) 2560 km (d) 64000 km

22. If the acceleration due to gravity at earth is g and mass of earth is 80 times that of moon and radius of earth is 4 times that of moon, the value of g at the surface of moon will be

(a) g (b) $g/20$ (c) $g/5$ (d) $320\ g$

23. A satellite is orbiting a planet at a certain height in a circular orbit. If the mass of the planet is suddenly reduced to half, the satellite would

(a) continue to revolve round the planet at the same speed

(b) falls freely on the planet

(c) orbit the planet at lesser speed

(d) escape from the planet

24. A satellite in a circular orbit around the earth has a kinetic energy E_k. What minimum amount of energy is to be added so that it escapes from the earth surface?

(a) $E_k/4$ (b) $E_k/2$ (c) E_k (d) $2E_k$

25. If R is the radius of a planet, g the acceleration due to gravity on its surface and G is the universal gravitational constant, the mean density of the planet is

(a) $\frac{4\pi G}{3gR}$ (b) $\frac{3gR}{4gG}$ (c) $\frac{3g}{4\pi GR}$ (d) $\frac{\pi Rg}{12G}$

EXERCISE – II

IIT-JEE-SINGLE CHOICE CORRECT

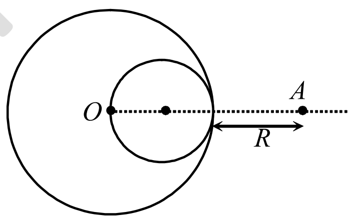
1 Two satellites S_1 and S_2 describe circular orbits of radii r and 2 r respectively around a planet. If the orbital angular velocity of S_1 is ω, the orbital angular velocity of S_2 is

(a) $\frac{\omega}{2\sqrt{2}}$ (b) $\frac{\omega\sqrt{2}}{3}$ (c) $\frac{\omega}{\sqrt{2}}$ (d) $\omega\sqrt{2}$

2 A satellite is launched into a circular orbit of radius R around the earth. A second satellite launched into an orbit of radius 1.01 R. The time period of the second satellite is larger than that of the first one by approximately

(a) 0.5% (b) 1.5% (c) 1% (d) 3.0%

3. A solid sphere of uniform density and radius R applies a gravitational force of attraction equal to F_1 on a particle placed at a distance $2R$ from the centre of the sphere. A spherical cavity of radius $(R/2)$ is now made in the sphere as shown in figure. The sphere with the cavity now applies a gravitational force F_2 on the same particle. The ratio (F_2/F_1) is



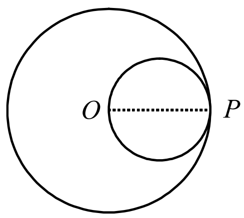
(a) (1/2) (b) (3/4) (c) (7/8) (d) (7/9)

4. Four particles of equal mass M move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is

(a) $\frac{GM}{R}$ (b) $\sqrt{\left(\frac{GM}{R}\right)}$

(c) $\sqrt{\left[\frac{GM}{R}\left(\frac{2\sqrt{2}+1}{4}\right)\right]}$ (d) $\sqrt{\left[\frac{GM}{R}(\sqrt{2}+1)\right]}$

5. A spherical cavity is made in a solid sphere of radius R and mass M such that its surface touches the surface of the sphere and passes through its centre. The work done in moving a particle of mass m inside the cavity from centre of sphere O to the point P, as shown in figure is



(a) $\frac{GMm}{R}$ (b) $\frac{3}{2}\frac{GMm}{R}$ (c) $\frac{GMm}{2R}$ (d) none

6. A uniform spherical shell gradually shrinks maintaining its shape. The gravitational potential at the centre

(a) increases (b) decreases (c) remains constant (d) oscillates

7. A body of mass m rises to a height $h = R/5$ from the earth's surface where R is radius of the earth. If g is acceleration due to gravity at the earth's surface, the increase in potential energy is

(a) mgR (b) $(4/5)\ mgR$ (c) $(1/6)\ mgR$ (d) $(6/7)\ mgR$

8. Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/2}$, the T^2 is proportional to

(a) R^3 (b) $R^{7/2}$ (c) $R^{3/2}$ (d) $R^{3.75}$

9. A body is suspended from a spring balance kept in a satellite. The reading of the balance is W_1 when the satellite goes in an orbit of radius R and is W_2 when it goes in an orbit of radius 2 R.

(a) $W_1 = W_2$ (b) $W_1 < W_2$ (c) $W_1 > W_2$ (d) $W_1 \neq W_2$

10. A planet is revolving around the sun in elliptical orbit. Its closest distance from the sun is r and the farthest distance is R. If the orbital velocity of the planet closest to the sun be v, then what is the velocity at the farthest point?

(a) vr/R (b) vR/r (c) $v\left(\frac{r}{R}\right)^{1/2}$ (d) $v\left(\frac{R}{r}\right)^{1/2}$

11. A satellite is revolving around the earth with orbital speed v_0. if it stops suddenly, the speed with which it will strike the surface of earth would be (v_e = escape velocity of a particle on earth's surface)

(a) $\frac{v_e^2}{v_0}$ (b) v_0 (c) $\sqrt{v_e^2 - v_0^2}$ (d) $\sqrt{v_e^2 - 2v_0^2}$

12. A body is fired with a velocity v (greater than orbital velocity and less than escape velocity) at an angle 30^0 with the radius vector. If at the highest point the speed of the body is $v/4$, then the maximum height (from surface of earth) attained by the body is equal to (R = radius of earth)

(a) $\frac{v^2}{8g}$ (b) $2R$ (c) $\sqrt{2}R$ (d) none of these

13. A body of mass m is in equilibrium under the gravity of two identical fixed heavy point masses each of mass M kept separated through a distance L. With what minimum speed should it be projected to escape to infinity?

(a) $2\sqrt{\dfrac{2GM}{L}}$ (b) $\sqrt{\dfrac{2GM}{L}}$ (c) $\sqrt{\dfrac{GM}{L}}$ (d) none of these

14. The gravitational field in a region is given by $\vec{E} = (2\vec{i} + 3\vec{j})$ N/kg. Then the work done by the gravitational field when a particle is moved on the line $3y + 2x = 5$ is

(a) 14 N (b) 35 N (c) 20 N (d) zero

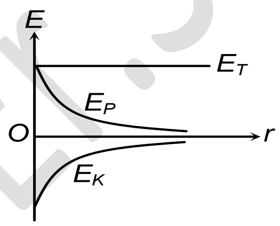
15. A body is projected vertically upwards from the surface of a planet of radius R with a velocity equal to half of the escape velocity for that planet. The maximum height attained by the body above the planet surface is

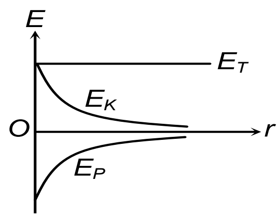
(a) $\dfrac{R}{3}$ (b) $\dfrac{R}{2}$ (c) $\dfrac{R}{4}$ (d) $\dfrac{R}{5}$

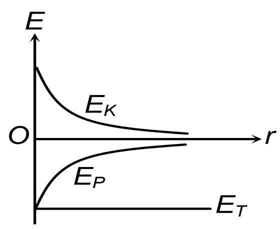
16. If three uniform spheres, each having mass M and radius r, are kept in such a way that each touches the other two, the magnitude of the gravitational force on any sphere due to the other two is

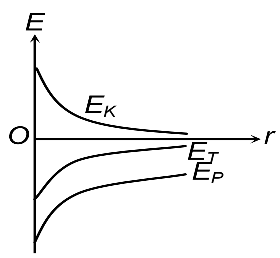
(a) $\dfrac{GM^2}{4r^2}$ (b) $\dfrac{2GM^2}{r^2}$ (c) $\dfrac{2GM^2}{4r^2}$ (d) $\dfrac{\sqrt{3}GM^2}{4r^2}$

17. The correct graph representing the variation of total energy (E_T), kinetic energy (E_K) and potential energy (E_P) of a satellite with its distance (r) from the surface of earth is

(a)


(b)


(c)


(d)


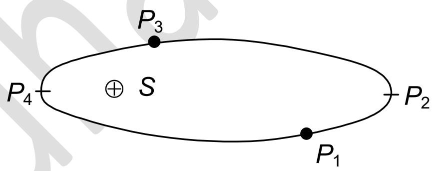
18. Two massive particles of mass M and m ($M > m$) are separated by a distance l. They rotate with equal angular velocity under their gravitational attraction force. The linear speed of the particle of mass m is

(a) $\sqrt{\frac{GMm}{(M+m)l}}$ (b) $\sqrt{\frac{GM^2}{(M+m)l}}$ (c) $\sqrt{\frac{GM}{l}}$ (d) $\sqrt{\frac{Gm^2}{(M+m)l}}$

19. A tunnel is dug along diameter inside the surface of earth and a particle is projected from the centre of tunnel. The minimum velocity of particle such that it escapes out from the earth's gravitation field is (Radius of earth = R_e)

(a) $\sqrt{2gR_e}$ (b) $\sqrt{\frac{3}{2}gR_e}$ (c) $\sqrt{3gR_e}$ (d) $\sqrt{\frac{5}{2}gR_e}$

20. The figure shows a planet in elliptical orbit around the sun S. Where is the kinetic energy of the planet maximum?



(a) P_1 (b) P_2

(c) P_3 (d) P_4

ONE OR MORE THAN ONE CHOICE CORRECT

1. A satellite of mass m_s is revolving in a circular orbit of radius r_s around the earth of mass M. If magnitude of its total energy is E, then

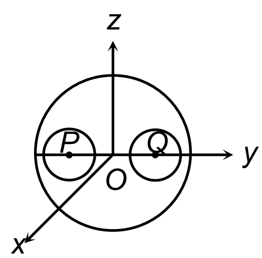
(a) its angular momentum is $\sqrt{2Em_s r_s^2}$

(b) its angular momentum is $\sqrt{2Em_s r_s}$

(c) its angular momentum is $(GMm_s^2 r_s)^{\frac{1}{2}}$

(d) zero

2. A solid sphere of uniform density and mass M kg has radius 4 meter. Its centre is at the origin of the coordinate system. The two spheres of radii 1m are taken out so that their centres are at $P(0,-2,0)$and $Q(0,2,0)$ respectively, leaving behind spherical cavities as shown in figure.



(a) Gravitational field at the origin of the co-ordinate axes is zero.

(b) Gravitational field at the centre of the cavity is $\frac{31GM}{1024}$ m/s^2.

(c) The gravitational potential is the same at all point on the circle $y^2 + z^2 = 36$

(d) The gravitational potential is the same at all point on the circle $x^2 + z^2 = 4$

3. A shell of mass m_2 radius r_2, lies inside and is concentric with a larger uniform shell of mass m_1, radius r_1. If E_P is the gravitational field at point P at distance r from the common centre then,

(a) $E_p = G\frac{(m_1 + m_2)}{r^2}$, for $r > r_1$ and $r > r_2$

(b) $E_p = \frac{Gm_2}{r^2}$, for $r < r_1$ and $r > r_2$

(c) $E_P = 0$ for $r < r_2$

(d) $E_P \neq 0$ for $r < r_2$

4. A "double star" is a composite system of two stars rotating about their centre of mass under their mutual gravitational attraction. Let us consider such a double star which has two starts of mass m and $2m$ at separation l. If T is the time period of rotation and L is angular momentum of the system about their centre of mass.

(a) $T = 2\pi\sqrt{\frac{l^3}{3mG}}$

(b) $T = 2\pi\sqrt{\frac{l^3}{2mG}}$

(c) $T = \frac{4m\pi l^2}{3L}$

(d) $T = \frac{4m\pi l^2}{9L}$

5. The magnitudes of gravitational field at distance r_1 and r_2 from the centre of a uniform sphere of radius R and mass M are F_1 and F_2, respectively then,

(a) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$

(b) $\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$ if $r_1 > R$ and $r_2 > R$

(c) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 > R$ and $r_2 > R$

(d) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$ if $r_1 < R$ and $r_2 < R$

6. A sky lab of mass 2×10^3 kg is first taken from the surface of earth in a circular orbit of radius $2R$ (from centre of earth). Then it is shifted to 2nd orbit of radius $3R$, R is radius of earth = 6.4×10^3 km, g = 10 m/s^2. The minimum work done to launch the lab in the first orbit (radius $2R$) is w_1 and work done shift from first orbit to 2nd orbit is w_2.

(a) $w_1 > w_2$

(b) $w_1 = 9.6 \times 10^{10}$ J

(c) $w_2 = 11 \times 10^{10}$ J

(d) $\frac{w_1}{w_2} = 8$

7. A satellite revolving around earth experiences a smal, air resistance due to earth atmosphere. Due to this satellite follows a spiral path towards the earth

(a) its potential energy decrease
(b) its kinetic energy decreases
(c) satellite will eventually fall on earth
(d) its speed increases

8. A planet of mass m moves along an ellipse around the sun of mass M (the sun is at one focus) so that its maximum and minimum distances from the sun are r_1 and r_2 and velocities at these points are v_1 and v_2 respectively. Then

(a) $v_1r_1 = v_2r_2$

(b) total energy of planet is always constant

(c) angular momentum of planet is $m\sqrt{\frac{2GM(r_1r_2)}{(r_1+r_2)}}$

(d) $v_2r_1 = v_1r_2$

9. If potential at the surface of uniform sphere of mass M and radius R is zero. Then

(a) potential at its centre would be $-\frac{GM}{2R}$

(b) potential at infinity would be $\frac{GM}{R}$

(c) potential at centre would be $-\frac{GM}{R}$

(d) potential at infinity would be $\frac{GM}{2R}$

10. A uniform solid sphere of mass M and radius a is surrounded symmetrically by a uniform thin spherical shell of equal mass and radius $2a$.

(a) the gravitational field at a distance $\frac{3}{2}$ a from the centre is $\frac{4}{9}\frac{GM}{a^2}$

(b) the gravitational field intensity at a distance $\frac{5}{2}$ a from the centre is $\frac{8}{25}\frac{GM}{a^2}$

(c) the gravitational field intensity at a distance $\frac{3}{2}$ a from the centre is $\frac{GM}{a^2}$

(d) none of these

EXERCISE – III

Note: Each statement in column – I has one or more than one match in column –II.

1. Let V and E denote the gravitational potential and gravitational field at a point. Then, match the following.

Column – I	Column - II
I. $E = 0$	**A.** At centre of a spherical shell
II. $V = 0$	**B.** At centre of a solid sphere
III. $V = 0, E = 0$	**C.** At centre of a circular ring
IV. $V \neq 0, E \neq 0$	**D.** At any point inside a solid sphere other than its centre
	E. At infinity

REASONING TYPE

Directions: Read the following questions and choose

(A) **If both the statements are true and statement-2** is the correct explanation of **statement-1**.

(B) **If both the statements are true but statement-2** is not the correct explanation of **statement-1**.

(C) **If statement-1** is True and **statement-2** is False.

(D) **If statement-1** is False **and statement-2** is True.

1. **Statement-1**: If a particle is dropped from *a* height h it collides to the surface of earth with a velocity $\sqrt{2gh}$.

Statement-2: When the gravity is assume to be uniform then change in potential energy is proportional to the distance moved.

(a) (A) (b) (B) (c) (C) (d) (D)

2. **Statement-1**: The total energy of a closed system is always negative in gravitational field.

Statement-2: Modulus of kinetic energy is equal to total energy in a closed system.

(a) (A) (b) (B) (c) (C) (d) (D)

3. **Statement-1**: If a smooth tunnel is dug across a diameter of earth and a particle is released from the surface of earth. Particle oscillates simple harmonically along it.

Statement-2: Component of gravitational force along the length of dug is directly proportional to the negative displacement from the middle point of tunnel.

(a) (A) (b) (B) (c) (C) (d) (D)

4. **Statement-1**: If the radius of earth is decreased keeping its mass constant, effective value of gravitational intensity increases at pole while decreases at equator.

Statement-2: Value of g on the surface of earth is given by $g = \frac{GM}{R^2}$.

(a) (A) (b) (B) (c) (C) (d) (D)

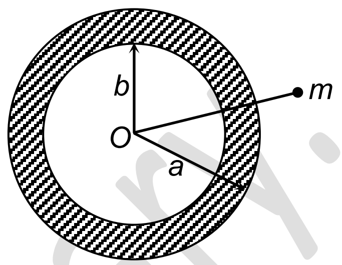
5. **Statement-1**: For a system of masses at some finite distance gravitational field can be zero but gravitational potential can not be zero.

Statement-2: Gravitational field is a vector quantity while gravitational potential is a scalar quantity.

(a) (A) (b) (B) (c) (C) (d) (D)

LINKED COMPREHENSION TYPE

A sphere of density ρ and radius *a* has a concentric cavity of radius *b* as shown in the figure.



1. Gravitational force *F* exerted by the sphere on the particle of mass *m*, located at a distance *r* from the centre of sphere as a function *r* when $b < r < a$.

(a) $F_r = 0$

(b) $F_r = \frac{4}{3}\pi G\rho m\left(\frac{a^3 - b^3}{r^2}\right)$

(c) $F_r = \frac{4}{3}\pi G\rho m\left(r - \frac{b^3}{r^2}\right)$

(d) none of these

2. Gravitational potential energy as a function of *r* where *r* is the distance from the centre of the sphere. When $0 < r < b$

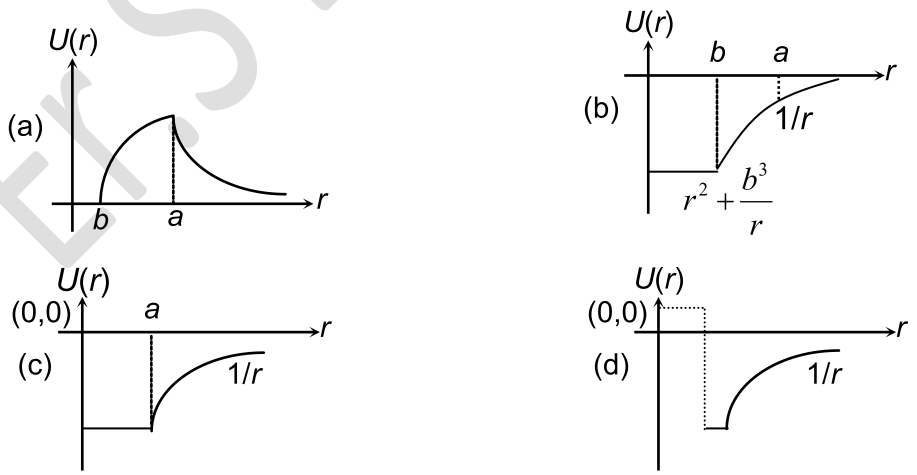
(a) $U(r) = -2\pi G\rho m(a^2 - b^2)$

(b) $U(r) = \frac{-4\pi G\rho m}{3r^2}(a^3 - b^3)$

(c) $U(r) = -\frac{2\pi G\rho m}{3r}(3ra^2 - 2b^2 - r^3)$

(d) none of these

3. Which one is correct sketch for potential as a function of *r*



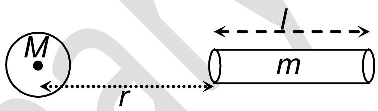
EXERCISE – IV

SUBJECTIVE PROBLEMS

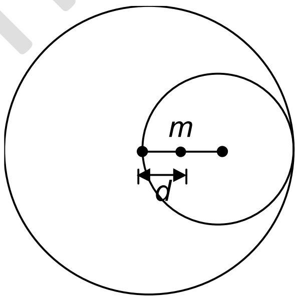
1. A boy can jump over a maximum horizontal distance 40m wide on earth. If mean density of an imaginary panel is twice that of earth. Calculate its maximum possible radius so that he may escape from it by jumping.

2. A mass 6×10^{24} kg (= mass of earth) is to be compressed in a sphere in such a way that the escape velocity from its surface is 3×10^8 m/s (equal to that of the velocity of light). What should be the radius of the sphere (in mm)?

3. Find gravitational force of attraction between a uniform sphere of mass $M = 2 \times 10^3$ kg and a thin rod of mass $m = 10^3$kg and length l = 1 cm, kept such that the distance between centre of sphere and edge of rod is r = 1cm.



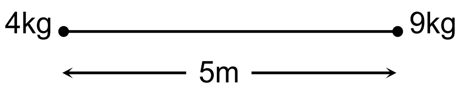
4. A spherical cavity is made in a lead sphere of radius R = 1m such that its radius is $R/2$ and passes through its centre. The mass of the lead sphere before hollowing was M =2kg. The force of attraction that this sphere would exert on a particle of mass m =1kg, which lies at a distance d from the centre of the lead sphere along the straight line joining the centres of the sphere and the centre of cavity as shown in the figure is given by xG. Find x



5. The value for the gravitational field due to a uniform rod of length L = 3m and mass M = 5 kg at a point on its perpendicular bisector at a distance d = 1m from the centre is given as xG. Find x

6. An artificial satellite of mass m of a planet of mass M revolves in a circular orbit whose radius is 9 times the radius of the planet. During its motion the satellite experiences a slight resistance due to cosmic dust which is given by $F = 2v^2$. If the time for which it stays in orbit before falling on the planet's surface is $xm\sqrt{\frac{R}{GM}}$ then, find the value of x.

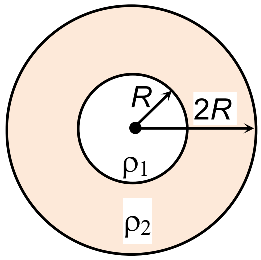
7. The gravitational potential of a system of two particles 4 kg and 9 kg separated by a distance of 5 m at a point where the field intensity is zero is $-xG$(J/kg). Find x



8. In a double mass system, two masses (one of mass m and the other of $2m$) d distance apart rotate about their common centre of mass. Find the value of the period of revolution. Show that the ratio of their angular momenta about the centre of mass is the same as the ratio of their kinetic energies.

($G = 2/3 \times 10^{-10}$ Nm²/kg², $m = 1.5 \times 10^{20}$ kg, $d = \frac{4}{\pi^{2/3}} \times 10^4$ m)

9. The density of the core of a planet is ρ_1 and that of the outer shell is ρ_2. The radii of the core and that of the planet are R and $2R$ respectively. Gravitational acceleration at the surface of the planet is same as at a depth R. Find the ratio $\frac{3\rho_1}{\rho_2}$.



10. Distance between the centres of two massive spheres is 10 a. The masses of these sphere are M and 16 M and their radii a and $2a$ respectively. A body of mass m is fired straight from the surface of the larger sphere towards the smaller sphere. What should be its minimum initial speed to reach the surface of the small sphere?

($G = 2/3 \times 10^{-10}$ Nm^2/kg^2, $M = 3 \times 10^{21}$kg, $a = 1 \times 10^6$m)

ANSWERS

EXERCISE – I

NEET- SINGLE CHOICE CORRECT

1. (b)	2. (d)	3. (a)	4. (c)	5. (c)
6. (c)	7. (d)	8. (a)	9. (a)	10. (c)
11. (b)	12. (a)	13. (d)	14. (c)	15. (c)
16. (d)	17. (d)	18. (c)	19. (b)	20. (c)
21. (b)	22. (c)	23. (d)	24. (c)	25. (c)

EXERCISE – II

IIT-JEE- SINGLE CHOICE CORRECT

1. (a)	2. (b)	3. (d)	4. (c)	5. (c)
6. (b)	7. (c)	8. (b)	9. (a)	10. (a)
11. (d)	12. (d)	13. (a)	14. (d)	15. (a)
16. (d)	17. (d)	18. (b)	19. (c)	20. (d)

ONE OR MORE THAN ONE CHOICE CORRECT

1. (a,c)	2. (a,b,d)	3. (a,b,c)	4. (a,c)	5. (a,b)
6. (a,b)	7. (a,c,d)	8. (a,b,c)	9. (a,b)	10. (a,b)

EXERCISE – III

MATCH THE FOLLOWING

1. I – A,B,C,E; II – E; III – E; IV- D

REASONING TYPE

1. (d)	2. (b)	3. (a)	4. (d)	5. (a)

LINKED COMPREHENSION TYPE

1. (c)	2. (a)	3. (b)

EXERCISE – IV

SUBJECTIVE PROBLEMS

1. $R_e = 6400$ km

2. 9 mm

3. 1 N

4. x = 1

5. x = 2

6. $x = 1$

7. 5

8. $T_m = T_{2m} = 92$ s

9. 7

10. 1500 m/s

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 1

Q.1 If R is the radius of the earth and g the acceleration due to gravity on the earth's surface, the mean density of the earth is-
(1) $4\pi G/3gR$ (2) $3\pi R/4gG$
(3) $3g/4\pi RG$ (4) $\pi Rg/12G$

Q.2 The weight of an object in the coal mine, sea level, at the top of the mountain are W_1, W_2 and W_3 respectively, then-
(1) $W_1 < W_2 > W_3$ (2) $W_1 = W_2 = W_3$
(3) $W_1 < W_2 < W_3$ (4) $W_1 > W_2 > W_3$

Q.3 The height above surface of earth where the value of gravitational acceleration is one fourth of that at surface, will be-
(1) $R_e/4$ (2) $R_e/2$
(3) $3R_e/4$ (4) R_e

Q.4 At the surface of a certain planet acceleration due to gravity is one quarter of that on earth. If a brass ball is transported to this planet, then which one of the following statements is not correct ?
(1) the mass of the brass ball on this planet is a quarter of its mass as measured on the earth
(2) the weight of the brass ball on this planet is a quarter of the weight as measured on the earth
(3) the brass ball has same mass on the another planet as on the earth
(4) the brass ball has the same volume on the other planet as on earth

Q.5 The weight of a person in a lift accelerating upwards-
(1) is zero (2) decrease
(3) increases (4) remains

Q.6 The decrease in the value of g on going to a height $\frac{R}{2}$ above the earth's surface will be -
(1) g/2 (2) $\frac{5g}{9}$
(3) $\frac{4g}{9}$ (4) $\frac{g}{3}$

Q.7 If the earth suddenly stops rotating, the value of g at any place will -
(1) remain same
(2) decrease
(3) increase
(4) none of these

Q.8 If the rotational motion of earth increases, then the weight of the body -
(1) will remain same (2) will increase
(3) will decrease (4) none of these

Q.9 If 'R' is the radius of earth and 'g' the acceleration due to gravity then mass of earth will be :
(1) $\frac{gR^2}{G}$ (2) $\frac{g^2R}{G}$ (3) $\frac{Rg}{G}$ (4) $\frac{GR^2}{g}$

Q.10 The dimensions of G are -
(1) ML^3T^{-2} (2) $M^{-1}LT^{-2}$
(3) $M^{-1}L^3T^{-2}$ (4) $M^{-1}L^3T^{-2}$

Q.11 Upto which distance the gravitational law is applicable-
(1) 10^8m (2) 1m
(3) 10^{-10}m (4) at all distances.

Q.12 On doubling the distance between two masses the gravitational force between them will -
(1) remain unchanged
(2) become one-fourth
(3) become half
(4) become double

Q.13 Newton's law of gravitation is true for :
(1) only uncharged particles
(2) only for planets
(3) only for heavenly bodies
(4) all the bodies

Q.14 The first successful determination of G in the laboratory was carried out by
(1) Sir Airy (2) Maskylene
(3) Cavendish (4) Faraday

Q.15 The gravitational force between two bodies is
(1) repulsive at short distances
(2) attractive at large distances
(3) repulsive at large distances
(4) attractive at all distances

Q.16 The weight of a body at the centre of the earth will -
(1) be greater than that at earth's surface
(2) be equal to zero
(3) be less than that at earth's surface.
(4) become infinite.

Q.17 Which of the following formula is correct- (d is the density of earth)
(1) $g = \frac{4}{3}\pi Gd$ (2) $g = \frac{4}{3}\pi RGd$
(3) $g = \frac{4}{3}\frac{\pi dG}{R}$ (4) $g = \frac{4}{3}\frac{\pi dG}{R^2}$

Q.18 A bomb explodes on the moon. On the earth-
(1) we will hear the sound after 10 minutes
(2) we will hear the sound after 2 hours 18 minutes
(3) we will hear the sound after 37 minutes.
(4) we can not hear the sound of explosion

Q.19 Newton's law of gravitation is called universal law because -
(1) force is always attractive
(2) it is applicable to lighter and heavier bodies
(3) it is applicable at all times
(4) it is applicable at all places of universe for all distances between all particles.

Q.20 If the acceleration due to gravity inside the earth is to be kept constant, then the relation between the density d and the distance r from the centre of earth will be -
(1) $d \propto r$ (2) $d \propto r^{1/2}$
(3) $d \propto \frac{1}{r}$ (4) $d \propto \frac{1}{r^2}$

Q.21 On increasing the angular velocity of the earth, the value of g in India -
(1) will decrease
(2) will increase
(3) will remain unchanged
(4) can increase or decrease.

Q.22 The value of g on the surface of earth is 9.8 m/s^2 and the radius of earth is 6400km. The average density of earth in kg/m^3 will be -
(1) 5.29×10^3 (2) 2.64×10^3
(3) 7.60×10^3 (4) 1.46×10^3

Q.23 The acceleration due to gravity of that planet whose mass and radius are half those of earth, will be - (g is acceleration due to gravity at earth's surface)
(1) 2g (2) g
(3) g/2 (4) g/4

Q.24 There is a body lying on the surface of the earth and suppose the earth suddenly loses its power of attraction then-
(1) the mass of the body will be reduced to zero
(2) the weight of the body will be reduced to zero
(3) both will be reduced to zero
(4) it becomes infinity

Q.25 A body is in a state of rest at infinite distance. If it is made a satellite of earth, which of the following physical quantities will be reduced -
(1) gravitational force (2) kinetic energy
(3) potential energy (4) mass

Q.26 Two different masses are dropped from same heights, then just before these strike the ground, the following is same :
(1) kinetic energy
(2) potential energy
(3) linear momentum
(4) Acceleration

Q.27 A body of mass m rises to height h = R/5 from the earth's surface, where R is earth's radius. If g is acceleration due to gravity at earth's surface, the increase in potential energy is-
(1) mgh (2) $\frac{5}{6}$ mgh
(3) $\frac{3}{5}$ mgh (4) $\frac{6}{7}$ mgh

Q.28 A body of mass m is taken to a height h from the surface of the earth (radius R) The gain in its gravitational potential energy is -
(1) mgR (2) mgh
(3) $mgh\left(\frac{R}{R+h}\right)$ (4) $mgh\left(\frac{R+h}{R}\right)$

Q.29 Binding energy of earth-moon system can be expressed as -
(1) $GM_eM_m/2r$ (2) $-GM_eM_m/r$
(3) GM_eM_m/r (4) $-GM_eM_m/2r$

Q.30 Weight of a person is 800 Newton. If he runs 4m in vertical ladder in 2 seconds then the needs a power of-
(1) 3200 kW (2) 3.2 kW
(3) 1.6 kW (4) zero

Q.31 A planet has mass 1/10 of that of earth, while radius is 1/3 that of earth. If a person can throw a stone on earth surface to a height of 90m, then he will be able to throw the stone on that planet to a height-
(1) 90m (2) 40 m
(3) 100 m (4) 45 m

Q.32 Work done in taking a body of mass m to a height nR above surface of earth will be - (R = radius of earth)
(1) mgnR (2) mgR $\left(\frac{n}{n+1}\right)$
(3) mgR $\frac{(n+1)}{n}$ (4) $\frac{mgR}{n(n+1)}$

Q.33 Mass of a planet is 5×10^{24}kg and radius is 6.1×10^{6} m. The energy needed to send a 2kg body from its surface in space is-
(1) 9 J (2) 10 J
(3) 2.2×10^{8} J (4) 1.1×10^{8} J

Q.34 The potential energy due to gravitational field of earth will be maximum at -
(1) infinite distance
(2) the poles of earth
(3) the centre of earth
(4) the equator of earth

Q.35 A particle falls on earth : (i) from infinity (ii) from a height 10 times the radius of earth. The ratio of the velocities gained on reaching at the earth's surface is
(1) $\sqrt{11} : \sqrt{10}$ (2) $\sqrt{10} : \sqrt{11}$
(3) 10 : 11 (4) 11 : 10

Q.36 The escape velocity is -
(1) 2gR (2) gR
(3) $\sqrt{gR}$ (4) $\sqrt{2gR}$

Q.37 A missile is launched with a velocity less than the escape velocity. The sum of kinetic energy and potential energy will be -
(1) positive
(2) negative
(3) negative or positive, uncertain
(4) zero

Q.38 There is no atmosphere on moon because -
(1) it is near the earth
(2) it is orbiting around the earth
(3) there was no gas at all
(4) the escape velocity of gas molecules is less than their root-mean square velocity

Q.39 The relation between the escape velocity from the earth and the velocity of a satellite orbiting near the earth's surface is -
(1) $v_e = v$
(2) $v_e = v\sqrt{2}$
(3) $v_e = 2v$

(4) $v_e = v/\sqrt{2}$

Q.40 The escape velocity from the earth does not depend upon -
(1) mass of earth
(2) mass of the body
(3) radius of earth
(4) acceleration due to gravity

Q.41 If the kinetic energy of a satellite orbiting around the earth is doubled then -
(1) the satellite will escape into the space.
(2) the satellite will fall down on the earth
(3) radius of its orbit will be doubled
(4) radius of its orbit will become half.

Q42 The escape velocity from a planet is v_0. The escape velocity from a planet having twice the radius but same density will be -
(1) 0.5 v_0 (2) v_0
(3) $2v_0$ (4) $4v_0$

Q.43 The potential energy of a body of mass 3kg on the surface of a planet is 54 joule. The escape velocity will be -
(1) 18m/s (2) 162 m/s
(3) 36 m/s (4) 6 m/s

Q.44 A body of mass m is situated at a distance $4R_e$ above the earth's surface, where R_e is the radius of earth. How much minimum energy be given to the body so that it may escape -
(1) mgR_e (2) $2mgR_e$
(3) $\frac{mgR_e}{5}$ (4) $\frac{mgR_e}{16}$

Q.45 If the radius of earth is to decrease by 4% and its density remains same, then its escape velocity will -
(1) remain same (2) increase by 4%
(3) decrease by 4% (4) increase by 2%

Q.46 The velocity of a satellite at a height h above the earth's surface is -
(1) $\sqrt{\frac{GM}{R}}$ (2) $\sqrt{\frac{GM}{R^2}}$
(3) $\sqrt{\frac{GM}{h}}$ (4) $\sqrt{\frac{GM}{R+h}}$

Q.47 Kinetic energy of a satellite will be -
(1) $\frac{GmM}{2r^2}$ (2) $\frac{GmM}{2r}$
(3) $\frac{GmM}{r^2}$ (4) $\frac{GmM}{r}$

Q.48 Binding energy of moon and earth is -
(1) $\frac{GM_eM_m}{r_{em}}$ (2) $\frac{GM_eM_m}{2r_{em}}$
(3) $-\frac{GM_eM_m}{r_{em}}$ (4) $-\frac{GM_eM_m}{2r_{em}}$

Q.49 A satellite is moving in a circular orbit around earth with a speed v. If its mass is m, then its total energy will be -

(1) $\frac{3}{4}$ mv^2 (2) mv^2

(3) $\frac{1}{2}mv^2$ (4) $-\frac{1}{2}mv^2$

Q.50 A satellite of earth is moving in its orbit with a constant speed v. If the gravity of earth suddenly vanishes, then this satellite will -

(1) continue to move in the orbit with velocity v.
(2) start moving with velocity v in a direction tangential to the orbit
(3) fall down with increased velocity
(4) be lost in outer space.

Q.51 The ratio of kinetic energy of a body orbiting near the earth's surface and the kinetic energy of the same body escaping the earth's gravitational field is -

(1) 1 (2) 2

(3) $\sqrt{2}$ (4) 0.5

Q.52 The ratio of kinetic and potential energies of a satellite is -

(1) 1 : 4 (2) 4 : 1

(3) 1 : 2 (4) 2 : 1

Q.53 If mass of earth is 5.98×10^{24} kg and earth moon distance is 3.8×10^5km, the orbital period of moon, in days is -

(1) 27 days (2) 2.7 days

(3) 81 days (4) 8.1 days

Q.54 The ratio of distances of satellites A and B above the earth's surface is 1.4 :1, then the ratio of energies of satellites B and A will be -

(1) 1.4 : 1 (2) 2 : 1

(3) 1 : 3 (4) 4 : 1

Q.55 A satellite of earth can move only in those orbits whose plane coincides with -

(1) the plane of great circle of earth
(2) the plane passing through the poles of earth
(3) the plane of a circle at any latitude of earth
(4) none of these

Q.56 A body is dropped by a satellite in its geo-stationary orbit -

(1) it will burn on entering into the atmosphere
(2) it will remain in the same place with respect to the earth.
(3) it will reach the earth in 24 hours
(4) it will perform uncertain motion

Q.57 An earth satellite is moved from one stable circular orbit to another higher stable circular orbit. Which one of the following quantities increases for the satellite as a result of this change-

(1) gravitational force
(2) gravitational potential energy
(3) centripetal acceleration
(4) Linear orbital speed

Q.58 A geostationary satellite -
(1) Revolves about the polar axis
(2) Has a time period less than that of the near earth satellite
(3) Moves faster than a near earth satellite
(4) Is stationary in the space

Q.59 One satellite is revolving round the earth in an elliptical orbit. Its speed will -
(1) be same at all the points of orbit
(2) be maximum at the point farthest from the earth
(3) be maximum at the point nearest from the earth
(4) depend on mass of satellite.

Q.60 The orbital velocity of an artificial satellite in a circular orbit just above the earth's surface is υ. For a satellite orbiting at an altitude of half of the earth's radius, the orbital velocity is –
(1) υ (2) $\sqrt{\frac{3}{2}}\,\upsilon$ (3) $\sqrt{\frac{2}{3}}\,\upsilon$ (4) $\frac{2}{3}\,\upsilon$

Q.61 Two satellites of masses m_1 and m_2 ($m_1 > m_2$) are revolving round the earth in circular orbits of radius r_1 and r_2 ($r_1 > r_2$) respectively. Which of the following statements is true regarding their speed υ_1 and υ_2?
(1) $\upsilon_1 = \upsilon_2$ (2) $\upsilon_1 < \upsilon_2$
(3) $\upsilon_1 > \upsilon_2$ (4) $\frac{\upsilon_1}{r_1} = \frac{\upsilon_2}{r_2}$

Q.62 Orbital velocity of earth's satellite near the surface is 7 Km/sec. When the radius of the orbit is 4 times than that of earth's radius then orbital velocity in that orbit is -
(1) 3.5 Km/sec (2) 7 Km/sec
(3) 14 Km/sec (4) 72 Km/sec

Q.63 If the earth-sun distance is held constant and the mass of the sun is doubled, then the period of revolution of the earth around the sun will change to-
(1) 2 years (2) $\frac{1}{2}$ years
(3) $\frac{1}{\sqrt{2}}$ years (4) $\sqrt{2}$ years

Q.64 The velocity of a satellite orbiting near the earth's surface is -
(1) $\sqrt{GR}$ (2) $\sqrt{gR}$
(3) $\sqrt{\frac{GM}{R^2}}$ (4) $\sqrt{2gR}$

Q.65 A satellite of mass m is revolving around the earth of mass M in a path of radius r, the angular velocity of the satellite will be -
(1) $M\sqrt{Gr}$ (2) $\sqrt{\frac{GM}{r}}$
(3) $M\sqrt{\frac{G}{r}}$ (4) $\sqrt{\frac{GM}{r^3}}$

Q.66 A satellite moves in a path of radius r with a velocity v, the mass of earth will be -

(1) $\frac{vr}{G}$ (2) $\frac{v^2 r}{G}$

(3) $\frac{Gr}{v^2}$ (4) $\frac{G}{vr^2}$

Q.67 Following quantity is conserved in the motion of a satellite -

(1) angular velocity (2) linear velocity

(3) angular momentum (4) linear momentum

Q.68 In an artificial satellite a person will have -

(1) zero mass (2) zero weight

(3) some weight (4) infinite weight

Q.69 If a satellite is orbiting very near to the earth's surface than its orbital velocity depends upon-

(1) mass of the satellite.

(2) mass of earth only

(3) radius of earth only

(4) mass and radius of earth

Q.70 The period of revolution of a communication satellite of earth is -

(1) zero (2) 4.8 hours

(3) 36 hours (4) 24 hours

Q.71 The formula of period of revolution of a satellite is -

(1) $T = 2\pi\sqrt{\frac{R}{GM}}$ (2) $T = 2\pi\sqrt{\frac{R^2}{GM}}$

(3) $T = 2\pi\sqrt{\frac{GM}{R^3}}$ (4) $T = 2\pi\sqrt{\frac{R^3}{GM}}$

Q.72 An astronaut feels weightlessness because -

(1) gravity is zero there

(2) atmosphere is not there

(3) energy is zero in the chamber of a rocket.

(4) the fictitious force in rotating frame of reference cancels the effect of weight.

Q.73 Orbital velocity of INSAT 1-B is nearly -

(1) 11.2 km/s (2) 3 km/s

(3) 2.6 km/s (4) 1.8 km/s

Q.74 Two artificial satellites A and B are at a distances r_A and r_B above the earth's surface. If the radius of earth is R, then the ratio of their speeds will be -

(1) $\left(\frac{r_B + R}{r_A + R}\right)^{1/2}$ (2) $\left(\frac{r_B + R}{r_A + R}\right)^{2}$

(3) $\left(\frac{r_B}{r_A}\right)^{2}$ (4) $\left(\frac{r_B}{r_A}\right)^{1/2}$

Q.75 If the distance between sun and earth is made 3 times of the present value then gravitational force between them will become :

(1) 9 times (2) $\frac{1}{9}$ times
(3) $\frac{1}{3}$ times (4) 3 times

Q.76 According to Kepler the period of revolution of a planet (T) and its mean distance from the sun (r) are related by the equation-
(1) $T^2 r$ = constant (2) $T^2 r^3$ = constant
(3) $T^2 r^{-3}$ = constant (4) $T r^3$ = constant

Q.77 If the earth is to be at half of the present distance from sun, then number of days in one year would be-
(1) 92 days (2) 129 days
(3) 183 days (4) 365 days

Q.78 If a body is carried from surface of earth to moon, then -
(1) the weight of a body will continuously increase
(2) the mass of a body will continuously increase
(3) the weight of a body will decrease first, become zero and then increase,
(4) the mass of a body will decrease first, become zero and then increase.

Q.79 The radii of paths of two planets moving around the sun are R_1 and R_2 and their periods are T_1 and T_2 respectively, T_1/T_2 will be -

(1) $\left(\frac{R_2}{R_1}\right)^{3/2}$ (2) $\left(\frac{R_1}{R_2}\right)^{3/2}$
(3) $\left(\frac{R_2}{R_1}\right)^{1/2}$ (4) $\left(\frac{R_1}{R_2}\right)^{1/2}$

Q.80 The paths of planets moving around the sun are -
(1) circular (2) elliptical
(3) parabolic (4) hyperbolic

Q.81 Orbit traced out by a planet around the sun is in general-
(1) circular (2) elliptical
(3) parabolic (4) none of the above

Q.82 The period of revolutions of two satellites are 3 hours and 24 hours. The ratio of their velocities will be -
(1) 1 : 8 (2) 1 : 2 (3) 2 : 1 (4) 4 : 1

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 2

Q.1 Gravitation on moon is 1/6th of that on earth. When a balloon filled with hydrogen is released on moon then this-

(1) will rise with an acceleration less than $\left(\frac{g}{6}\right)$

(2) will rise with acceleration $\left(\frac{g}{6}\right)$

(3) will fall down with an acceleration less than $\left(\frac{5g}{6}\right)$

(4) will fall down with acceleration $\left(\frac{g}{6}\right)$

Q.2 A planet is moving in an elliptical orbit. If T, U, E and L are its kinetic energy, potential energy, total energy and magnitude of angular momentum respectively, then which of the following statements is true-
(1) T is conserved
(2) U is always positive
(3) E is always negative
(4) L is conserved but the direction of vector $\vec{L}$ will continuously change

Q.3 Distances of two planets from the sun are 10^{13} and 10^{12} meter respectively. Ratio of their time periods is-
(1) $\frac{10}{\sqrt{10}}$
(2) $10\sqrt{10}$
(3) 100
(4) $\frac{1}{\sqrt{10}}$

Q.4 The gravitational force between two bodies is directly proportional to $\frac{1}{R}$ (not $\frac{1}{R^2}$), where 'R' is the distance between the bodies. Then the orbital speed for this force in circular orbit is proportional to-
(1) $1/R^2$ (2) R^0
(3) R (4) 1/R

Q.5 During the journey of space ship from earth to moon and back, the maximum fuel is consumed-
(1) Against the gravitation of earth in return journey
(2) Against the gravitation of earth in onward journey
(3) Against the gravitation of moon while reaching the moon
(4) None of the above

Q.6 Escape velocity from the surface of the earth is-
(1) 12.1 km/sec (2) 1.12 m/sec
(3) 112 km/sec (4) 11.2 km/sec

Q.7 What will be velocity of a satellite revolving around the earth at a height h above surface of earth if radius of earth is R-

(1) $R^2\sqrt{\frac{g}{R+h}}$ (2) $R\frac{g}{(R+h)^2}$

(3) $R\sqrt{\frac{g}{R+h}}$ (4) $R\sqrt{\frac{R+h}{g}}$

Q.8 A satellite is put in a circular orbit of radius R. Another satellite is thrown in a circular orbit of radius 1.01 R. The percentage difference between the periodic time of second satellite with respect to the first will be-

(1) 1% increased (2) 1% decreased

(3) 1.5% increased (4) 1.5% decreased

Q.9 If the gravitational acceleration at surface of earth is g, then increase in potential energy is lifting an object of mass m to a height equal to the radius R of earth will be-

(1) $\frac{mgR}{2}$ (2) 2 mgR (3) mgR (4) $\frac{mgR}{4}$

Q.10 The gravitational potential energy is maximum at-

(1) infinity

(2) the earth's surface

(3) the centre of earth

(4) twice the radius of the earth

Q.11 The escape velocity of a particle of mass (m) is proportional to-

(1) m^2 (2) m^0

(3) m^3 (4) 1/m

Q.12 Escape velocity of a body from earth is 11.2 km/s. Escape velocity, when thrown at an angle of 45º from horizontal will be-

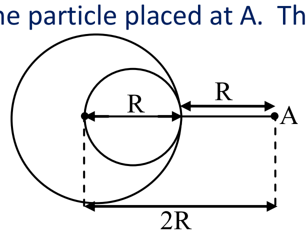
(1) 11.2 km/s

(2) 22.4 km/s

(3) $11.2/\sqrt{2}$ km/s

(4) $11.2\sqrt{2}$ km/s

Q.13 A solid sphere of uniform density and radius R applies gravitational force of attraction equal to F_1 on a particle placed at A, distant 2R from the centre of the sphere. A spherical cavity of radius R/2 is now made in the sphere as shown in the figure. The sphere with cavity now applies a gravitational force F_2 on the same particle placed at A. The ratio F_2/F_1 will be-



(1) $\frac{1}{2}$ (2) $\frac{3}{4}$ (3) $\frac{7}{8}$ (4) $\frac{7}{9}$

Q.14 A satellite orbiting around earth in an orbit of radius R is shifted to an orbit of radius 2R. Time taken for one revolution will become-

(1) 8 times (2) 2 times

(3) 2.5 times (4) 2.8 times

Q.15 The radius of earth is about 6400 km and that of mars is 3200 km. The mass of the earth is 10 times the mass of mars. An object weight 200N on the surface of earth. Its weight on the surface of mars will be-
(1) 80 N (2) 40 N
(3) 20 N (4) 8 N

Q.16 The earth (M_e = 6 × 10^{24} kg) is revolving round the sun in an orbit of radius (1.5 × 10^8) km with angular velocity of (2 × 10^{-7}) rad/s. The force (in newton) exerted on the earth by the sun will be-
(1) 36 × 10^{21} (2) 16 × 10^{24}
(3) 25 × 10^{16} (4) Zero

Q.17 The acceleration due to gravity g and mean density of earth ρ are related by which of the following relations ? [G = gravitational constant and R = radius of earth]
(1) $\rho = \frac{4\pi g R^2}{3G}$ (2) $\rho = \frac{4\pi g R^3}{3G}$
(3) $\rho = \frac{3g}{4\pi G R}$ (4) $\rho = \frac{3g}{4\pi G R^3}$

Q.18 What will be the formula of the mass in terms of g, R and G ? [R = radius of earth]
(1) $\frac{g^2 R}{G}$ (2) $\frac{GR^2}{g}$
(3) $\frac{GR}{g}$ (4) $\frac{gR^2}{G}$

Q.19 A ball is dropped from a space craft revolving around the earth at a height of 120 km. What will happen to the ball ?
(1) It will continue to move with the same velocity along the original orbit of space craft
(2) It will move with the same speed, tangentially to the space craft
(3) It will fall down to the earth gradually
(4) It will go very far in the space

Q.20 Escape velocity of a body from the surface of earth is 11.2 km/sec. If the mass of earth becomes double of its present mass and radius becomes half of its present radius, then escape velocity will become-
(1) 5.6 km/sec
(2) 11.2 km/sec
(3) 22.4 km/sec
(4) 44.8 km/sec

Q.21 If resistive force is negligible then escape velocity of an object of mass m will be-
(1) $\sqrt{\frac{2GM}{R}}$ (2) $\sqrt{\frac{GM}{R}}$
(3) $\sqrt{\frac{2GM}{mR}}$ (4) $\sqrt{\frac{GM}{mR}}$

Q.22 Two artificial satellites of masses m_1 and m_2 are moving with speeds v_1 and v_2 in orbits of radii r_1 and r_2 respectively. If $r_1 > r_2$ then which of the following statements is true-
(1) $v_1 = v_2$ (2) $v_1 > v_2$
(3) $v_1 < v_2$ (4) $v_1/r_1 = v_2/r_2$

Q.23 Escape velocity from earth is 11.2 km/sec. Another planet of same mass has radius $\frac{1}{4}$ times that of earth. What is the escape velocity from this planet ?
(1) 11.2 km/sec (2) 44.8 km/sec
(3) 22.4 km/sec (4) 5.6 km/sec

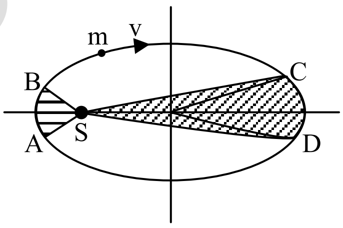
Q.24 A body attains a height equal to the radius of the earth when projected from earth surface. The velocity of the body with which it was projected is-
(1) $\sqrt{\frac{GM}{R}}$ (2) $\sqrt{\frac{2GM}{R}}$
(3) $\sqrt{\frac{5}{4}\frac{GM}{R}}$ (4) $\sqrt{\frac{3GM}{R}}$

Q.25 Imagine a new planet having the same density as that of earth but it is 3 times bigger than the earth in size. If the acceleration due to gravity on the surface of earth is g and that on the surface of the new planet is g', then-
(1) g' = 3g (2) g' = g/9
(3) g' = 9g (4) g' = 27g

Q.26 Escape velocity for a projectile at earth's surface is V_e. A body is projected from earth's surface with velocity $2V_e$. The velocity of the body when it is at infinite distance from the centre of the earth is-
(1) V_e (2) $2V_e$
(3) $\sqrt{2}\, V_e$ (4) $\sqrt{3}\, V_e$

Q.27 For a satellite moving in an orbit around the earth, the ratio of kinetic energy to potential energy is-
(1) 2 (2) 1/2 (3) $\frac{1}{\sqrt{2}}$ (4) $\sqrt{2}$

Q.28 The figure shows elliptical orbit of a planet m about the sun S. The shaded area SCD is twice the shaded area SAB. If t_1 is the time for the planet to move from C to D and t_2 is the time to move from A to B then-



(1) $t_1 = t_2$ (2) $t_1 > t_2$
(3) $t_1 = 4t_2$ (4) $t_1 = 2t_2$

Q.29 Escape velocity of a body when projected from the earth's surface is 11.2 km/sec. If it is projected at angle of 50º from the horizontal, the escape velocity is-
(1) 12.8 km/sec (2) 16.2 km/sec
(3) 11.2 km/sec (4) 11.8 km/sec

Q.30 Knowing that the mass of the moon is 1/81 times that of earth and its radius is ¼ the radius of earth. If the escape velocity at the surface of the earth is 11.2 km/sec. Then the value of escape velocity at the surface of the moon is-
(1) 2.5 km/sec (2) 0.14 km/sec
(3) 5 km/sec (4) 8 km/sec

Q.31 What is the dimensional formula of gravitational constant ?
(1) $[ML^2T^{-2}]$ (2) $[ML^{-1}T^{-1}]$
(3) $[M^{-1}L^3T^{-2}]$ (4) None of these

Q.32 If the mass of moon is M/81, where M is the mass of earth, find the distance of the point from the moon, where gravitation field due to earth and moon cancel each other. Given that distance between earth and moon is 60R where R is the radius of earth-
(1) 4R (2) 8R (3) 2R (4) 6R

Q.33 The escape velocity from the earth is 11.2 km/sec. The escape velocity from a planet having twice the radius and the same mean density as the earth, is-
(1) 11.2 km/sec (2) 22.4 km/sec
(3) 15.00 km/sec (4) 5.8 km/sec

Q.34 If v_0 be orbital velocity of a satellite in a circular orbital close to the earth's surface and v_e is escape velocity from earth, then relation between the two is-
(1) $v_e = 2v_0$ (2) $v_e = \sqrt{3}v_0$
(3) $v_e = v_0\sqrt{2}$ (4) $v_0 = v_e$

Q.35 A satellite is launched into a circular orbit of radius R around the earth. While a second satellite launched into an orbit of radius 1.01 R. The period of the second satellite is longer than the first one by approximately-
(1) 3.0 % (2) 1.5 % (3) 0.7 % (4) 1.0 %

Q.36 The velocity with which a projectile must be fired so that it escape earth's gravitation does not depend on-
(1) mass of the earth
(2) mass of the projectile
(3) radius of the projectile's orbit
(4) gravitational constant

Q.37 The condition for a uniform spherical mass m of radius r to be a black hole is-
[G = gravitational due to gravity]
(1) $\left(\frac{2Gm}{r}\right)^{1/2} \le c$ (2) $\left(\frac{2Gm}{r}\right)^{1/2} = c$
(3) $\left(\frac{2Gm}{r}\right)^{1/2} \ge c$ (4) $\left(\frac{gm}{r}\right)^{1/2} \ge c$

Q.38 A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them, to take the particle far away from the sphere-
(you may take $G = 6.67 \times 10^{-11}$ Nm^2/kg^2)
(1) 13.34×10^{-10} J (2) 3.33×10^{-16} J
(3) 6.67×10^{-9} J (4) 6.67×10^{-10}

Q.39 Two bodies of masses m_1 and m_2 are initially at rest at infinite distance apart. They are then allowed to move towards each other under mutual gravitational attraction. Their relative velocity of approach at a separation distance r between them is-

(1) $\left[2G\frac{(m_1-m_2)}{r}\right]^{1/2}$ (2) $\left[\frac{2G}{r}(m_1+m_2)\right]^{1/2}$

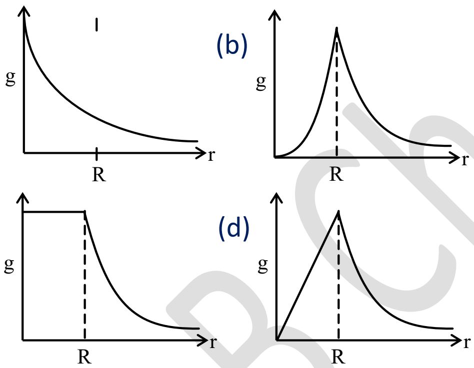
(3) $\left[\frac{r}{2G(m_1m_2)}\right]^{1/2}$ (4) $\left[\frac{2G}{r}m_1m_2\right]^{1/2}$

Q.40 If the orbital velocity of moon is increased by 41.4% of its present value, then the-
(1) moon will orbit around earth with double velocity
(2) radius of moon's orbit will become double
(3) moon will become stationary satellite
(4) moon will leave its orbit and escape into space

Q.41 The radii of circular orbits of two satellite A and B of the earth, are 4R and R, respectively speed of satellite A is 3V, then the speed satellite B will be –
(1) 3V/4 (2) 6 V
(3) 12 V (4) 3 V/2

Q.42 The dependence of acceleration due to gravity 'g' on the distance 'r' from the centre of the earth, assumed to be a sphere of radius R of uniform density is as shown in figure below-

(a) (b) (c) (d)



The correct figure is
(1) a (2) b
(3) c (4) d

Q.43 The additional kinetic energy to be provided to a satellite of mass m revolving around a planet of mass M, to transfer it from a circular orbit of radius R_1 to another of radius R_2 ($R_2 > R_1$) is –

(1) GmM $\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

(2) 2 GmM $\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

(3) $\frac{1}{2}$ GmM $\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

(4) GmM $\left(\frac{1}{R_1^2}-\frac{1}{R_2^2}\right)$

Q.44 A body projected vertically from the earth reaches a height equal to earth's radius before returning to the earth. The power exerted by the gravitational force is greatest :
(1) at the instant just after the body is projected. (2) at the highest position of the body
(3) at the instant just before the body hits the earth.
(4) it remains constant all through.

Q.45 A planet moving along an elliptical orbit is closest to the sun at a distance r_1 and farthest away at a distance of r_2. If v_1 and v_2 are the linear velocities at these points respectively, then the ratio $\frac{v_1}{v_2}$ is :
(1) r_1/r_2 (2) $(r_1/r_2)^2$ (3) r_2/r_1 (4) $(r_2/r_1)^2$

Q.46 A particle of mass m is thrown upwards from the surface of the earth, with a velocity u. The mass and the radius of the earth are, respectively, M and R. G is gravitational constant and g is acceleration due to gravity on the surface of the earth. The minimum value of u so that the particle does not return back to earth is :
(1) $\sqrt{\frac{2GM}{R^2}}$ (2) $\sqrt{\frac{2GM}{R}}$
(3) $\sqrt{\frac{2gM}{R^2}}$ (4) $\sqrt{2gR^2}$

Q.47 A particle of mass M is situated at the centre of a spherical shell of same mass and radius a. The magnitude of the gravitational potential at a point situated at a/2 distance from the centre, will be :
(1) $\frac{GM}{a}$ (2) $\frac{2GM}{a}$
(3) $\frac{3GM}{a}$ (4) $\frac{4GM}{a}$

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 3

Q.1 There is no atmosphere on moon because-
(1) The root mean square velocity of atoms is more than escape velocity
(2) The root mean square velocity of atoms is less than escape velocity
(3) There is no oxygen
(4) None of the above

Q.2 If the radius of earth shrinks by 1% maintaining its mass, then 'g' at the earth surface will-
(1) Decrease
(2) Increase
(3) Remain same
(4) Increase or decrease

Q.3 If R is the average radius of earth, ω is its angular velocity about its axis and g is the gravitational acceleration on the surface of earth then the cube of the radius of orbit of a geostationary satellite will be equal to-
(1) $\frac{R^2 g}{\omega}$ (2) $\frac{R^2\omega^2}{g}$
(3) $\frac{Rg}{\omega^2}$ (4) $\frac{R^2 g}{\omega^2}$

Q.4 If the radius of a planet becomes half, where as its mass remains unchanged, then g becomes-
(1) Half (2) Doubled
(3) Unchanged (4) Four times

Q.5 Orbital radius of a satellite S of earth is four times that of a communication satellite C. Period of revolution of S is-
(1) 4 days (2) 8 days
(3) 16 days (4) 32 days

Q.6 A missile is launched with a velocity less than the escape velocity. Sum of its kinetic energy and potential energy is-
(1) Positive
(2) Negative
(3) May be negative or positive depending upon its initial velocity
(4) Zero

Q.7 Time period of a satellite revolving round a planet in an orbit of radius R is T. Periodic time of a satellite moving in an orbit of radius 9R will be-
(1) 27 T (2) 81 T
(3) 729 T (4) 3 T

Q.8 More amount of sugar is obtained in 1kg weight-
(1) At north pole
(2) At equator
(3) Between pole and equator
(4) At south pole

Q.9 If a satellite is revolving very close to the surface of earth, then its orbital velocity does not depend upon-
(1) Mass of satellite (2) Mass of earth
(3) Radius of earth (4) Orbital radius

Q.10 If the escape velocity from the surface of earth is v_e and velocity of a satellite revolving near the surface of earth is v then-

(1) $v = \sqrt{2}\, v_e$ (2) $v_e = 2v$
(3) $v_e \approx \sqrt{2v}$ (4) $v_e \approx \sqrt{2}\, v$

Q.11 Two identical satellites are at the heights R and 7R from the earth's surface. Then which of the following statement is incorrect- (R = radius of the earth)
(1) Ratio of total energy of both is 5
(2) Ratio of kinetic energy of both is 4
(3) Ratio of potential energy of both 4
(4) Ratio of total energy of both is 4

Q.12 Imagine a body revolving around a big star in a circular orbit of radius R with time period T. If the force of attraction between star and the body is proportional to $R^{-5/2}$, then T^2 will be proportional to-
(1) R^3 (2) $R^{7/2}$ (3) $R^{3/2}$ (4) $R^{3.75}$

Q.13 The minimum projection velocity of a body from the earth's surface so that it becomes the satellite of the earth ($R_e = 6.4 \times 10^6$ m)
(1) 11×10^3 m/s (2) 8×10^3 m/s
(3) 6.4×10^3 m/s (4) 4×10^3 m/s

Q.14 Geostationary satellite-
(1) is situated at a great height above the surface of earth
(2) moves in equatorial plane
(3) have time period of 24 hours
(4) have time period of 24 hours and moves in equatorial plane

Q.15 A planet whose mass and radius are both half of that of earth consists of a satellite. Acceleration due to gravity (g) at its surface should be-
(1) 29.4 m/sec^2 (2) 19.6 m/sec^2
(3) 9.8 m/sec^2 (4) 4.9 m/sec^2

Q.16 A satellite of mass m moves around the earth along a circular path of radius r. Let m_e is the mass of the earth and R_e is its radius. The linear speed of the satellite depends upon-
(1) m_e and r (2) m_e only
(3) r only (4) m, R_e and r

Q.17 If the radius of earth is reduced by 2% keeping its mass constant, then the weight of the body on its surface will-
(1) increase (2) decrease
(3) remain same (4) either (2) or (3)

Q.18 An earth's satellite is moving in a circular orbit with a uniform speed v. If the gravitational force of the earth suddenly disappears, the satellite will-
(1) vanish into outer space
(2) continue to move with velocity v in original orbit
(3) fall down with increasing velocity
(4) fly off tangentially from the orbit with velocity v

Q.19 The distance of a geostationary satellite from the centre of earth (radius R = 6400 km) is nearly-
(1) 18 R (2) 10 R (3) 7 R (4) 5 R

Q.20 The maximum and minimum distances of a comet from the sun are 8×10^{12} m and 1.6×10^{12} m respecting. If its velocity when it is nearest to the sun is 60 m/sec then what will be its velocity in m/s when it is farthest ?

(1) 12 (2) 60 (3) 112 (4) 6

Q.21 The gravitational potential energy of a body at a distance r from the centre of the earth is U. The force at that point is-

(1) $\frac{U}{r^2}$ (2) $\frac{U}{r}$

(3) Ur (4) Ur^2

Q.22 If the spinning speed of the earth is increased, then the weight of the body at the equator-

(1) does not change
(2) doubles
(3) decreases
(4) increases

Q.23 When the radius of earth is reduced by 1% without changing the mass, then the acceleration due to gravity will-

(1) increase by 2%
(2) decrease by 1.5%
(3) increase by 1%
(4) decrease by 1%

Q.24 A particle falls from infinity to the earth. Its velocity on reaching the earth surface is-

(1) 2Rg (2) Rg

(3) $\sqrt{Rg}$ (4) $\sqrt{2Rg}$

Q.25 Weight of a body of mass m decreases by 1% when it is raised to height h above the earth's surface. If the body is taken to a depth h in a mine, then in its weight will-

(1) decrease by 0.5% (2) decrease by 2%

(3) increase by 0.5% (4) increase by 1%

Q.26 The escape velocity from the earth is 11.2 km/sec. The mass of another planet is 100 times of mass of earth and its radius is 4 times the radius of earth. The escape velocity for the planet is-

(1) 56.0 km/sec (2) 280 km/sec

(3) 112 km/sec (4) 11.2 km/sec

Q.27 Acceleration due to gravity at earth's surface is 'g' m/s^2. Find the effective value of acceleration due to gravity at a height of 32 km from sea level- (R_e = 6400 km)

(1) 0.5 g m/s^2 (2) 0.99 g m/s^2

(3) 1.01 g m/s^2 (4) 0.90 g m/s^2

Q.28 Near the earth's surface time period of a satellite is 1.4 hrs. Find its time period if it is at the distance '4R' from the centre of earth.

(1) 32 hrs. (2) $\left(\frac{1}{8\sqrt{2}}\right)$ hrs.

(3) $8\sqrt{2}$ hrs. (4) 16 hrs.

Q.29 The mass of the moon is 1% of mass of the earth. The ratio of gravitational pull of earth on moon to that of moon on earth will be-

(1) 1 : 1 (2) 1 : 10
(3) 1 : 100 (4) 2 : 1

Q.30 A planet revolves around the sun in an elliptical orbit. If v_p and v_a are the velocities of the planet at the perigee and apogee respectively, then the eccentricity of the elliptical orbit is given by-

(1) $\frac{v_p}{v_a}$ (2) $\frac{v_a - v_p}{v_a + v_p}$
(3) $\frac{v_p + v_a}{v_p - v_a}$ (4) $\frac{v_p - v_a}{v_p + v_a}$

Q.31 A communication satellite of earth which takes 24 hrs. to complete one circular orbit eventually has to be replaced by another satellite of double mass. If the new satellites along has an orbital time period of 24 hrs, then what is the ratio of the radius of the new orbit to the original orbit ?

(1) 1 : 1 (2) 2 : 1
(3) $\sqrt{2}$: 1 (4) 1 : 2

Q.32 When you move from equator to pole, the value of acceleration due to gravity (g) –

(1) increases
(2) decreases
(3) remains the same
(4) first increases then decreases

Q.33 If earth revolves around the sun in an orbit of double radius of its present radius, then the year on earth will be of-

(1) $\frac{365}{4}$ days (2) $365 \times 2\sqrt{2}$ days
(3) $\frac{365}{2}$ days (4) 365 × 4 days

Q.34 Mars has a diameter of approximately 0.5 of that of earth, and mass of 0.1 of that of earth. The surface gravitational field strength on mars as compared to that on earth is greater by a factor of-

(1) 0.1 (2) 0.2
(3) 2.0 (4) 0.4

Q.35 The change in the value of 'g' at a height 'h' above the surface of the earth is same as at a depth 'd'. If 'd' and 'h' are much smaller than the radius of earth, then which one of the following is correct ?

(1) d = h (2) d = 2h
(3) d = $\frac{3h}{2}$ (4) d = $\frac{h}{2}$

Q.36 Average density of the earth

(1) does not depend on g
(2) is a complex function of g
(3) is directly proportional to g
(4) is inversely proportional to g

Q.37 A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere (you may take G = 6.67×10^{-11} Nm^2 / kg^2)

(1) 13.34×10^{-10} J (2) 3.33×10^{-10} J
(3) 6.67×10^{-9} J (4) 6.67×10^{-10} J

Q.38 A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be

(1) 11 km s^{-1} (2) 110 km s^{-1}
(3) 0.11 km s^{-1} (4) 1.1 km s^{-1}

Q.39 The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth, is -

(1) $\frac{R}{\sqrt{2}}$ (2) R/2
(3) $\sqrt{2}$ R (4) 2R

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 4

These questions consists of two statements each, printed as Assertion and Reason. While answering these questions you are required to choose any one of the following four responses.
(A) If both Assertion & Reason are true & the Reason is a correct explanation of the Assertion.
(B) If both Assertion and Reason are true but Reason is not a correct explanation of the Assertion.
(C) If Assertion is true but the Reason is false.
(D) If Assertion & Reason both are false.

Q.1 **Assertion :** Two satellites A and B are in the same orbit around the earth, B being behind A. B cannot overtake A by increasing its speed.
Reason : It will then go into a different orbit.
(1) A (2) B (3) C (4) D

Q.2 **Assertion :** A geostationary satellite rotates in a direction from west to east.
Reason : At midnight when the sun is directly below, it pulls on an object in the same direction as the pull of the earth on that object. At noon when the sun is directly above, it pulls on an object in a direction opposite to the pull of the earth.
(1) A (2) B (3) C (4) D

Q.3 **Assertion :** The acceleration of a particle near the earth surface differs slightly from the gravitational acceleration $a_g = GM/R^2$.
Reason : The earth is not a uniform sphere and because the earth rotates.
(1) A (2) B (3) C (4) D

Q.4 **Assertion :** Kepler's law of areas is equivalent to the law of conservation of angular momentum.
Reason : $\frac{dA}{dt} = \frac{L}{2m}$ = constant
(1) A (2) B (3) C (4) D

Q.5 **Assertion :** For circular orbits, the law of periods is $T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$, where M is the mass of sun.
Reason : The square of the period **T** of any planet about the sun is proportional to the cube of the semi-major axis **a** of the orbit.
(1) A (2) B (3) C (4) D

Q.6 **Assertion :** If rotation of earth is suddenly stops then acceleration due to gravity will increase at all places on the earth.
Reason : At height h from the surface of earth, acceleration due to gravity is $g_h = g\left(1-\frac{2h}{R_e}\right)$.
(1) A (2) B (3) C (4) D

Q.7 **Assertion :** Earth is continuously pulling moon towards its centre but moon does not fall to earth.
Reason : Attraction of sun on moon is greater than that of earth on moon.
(1) A (2) B (3) C (4) D

Q.8 **Assertion :** If the earth stops rotating about its axis. The value of the weight of the body at equator will decrease.
Reason : The centripetal force will not acting on the body at the equator.
(1) A (2) B (3) C (4) D

Q.9 **Assertion :** Gravitational potential is maximum at infinite.
Reason : Gravitational potential is the amount of work done to shifting a unit mass from infinity to a given point in gravitational attraction force field.
(1) A (2) B (3) C (4) D

Q.10 **Assertion :** A person feels weightlessness in an artificial satellite of the earth. However a person on the moon (natural satellite) feels his weight.
Reason : Artificial satellite in a freely falling body and on the moon surface, the weight is mainly due to moon's gravitational attraction.
(1) A (2) B (3) C (4) D

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 1 (ANSWERS)

Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	3	1	4	1	3	2	3	3	1	3	4	2	4	3	4	2	2	4	4	3
Q.No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans.	1	1	1	2	3	4	2	3	1	3	3	2	4	1	1	4	2	4	2	2
Q.No.	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Ans.	1	3	4	3	3	4	2	2	4	2	4	3	1	1	1	2	2	1	3	3
Q.No.	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
Ans.	2	1	4	2	4	2	3	2	4	4	4	4	2	1	2	3	2	3	2	2
Q.No.	**81**	**82**																		
Ans.	2	3																		

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 2 (ANSWERS)

Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	4	3	2	2	2	4	3	3	1	1	2	1	4	4	1	1	3	4	1	3
Q.No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans.	1	3	3	1	1	4	2	4	3	1	3	4	2	3	2	2	3	4	2	4
Q.No.	**41**	**42**	**43**	**44**	**45**	**46**	**47**													
Ans.	2	4	3	3	3	2	3													

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 3 (ANSWERS)

Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	1	2	4	4	2	2	1	2	1	4	1	2	2	4	2	1	1	4	3	1
Q.No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	
Ans.	2	3	1	4	1	1	2	3	1	4	1	1	2	4	2	1	4	2	4	

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 4 (ANSWERS)

Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	1	2	1	1	1	4	3	4	1	1