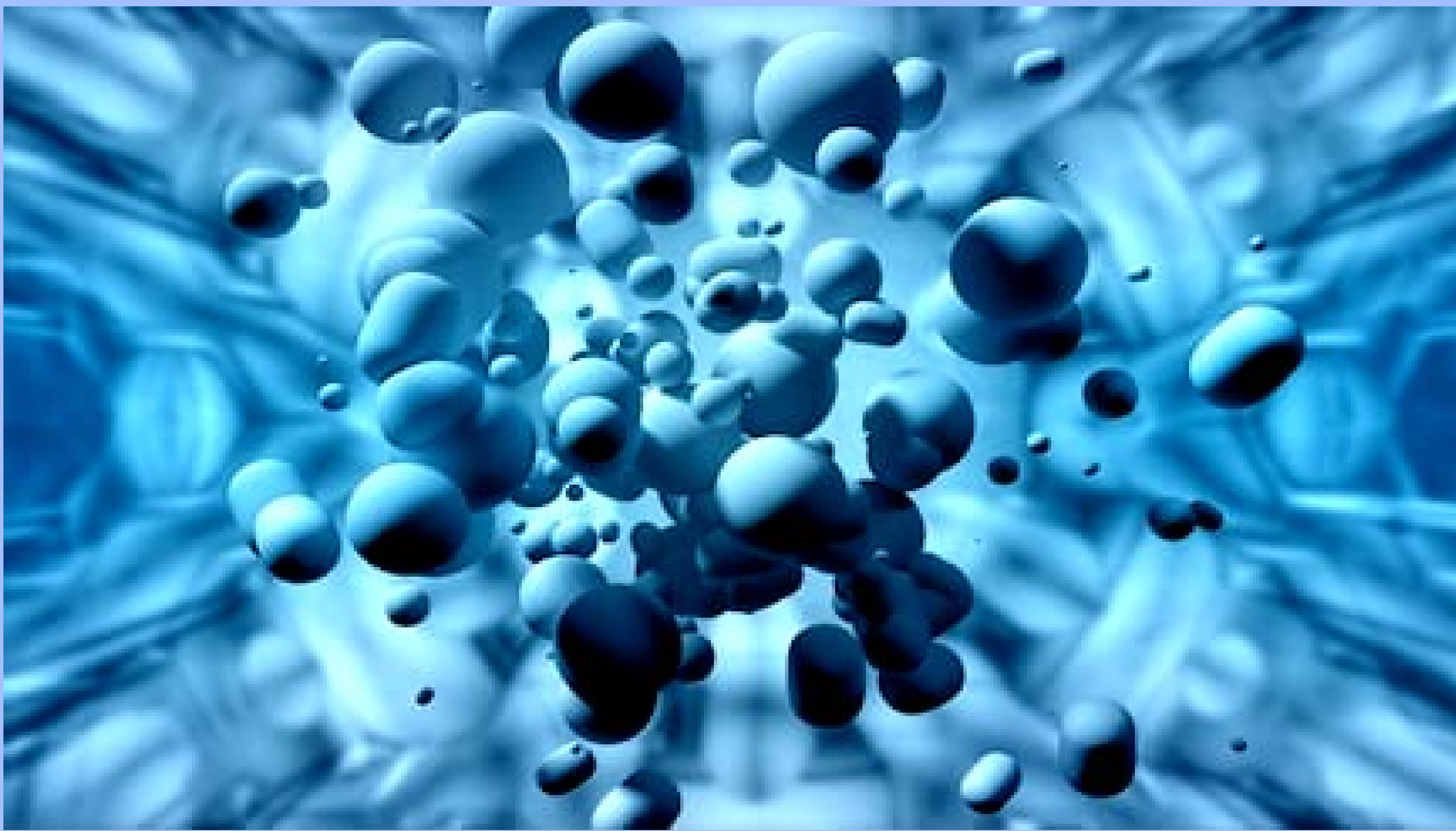
PHYSICS

for JEE, NEET & BOARD

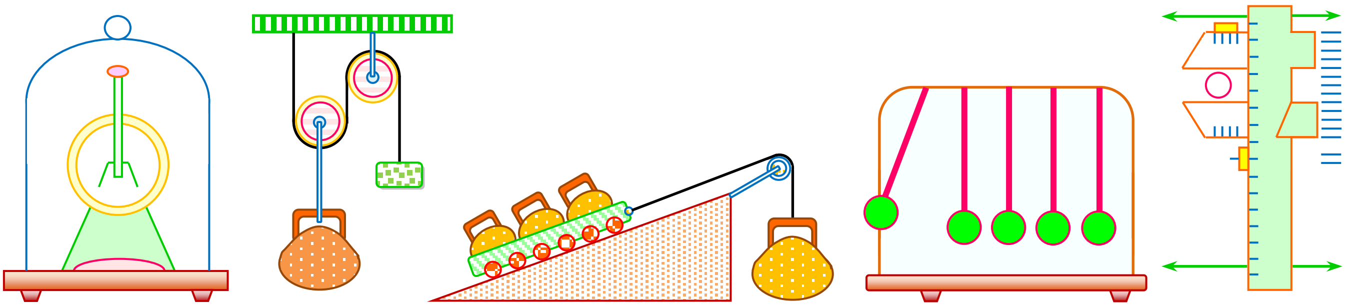
11th

PROPERTIES OF BULK MATTER



Key Features

- All-in one Study Material (for Boards/IIT/Medical/Olympiads)
- Multiple Choice Solved Questions for Boards & Entrance Examinations
- Concise, conceptual & trick – based theory
- Magic trick cards for quick revision & understanding
- NCERT & Advanced Level Solved Examples



PHYSICS BOOKLET FOR JEE NEET & BOARDS

India's First Colour Smart Book

PROPERTIES OF BULK MATTER

1 ELASTICITY

In our study of mechanics so far, we have considered the objects as rigid, i.e., whatever force we apply on objects, they remain undeformed. In reality, when an external force is applied to a body, the body gets deformed in shape or size or both. Also if we remove the external force the deformed body tends to regain its original shape and size. The property by virtue of which a deformed body tends to regain its original shape and size after removal of deforming force is called elasticity. Let us consider two possibilities:

(i) *If the body regains its original size and shape it is said to be perfectly elastic.*

(ii) *If the body does not have any tendency to regain its original shape or size once deforming forces* are removed it is said to be perfectly plastic.

In nature, actual behaviour of bodies lies between these two extreme limits.

1.1 STRESS

When external forces are applied, the body is distorted. i.e. the different portions of the body move relative to each other. Due to these displacements atomic forces (called restoring forces) are set up inside the body to restore the original form. The restoring force per unit area set up inside the body is called stress. This is measured by the magnitude of the deforming force acting per unit area of the body when equilibrium is established. If F is the force applied to an area of cross-section A, then

$$\text{stress} = \frac{F}{A}. \qquad \dots (1)$$

The unit of stress in S.I system is N/m^2 when the stress is normal to the surface, this is called **normal stress**. The normal stress produces a change in length or a change in volume of a body. The normal stress to a wire or body may be compressive or tensile (expansive) according as it produces a decrease or increase in length of a wire or volume of a body. When the stress is tangential to a surface, it is called **tangential (shearing) stress.**

Illustration 1

Question: **A rectangular bar having a cross-sectional area of 70m^2 has a tensile force of 14 kN applied to it. Determine the stress in the bar.**

Solution: Cross-sectional area A = 70 m^2

Tensile force F = 14 kN = 14 × 10^3 N

Stress in the bar = $\dfrac{\text{Force}}{\text{Area}} = \dfrac{14 \times 10^3 \text{N}}{70 \text{ m}^2}$ = **200 N**

1.2 STRAIN

The external forces acting on the body cause a relative displacement of its various parts. A change in length, volume or shape takes place. The body is then said to be strained. The relative change produced in the body under a system of force is called strain.

$$\text{Strain } (e) = \frac{\text{change in dimension}}{\text{original dimension}} \quad \dots (2)$$

Strain has no dimensions as it is a pure number. The change in length per unit length is called **linear strain**. The change in volume per unit volume is called **volume strain.** If there is a change in shape the strain is called **shearing strain.** This is measured by the angle through which a line originally normal to the fixed surface is turned.

Illustration 2

Question: **A wire of length 2.5 m has a percentage strain of 0.12% under a tensile force. Determine the extension in the wire.**

Solution: Original length of wire $L = 2.5$ m

Strain $= 0.12\ \%$

$$\text{Strain} = \frac{\text{Extension in length}}{\text{Original length}} = \frac{\Delta L}{L}$$

$$\frac{\Delta L}{L} = \frac{0.12}{100}$$

$$\Delta L = \frac{0.12}{100} \times L = \frac{0.12}{100} \times 2.5 = 3 \times 10^{-3} \text{ m}$$

Extension = **3 mm**.

1.3 HOOKE'S LAW AND MODULI OF ELASTICITY

According to Hooke's law, "within elastic limits stress is proportional to strain".

i.e. $$\frac{\text{stress}}{\text{strain}} = \text{constant} = \lambda \quad \dots (3)$$

where λ is called modulus of elasticity. Depending upon different types of strain the following three moduli of elasticity are possible.

(i) Young's modulus: When a wire or rod is stretched by a longitudinal force the ratio of the longitudinal stress to the longitudinal strain within the elastic limits is called Young's modulus.

$$\text{Young's modulus } (Y) = \frac{\text{Longitudinal stress}}{\text{Linear strain}}$$

Consider a wire or rod of length L and radius r under the action of a stretching force F applied normal to its faces. Suppose the wire suffers a change in length l then

$$\text{Longitudinal stress} = \frac{F}{\pi r^2}$$

$$\text{Linear strain} = \frac{l}{L}$$

$$\therefore \text{ Young's modulus } (Y) = \frac{\frac{F}{\pi r^2}}{\frac{l}{L}}$$

$$Y = \frac{FL}{\pi r^2 l} \quad \dots (4)$$

(ii) Bulk modulus: When a solid or fluid (liquid or gas) is subjected to a uniform pressure all over the surface, the shape remains the same, but there is a change in the volume. The force per unit area applied normally and uniformly over the surface is called normal stress. The change in volume per unit volume is called volume or bulk strain. The ratio

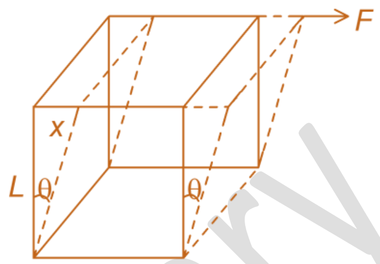
$$\frac{\text{Volume stress or normal stress}}{\text{Volume strain}}$$ is called bulk modulus (B).

In symbols $B = -\dfrac{\frac{F}{A}}{\frac{\Delta V}{V}} = -\dfrac{F.V}{A\,\Delta A}$ … (5)

The reciprocal of bulk modulus is called compressibility.

$\therefore$ compressibility $= \dfrac{1}{\text{bulk modulus}}$ … (6)

(iii) Modulus of Rigidity: According to definition, the ratio of shearing stress to shearing strain is called modulus of rigidity (η). In this case the shape of the body changes but its volume remains unchanged. Consider the case of a cube fixed at its lower face and acted upon by a tangential force F on its upper surface of area A as shown in figure.



$\therefore$ shearing stress $= \dfrac{F}{A}$

shearing strain $= \theta = \left(\dfrac{x}{L}\right)$

$\therefore \quad \eta = \dfrac{F}{A\theta} = \dfrac{FL}{Ax}$ … (7)

Poisson's Ratio: This is the name given to the ratio of lateral strain to the longitudinal strain. For example, consider a force F applied along the length of the wire which elongates the wire along the length while it contracts radially. Then the longitudinal strain $= \dfrac{\Delta L}{L}$ and Lateral strain $= \dfrac{\Delta r}{r}$, where r is the original radius and Δr is the change in radius.

$\therefore$ Poisson's ratio (σ) $= -\dfrac{\frac{\Delta r}{r}}{\frac{\Delta L}{L}}$. … (8)

The negative sign indicates that change in length and radius are of opposite sign. The theoretical value of σ lies between 1 and 0.5 while the practical value of σ lies between 0 and 0.5.

Illustration 3

Question: **A wire is stretched by 2 mm when a force of 250 N is applied. Determine the force that would stretch the wire by 5 mm assuming that the elastic limit is not exceeded.**

Solution: According to Hooke's law, within elastic limits,
Extension x is proportional to force F
or $x \propto F$
or $x = kF$ where k is a constant.
It is given that when $x = 2$ mm, $F = 250$ N
When $x = 5$ mm, $\dfrac{x\,mm}{2mm} = \dfrac{F}{250}$

or the force required $F = \dfrac{5}{2} \times 250 =$ **625 N**

1.4 STRESS-STRAIN RELATIONSHIP FOR A WIRE SUBJECTED TO LONGITUDINAL STRESS

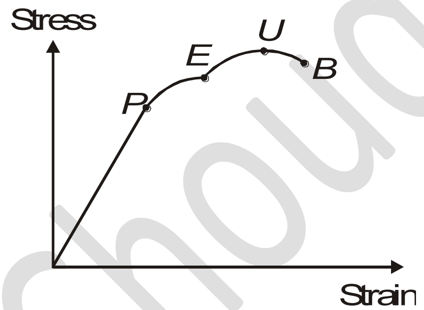
Consider a long wire (made of steel) of cross sectional area A and original length L in equilibrium under the action of two equal and opposite variable force F as shown in figure.

Due to the applied forces the length gets changed to $L + l$.

Then, longitudinal stress = $\frac{F}{A}$ and

longitudinal strain $= \frac{l}{L}$

The graph between the stress and strain, as the elongation gradually increases is shown in figure. Initially for small value of deformation stress is proportional to strain upto point P called as **proportional limit**. Till this point Hook's law is valid and Young's modulus is defined. If deformation further increases, the curve becomes non-linear but still it has elastic property till point E. This point E is called as **elastic limit**. After elastic limit yielding of wire starts taking place and body starts gaining some permanent deformation.



That is on the removal of force body remains deformed. If deformation is further increased, the wire breaks at point B called as breaking point. The stress corresponding to this point is called as breaking stress. Before reaching breaking point the curve has a point where tangent to this curve has zero slope. This point corresponds to maximum stress that the body can sustain. This point is called as ultimate point and stress at this point is called as ultimate strength of wire.

1.5 ELASTIC ENERGY STORED IN A DEFORMED BODY

The elastic energy is measured in terms of work done in straining the body within its elastic limit.

Let F be the force applied across the cross-section A of a wire of length L. Let l be the increase in length. Then

$$Y = \frac{\frac{F}{A}}{\frac{l}{L}} = \frac{FL}{Al} \quad \text{or} \quad F = \frac{YAl}{L}$$

If the wire is stretched through a further distance $d\,l$ the work done

$$= F \times d\;l = \frac{YAl}{L}dl$$

Total work done in stretching the wire from original length L to a length $L + l$

(i.e. from $l = 0$ to $l = l$)

$$W = \int_0^l \frac{YA \cdot l}{L} \cdot dl$$

$$= \frac{YA}{L}\cdot\frac{l^2}{2} = \frac{1}{2}(AL)\left(\frac{Yl}{L}\right)\left(\frac{l}{L}\right)$$

$$\Rightarrow W = \frac{1}{2} \times \text{volume} \times \text{stress} \times \text{strain} \qquad \dots (9)$$

Illustration 4

Question: **The rubber cord of catapult has a cross-sectional area 1 mm^2 and total unstretched length 10 cm. It is stretched to 12 cm and then released to project a body of mass 5 gm. Taking the Young's modulus of rubber as 5 × 10^8 N/m^2, calculate the velocity of projection.**

Solution: It can be assumed that the total elastic energy of catapult is converted into kinetic energy of body without any heat loss.

Elastic energy $= \frac{1}{2} \times \text{load} \times \text{extension}$

Extension $= 12 - 10 = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$

$$\therefore Y = \frac{FL}{A\Delta L}$$

$$F = \frac{5\times10^8 \times 1\times10^{-6} \times 2\times10^{-2}}{10\times10^{-2}} = 100 \text{ N}$$

If v is the velocity of projection,

Elastic energy of catapult = Kinetic energy of missile

$$\frac{1}{2} \times \text{load} \times \text{extension} = \frac{1}{2}mv^2$$

$$\frac{1}{2}\times 100\times 2\times 10^{-2} = \frac{1}{2}\times 5\times 10^{-3}\times v^2$$

$$v^2 = \frac{100\times10^{-2}\times 2}{5\times10^{-3}} = 400$$

$$v = \mathbf{20 \text{ m/s}}$$

PROFICIENCY TEST - 1

The following questions deal with the basic concepts of this section. Answer the following briefly. Go to the next section only if your score is at least 80%. Do not consult the Study Material while attempting these questions.

1. Which is more elastic?

(a) rubber or steel

(b) air or water

2. A solid sphere of radius R made of a material of bulk modulus B is surrounded by a liquid in a cylindrical container. A massless piston of area A floats on the surface of the liquid. Find the percentage fractional change in the radius of the sphere (dR/R) when a mass M is placed on the piston to compress the liquid. ($Mg = 0.03\ AB$).

3. When a wire of 0.4 cm diameter is loaded with 25 kg weight the length of wire 100 cm is found to expand to 102 cm. Calculate the Young's modulus of the material of wire.

4. Find the natural length of a rod if its length is L_1 = 5m under tension T_1 =10 N and L_2 = 6m under tension T_2 =20 N within limits of elasticity.

ANSWERS TO PROFICIENCY TEST - 1

1. (a) steal

(b) water

2. 1 %

3. 10^{10} dyne/cm^2

4. 4 m

2 FLUID STATICS

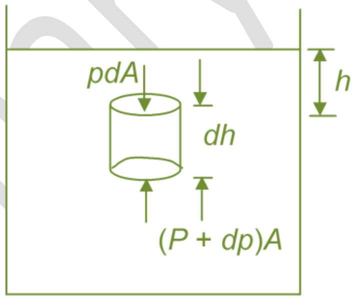
Fluid statics is the branch of mechanics, which deals with the forces on fluids (liquids and gases) at rest. As far back as 250 B.C. Archimedes, the famous Greek philosopher stated in his works that liquids exert an upward buoyant force on solids immersed in them causing an apparent reduction in their weights. It was about the end of 17th century that Pascal, a French scientist explained the fundamental principles of the subject in a clear manner. The consequences of Pascal's work are far-reaching for it forms the basis of several practical appliances like the fluid pumps, hydraulic presses and brakes, pneumatic drills, etc.

2.1 THRUST AND PRESSURE

A **perfect fluid** resists forces normal to its surface and offers no resistance to forces acting tangential to its surface. A heavy log of wood can be drawn along the surface of water with very little effort because the force applied on the log of wood is horizontal and parallel to the water surface. Thus a fluid is capable of exerting normal stresses on a surface with which it is in contact.

Force exerted perpendicular to a surface is called thrust and thrust per unit area is called pressure.

2.2 VARIATION OF PRESSURE WITH HEIGHT



In all fluids at rest the pressure is a function of vertical dimension. To determine this consider the forces acting on a vertical column of fluid of cross sectional-area dA as shown in figure. The positive direction of vertical measurement h is taken downward. The pressure on the upper side is P, and that on the lower face is $P + dP$. The weight of the element is $\rho g dh dA$. The normal forces on the vertical surfaces of the column do not affect the balance of forces in the vertical direction. Equilibrium of the fluid element in the vertical direction requires.

$$p dA + \rho g dA dh - (p + dp)\, dA = 0$$

$$\Rightarrow \quad dP = \rho g dh$$

This differential relation shows that the pressure in a fluid increases with depth or decreases with increased elevation. Above equation holds for both liquids and gases and agrees with our common observations of air and water pressure.

Liquids are generally treated as incompressible and we may consider their density ρ constant for every part of the liquid. With ρ as constant equation may be integrated as it stands, and the result is

$$P = P_0 + \rho g h \quad \ldots \textbf{(10)}$$

The pressure P_0 is the pressure at the surface of the liquid where $h = 0$.

2.3 CHARACTERISTICS OF FLUID PRESSURE

(i) Pressure at a point acts equally in all directions.

(ii) Liquids at rest exerts lateral pressure, which increases with depth.

(iii) Pressure acts normally on any area in whatever orientation the area may be held.

(iv) Free surface of a liquid at rest remains horizontal.

(v) Pressure at every point in the same horizontal line is the same inside a liquid at rest.

(vi) Liquid at rest stands at the same height in communicating vessels.

Illustration 5

Question: **What is the ratio of pressure at the bottom to that of atmospheric pressure of a tank filled with water upto a height of 10 m? Atmospheric pressure is equal to 10 m of water. ($P_0 = 10^5$ N/m^2)**

Solution: Pressure at the bottom of the vessel

= atmospheric pressure + pressure due to water column
= 10 m of water + 10 m of water
= 20 m of water
= $h\rho g$

$\therefore \quad P/P_0 = \mathbf{2}$

Illustration 6

Question: **If the atmospheric pressure is 76 cm of mercury at what depth of liquid the pressure will be equal to 2 atmosphere?**
Density of mercury = 13600 kg/m³, density of liquid = 136 kg/m³.

Solution: Let the pressure be 2 atm at a depth h below the water surface.
Of this pressure, one atmosphere is due to the atmospheric pressure over the surface of water and hence the pressure due to the water column alone = 1 atm.

$$= 76 \text{ cm of mercury}$$
$$= (0.76)(13600)(9.8) \text{ N/m}^2$$

Now a height of h m of water column produces this pressure and we are required to find this height h.

$$(h\rho g)_{water} = (h\rho g)_{mercury}$$
$$h \times 136 \times 9.8 = 0.76 \times 13600 \times 9.8$$
$$h = \frac{0.76 \times 13600 \times 9.8}{136 \times 9.8} = 76 \text{ m}$$

Hence the height of water barometer corresponding to standard atmospheric pressure is **76 m**.

2.4 FORCE DUE TO FLUID ON A PLANE SUBMERGED SURFACE

A surface submerged in liquid such as bottom of a tank or wall of a tank or gate valve in a dam, is subjected to pressure acting normal to its surface and distributed over its area. In problems where the resultant forces are appreciable, we must determine the resultant force due to distribution of pressure on the surface and the position at which resultant force acts.

For system, which is open to earth's atmosphere, the atmospheric pressure P_0 acts over all surfaces and hence, yields a zero resultant. In such cases, then we need to consider only fluid pressure $P = \rho gh$ which is increment above atmospheric pressure.

In general pressure at different point on the submerged surface varies so to calculate resultant force, we divide the surface into number of elementary areas and we calculate force on it first by treating pressure as constant then we integrate it to get net force.

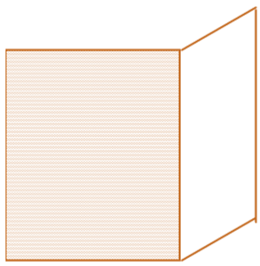
i.e., $F_R = \int PdA$

The point of application of resultant force must be such that the moment of the resultant force about any axis is equal to the moment of the distributed force about the axis.

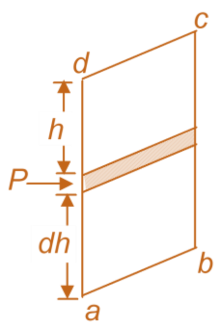
Illustration 7

Question: **A liquid is filled upto the top in a rectangular tank of square cross section. The sides of cross section is a = 1m and height of the tank is H = 3m. If density of liquid is ρ =100 kgm³ find force on the bottom of the tank and on one of its wall. Also calculate the position of point of application of the force on the wall.**

Solution: Force on the bottom of tank: pressure at the bottom of tank due to liquid is uniform of magnitude ρgh.
Area of the bottom of the tank = a^2
$\therefore$ force on the bottom = pressure × area
= ρgHa^2 = **3000 N**
Force on the wall and its points of application:



Force on the wall of the tank can't be calculated using pressure × area as pressure is not uniform over the surface. The problem can be solved by finding force dF on a thin strip of thickness dh at a depth h below the free surface and then taking its integral.



Pressure of liquid at depth h,

$$P = \rho gh$$
Area of thin strip $= adh$
$\therefore$ Force of the strip, $dF = P\,adh$

$$= \rho gahdh$$

$\therefore$ Total force on the wall = $\int_{O}^{H} \rho gahdh$

$$F = \rho ga\frac{H^2}{2} = \mathbf{4500\ N}$$

The point of application of force on the wall can be calculated by equating the moment of resultant force about any line say dc to the moment of distributed force about the same line dc.

Moment of dF about line cd $= dFH$

$= \rho gh.\ adh.\ h$

$\therefore$ Net moment of distributed forces = $\rho ga\int_{O}^{H} h^2 dh$

$$= \rho ga\frac{H^3}{3}$$

Let the point of application of net force is at a depth x from the line cd

Then torque of resultant force about line $cd = Fx = \rho ga\frac{H^2}{2}x$

From the above discussion,

$$\rho ga\frac{H^2}{2}x = \rho ga\frac{H^3}{3}$$

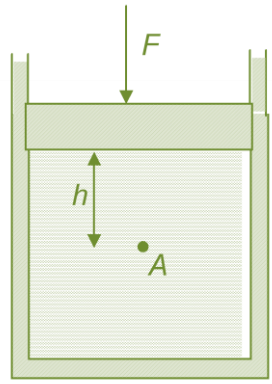
$$\Rightarrow \quad x = \frac{2H}{3} = \mathbf{2\ m}$$

Hence, resultant force on the vertical wall of the tank will act at a depth **2m** from the free surface of water or at a height of **1m** from the bottom of tank.

2.5 PASCAL'S LAW

When we squeeze a tube of toothpaste, the toothpaste comes out of the tube from its opening. This is an example of Pascal's law. When we squeeze the tube, pressure is applied on the tube and this pressure is transmitted everywhere in the tube and forces the toothpaste out of the tube. According to Pascal, "Pressure applied to a fluid confined in a vassel is transmitted to the entire fluid without being diminished in magnitude".

To prove Pascal's law consider a fluid having density ρ in a cylinder fitted with a movable piston. A point A is at the depth h in the fluid. If no force is applied to the piston pressure just below the piston will have same value say P_0. So according to previous discussion pressure at point A will be $P_1 = P_0 + \rho gh$. Now if a force F is applied on the piston, pressure just below the piston will have its value initial pressure plus the increased pressure say $P_0 + P_{ext}$. So new pressure at A can be written as $P_2 = P_0 + P_{ext} + \rho gh$. Therefore increase in the pressure at A will be $P_2 - P_1 = P_{ext}$. That is the increased pressure at the top gets transmitted to A also.

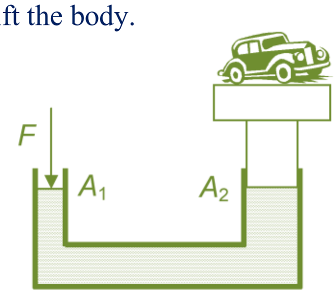


Pascal's law has many important applications and one of the interesting application is in Hydraulic lever. Hydraulic lever is used to lift heavy bodies like automobile by using less effort. It is made of two interconnected vertical cylinders of different cross sectional area A_1 and A_2 and a force F is applied to smaller piston causes much greater force on the larger piston, which can lift the body.

The pressure increased below the smaller piston $= \frac{F}{A_1}$

The same increment of pressure will take place below the larger piston also.

So force transmitted to larger piston = $A_2\frac{F}{A_1}$.



$\therefore$ If the body lifts due to application of F

$$A_2 \frac{F}{A_1} = Mg$$

$$\therefore \qquad F = Mg\frac{A_1}{A_2}$$

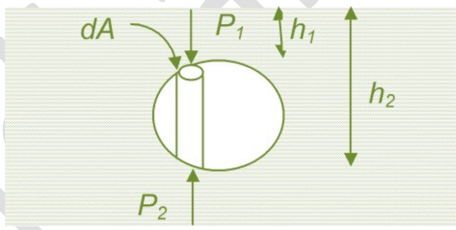
The ratio $\frac{A_1}{A_2}$ is smaller than 1 and thus applied force can be much smaller than the weight Mg that is lifted.

2.6 BUOYANCY AND ARCHIMEDE'S PRINCIPLE

If an object is immersed in or floating on the surface of, a liquid, it experiences a net vertically upward force due to liquid pressure. This force is called as Buoyant force or force of Buoyancy and it acts from the center of gravity of the displaced liquid. According to Archimede's the magnitude of force of buoyancy is equal to the weight of the displaced liquid.

To prove Archimede's principle consider a body totally immersed in a liquid as shown in the figure.

The vertical force on the body due to liquid pressure may be found most easily by considering a cylindrical volume similar to the one shown in figure.



The net vertical force an the element is

$$dF = (P_2 - P_1)\, dA$$

$$= \left[\left(P_0 + \rho\int gh_2\right) - \left(P_0 + \rho\int gh_1\right)\right] dA$$

$$= \rho g \int (h_2 - h_1)\, dA$$

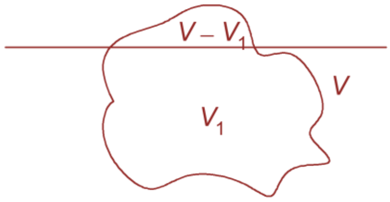
but $(h_2 - h_1)\, dA = dV$, the volume of element. Thus

Net vertical force on the body $F = \int \rho g dV = V\rho g$

$\therefore$ Force of Buoyancy $= V\rho g$

2.7 EXPRESSION FOR THE IMMERSED VOLUME OF FLOATING BODY

Let a solid of volume V and density ρ floats in a liquid of density σ with a volume V_1 immersed inside the liquid.



The weight of the floating body $= V\rho g$

The weight of the displaced liquid $= V_1\sigma g$

For the equilibrium of the floating body

$$V_1\sigma g = V\rho g$$

or $$\frac{V_1}{V} = \frac{\rho}{\sigma}$$

or $$V_1 = \frac{\rho}{\sigma} \times V$$

$$\text{Immersed volume} = \frac{\text{density of solid}}{\text{density of liquid}} \times \text{volume of solid}$$

$$= \frac{\textbf{mass of solid}}{\textbf{density of liquid}}$$

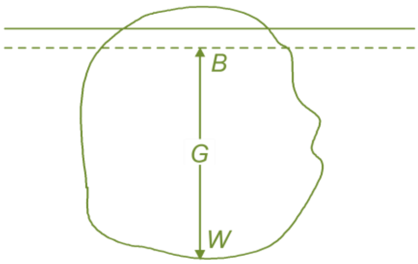
From the above relations it is clear that the density of the solid must be less than the density of the liquid to enable it to float freely in the liquid. However a metal vessel may float in water though the density of metal is much higher than that of water. The reason in that the floating bodies are hollow inside and hence they have a large displaced volume. When they float in water the weight of the displaced water is equal to the weight of the body.

2.8 LAWS OF FLOATION

The principle of Archimedes may be applied to floating bodies to give the laws of floatation as follows:

(i) When a body floats freely in a liquid the weight of the body is equal to the weight of the liquid displaced.

(ii)The centre of gravity of the displaced liquid (called the centre of buoyancy) lies vertically above or below the centre of gravity of the body.

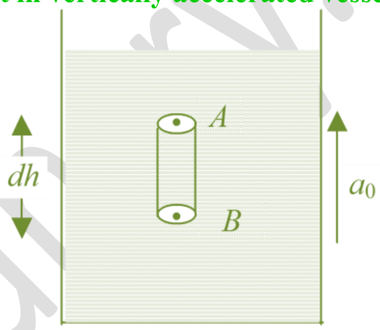


2.9 LIQUID IN ACCELERATED VESSEL

A Liquid in accelerated vessel can be considered as in the rigid body motion i.e. motion without deformation as though it were a solid body. As in case of static liquid to determine the pressure variation we apply Newton's law, the same is applicable in case of liquid in accelerated vessel also.

2.9.1 Variation of pressure and force of buoyancy in a liquid kept in vertically accelerated vessel

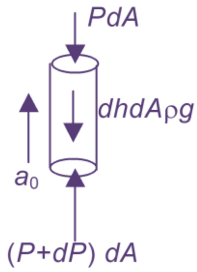
Consider a liquid of density ρ kept in a vessel moving with acceleration a_0 in upward direction.



Let *A* and *B* are two points separated vertically by a distance *dh*. The forces acting an *a* vertical liquid column of cross sectional area *dA* are shown in the figure. For the vertical motion of this liquid column,

$$(P + dP)\, dA - PdA - (dh)dA\rho g = (dh)dA\rho a_0$$

$$\Rightarrow dP = \rho\,(g + a_0)\, dh$$



If pressure at the free surface of liquid is P_0 them pressure *P* at a depth *h* from the free surface is given by

$$\int_{P_0}^{P} dP = \int_0^h \rho(g + a_0)\, dh$$

$$\Rightarrow P = P_0 + \rho\,(g + a_0)\, h$$

On the basis of similar calculation, pressure at any depth *h* from free surface in case of liquid in downward accelerated vessel can be written of

$$P = P_0 + \rho\,(g - a_0)\, h$$

We can generalize the above results and conclude that in liquid for a vertically accelerated vessel the pressure at any depth *h* below the free surface,

$$P = P_0 + \rho g_{eff}\, h$$

Where $g_{eff} = g + a_0$ in case of upward acceleration and

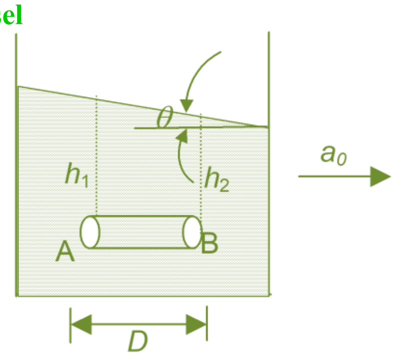
$g_{eff} = g - a_0$ in case of downward acceleration

Also we can say the force of buoyancy F_B on a body in liquid in vertically accelerated vessel is given by $F_B = V\rho g_{eff}$

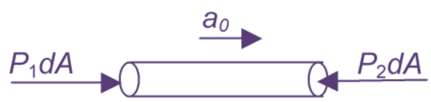
Where *V* is volume of liquid displaced by the body.

2.9.2 Shape of free surface of liquid in horizontally accelerated vessel

When a vessel filled with liquid accelerates horizontally. We observe its free surface inclined at some angle with horizontal. To find angle θ made by free surface with horizontal consider a horizontal liquid column including two points, *A* and *B* at depths h_1 and h_2 from the inclined free surface of liquid as shown in figure. *D* is length of liquid column. Horizontal forces acting on the liquid column are as shown in diagram, for horizontal motion of column we can write,



$P_1dA - P_2\,dA = \rho(dA)\,D(a_0)$
$(h_1 - h_2)\,g = D\,a_0$



$$\Rightarrow \quad \frac{h_1 - h_2}{D} = \frac{a_0}{g} \Rightarrow \quad \tan\theta = \frac{a_0}{g} \quad \Rightarrow \quad \theta = \tan^{-1}\frac{a_0}{g}$$

Illustration 8

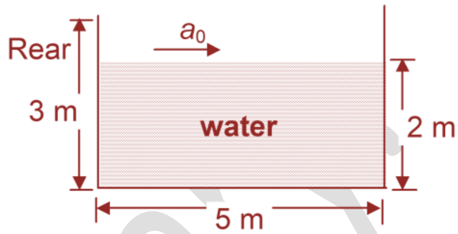
Question: **An open rectangular tank 5 m × 4 m × 3 m high containing water upto a height of 2 m is accelerated horizontally along the longer side.**

(a) Determine the maximum acceleration that can be given without spilling the water.

(b) Calculate the percentage of water spilt over, if this acceleration is increased by 20%.

(c) If initially, the tank is closed at the top and is accelerated horizontally by 9 m/s², find the gauge pressure at the bottom of the front wall of the tank.

(Take g = 10 m/s²).

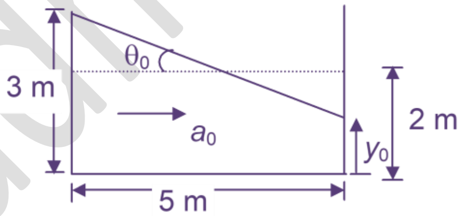


Solution: (a) Volume of water inside the tank remains constant

$$\frac{3+y_0}{2}5\times 4 = 5\times 2\times 4$$

or $y_0 = 1$ m

$\therefore\ \tan\theta_0 = \dfrac{3-1}{5} = \mathbf{4.0}$



since, $\tan\theta_0 = \dfrac{a_0}{g}$, therefore $a_0 = 0.4\,g = \mathbf{4\ m/s^2}$

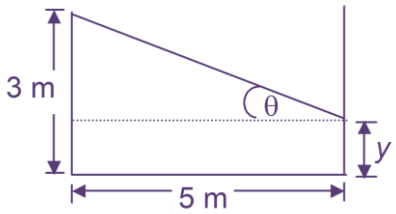
(b) When acceleration increased by 20%

$a = 1.2\,a_0 = 0.48\,g$

$\therefore\ \tan\theta = \dfrac{a}{g} = 0.48$

Now, $y = 3 - 5\tan\theta = 3 - 5\,(0.48) = 0.6$ m

$$= \frac{4\times 2\times 5 - \frac{(3+0.6)}{2}\times 5\times 4}{2\times 5\times 4} = 0.1$$



percentage of water spilt over = **10%**

(c) $a' = 0.9\,g$

$\tan\theta' = \dfrac{a'}{g} = 0.9$

volume of air remains constant

$4\times\dfrac{1}{2}yx = (5)\,(1)\times 4$

since $y = x\tan\theta'$

$\therefore\ \dfrac{1}{2}x^2\tan\theta' = 5$

or $x = 3.33$ m; $y = \mathbf{3.0\ m}$

Gauge pressure at the bottom of the Front wall p_f = **zero**

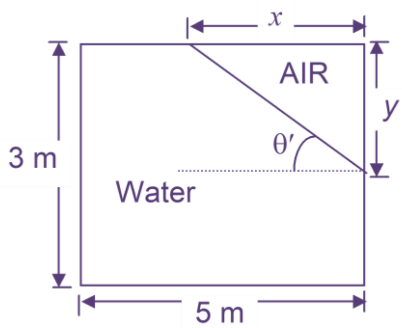


Illustration 9

Question: **When a boulder of mass 240 kg is placed on a floating iceberg it is found that the iceberg just sinks. What is the mass of the iceberg? Take the relative density of ice as 0.9 and that of sea-water as 1.02.**

Solution: Let M be the weight of iceberg in kg. Then its volume

$$V = \frac{M}{0.9 \times 10^3} m^3 = \frac{M}{900} m^3$$

When this volume just sinks under sea-water, the mass of water displaced = $V \times 1020$ kg.
From the principle of buoyancy, this weight must be equal to the weight of iceberg and the boulder on it.

$$M + 240 = \frac{M}{900} \times 1020 \quad = M \times \frac{102}{90}$$

or $$M\left(\frac{102}{90} - 1\right) = 240$$

or $$M = \frac{240 \times 90}{12} = 1800 \text{ kg}$$

The mass of the iceberg = **1800 kg**

PROFICIENCY TEST - 2

The following questions deal with the basic concepts of this section. Answer the following briefly. Go to the next section only if your score is at least 80%. Do not consult the Study Material while attempting these questions.

1. What is the pressure (in 10^4 N/m^2) at the bottom of a tank filled with water upto a height of 5 m? Atmospheric pressure is equal to 10 m of water.

2. A tank with a base which is the square of side 50 cm contains a liquid of relative density 1.2 to a height of 80 cm. Calculate the thrust (i) on the horizontal base and (ii) on a vertical side.

3. If water be used to construct a barometer, what would be the height of water column at standard atmospheric pressure (75 cm of mercury)? Density of mercury = 13600 kg/m^3.

4. A piece of brass of relative density 8 weighing 400 gram in air is suspended by means of a thread in an oil of relative density 2. Find the tension in the thread.

5. A boat floating in a water tank is carrying a number of large stones. If the stones were unloaded into water what will happen to the water level? Give the reason in brief.

6. A piece of ice is floating in a vessel containing water and inside the ice is a bubble of air. Will the level of water in the vessel be altered when the ice melts? Why?

7. An ice cube containing a piece of cork floats in water contained in a beaker. Will there be any change in level when the ice melts? Explain.

8. Does Archimedes principle hold (a) in a vessel under free fall? (b) in a satellite moving in a circular orbit? Explain.

9. A stone of mass 3 kg and relative density 2.5 is immersed in a liquid of relative density 1.25. Calculate the resultant upward thrust exerted on the stone by the liquid and the weight of stone in liquid.

ANSWERS TO PROFICIENCY TEST - 2

1. 15

2. (i) 2352 N
(ii) 1882 N

3. 1020 cm

4. 3 N

5. water level in the tank will fall.

6. no change

7. no change

8. No

9. 15 N, 15 N

3 FLUID DYNAMICS

3.1 RATE OF FLOW

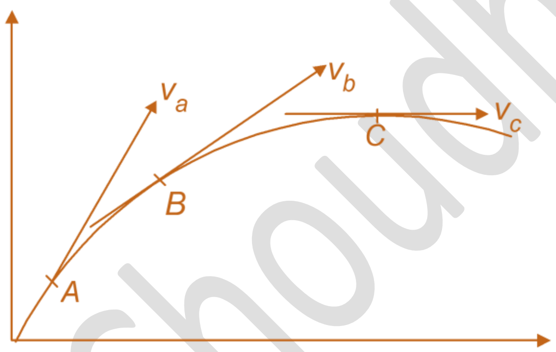
The rate of flow of a liquid is expressed in terms of its volume that flows out from an orifice of known cross-sectional area in unit time. All liquids are practically incompressible and the rate of flow through any section is the same. The rate of flow of a liquid hence is defined as the volume of it that flows across any section in unit time. For example, if the velocity of flow of a liquid is v in a direction perpendicular to two sections A and B of area A and distant l apart and if t be the time taken by the liquid to flow from A to B, we have $vt = l$. Obviously the volume of liquid flowing through section AB is equal to cylindrical column $AB = lA$ or vtA.

$$\therefore \quad \text{Rate of flow of liquid} = \frac{vtA}{t} = vA$$

Sometimes the rate of flow of liquid is expressed in terms of mass of liquid flowing across any section in unit time $vA\rho$ where ρ is the density of liquid.

3.2 STREAMLINES

In steady flow the velocity v at a given point does not change with time. Consider the path ABC of a fluid particle in a steady flow and let the velocities be v_a, v_b, v_c at A, B and C respectively as shown in the figure.

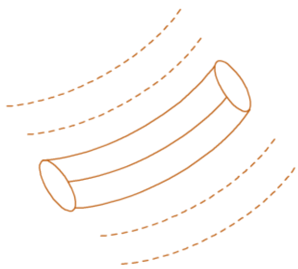


Because v_a does not change with time any particle passing through A will have the same velocity v_a and follow the same path ABC. This curve is called a streamline. A streamline is parallel to the velocity of fluid particle at every point. No two streamlines will ever cross each other. Otherwise any particle at the point of intersection will have two paths to go so that the flow will no more be steady. In non-steady flows the pattern of streamline changes with time. In steady flow if we draw a family of streamlines the tangent to the streamline at any point gives the direction of instantaneous velocity of the fluid at that path of motion. Thus as long as the velocity pattern does not change with time, the streamlines are also the paths along which the particles of the fluid move. In this type of flow the fluid between two surfaces formed by sets of adjacent streamlines is confined to the region between these two surfaces. This is so since the velocity vector has no component perpendicular to any streamline. The flow may be considered to occur in sheets or layers. Hence it is also sometimes called laminar flow. This type of flow is possible only if the velocity is below a certain limiting value called critical velocity. Above this velocity the motion is said to be unsteady or turbulent. The streamlines may be straight or curved depending on to the lateral pressure as it is the same throughout or different.

3.3 TUBE OF FLOW

In a fluid in steady flow, if we select a finite number of streamlines to form a bundle like the streamline pattern shown in figure below the tubular region is called a tube of flow.

The tube of flow is bounded by streamlines so that no fluid can flow across the boundaries of the tube of flow and any fluid that enters at one end must leave at the other end.



3.4 EQUATION OF CONTINUITY

Consider a thin tube of flow as in the figure. Let A_1 and A_2 be the area of cross-section of the tube perpendicular to the streamlines at P and Q respectively. If v_1 is the velocity of fluid particle at P and v_2 the velocity at Q, then the mass of liquid entering A_1 in a small interval of time Δt is given by $\Delta m_1 = \rho_1 A_1 v_1 \Delta t$.

The mass flux i.e., the mass of fluid flowing per unit time is

$$\frac{\Delta m_1}{\Delta t} = \rho_1 A_1 v_1$$

Let $\Delta t \to 0$ so that v and A does not vary appreciably. The mass flux at P is given by $\rho_1 A_1 v_1$. Similarly the mass flux at $Q = \rho_2 A_2 v_2$ where ρ_1 and ρ_2 are the densities of the fluid at P and Q respectively. Since no fluid crosses the wall of the tube of flow, the mass of fluid crossing each cross-section of the tube per unit time must be the same

$\therefore \quad \rho_1 A_1 v_1 = \rho_2 A_2 v_2$

or $\quad \rho A v = \text{constant}$

This equation expresses the law of conservation of mass in fluid dynamics and is called the **Equation of Continuity**. If the fluid is incompressible as in most liquids ρ = constant

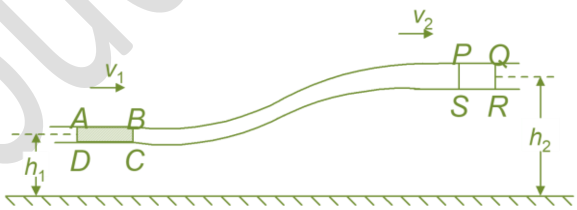
$\therefore \quad A_1 v_1 = A_2 v_2$ or Av = constant. … **(11)**

The product Av represents the rate of flow of fluid. Hence, v is inversely proportional to the area of cross-section along a tube of flow. Therefore, widely spaced streamlines indicate regions of low speed and closely spaced streamlines indicate regions of high speed.

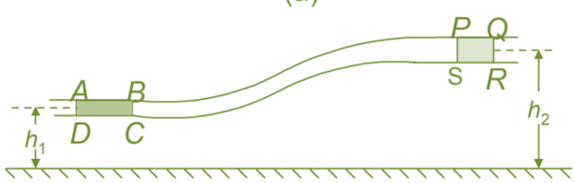
3.5 BERNOULLI'S EQUATION

It is a fundamental equation in fluid mechanics and can be derived from the work-energy theorem. The theorem states that the work done by all the forces acting on a system is equal to the change in kinetic energy of the system.

Consider a steady, irrotational, non-viscous, incompressible flow of a fluid through a pipe or tube of flow as shown in the figure. A portion of the fluid flowing through a section of pipe line from the position shown in figure (a) to that shown in figure (b).



(a)



(b)

The pipe line is horizontal at A and P and are at heights h_1 and h_2 respectively from a reference plane. The pipe line gradually widens from left to right. Let P_1, A_1, v_1 and P_2, A_2, v_2 be the pressures, areas of cross-section and velocities at A and P respectively.

Suppose the fluid which escapes the volume $ABCD = A_1 \Delta l_1$ flows out at P after some time and occupies $PQRS$ which is equal to $A_2 \Delta l_2$ as indicated in (a) and figure (b) respectively. The forces acting on the fluid are the pressure forces $P_1 A_1$ and $P_2 A_2$, the force of gravity at A and P. The work done on the system by the above forces is as follows:

(i) The work done on the system by pressure force $P_1 A_1$ which is $P_1 A_1 \Delta l_1$

(ii) The work done on the system by pressure force $P_2 A_2$ that is $-P_2 A_2 \Delta l_2$, the negative sign implying that positive work is done by the system.

(iii) The gravitational work done by the system in lifting the mass m inside from the height h_1 to height h_2 is given by $mg\,(h_2 - h_1)$ or the work done on the system will be $-\,mg(h_2 - h_1)$

Hence the work done by all the forces on the system is the sum of all the above terms.

$$W = P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - mg\,(h_2 - h_1)$$

Since $A_1\Delta l_1 = A_2\Delta l_2$ we can put $A_1\Delta l_1 = A_2\Delta l_2 = \frac{m}{\rho}$ where ρ is the density of the fluid

$$\therefore\ W = (P_1 - P_2)\frac{m}{\rho} - mg\,(h_2 - h_1)$$

The change in kinetic energy of the fluid is $\Delta K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

From the work-energy theorem we can put the work done on the system is equal to the change in kinetic energy of the system.

$$(P_1 - P_2)\frac{m}{\rho} - mg(h_2 - h_1) = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

On multiplying both sides of equation by ρ/m and rearranging we get

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Since the subscripts 1 and 2 refer to any two locations on the pipeline, we can write in general

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant} \qquad \ldots (12)$$

The above equation is called **Bernoulli's equation** for steady non-viscous incompressible flow. Dividing the above equation by ρg we can rewrite the above as

$$h + \frac{v^2}{2g} + \frac{P}{\rho g} = \text{constant, which is called total head.}$$

Out of the three components of the total head on the left hand side of the above equation the first term '*h*' is called elevation head or gravitational head, the second term $\frac{'v^2'}{2g}$ is called velocity head and the third term $\frac{'P'}{\rho g}$ is called pressure head.

Now if the liquid flows horizontally its potential energy remains constant so that the Bernoulli's equation becomes $P + \frac{1}{2}\rho v^2$ = constant. Further if the fluid is at rest or v = 0, P + ρgh = constant.

The pressure ($P + \rho gh$) which would be present also when there is flow is called the static pressure while the term $\frac{1}{2}\rho v^2$ is called the dynamic pressure. The equation shows that for an ideal liquid the velocity increases when pressure decreases and vice-versa.

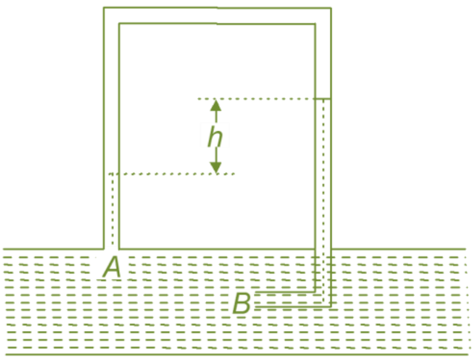
3.6 APPLICATIONS OF BERNOULLI'S THEOREM

We shall consider a few examples wherein Bernoulli's principle is employed.

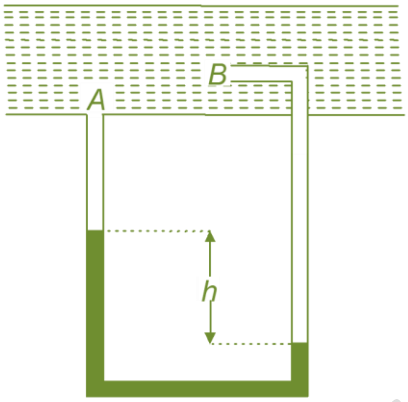
Filter pump: In a filter pump water flows suddenly from a wide tube into a narrow one. The velocity is increased and pressure reduced far below the atmospheric pressure. The receiver to be exhausted is connected by a side tube with this region of low pressure. Air is carried along with water and in a short time air in the receiver is reduced to a pressure, which is slightly greater than the vapour pressure of water.

Atomizer or sprayer: In flowing out of a narrow tube into the atmosphere or a wider tube, air produces a suction effect. In a sprayer a strong jet of air on issuing from a nozzle of a tube lowers the pressure over another tube, which dips in a liquid. The liquid is sucked up and mixed with air stream and thus a fine mist is produced.

Pitot tube: This device is used to measure the flow speed of liquids and gases in pipes. It consists of a manometer tube connected into pipeline as shown in figure. One of the ends of the manometer tube *A* is connected with its plane of aperture parallel to the direction of flow of fluid while the other end *B* is perpendicular to it.



(a) For liquid



(b) For Gases

The pressure and velocity on the left arm of the manometer opening A remains the same as they are elsewhere and is equal to static pressure P_a. Since the velocity of fluid at B is reduced to zero the pressure is the full arm pressure. Applying Bernoulli's principle, $P_a + \frac{1}{2}\rho v^2 = P_b$ which shows $P_b > P_a$. If h is the difference in height of liquid in the manometer and ρ' the density of manometric liquid,

$$P_a + \rho' gh = P_b$$

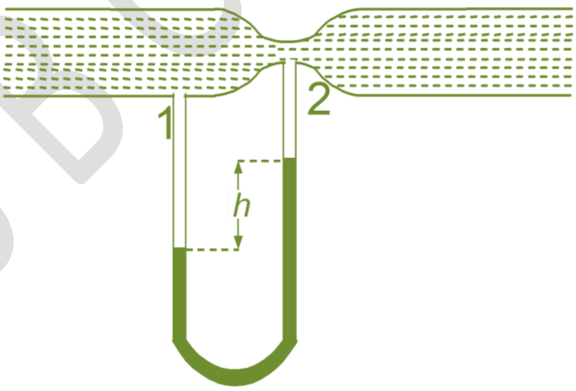
$$\therefore \quad \frac{1}{2}\rho v^2 = \rho' gh$$

In case of Pitot tube $\rho = \rho'$

$$\therefore \quad v^2 = 2gh$$

In case of gas Pitot tube $v = \sqrt{\frac{2gh\rho'}{\rho}}$ … **(13)**

Venturimeter: This is a device based on Bernoulli's principle used for measuring the flow of a liquid in pipes. A liquid of density ρ flows through a pipe of cross sectional area A. At the constricted part the cross sectional area is a. A manometer tube with a liquid say mercury having a density ρ' is attached to the tube as shown in figure.



If P_1 is the pressure at point 1 and P_2 the pressure at point 2 we have

$\frac{P_1}{\rho} + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + \frac{1}{2}v_2^2$ where v_1 and v_2 are the velocities at these points respectively.

We have $Av_1 = av_2$

$$\therefore \quad v_1 = \frac{av_2}{A}$$

$$\therefore \quad \frac{1}{2}v_1^2 - \frac{1}{2}v_2^2 = \frac{P_2}{\rho} - \frac{P_1}{\rho} = \frac{P_2 - P_1}{\rho}$$

$$\therefore \quad v_1^2 - \frac{A^2 v_1^2}{a^2} = \frac{2(P_2 - P_1)}{\rho},$$

$$v_1^2\left(1-\frac{A^2}{a^2}\right)=\frac{2(P_2-P_1)}{\rho}$$

$$v_1^2=\frac{2(P_2-P_1)}{\left(1-\frac{A^2}{a^2}\right)\rho}=\frac{2a^2(P_2-P_1)}{(a^2-A^2)\rho}$$

If $a < A$,

$$v_1^2=\frac{2a^2(P_1-P_2)}{(A^2-a^2)\rho}$$

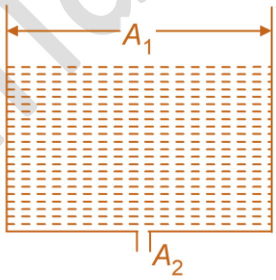
$$v_1=\sqrt{\frac{2(P_1-P_2)a^2}{(A^2-a^2)\rho}}$$

Volume of liquid flowing through the pipe per sec. = $Av_1 = Q$

$$Q=Aa\sqrt{\frac{2(P_1-P_2)}{(A^2-a^2)\rho}} \qquad \ldots (14)$$

Speed of Efflux

As shown in figure a tank of cross-sectional area A_1, filled to a depth h with a liquid of density ρ. There is a hole of cross-sectional area A_2 at the bottom and the liquid flows out of the tank through the hole $A_2 << A_1$



Let v_1 and v_2 be the speeds of the liquid at A_1 and A_2.

As both the cross-sections are open to the atmosphere, the pressures there equals the atmospheric pressure P_0. If the height of the free surface above the hole is h,

Bernoulli's equation gives

$$P_0+\frac{1}{2}\rho v_1^2+\rho gh=P_0+\frac{1}{2}\rho v_2^2$$

By the equation of continuity,

$A_1v_1 = A_2v_2$

$$P_0+\frac{1}{2}\rho\left[\frac{A_2}{A_1}\right]^2 v_2^2+\rho gh=P_0+\frac{1}{2}\rho v_2^2$$

$$\left[1-\left(\frac{A_2}{A_1}\right)^2\right]v_2^2=2gh$$

$$\Rightarrow v_2=\sqrt{\frac{2gh}{1-\left(\frac{A_2}{A_1}\right)^2}} \qquad \ldots (15)$$

If $A_2 <<< A_1$, the equation reduces to $v_2=\sqrt{2gh}$ … (16)

The speed of efflux is the same as the speed a body would acquire in falling freely through a height h. This is known as Torricelli's theorem.

Illustration 10

Question: **A tube having its two limbs bent at right angles to each other is held with one end dipping in a stream and opposite to the direction of the flow. If the speed of stream is 10 m/sec, find the height to which the water rises in the vertical limb of the tube.**

Solution: Clearly the flow of water will be stopped by the tube dipping in the stream and facing the flow so that the loss of kinetic energy per unit mass of water is $\frac{v^2}{2}$. This will therefore be the gain in pressure energy i.e., $\frac{P}{\rho}$.

$$\therefore \quad \frac{P}{\rho} = \frac{v^2}{2} \quad \text{or} \quad P = \frac{v^2\rho}{2}$$

If h is the height to which water rises in the tube

$$P = h\rho g \quad \text{or} \quad h\rho g = \frac{v^2\rho}{2}$$

$$\therefore \quad h = \frac{v^2}{2g} = \frac{10^2}{2\times 10} = \mathbf{5\ m}$$

Illustration 11

Question: **A Pitot tube is fixed in a main of diameter $2\sqrt{\pi}$ m and the difference in pressure indicated by gauge is 5 m of water column. Find the volume of water passing through the main in one minute. ($\pi^2 = 10$)**

Solution:

Radius of main $= \sqrt{\pi}$ m

Area of cross-section $= \pi^2$ m^2

Loss of kinetic energy $= \frac{1}{2}v^2$

Gain of pressure energy $= \frac{P}{\rho} = hg$

$= 5 \times 10$ J

$$\therefore \quad \frac{1}{2}v^2 = 5 + 10$$

$V/t = Av = 10 \times 10 = \mathbf{100\ m^3/s}$

Illustration 12

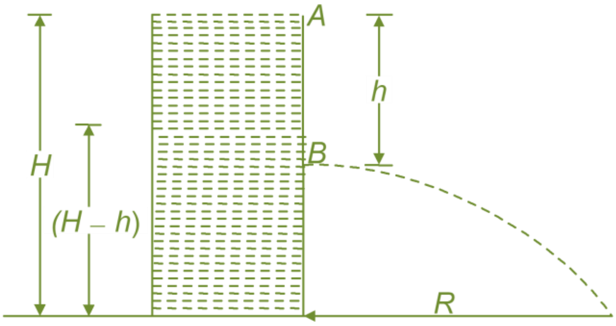
Question: **Water stands at a depth H = 2m in a tank whose side walls are vertical. A hole is made at one of the walls at a depth h below the water surface. What is the maximum possible range of the water that emerges from the hole?**

Solution: Applying Bernoulli's theorem at points A and B,

$$P_A + \frac{1}{2}\rho v_A^2 + \rho g H_A = P_B + \frac{1}{2}\rho v_B^2 + \rho g H_B$$

$$P + 0 + \rho g H = P + \frac{1}{2}\rho v^2 + \rho g(H-h)$$

$$\therefore \ v^2 = 2gh$$



The vertical component of velocity of water emerging from hole B is zero. Therefore time taken (t) by the water to fall through a distance $(H - h)$ is given by

$$H - h = \frac{1}{2}gt^2 \text{ or } t = \sqrt{\frac{2(H-h)}{g}}$$

Required horizontal range $R = vt$

$$= \sqrt{2gh}\sqrt{\frac{2(H-h)}{g}}$$

$$= 2\sqrt{h(H-h)}$$

The range R is maximum when $\frac{dR}{dh} = 0$

$$2\times\frac{1}{2}(Hh-h^2)^{-\frac{1}{2}}(H-2h) = 0$$

This gives $h = \frac{H}{2}$

$\therefore$ Maximum possible range $= 2\sqrt{\frac{H}{2}\times\left(H-\frac{H}{2}\right)}$

$=$ **2 m**

PROFICIENCY TEST - 3

The following questions deal with the basic concepts of this section. Answer the following briefly. Go to the next section only if your score is at least 80%. Do not consult the Study Material while attempting these questions.

1. Water flows in a horizontal pipe whose one end is closed with a valve and the reading of a pressure gauge attached to the pipe is 3×10^5 N/m^2. This reading of the pressure gauge falls to 1×10^5 N/m^2 when the valve is opened. Calculate the speed of water flowing in the pipe.

2. Water is flowing steadily through a horizontal pipe of non-uniform cross-section. If the pressure of water is 4×10^4 N/m^2 at a point where the cross-section is 0.02 m^2 and velocity of flow is 2 m/s. What is the pressure (in KPa) at a point where the cross-section reduces to 0.01 m?

3. Water flows at 1.2 m/s through a hose of diameter 1.59 cm. how long does it take to fill a cylindrical pool of radius 2 m to a height of 1.25 m?

4. A 40-m/s wind blows past of roof of dimensions 10 m × 15 m. Assuming that the air under the roof is at rest, what is the net force (in kN) on the roof?
(density of air = 1.29 kg/m^3)

5. Water enters a basement inlet pipe of radius 1.5 cm at 40 cm/s. It flows through a pipe of radius 0.5 cm at a height of 35 m at a gauge pressure of 0.2 atm. (a) what is the speed of the water at the highest point? (b) what is the gauge pressure at the basement?

ANSWERS TO PROFICIENCY TEST - 3

1. 20 m/s

2. 34 kPa

3. 1100 min

4. 155 kN

5. (a) 360 m/s
(b) 370 kPa

4 SURFACE TENSION

4.1 INTRODUCTION

The surface of a liquid behaves somewhat like a stretched elastic membrane. Just as a stretched elastic membrane has a natural tendency to contract and occupy a minimum area, so also the surface of a liquid has got the natural tendency to contract and occupy a minimum possible area as permitted by the circumstances of the liquid mass. We have many evidences in support of this fact, which we discuss now

(i) We very often find that a small quantity of a liquid spontaneously takes a spherical shape, e.g., rain drops, small quantities of mercury placed on a clean glass plate etc. Now, for a given volume, a sphere has the least surface area. Thus, this fact shows that a liquid always tends to have the least surface area.

(ii) If we immerse a camel-hair brush in water, its hairs spread out, but the moment it is taken out of water, they all cling together as though bound by some sort of elastic thread.

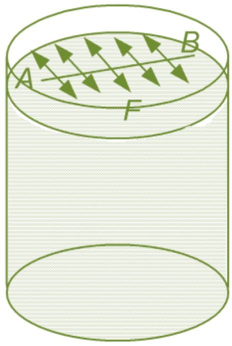
Experimental demonstration: Make a circular loop of wire and dip it in a soap solution. Take it out of the solution and find a thin film of soap solution across the loop. Place a moistened cotton loop gently on the film. The thread will lie on the film in an irregular manner as shown in figure (a).

(a) (b)

Now prick the film inside the cotton thread loop by a pin. At once the thread will spontaneously take the shape of a circle as shown in figure (b). The thread has a fixed perimeter. For a given perimeter a circle has got the maximum area. Hence the remaining portion of the film occupies the minimum area.

4.2 DEFINITION

The above evidences and experiment prove beyond doubt that the surface of a liquid behaves like a stretched elastic membrane having a natural tendency to contract and occupy a minimum possible area as permitted by the circumstances of the liquid mass. It is defined as ***"the property of the surface of a liquid by virtue of which it tends to contract and occupy the minimum possible area is called surface tension and is measured by the force per unit length of a line drawn on the liquid surface, acting perpendicular to it and tangentially to the surface of the liquid"***.



Let an imaginary line AB on the surface of liquid of length L as shown in figure and force on this line is F, So surface tension

$$T = \frac{F}{L} \qquad \dots (1)$$

Surface tension has dimensions $[MT^{-2}]$ and Its unit is Newton per metre (Nm^{-1}).
It depends on temperature. The surface tension of all liquids decreases linearly with temperature
It is a scalar quantity and become zero at a critical temperature.

Illustration 13

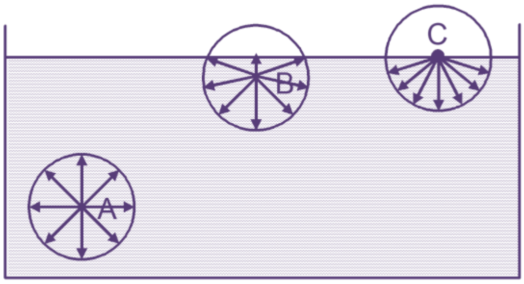
Questions: **Calculate the force required to take away a flat circular plate of radius 4 cm from the surface of water, surface tension of water being 75 dyne cm^{-1}.**

Solution: Length of the surface = circumference of the circular plate
= $2\pi r = (8\pi)$ cm
Required force $72 \times 8\pi$ = **1884 dyne**

4.3 MOLECULAR THEORY OF SURFACE TENSION

The surface tension of a liquid arises out of the attraction of its molecules. Molecules of a fluid (liquid and gas) attract one another with a force. It depends on the distance between molecules. The distance up to which the force of attraction between two molecules is appreciable is called the molecular range, and is generally of the order of 10^{-9} metre. A sphere of radius equal to the molecular range drawn around a molecule is called sphere of influence of the molecules lying within the sphere of influence.

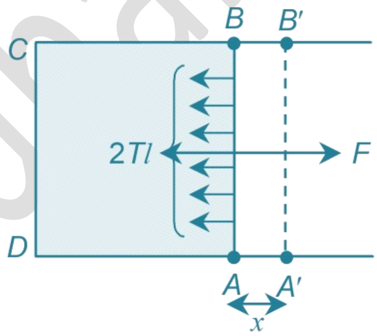
Consider three molecules A, B and C having their spheres of influence as shown in the figure. The sphere of influence of A is well inside the liquid, that of B partly outside and that of C exactly half of total.



Molecules like A do not experience any resultant force, as they are attracted equally in all directions. Molecules like B or C will experience a resultant force directed inward. Thus the molecules well inside the liquid will have only kinetic energy but the molecules near the surface will have kinetic energy as well as potential energy which is equal to the work done in placing them near the surface against the force of attraction directed inward.

4.4 SURFACE ENERGY

Any strained body possesses potential energy, which is equal to the work done in bringing it to the present state from its initial unstrained state. The surface of a liquid is also a strained system and hence the surface of a liquid also has potential energy, which is equal to the work done in creating the surface. This energy per unit area of the surface is called surface energy.



To derive an expression for surface energy consider a wire frame equipped with a sliding wire AB as shown in figure. A film of soap solution is formed across $ABCD$ of the frame. The side AB is pulled to the left due to surface tension. To keep the wire in position a force F, has to be applied to the right. If T is the surface tension and l is the length of AB, then the force due to surface tension over AB is $2lT$ to the left because the film has two surfaces (upper and lower).

Since the film is in equilibrium, $F = 2lT$

Now, if the wire AB is pulled to the right, energy will flow from the agent to the film and this energy is stored as potential energy of the surface created just now. Let the wire be pulled slowly through x.

Then the work done = energy added to the film from the agent

$$= F.x = 2lTx$$

Potential energy per unit area (Surface energy) of the film = $\dfrac{2lTx}{2lx} = T$ … **(2)**

Thus surface energy of the film is numerically equal to its surface tension.

Its unit is joule per square metre (Jm^{-2}).

Illustration 14

Question: **Calculate the work done in blowing a soap bubble of radius 10 cm, surface tension being 0.06 Nm^{-1}. What additional work will be done in further blowing it so that its radius is doubled?**

Solution: In case of a soap bubble, there are two free surfaces.

$\therefore$ Work done in blowing a soap bubble of radius R is given by,

$W = 2 \times 4\pi R^2 \times T$, where T is the surface tension of the soap solution.

where $R = 0.1$ m, $T = 0.06\ Nm^{-1}$

$\therefore$ $W' = 8\pi\,(0.1)^2 \times 0.06$ J = 1.51 J

Similarly, work done in forming a bubble of radius 0.2 m is,

$W = 8\pi\,(0.2)^2 \times 0.06$ J = 6.03 J

$\therefore$ Additional work done in doubling the radius of the bubble is given by,

$W' - W = 6.03$ J $- 1.51$ J $= 4.52$ J **= 4520 Mj**

Illustration 15

Question: **A film of soap is formed on a rectangular frame of length 10 cm dipping into a soap solution. The frame hangs from the arm of a balance. An extra weight of 0.42 kg must be placed in the opposite pan to balance the pull of the frame. Calculate the surface tension of the soap solution.**

Solution: Since the film has two surface, the force of surface tension will be given by

$F = 2 \times 10 \times T = 20\,T$,

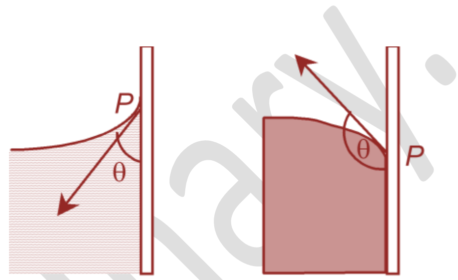
where T is surface tension of soap solution.

The force is balanced by 0.42 kg wt

$$\therefore \quad 0.42 \times 10 = \frac{20T}{100} \qquad T = \frac{0.42 \times 1000}{20} = \mathbf{21\ N}$$

4.5 ANGLE OF CONTACT

When a solid body in the form of a tube or plate is immersed in a liquid, the surface of the liquid near the solid is, in general, curved. It is defined as *the angle between the tangents to the liquid surface and the solid surface at the point of contact, for that pair of solid and liquid is called* ***angle of contact***.



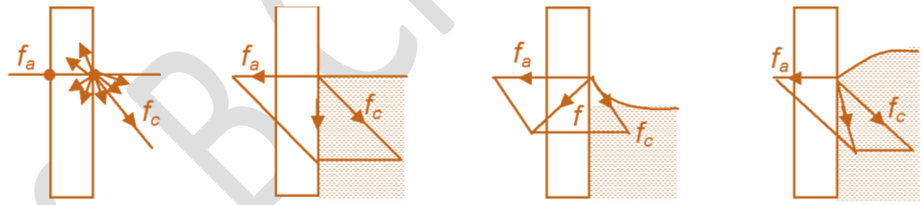
(a) (b)

For example when glass strip is dipped in water and mercury as shown in figure (a) and (b) respectively, the angle θ is angle of contact, which is acute in case of water and obtuse in case of mercury.

4.6 ADHESIVE FORCE AND COHESIVE FORCE

The angle of contact arises due to adhesive and cohesive forces on the molecules of the liquid which lie near the solid surface. Forces of attraction between molecules of different substances are called **adhesive forces** and the forces of attraction between molecules of the same substance are called **cohesive forces**.

Consider a liquid molecule near the solid surface. The molecules of the solid wall attract this molecule.



(a) (b) (c) (d)

These forces are adhesive and are distributed over 180° and hence their resultant acts at right angles to the solid wall and the forces of cohesion, i.e., attraction by liquid molecules, are distributed over 90° and hence their resultant will be inclined at 45° to the solid wall as shown in figure (a) and (b). Let f_a be the resultant adhesive force and f_c be the resultant cohesive force. Then the angle between f_a and f_c is 135°. Let f be their resultant making angle θ with f_a. Then

$$\tan\theta = \frac{f_c \sin 135°}{f_a + f_c \cos 135°} = \frac{f_c}{\sqrt{2}\,f_a - f_c} \qquad \ldots (3)$$

Case I: If $\sqrt{2}\,f_a = f_c$, then tan θ = ∝ or θ = 90°, i.e., resultant will lie along the solid surface.

Case II: If $\sqrt{2}\,f_a > f_c$, tanθ is positive and hence θ < 90°, i.e., the resultant lies inside the solid as in figure(c).

Case III: If $\sqrt{2}\,f_a < f_c$, tanθ is negative and hence θ > 90°, i.e., the resultant lies inside the liquid as shown in figure (d).

A liquid cannot permanently withstand a shearing force, as it has no rigidity. Hence the free surface of a liquid will be at right angles to the resultant force.

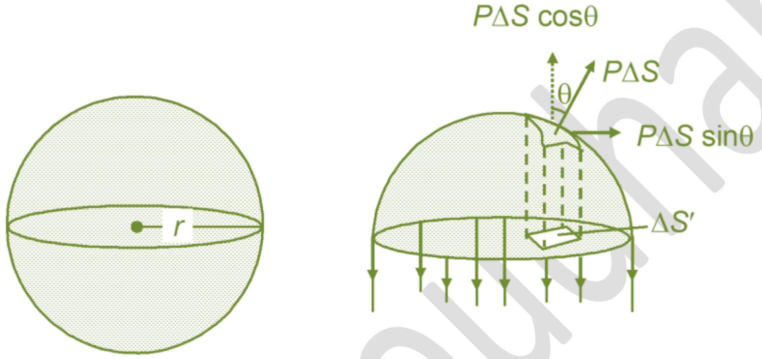
Thus, in the first case, when the resultant force f acts along the solid surface, the liquid surface is at right angles to the solid surface.

In the second case when the cohesive force is smaller than the adhesive force such that $f_c < \sqrt{2} f_a$, the resultant is inside the solid and so the liquid surface is inclined to the solid surface at a small angle. The free surface of the liquid is, in consequence, concave upwards near the wall. The concavity of the surface decreases gradually and the free surface becomes horizontal at a large distance from the wall. In this case it is said that the liquid wets the solid, e.g., water wets glass.

In the third case when the cohesive force (f_c) is larger than the adhesive force such that $f_c > \sqrt{2} f_a$ the resultant lies inside the liquid and so the liquid surface is inclined to the solid surface at a large angle. Thus in this case the liquid surface is convex upwards near the wall. In this case it is said that the liquid surface does not wet the solid, e.g., mercury and glass.

4.7 EXCESS PRESSURE

The pressure inside a liquid drop or a soap bubble must be in excess of the pressure outside the bubble drop because without such pressure difference a drop or a bubble cannot be in stable equilibrium. Due to surface tension the drop or bubble has got the tendency to contract and disappear altogether. To balance this, there must be an excess of pressure inside the bubble.



To obtain a relation between the excess of pressure and the surface tension, consider a water drop of radius r and surface tension T. Divide the drop into two halves by a horizontal plane passing through its center as shown in figure and consider the equilibrium of one-half, say, the upper half. The forces acting on it are:

(i) forces due to surface tension distributed along the circumference of the section.
(ii) outward thrusts on elementary areas of it due to excess pressure.

Obviously, both the types of forces are distributed. The first type of distributed forces combine into a force of magnitude $2\pi r \times T$. To find the resultant of the other type of distributed forces, consider an elementary area ΔS of the surface. The outward thrust on $\Delta S = p\,\Delta S$ where p is the excess of the pressure inside the bubble. If this thrust makes an angle θ with the vertical, then it is equivalent to $\Delta Sp \cos\theta$ along the vertical and $\Delta Sp \sin\theta$ along the horizontal. The resolved component $\Delta Sp \sin\theta$ is ineffective as it is perpendicular to the resultant force due to surface tension. The resolved component $\Delta Sp \cos\theta$ contributes to balancing the force due to surface tension.

The resultant outward thrust $= \Sigma \Delta Sp \cos\theta$

$= p\Sigma\Delta S \cos\theta$

$= p\Sigma\Delta S'$ where $\Delta S' = \Delta S \cos\theta$

= area of the projection of ΔS on the horizontal dividing plane

$= p \times \pi r^2 \quad (\because \Sigma\Delta S' = \pi r^2)$

For equilibrium of the bubble we have

$$\pi r^2 p = 2\pi r . T$$

$$\text{or,}\quad p = \frac{2T}{r} \qquad \ldots (4)$$

If it is a **soap bubble**, the resultant force due to surface tension is $2\pi r . 2T$, because a bubble has two surfaces. Hence for the equilibrium of a bubble we have

$$\pi r^2 p = 4\pi r T$$

$$\text{or,}\quad p = \frac{4T}{r} \qquad \ldots (5)$$

Illustration 16

Question: **Find the difference in the air pressure between the inside and outside of a soap bubble, 5 mm in diameter. Assume surface tension to be 1.6 Nm^{-1}.**

Solution: Excess pressure = $\frac{4T}{r}$, where r is radius of curvature of the surface.

Required pressure difference = $\frac{4\times 1.6}{2.5\times 10^{-3}}$

= **2560 Nm^{-2}**

Illustration 17

Question: **If a number of little droplets of water, all of the same radius r coalesce to form a single drop of radius R, Show that the rise in temperature of the water is given by, $\frac{3\sigma}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$ where J is the mechanical equivalent of heat, and σ is the surface tension of water. (Given $\frac{\sigma}{J}=10$ unit, r = 1m, R = 2m)**

Solution: Let n be the number of droplets, each of radius r cm that coalesce to form a single drop of radius R cm.

$\therefore$ Decrease in the surface area = $4\pi r^2 n - 4\pi R^2$

Decrease in the surface energy = $(4\pi r^2 n - 4\pi R^2)\,\sigma$

$\therefore$ Heat energy produced in the drop = $\frac{4\pi\sigma}{J}(nr^2 - R^2)$

Suppose the whole of heat energy is used to raise the temperature of the resultant drop by θ, therefore,

$$mS\theta = \frac{4\pi\sigma}{J}(nr^2 - R^2),$$

where m is the mass of the drop having specific heat S.

$$m = \frac{4}{3}\pi R^3 \times 1 \text{ (density = 1 gm/cc)}$$

S = 1 cal/g°C for water

$$\therefore \quad \theta = \frac{4\pi\sigma}{J}(nr^2 - R^2)\times\frac{3}{4\pi R^3} = \frac{3\sigma}{J}\left(\frac{nr^2}{R^3}-\frac{1}{R}\right) \quad \ldots\text{(i)}$$

Since volume remains the same,

$$n\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \quad \text{or} \quad n = \left(\frac{R}{r}\right)^3$$

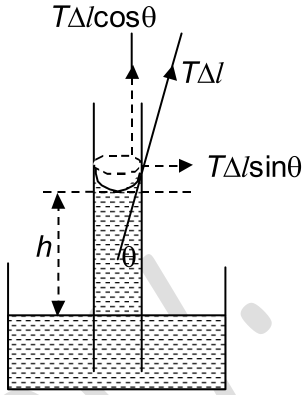
Putting this value in (i), we have $\theta = \frac{3\sigma}{J}\left(\frac{R^3}{r^3}\times\frac{r^2}{R^3}-\frac{1}{R}\right) = \frac{3\sigma}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$

Thus the rise in temperature is given by, $\theta = \frac{3\sigma}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$ = **15 SI unit**

4.8 CAPILLARY ACTION

When a glass tube of very fine bore called a capillary tube is dipped in a liquid (like water,) the liquid immediately rises up into it due to the surface tension. This phenomenon of rise of a liquid in a narrow tube is known as capillarity.

Suppose that a capillary tube of radius r is dipped vertically in a liquid. The liquid surface meets the wall of the tube at some inclination θ called the angle of contact. Due to surface tension a force, $\Delta l T$ acts on an element Δl of the circle of contact along which the liquid surface meets the solid surface and it is tangential to the liquid surface at inclination θ to the wall of the tube. (The liquid on the wall of the tube exerts this force. By the third law of motion, the tube exerts the same force on the liquid in the opposite direction.) Resolving this latter force along and perpendicular to the wall of the tube, we have $\Delta l T\cos\theta$ along the tube vertically upward and $\Delta l T\sin\theta$ perpendicular to the wall. The latter component is ineffective. It simply compresses the liquid against the wall of the tube. The vertical component $\Delta l T\cos\theta$ pulls the liquid up the tube.



The total vertical upward force = $\Sigma \Delta l T\cos\theta$

$$= T\cos\theta\Sigma\Delta l \quad = T\cos\theta\, 2\pi r$$

$$(\because \Sigma\Delta l = 2\pi r).$$

Due to this upward pull liquid rises up in the capillary tube till it is balanced by the downward gravitational pull. If h is the height of the liquid column in the tube up to the bottom of the meniscus and v is the volume of the liquid above the horizontal plane touching the meniscus at the bottom, the gravitational pull, i.e., weight of the liquid inside the tube is $(\pi r^2 h + v)\rho g$.

For equilibrium of the liquid column in the tube

$$2\pi r T\cos\theta = (\pi r^2 h + v)\rho g.$$

If volume of the liquid in meniscus is negligible then,

$$2\pi r T\cos\theta = (\pi r^2 h)\rho g.$$

$$h = \frac{2T\cos\theta}{r\rho g} \qquad \dots (6)$$

The small volume of the liquid above the horizontal plane through the lowest point of the meniscus can be calculated if θ is given or known. For pure water and glass θ = 0° and hence the meniscus is hemispherical.

∴ v = volume of the cylinder of height r – volume of hemisphere.

$$= \pi r^3 - \frac{1}{2}\frac{4\pi}{3}r^3$$

$$= \pi r^3 - \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^3$$

∴ For water and glass

$$2\pi r T = \left(\pi r^2 h + \frac{\pi r^3}{3}\right)\rho g$$

$$2T = r\left(h + \frac{r}{3}\right)\rho g$$

$$h = \frac{2T}{r\rho g} - \frac{r}{3}$$

For a given liquid and solid at a given place as ρ, T, θ and g are constant,

∴ hr = constant

i.e., lesser the radius of capillary greater will be the rise and vice-versa.

Illustration 18

Question: **Water rises to a height of 10 cm in a certain capillary tube. The level of mercury in the same tube is depressed by 3.42 cm. Calculate the percentage ratio of the surface tensions of water and mercury. Specific gravity of mercury is 13.6 g/cc and angle of contact for water and mercury with capillary are zero and 135° respectively.**

Solution: Using the capillarity relation, $T = \dfrac{r\,h\,\rho\,g}{2\cos\theta}$

$$T_1 \text{ (for water)} = \frac{r \times 1 \times g \times 10}{2\cos\theta} = 5rg$$

$$T_2 \text{ (for mercury)} = \frac{r \times 13.6 \times g \times (-3.42)}{2\cos 135^\circ}$$

$$= \frac{r \times 13.6 \times g \times (-3.42)}{2 \times \left(\frac{-1}{\sqrt{2}}\right)}$$

$$= 32.9\ rg$$

$$\%\ \text{ration} = \frac{T_1}{T_2} = \frac{5\ rg}{32.9\ rg} = 100 = \mathbf{15\ \%}$$

5 VISCOSITY

If water in a tube is whirled and then left to itself, the motion of the water stops after some time. This is a very common observation. What stops the motion? There is no external force to stop it. A natural conclusion is, therefore, that whenever there is relative motion between parts of a fluid, internal forces are set up in the fluid, which oppose the relative motion between the parts in the same way as forces of friction operate when a block of wood is dragged along the ground. This is why to maintain relative motion between layers of a fluid an external force is needed.

"This property of a fluid by virtue of which it oppose the relative motion between its different layers is known as viscosity and the force that is into play is called the viscous force".

Consider the slow and steady flow of a fluid over a fixed horizontal surface. Let v be the velocity of a thin layer of the fluid at a distance x from the fixed solid surface. Then according to Newton, the viscous force acting tangentially to the layer is proportional to the area of the layer and the velocity gradient at the layer. If F is the viscous force on the layer, then

$F \propto A$ where A is the area of the layer

$$\propto -\frac{dv}{dx}$$

The negative sign is put to account for the fact that the viscous force is opposite to the direction of motion.

$$\Rightarrow \quad F = -\eta A \frac{dv}{dx} \qquad \ldots (7)$$

where η is a constant depending upon the nature of the liquid and is called the coefficient of viscosity and

Velocity gradient $= \dfrac{dv}{dx}$

If $A = 1$ and $\dfrac{dv}{dx} = 1$, we have $F = -\eta$

Thus the coefficient of viscosity of a liquid may be defined as *the viscous force per unit area of the layer where velocity gradient is unity.*

The coefficient of viscosity has the dimension $[ML^{-1}T^{-1}]$ and its unit is Newton second per square metre (Nsm^{-2}) or kilograme per metre per second ($kgm^{-1}s^{-1}$). In CGS the unit of viscosity is Poise

Illustration 19

Question: **A metal plate 1 m^2 in area rests on a layer of castor oil (η = 1 decapoise) 0.1 cm thick. Calculate the horizontal force required to move the plate with a speed of 3 cm/s.**

Solution: $F = -\eta A \dfrac{dv}{dx}$

where η = 15.5 poise A = 100 cm^2

$$\frac{dv}{dx} = \frac{3}{0.1} = 30\text{s}^{-1}$$

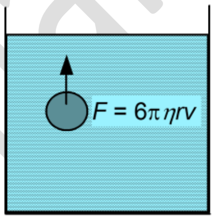
$$F = -1 \times 1 \times 30 = -30$$

So force required = **30 N**

5.1 STOKE'S LAW

When a solid moves through a viscous medium, its motion is opposed by a viscous force depending on the velocity and shape and size of the body. The energy of the body is continuously decreases in overcoming the viscous resistance of the medium. This is why cars, aeroplanes etc. are shaped streamline to minimize the viscous resistance on them.

The viscous drag on a spherical body of radius r, moving with velocity v, in a viscous medium of viscosity η is given by



$$F_{\text{viscous}} = 6\pi\eta rv.$$

This relation is called **Stokes' law**

This law can be deduced by the method of dimensions. By experience we guess that the viscous force on a moving spherical body may depend on its velocity, radius and coefficient of viscosity of the medium. We may then write

$$F = k\, v^a r^b \eta^c$$

where k is a constant (dimensionless) and a, b and c are the constants to be determined.

By taking dimensions of both sides, we have

$$MLT^{-2} = (LT^{-1})^a L^b (ML^{-1}T^{-1})^c$$

or, $$MLT^{-2} = M^c L^{a+b-c} T^{-a-c}$$

Equating powers of M, L and T we have

$$c = 1$$

$$a + b - c = 1$$

and $$-a - c = -2$$

Solving we have $a = 1$, $b = 1$ and $c = 1$

$$F = k\eta rv$$

Experimentally k is found to be 6π;

$$F = 6\pi\eta rv. \qquad \ldots \textbf{(15)}$$

Illustration 20

Question: **An air bubble of diameter 2 cm rises through a long cylindrical column of a viscous liquid, and travels at 0.21 cms^{-1}. If the density of the liquid is 189 kg m^{-3}, find its coefficient of viscosity. Ignore the density of the air.**

Solution: Weight of the bubble is equal to the viscous force.

$$\frac{4}{3}\pi r^3 \rho g = 6\pi\eta rv \quad \text{or} \quad \eta = \frac{2r^2 g\rho}{9v} \qquad \ldots\text{(i)}$$

Given: $r = 10^{-2}$ m

$v = 0.21 \times 10^{-2}$ m/s $g = 10$ m/s^2

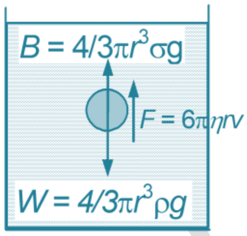
Substituting these values in (i) we have,

$$\eta = \frac{2\times(10^{-2})^2\times189\times10}{9\times0.21\times10^{-2}} \approx \mathbf{20\ kgm^{-1}\,s^{-1}}$$

5.2 TERMINAL VELOCITY

Let the body be driven by a constant force. In the beginning the viscous drag on the body is small. Velocity is small and so the body is accelerated through the medium by the driving force. With the increase of velocity of the body the viscous drag on it will also increase and eventually when it becomes equal to the driving force, the body will acquire a constant velocity. This velocity is called the terminal velocity of the body.

Consider the downward motion of a spherical body through a viscous medium such as a ball falling through liquid. If r is the radius of the body, ρ the density of the material of the body and σ is the density of the liquid then,



the weight of the body = $\frac{4\pi}{3}r^3\rho g$,downwards

and the buoyancy of the body = $\frac{4\pi}{3}r^3\sigma g$,upwards

The net downward driving force = $\frac{4\pi}{3}r^3(\rho-\sigma)g$

If v is the terminal velocity of the body, then the viscous force on the body is

$$F = 6\pi\eta rv$$

For no acceleration of the body we have

$$6\pi\eta rv = \frac{4\pi}{3}r^3(\rho-\sigma)g \quad \text{or,} \quad v = \frac{2}{9}.\frac{r^2g(\rho-\sigma)}{\eta} \quad \ldots (16)$$

Illustration 21

Question: **Eight spherical drops of equal size fall vertically through air with a terminal velocity of 0.1 ms^{-1}. What would be the velocity if these eight drops were to combine to form one large spherical drop?**

Solution: Let r_1 be the radius of each small drop, and r_2 that of the bigger drop. As the volume remains constant,

$$\frac{4}{3}\pi r_2^3 = \frac{4}{3}\pi r_1^3 \times 8$$

or, $$\frac{r_1}{r_2} = \frac{1}{2}$$

Since the terminal velocity is proportional to the square of the radius of the drop,

$$\frac{v_1}{v_2} = \left(\frac{r_1}{r_2}\right)^2 = \frac{1}{4}$$

$\therefore$ $v_2 = 4 \times 0.1 = 0.40$ ms^{-1} = **40 cm/s**

Thus the terminal velocity of the bigger drop is 40cm/s

Illustration 22

Question: **A gas bubble of diameter 2 cm rises steadily through a solution of density 1750 kgm^{-3} at the rate of 0.35 cm/s. Calculate the coefficient of viscosity of the solution.**

Solution: We have, $v = \frac{2}{9}.\frac{r^2g(\rho-\sigma)}{\eta}$

Here, ρ = density of air is negligible

$\therefore$ $$v = -\frac{2}{9}.\frac{r^2g\sigma}{\eta}$$

The negative sign shows that the velocity is upward

$\therefore$ $$\eta = \frac{2}{9}.\frac{r^2g\sigma}{v} = \frac{2}{9}.\frac{0.01^2 \times 9.8 \times 1750}{0.0035}$$

= **109 kgm^{-1} s^{-1} or Nsm^{-2}**

PROFICIENCY TEST - 4

The following questions deal with the basic concepts of this section. Answer the following briefly. Go to the next section only if your score is at least 80%. Do not consult the Study Material while attempting these questions.

1. A water drop of radius 10^{-1} m is broken into 1000 equal droplets. Calculate the gain in surface energy. Surface tension of water is 1.366 N/m.

2. Find the excess pressure inside a mercury drop of radius 2 mm. The surface tension of mercury = 0.464 N/m

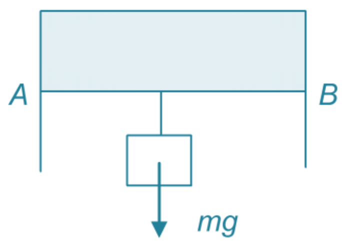
3. A 0.03 cm liquid column balances the excess pressure inside a soap bubble of radius 75 mm. Determine the density of the liquid. Surface tension of soap solution is 0.09 N/m.

4. A capillary tube of radius 0.2 mm is dipped vertically in water. Find the height of the water column raised in the tube. Surface tension of water = 0.75 N/m and density of water = 1000 kg/m^3. Take g = 10 ms^{-2}.

5. An air bubble of diameter 2 mm rises steadily through a solution of density 1750 kg/m^3 at the rate of 0.35 cm/s. Calculate the coefficient of viscosity of the solution. The density of air is negligible.

6. Find the terminal velocity of a liquid drop of radius 0.03 mm. The coefficient of viscosity of air is 2×10^{-5} N-s/m^2 and its density is 1.2 kg/m^3. Density of liquid = 1001.2 kg/m^3. g = 10 ms^{-2}.

7. A light wire *AB* of length 10 cm. can slide on a vertical frame as shown in figure. There is a film of soap solution trapped between the frame and the wire. Find the mass of the block that should be suspended from the wire to keep it in equilibrium neglect friction. Surface tension of soap solution = 25 dyne/c. Take g = 10 ms^{-2}.



8. A flat plate of area 1000 sq. cm. is separated from a large plate by a thin layer of glycerine 1 mm thick. If the viscous coefficient of glycerine is 2 kgm^{-1}s^{-1}, what force is required to keep the plate moving with a velocity of 0.01 ms^{-1}?

ANSWERS TO PROFICIENCY TEST - 4

1. 85 mJ

2. 464 N/m^2

3. 1600 kg/m^3

4. 75 m

5. $\eta \approx 11$ poise

6. 10 cm/s

7. 500 mg

8. 2 N

SOLVED OBJECTIVE EXAMPLES

***Example* 1:**

Two steel wires of length 1 m and 2 m have diameters 1 mm and 2 mm respectively. If they are stretched by forces of 40 N and 80 N respectively their elongation will be in the ratio of

(a) 1 : 1 **(b) 2 : 1** **(c) 4 : 1** **(d) 1 : 8**

Solution:

Young's modulus: $Y = \dfrac{F_1/A_1}{\Delta L_1/L_1} = \dfrac{F_2/A_2}{\Delta L_2/L_2}$

$\Rightarrow \quad \dfrac{\Delta L_1}{\Delta L_2} = \dfrac{F_1 L_1}{F_2 L_2} \times \dfrac{A_2}{A_1}$

$= \dfrac{40 \times 1}{80 \times 2} \times \dfrac{2^2}{1^2} = \mathbf{1}$

$\therefore$ **(a)**

***Example* 2:**

The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied to all?

(a) L = 50 cm; D = 0.05 mm **(b) L = 100 cm; D = 1 mm**
(c) L = 200 cm; D = 2 mm **(d) L = 300 cm; D = 3 mm**

Solution:

Extension: $\Delta L = \dfrac{FL}{AY} \propto \dfrac{L}{A}$ ($\because$ F and Y are common)

$\dfrac{L}{A}$ is maximum in case of (a)

$\therefore$ **(a)**

***Example* 3:**

Two wires of different materials support a massive object suspended from the centre of a uniform horizontal bar with its ends connected to two vertical wires of equal length. What must be the ratio of the diameters of the wires so that the bar may remain horizontal when the object is removed? [Young's modulus of the wires are Y_1 and Y_2 respectively]

(a) $\dfrac{d_1}{d_2} = \dfrac{Y_1}{Y_2}$ **(b) $\dfrac{d_1}{d_2} = \dfrac{Y_2}{Y_1}$** **(c) $\dfrac{d_1}{d_2} = \sqrt{\dfrac{Y_1}{Y_2}}$** **(d) $\dfrac{d_1}{d_2} = \sqrt{\dfrac{Y_2}{Y_1}}$**

Solution:

Extension: $\Delta l = \dfrac{Fl}{AY}$

For the bar to remain horizontal, $\Delta l_1 = \Delta l_2$

$\Rightarrow A_1 Y_1 = A_2 Y_2$ ($\because$ F and ℓ are common)

$\Rightarrow d_1^2 Y_1 = d_2^2 Y_2$ ($\because$ A $\propto$ d)

$\dfrac{d_1}{d_2} = \sqrt{\dfrac{Y_2}{Y_1}}$

$\therefore$ **(d)**

***Example* 4:**

By what fraction is the volume of an aluminium sphere be reduced as it is lowered from the surface to the depth of 3 km in ocean? Bulk modulus of $Al = 8 \times 10^{10}$ N/m². At the surface of ocean $P = 10^5$ N/m². At a depth 3 km, P = 300 atmospheres.

(a) 20 % **(b) 10 %** **(c) 1 %** **(d) 0.04 %**

Solution:

$$\frac{\Delta V}{V} = \frac{\text{Change in pressure}}{\text{Bulk modulus}} = \frac{3 \times 10^7 - 10^5}{8 \times 10^{10}}$$

$$= \frac{299}{8} \times 10^{-5} \times 100\% = 0.0374\% \approx \mathbf{0.04\%}$$

$\therefore$ **(d)**

***Example* 5:**

A wire of length L, radius r, when stretched with a force F, changes in length by l. What will be the change in length in a wire of same material having length $2L$, radius $2r$ and stretched by a force $2F$?

(a) $\frac{l}{2}$ **(b) l** **(c) 2 l** **(d) 4L**

Solution:

$$Y = \frac{\frac{F}{A}}{\left(\frac{\Delta \ell}{\ell}\right)} \Rightarrow \frac{\left(\frac{F}{\pi r^2}\right)}{\left(\frac{\ell}{L}\right)} = \frac{\left(\frac{2F}{4\pi r^2}\right)}{\left(\frac{\Delta \ell}{2L}\right)}$$

$$\Rightarrow \quad \frac{1}{l} = \frac{1}{\Delta l}$$

$$\Rightarrow \quad \Delta l = l$$

$\therefore$ **(b)**

***Example* 6:**

The weight of a body in air is 100 N. How much will it weigh in water, if it displaces 400 cc of water?

(a) 90 N **(b) 94 N** **(c) 98 N** **(d) None of these**

Solution:

400 cc = 4×10^{-4} m³

$\therefore$ weight of body inside the water = $100 - 4 \times 10^{-4} \times 10^3 \times 10$

= 96 N

$\therefore$ **(d)**

***Example* 7:**

A mercury barometer reads 75 cm. If the tube be inclined by 60° from vertical the length of mercury in the tube will be

(a) 37.5 cm **(b) 150 cm** **(c)** $\frac{75\sqrt{3}}{2}$ **cm** **(d) 100 cm**

Solution:

We have

$\rho gh = \rho gh' \cos 60°$ (where h' is the length of column)

$\therefore$ $h' = 2h =$ **150 cm** inside the tube

$\therefore$ **(b)**

***Example* 8:**

The surface area of air bubble increases four times when it rises from bottom to the top of a water tank where the temperature is uniform. If the atmospheric pressure is 10 m of water, the depth of the water in the tank is

(a) 70 m **(b) 30 m** **(c) 80 m** **(d) 40 m**

Solution:

Area $A = 4\pi r^2$ $\therefore r \propto \sqrt{A}\,(or)\, r = k_1\sqrt{A}$

Volume $V = \frac{4}{3}\pi r^3$

$= \text{constant } k_2 \times r^3$

$= k_2 \times (k_1\sqrt{A})^3$

$= kA^{3/2}, k = k_1^3 k_2 = a$ constant

Applying Boyle's law,

$P_1V_1 = P_2V_2$

$P_2 = \frac{P_1V_1}{V_2}$

$= \frac{(10+h)kA_1^{3/2}}{kA_2^{3/2}}$

$= (10+h)\left(\frac{A_1}{A_2}\right)^{3/2} = (10+h)\left(\frac{1}{4}\right)^{3/2} = \frac{10+h}{8}$

Since $P_2 = 10$ m of water

$10 + h = 80$

$h =$ **70 m**

$\therefore$ **(a)**

***Example* 9:**

The volume of a body is V and its density is d'. Density of water is d and $d' > d$. The body is submerged inside water and is lifted through a height h in water. Which of the following is correct?

(a) The potential energy of the body increases by $hVdg$

(b) P.E. increases by $hV(d' - d)g$

(c) P.E. increases by $hV(d' + d)g$

(d) P.E. decreases

Solution:

The resultant downward force on the sinking particle $= Vd'g - Vdg = V(d' - d)g$. When it is lifted up work is done against this force, $W =$ **$hV(d' - d)g$ = P.E. gained**

$\therefore$ **(b)**

***Example* 10:**

Two circular metal plates of radius 1 m and 2 m are placed horizontally in a liquid at rest at the same depth. The ratio of thrusts on them is

(a) 1 : 2 **(b) 1 : 1** **(c) 1 : 4** **(d) 4 : 1**

Solution:

$\frac{F_1}{F_2} = \frac{P\,\pi(1)^2}{P\,\pi(2)^2} = \frac{\mathbf{1}}{\mathbf{4}}$

$\therefore$ **(c)**

***Example* 11:**

A cylinder is filled with non-viscous liquid of density *d* to *a* height h_0 and a hole is made at a height h_1 from the bottom of the cylinder. The velocity of liquid issuing out of the hole is

(a) $\sqrt{2gh_0}$ **(b)** $\sqrt{2g(h_0-h_1)}$ **(c)** $\sqrt{dgh_1}$ **(d)** $\sqrt{dgh_0}$

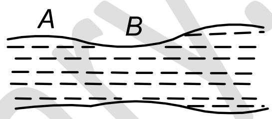
Solution:

We have $P_0 + \rho g(h_0 - h_1) = P_0 + \frac{1}{2}\rho V^2$

$\therefore v = \sqrt{2g(h_0 - h_1)}$

$\therefore$ **(b)**

***Example* 12:**



Water flows in a horizontal tube as shown in Figure. The pressure of water changes by 600 N/m² between *A* and *B* where the cross sections are 30 cm² and 15 cm². Find the rate of flow of water through the tube.

(a) 1890 cm³/sec (b) 1060 cm³/sec (c) 1540 cm³/sec (d) 1280 cm³/sec

Solution:

(a) Let the velocity at $A = v_A$ and that at $B = v_B$

By equation of continuity, $\frac{v_B}{v_A} = \frac{30}{15} = 2$

By Bernoulli's equation,

$$P_A + \frac{1}{2}\rho v_A^2 = P_B + \frac{1}{2}\rho v_B^2$$

$$P_A - P_B = \frac{1}{2}\rho(2v_A)^2 - \frac{1}{2}\rho v_A^2 \left(\because v_B = 2v_A\right)$$

$$= \frac{3}{2}\rho v_A^2$$

$$\Rightarrow \quad 600 = \frac{3}{2}\times 1000 \times v_A^2$$

$$v_A = \sqrt{0.4} = 0.63 \text{ m/s}$$

Rate of flow = (30 cm²) (63 cm/s) = **1890 cm³/s**

$\therefore$ **(a)**

***Example* 13:**

At two points on a horizontal tube of varying cross section carrying water, the radii are 1 cm and 0.4 cm. The pressure difference between these points is 4.9 cms of water. How much liquid flows through the tube per second?

(a) 100 c.c. per sec (b) 80 c.c. per sec (c) 50 c.c per sec (d) 70 c.c. per sec

Solution:

Since the tube is horizontal there is no pressure difference along the tube due to hydrostatic effects. Using Bernoulli's equation,

$$p_1 + \frac{1}{2}\rho_1 v_1^2 = p_2 + \frac{1}{2}\rho_2 v_2^2 \qquad \text{... (i)}$$

where ρ is the density of liquid, v its velocity, p its pressure and subscripts 1 and 2 refer to two points.

Also $A_1v_1 = A_2v_2$ by equation of continuity ... (ii)

$p_1 - p_2 = \rho g \times 4.9$ ... (ii)

From (1) and (3),

$$v_2^2 - v_1^2 = \frac{2(p_1 - p_2)}{\rho} = \frac{2\rho g \times 4.9}{\rho}$$

$= (2g) \times 4.9$

$= 2 \times 980 \times 4.9$

$v_2^2 - v_1^2 = 98^2\ cm^2/sec^2$ ... (iv)

Using (2), $\frac{v_1}{v_2} = \frac{A_2}{A_1} = \frac{\pi \times 0.4^2}{\pi \times 1^2} = 0.16$

Substituting $v_1^2 = 0.16^2 \times v_2^2$ in (4),

$v_2^2[1-(0.16)^2] = 98^2$

$$v_2 = \sqrt{\frac{98^2}{0.9744}}$$

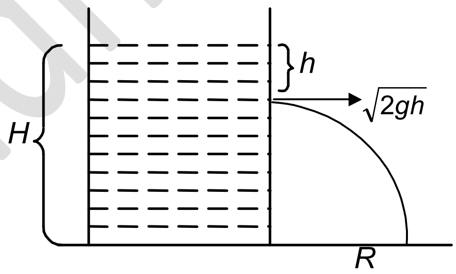
Quantity of water flowing $= A_1v_1 = A_2v_2 = \pi \times 0.4^2 \times \sqrt{\frac{98^2}{0.9744}}$

= 50 c.c per sec

$\therefore$ **(c)**

***Example* 14:**

Water stands at a depth *H* in a large open tank whose side walls are vertical. A hole is made in one of the walls at a depth *h* below the water surface and the water from the hole strikes the floor at a distance *R* from the foot of the wall. At what height above the bottom of the tank could a second hole be cut so that the stream emerging from it would have the same range *R*?



(a) $(H-h)$ **(b) $\sqrt{h(H-h)}$**

(c) h **(d) $\frac{h}{2}$**

Solution:

Velocity of efflux from hole $= \sqrt{2gh}$

Vertical distance travelled when it strikes the ground $= (H-h)$

$\therefore$ $(H-h) = \frac{1}{2}gt^2$

$$t^2 = \frac{2(H-h)}{g} \text{ or } t = \sqrt{\frac{2(H-h)}{g}}$$

Range $R = \sqrt{2gh} \cdot \sqrt{\frac{2(H-h)}{g}}$

$= 2\sqrt{h(H-h)}$

In the second case, the hole be made at a height x from bottom. Then velocity of efflux

$= \sqrt{2g(H-x)}$

$\therefore$ in this case the range $= \sqrt{\frac{2x}{g}} \cdot \sqrt{2g(H-x)}$

$= 2\sqrt{x(H-x)}$

If the ranges are to be the same, $x = \boldsymbol{h}$.

$\therefore$ **(c)**

***Example* 15:**

Water enters a house through a pipe 2 cm inside diameter at an absolute pressure 4 × 10⁵ Pa (about 4 atmospheres). The pipe leading to second floor bathroom 5 m above is 1 cm in diameter. When the flow velocity at the inlet pipe is 4 m/sec, find the pressure in the bathroom.

(a) 2×10^5 Pa (b) 2.3×10^5 Pa (c) 3×10^5 Pa (d) 3.3×10^5 Pa

Solution:

The flow velocity is obtained from the continuity equation

$$v_2 = \frac{A_1}{A_2} \cdot v_1$$

$$= \frac{\pi(1.0cm)^2}{\pi(0.5cm)^2} 4ms^{-1}$$

= 16 m/sec

The pressure is obtained from Bernoulli's equation

$$p_2 = p_1 - \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) - \rho g\,(y_2 - y_1)$$

$$= 4 \times 10^5 - \frac{1}{2} \times 1 \times 10^3\ (256 - 16) - 1 \times 10^3 \times 9.8 \times 5$$

$= 2.3 \times 10^5$ Pa

∴ **(b)**

***Example* 16:**

Water rises in a capillary tube to a height of 2.0 cm. In another capillary tube whose radius is one third of it, how much the water will rise?

(a) 5 cm (b) 3 cm (c) 6 cm (d) 9 cm

***Solution*:**

$$h = \frac{2T\cos\theta}{r\rho g}$$

$$hr = \frac{2T\cos\theta}{\rho g} = \text{constant}$$

$$h_1 r_1 = h_2 r_2$$

or $$h_2 = \frac{h_1 r_1}{r_2}$$

Substituting the values $h_2 = (2.0)(3) = 6.0$ cm

∴ **(c)**

***Example* 17:**

A ball of mass *m* and radius *r* is released in viscous liquid. The value of its terminal velocity is proportional to

(a) $(1/r)$ only (b) m/r (c) $(m/r)^{1/2}$ (d) m only

***Solution*:**

$$6\pi\eta r v = mg - F_{\text{thrust}} = mg - mg\frac{\sigma}{\rho}$$

where σ = density of liquid

and ρ = density of ball, $6\pi\eta r v = mg\left(1 - \frac{\sigma}{\rho}\right)$

$$v = \left(\frac{m}{r}\right)\frac{g(1 - \sigma/\rho)}{6\pi\eta}$$

$$\therefore \quad v \alpha \frac{m}{r}$$

$\therefore$ **(b)**

***Example* 18:**

The terminal velocity v of a small steel ball of radius r falling under gravity through a column of a viscous liquid of coefficient of viscosity η depends on mass of the ball m, acceleration due to gravity g, coefficient of viscosity η and radius r. Which of the following relations is dimensionally correct?

(a) $v \propto mgr/\eta$ **(b) $v \propto mg\eta r$** **(c) $v \propto mg/r\eta$** **(d) $v \propto \eta mg/r$**

***Solution*:**

$$v_T = \frac{mg\left(1-\frac{\sigma}{e}\right)}{6\pi\eta r} \Rightarrow v_T \, \alpha \, \frac{mg}{\eta r}$$

$\therefore$ **(c)**

***Example* 19:**

A small drop of water falls from rest through a large height h in air, the final velocity is

(a) almost independent of h **(b) proportional to $\sqrt{h}$**
(c) proportional to h **(d) inversely proportional to h**

***Solution*:**

Air acts as viscous medium and hence the finally attains the constant terminal velocity which is independent of h.

$\therefore$ **(a)**

***Example* 20:**

A spherical ball of radius 3.0×10^{-4} m and density 10^4 kg/m^3 falls freely under gravity through a distance h before entering a tank of water. If after entering the water the velocity of the ball does not change, the value of h is. (Viscosity of water is 9.8×10^{-6} N-s/m^2 and density of water is 10^3 kg/m^3)

(a) 1.65×10^3 m **(b)** 1.65×10^2 m **(c)** 1.05×10^3 m **(d)** 1.05×10^2 m

***Solution*:**

Before entering the water the velocity of ball is $\sqrt{2gh}$. If after entering the water this velocity does not change then this value should be equal to the terminal velocity. Therefore,

$$\sqrt{2gh} = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$$

$$h = 1.65 \times 10^3 \text{ m}$$

$\therefore$ **(a)**

***Example* 21:**

The amount of work done in increasing the size of a soap film 10 × 6 cm to 10 × 10 cm is (S.T. = 30 × 10^{-3} N/m)

(a) 2.4 × 10^{-2} J **(b) 1.2 × 10^{-2} J** **(c) 2.4 × 10^{-4} J** **(d) 1.2 × 10^{-4} J**

***Solution*:**

$W = 2T\,\Delta S$
$= 2 \times 30 \times 10^{-3} [10 \times 10 - 10 \times 6] \times 10^{-4} = 2 \times 30 \times 10^{-3} [100 - 60] = \mathbf{2.4 \times 10^{-4}}$ **J**

$\therefore$ **(c)**

***Example* 22:**

A spherical liquid drop of radius R is divided into eight equal droplets. If surface tension is T, then work done in the process will be

(a) $2\pi R^2T$ **(b) $3\pi R^2T$** **(c) $4\pi R^2T$** **(d) $2\pi RT^2$**

***Solution*:**

$$\frac{4}{3}\pi R^3 = \left(\frac{4}{3}\pi r^3\right)\times 8$$

$\therefore \quad r = \frac{R}{2}$

$W = T\Delta S = T(-4\pi R^2 + 8 \times 4\pi r^2) = \mathbf{4\pi R^2 T}$

$\therefore$ **(c)**

***Example* 23:**

Excess pressure inside a soap bubble is

(a) $\propto 1/r$ **(b) $\propto r$** **(c) $\propto \sqrt{r}$** **(d) independent of r**

***Solution*:**

$$P = \frac{4T}{r}$$

$$\therefore \quad P \,\alpha\, \frac{1}{r}$$

$\therefore$ **(a)**

***Example* 24:**

If two soap bubbles of radii r_1 and r_2 ($> r_1$) are in contact, the radius of their common interface is

(a) $r_1 + r_2$ **(b) $(r_1 + r_2)^2$** **(c) $\frac{r_1 r_2}{r_2 - r_1}$** **(d) $\sqrt{r_1 r_2}$**

***Solution*:**

For 1st bubble; $P_1 = P_0 + \frac{4T}{r_1}$

For 2nd bubble; $P_2 = P_0 + \frac{4T}{r_2}$

$\therefore$ Pressure inside smaller bubble will be greater than the larger bubble, so for interface

$$P = P_1 - P_2 = \frac{4T}{R} = 4T\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$\therefore \quad R = \frac{r_1 r_2}{r_2 - r_1}$$

$\therefore$ **(c)**

***Example* 25:**

On putting a capillary tube in a pot filled with water the level of water rises up to a height of 4 cm in the tube. If a tube of half the diameter is used, the water will rise to the height of nearly

(a) 2 cm **(b) 5 cm** **(c) 8 cm** **(d) 11 cm**

***Solution*:**

$\therefore \quad hr = \text{constant}$

$h_1 r_1 = h_2 r_2$

$$\therefore \quad h_2 = h_1\left(\frac{r_1}{r_2}\right) = 2h_1 = \mathbf{8\ cm}$$

$\therefore$ **(c)**

SOLVED SUBJECTIVE EXAMPLES

***Example* 1:**

A light rod of length 600 cm is suspended from the ceiling horizontally by means of two vertical wires of equal length tied to its ends. One of the wires is made of steel and is of cross-section 0.1 sq.cm and the other is made of brass and is of cross-section 0.2 sq.cm. Find the position along the rod at which a weight may be hung to produce.

(i) equal stress in both wires

(ii) equal strain in both wires

Y of steel $= 20 \times 10^{10}$ Nm^{-2}

Y of brass $= 10 \times 10^{10}$ Nm^{-2}

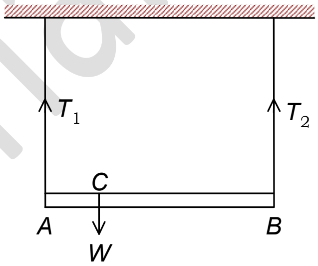
Solution:

For equilibrium $W = T_1 + T_2$ ... (i)

where W is the weight, and T_1 and T_2 are tensions in the wires.

Taking moments about C

$T_1\,(AC) = T_2\,(BC)$ ... (ii)



(i) If F_1 and F_2 are the elastic stresses in the two wires, their areas of cross-sections are a_1 & a_2 respectively then $T_1 = F_1a_1$ and $T_2 = F_2a_2$

Substituting in equation (2)

$F_1a_1\,(AC) = F_2a_2\,(BC)$

If the stresses in the two wires are equal,

$F_1 = F_2$

$a_1 \times AC = a_2 \times BC$

$0.1 \times AC = 0.2 \times BC$

$AC = 2\,BC$

$AC = \frac{2}{3} AB = \mathbf{4}$

The weight must be suspended at 4 m from steel wire.

(ii) Strain $= \frac{\text{increase in length}}{\text{original length}} = \frac{\Delta L}{L}$

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{\text{stress}}{\frac{\Delta L}{L}}$$

$\therefore$ stress $= Y \cdot \frac{\Delta L}{L}$

Tension = Stress × area = $a\,Y\,\frac{\Delta L}{L}$

Hence for two wires

$$T_1 = a_1\,Y_1\,\frac{\Delta L_1}{L}$$

$$T_2 = a_2\,Y_2\,\frac{\Delta L_2}{L}$$

Substituting in equation (2),

$$a_1Y_1\frac{\Delta L_1}{L}\cdot AC = a_2Y_2,\frac{\Delta L_2}{L}\cdot BC$$

If the strain in the two wires are equal

$$\frac{\Delta L_1}{L}=\frac{\Delta L_2}{L}$$

$$a_1\ Y_1\ (AC) = a_2\ Y_2\ (BC)$$

$$0.1 \times 20 \times 10^{10} \times AC = 0.2 \times 10 \times 10^{10} \times BC$$

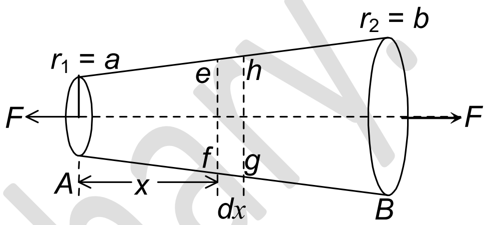
$$AC = BC$$

$\therefore \quad AC = \mathbf{3\ m}$

The weight must be suspended at the midpoint of the rod.

Example 2:

A slightly tapering wire of length L = 8 m and end radii a = 1 mm and b = 4 mm is subjected to stretching forces F = 10^3N as shown in Figure. If $Y = \pi \times 10^{11}$ N/m² is Young's modulus, calculate the extension produced in the wire. (π^2 = 10)



Solution:

Consider a small element of width dx, radius r situated at a distance x from A. The gradient in radius is uniform. Let it be k.

$$\therefore \quad k = \frac{b-a}{L} \qquad \text{... (i)}$$

If r is the radius of the elementary portion at ef and $(r + dr)$ at gh, then

$$k = \frac{dr}{dx} \qquad \text{... (ii)}$$

$$\therefore \quad \frac{dr}{dx} = \frac{b-a}{L} \qquad \text{... (iii)}$$

$$dx = \frac{dr}{k}$$

If dl is the extension produced in length dx due to stretching force F using the relation

$$Y = \frac{\frac{F}{A}}{\frac{\Delta L}{L}} \text{ we get } \Delta L = \frac{FL}{AY}$$

$$\therefore \quad d\,l = \frac{F\cdot dx}{\pi r^2 Y}$$

$$= \frac{F\cdot dr}{\pi r^2 Yk} \quad \text{since } dx = \frac{dr}{k}$$

The variable on R.H.S. is r which varies from a to b.

$$\therefore \text{ Total extension produced} = \int dl = \int_a^b \frac{Fdr}{\pi r^2 Yk}$$

$$l = \frac{F}{\pi Yk}\int_a^b \frac{dr}{r^2} = -\frac{F}{\pi Yk}\left[\frac{1}{r}\right]_a^b$$

$$= \frac{F}{\pi Yk}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$= \frac{F(b-a)}{\pi Ykab}$$

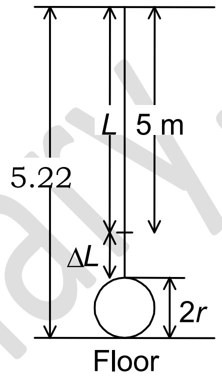
But $k = \dfrac{b-a}{L}$

$\therefore \quad l = \dfrac{F.L}{\pi Yab}$ **= 2 mm**

***Example* 3:**

A sphere of radius 0.1 m and mass 8π kg is attached to the lower end of a steel wire of length 5 m and diameter 10^{-3} m. The wire is suspended from 5.22 m high ceiling of a room. When the sphere is made to swing as a simple pendulum, it just grazes the floor at its lowest point. Calculate the velocity (in cm/s) of the sphere at the lowest position. Young's modulus of steel is 1.994×10^{11} N/m^2.

Solution:



The situation is shown in Figure. Let Δl be the extension of wire at mean position when oscillating and T is the tension.

$Y = \dfrac{T/A}{\Delta L / L}$ or $T = \dfrac{Y A \Delta L}{L}$

$\Delta L = 5.22 - (L + 2r)$

$= 5.22 - (5 + 2 \times 0.1) = 0.02$ m

$\therefore \; T = \dfrac{1.994 \times 10^{11} \times \pi (5 \times 10^{-4})^2 \times 0.02}{5} = 199.4\pi$ N.

At mean position $T - Mg = \dfrac{Mv^2}{R}$

R, the radius of circular path of oscillating sphere = 5.22 – 0.1 = 5.12 m

$Mg = 8\pi \times 9.8 = 78.4\pi$ N

$\therefore \; (199.4\pi - 78.4\pi) = \dfrac{8\pi v^2}{5.12}$

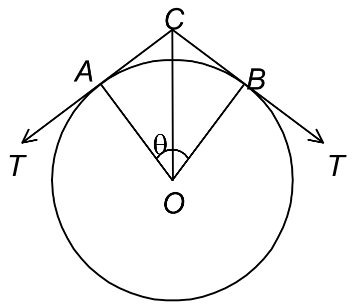
$v^2 = \dfrac{121 \times 5.12}{8} = 72.44$

v = **880 cm/s**

***Example* 4:**

A thin ring of radius R is made of a material, which has density ρ and Young's modulus Y. If the ring is rotated about its centre in its own plane with an angular velocity ω, find the small increase in its radius. (take $\dfrac{\rho\omega^2}{4} = 1$, $R = 10$ cm)

Solution:



Consider a small element AB of length dl. If a is the area of cross-section the mass will be $adl\rho$. The centrifugal force is equal to $a\, dl\, \rho \cdot \omega^2 R$

If the consequent tension in the wire is T, the radial component is $2T \sin \dfrac{\theta}{2}$. For small angle θ this can be written as $2T \cdot \dfrac{\theta}{2} = T\theta$

$\therefore \; T \cdot \theta = a\, dl\, \rho\, \omega^2 R$

$= a R\theta\, \rho\, \omega^2 R$

$T = a\, \rho\, \omega^2 R^2$

If the increase in radius is dR, the increase in circumference is $2\pi(R + dR) - 2\pi R = 2\pi dR$

Strain $= \dfrac{2\pi dR}{2\pi R} = \dfrac{dR}{R}$

Young's modulus $Y = \dfrac{\dfrac{T}{a}}{\dfrac{dR}{R}}$

$$T = a\,Y \cdot \frac{dR}{R}$$

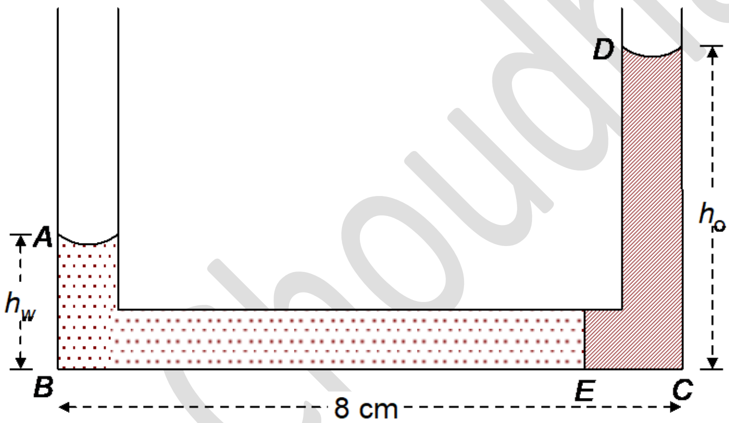
$$a\,Y \cdot \frac{dR}{R} = a\,\rho\,\omega^2 R^2$$

$$\therefore \quad dR = \frac{\rho\omega^2 R^2}{Y} = \mathbf{10\ mm}$$

***Example* 5:**

A tube of uniform cross-section consists of two vertical portions with a horizontal portion 8 cm long connecting their lower ends. Enough water to occupy 22 cm of the tube is poured into one branch and enough oil of specific gravity 0.8 to occupy 22 cm is poured into the other. Find the position of the common surface of the two liquids.

Solution:



Let AB be the vertical water column and CD the vertical oil column in the tube.
As the liquids in the tube are at rest, pressure at points B and C at the same horizontal level should be equal.
Let h_W cm be the height of water column AB and h_0 be the height of oil column CD.
Now considering pressures at points B and C

$$h_W d_W g = g_0 d_0 g$$

Where d_W and d_0 are the densities of water and oil respectively and it is given that

$$\frac{d_o}{d_w} = 0.8$$

$$\therefore \quad \frac{h_w}{h_o} = \frac{d_o}{d_w} = 0.8$$

$$\frac{h_w}{h_o} + 1 = 0.8 + 1$$

or $$\frac{h_w + h_o}{h_o} = 1.8$$

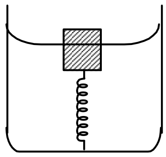
or $$\frac{22+22-8}{h_o} = 1.8$$

or $$h_0 = \frac{36}{1.8} = 20\text{ cm}$$

$$h_W = h_0 \times 0.8 = \mathbf{16\ cm}$$

The common surface E of the two liquids lies at the horizontal portion of the tube and is at a distance 6 cm from B.

***Example* 6:**



A cubical block of wood of edge 3 cm floats in water. The lower surface of the cube just touches the free end of a vertical spring fixed at the bottom of the pot. Find the maximum mass in gm that can be put on the block without wetting it. Density of wood = 800 kg/m^3 and spring constant of the spring = 50 N/m. Take g = 10 m/s^2.

Solution:

The specific gravity of the block = 0.8. Hence the height inside water = 3 cm × 0.8 = 2.4 cm. The height outside water = 3 cm – 2.4 cm = 0.6 cm. Suppose the maximum weight that can be put without wetting it is W. The block in this case is completely immersed in the water. The volume of the displaced water

= volume of the block = 27×10^{-6} m^3.

Hence, the force of buoyancy

= $(27 \times 10^{-6}\ \text{m}^3) \times (1000\ \text{kg/m}^3) \times (10\ \text{m/s}^2)$

= 0.27 N

The spring is compressed by 0.6 cm and hence the upward force exerted by the spring

= 50 N/m × 0.6 cm = 00.3 N.

The force of buoyancy and the spring force taken together balance the weight of the block plus the weight W put on the block. The weight of the block is

$W' = (27 \times 10^{-6}\text{m}) \times\text{I}\ (800\ \text{kg/m}^3) \times (10\ \text{m/s}^2)$

≈ 0.22 N

Thus, W = 0.27 N + 0.3 N – 0.22 N

= **0.35 N**

∴ ***m* = 35 gm**

***Example* 7:**

A wooden stick of length L= 1 m, radius $R = \sqrt{\pi}$ cm and density ρ = 1000 kg/m^3 has a small metal piece of mass m (of negligible volume) attached to its one end. Find the minimum value for the mass m (in terms of given parameters) that would make the stick float vertically in equilibrium in a liquid of density σ. $\left(\frac{\sigma}{\rho} = 4, \pi^2 = 10\right)$

Solution:

For the stick to be vertical for rotational equilibrium, centre of gravity should be below in a vertical line through the centre of buoyancy. For minimum m, the two will coincide.

Let h be the length of immersed portion. For translational equilibrium,

Wt. of rod + mass attached = force of buoyancy

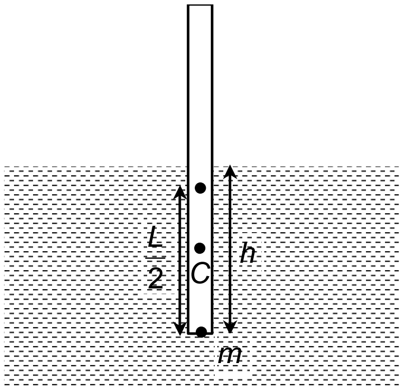
$$(M + m)g = \pi R^2 h\sigma g \qquad \text{... (i)}$$

where $M = \pi R^2 L\rho$.

The height of centre of mass from bottom

$$= \frac{(M)L/2 + m \times 0}{m+M} = \frac{ML}{2(m+M)}$$

For rotatory equilibrium and for minimum m, this should be equal to $h/2$.



$$\therefore \quad \frac{h}{2} = \frac{ML}{2(m+M)} \qquad \text{... (ii)}$$

$$\therefore \quad h = \frac{ML}{(m+M)}$$

Substituting for h in equation (i), we get

$$(M+m)g = \pi R^2 \sigma g . \frac{ML}{(m+M)}$$

$$(M+m)^2 = \pi R^2 \sigma . ML$$

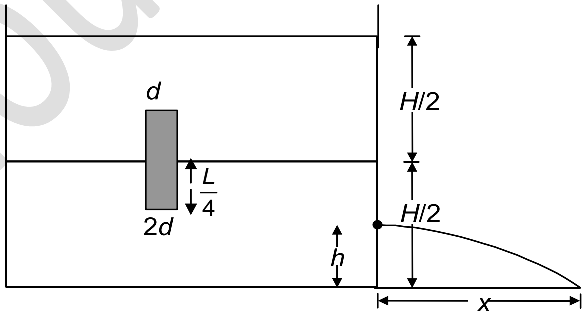
$$(M+m) = \sqrt{M\pi R^2 \sigma L} = \sqrt{\pi R^2 L\rho . \pi R^2 \sigma L}$$

$$m = \pi R^2 L \sqrt{\sigma\rho} - \pi R^2 L\rho$$

$$= \pi R^2 L\rho \left[\sqrt{\frac{\sigma}{\rho}} - 1\right] = \mathbf{1\ kg}$$

***Example* 8:**

A container of large uniform cross-sectional area A resting on a horizontal surface, holds two immiscible non-viscous and incompressible liquids of densities d = 4000 kg/m^3 and $2d$, each of height as shown in Figure. The lower density liquid is open to the atmosphere having pressure P_0.



(a) A homogeneous solid cylinder of length $L\left(L<\frac{H}{2}\right)$, cross-sectional area $\frac{A}{5}$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $\frac{L}{4}$ in the denser liquid.

Determine: the density D of the solid

(b) The cylinder is removed and the original arrangement is restored. A tiny hole of area S(S << A) is punched on the vertical side of the container at a height $h\left(h<\frac{H}{2}\right)$, $\left(H = \frac{29}{3}m,\ h = 1m\right)$

Determine: The horizontal distance x travelled by the liquid initially.

(Neglect the air resistance in these calculations).

Solution:

(a) The weight of the body = VDg. This must be equal to the weights of the displaced volumes of the two liquids.

i.e., Weight of the cylinder = Net Buoyant force

$VDg = V_1(2d)g + V_2(d)g$ ... (i)

$$LADg=\frac{L}{4}A\frac{(2d)g}{5}+\frac{3L}{4}\frac{Adg}{5}$$

Density of the solid $D = \frac{5d}{4}$ = **5000 kg/m³**

Total pressure at the bottom of the container

$$=\frac{\text{weight of liquid}+\text{weight of cylinder}}{A}+P_0$$

$$P=\frac{Ah_1d_1g+Ah_2d_2g+VDg}{A}+P_0 \;=\; \frac{d\frac{H}{2}gA+2d\frac{H}{2}gA+\frac{A5dLg}{5\times 4}}{A}+P_0$$

$$=\frac{(6H+L)dg}{4}+P_0$$

(b) Using Bernoulli's theorem, just inside and just outside orifice,

$$P_0+dg\frac{H}{2}+2dg\left(\frac{H}{2}-h\right)=\frac{1}{2}(2d)v^2+P_0$$

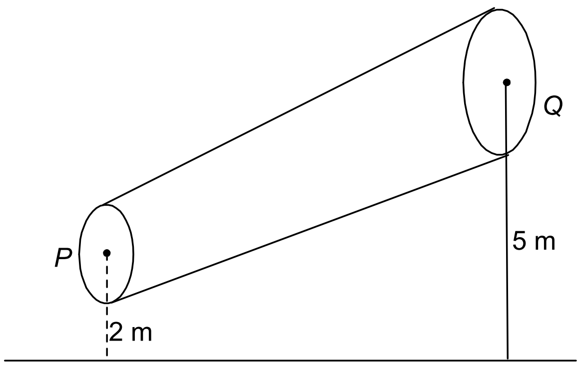
$$v=\sqrt{g\left[\frac{H}{2}+2\left(\frac{H}{2}-h\right)\right]}=\sqrt{g\left[\frac{3H}{2}-2h\right]}$$

The horizontal range $x = vt = \sqrt{g\left[\frac{3H}{2}-2h\right]}\sqrt{\frac{2h}{g}}$

$$=\sqrt{\left(\frac{3H}{2}-2h\right)(2h)}=\sqrt{(3H-4h)h}=\mathbf{5m}$$

***Example* 9:**

A non-viscous liquid of constant density 1000 kg/m³ flows in a streamline motion along a tube of variable cross-section. The tube is kept inclined in the vertical plane as shown in the Figure. The area of cross-section of the tube at two points *P* and *Q* at heights 2 m and 5 m are respectively 4×10^{-3} m² and 8×10^{-3} m². The velocity of the liquid at *P* is 1 m/s. Find the work done per unit volume by the pressure forces as the fluid flows from *P* to *Q*.



Solution:

Work done per unit volume by the pressure force = $\frac{1}{2}\rho\left(v_2^2-v_1^2\right)+\rho g\left(h_2-h_1\right)$

$= -375 + 2.94\times10^4$

$=$ **29025 J/m³**

***Example* 10:**

A tank having cross-sectional area $A = \pi$ m² is filled with water to a height *H*. If a hole of cross-sectional area $a = \sqrt{2}$ m² is made at the bottom of the tank, find the time taken by water level to decrease from $H_1 = 9$ m to $H_2 = 4$ m.

Solution:

Let h be the level of water at any instant. Then rate of decrease of water level is $\frac{-dh}{dt}$. Therefore

$$-A\frac{dh}{dt} = av \qquad = a\sqrt{2gh}$$

$$-\frac{dh}{dt} = \frac{a}{A}\sqrt{2gh}$$

$$-\int_{H_1}^{H_2}\frac{dh}{\sqrt{h}} = \frac{a}{A}\sqrt{2g}\int_0^t dt$$

Integrating, $2\left[\sqrt{H_1} - \sqrt{H_2}\right] = \frac{a}{A}\sqrt{2g}\cdot t$

$\therefore\ t = \frac{A}{a}\sqrt{\frac{2}{g}}\left(\sqrt{H_1} - \sqrt{H_2}\right)$ is the time taken for the level change = **1 s**

***Example* 11:**

A vessel, whose bottom has round holes with diameter of 1 mm is filled with water. Assuming that surface tension acts only at holes, find the maximum height to which the water can be filled in the vessel without leakage. Given that surface tension of water is 75×10^{-3} N/m and $g = 10$ m/s².

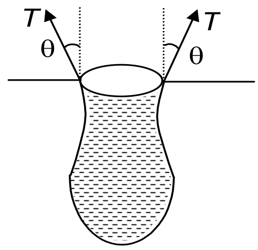
***Solution*:**

As shown in figure here the vertical force due to surface tension at the hole $T\cos\theta \times L = T\cos\theta \times 2\pi r$ will balance the weight mg, i.e., $\pi r^2 h\rho g$, i.e.,

$$T\cos\theta\ 2\pi r = \pi r^2 h\rho g$$

or, $\quad h = (2T\cos\theta/\rho r g)$

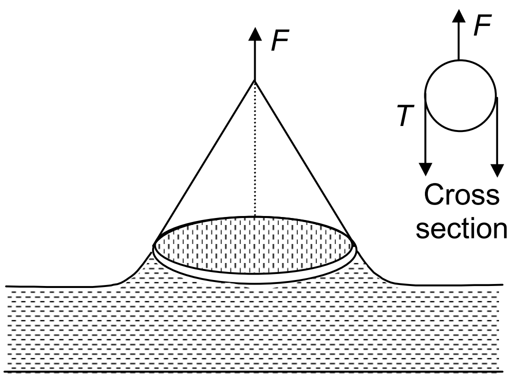
This h will be max when $\cos\theta = \max = 1$



So $(h)_{max} = \dfrac{2\times 75\times 10^{-3}}{10^3\times 5\times 10^{-4}\times 10} = 0.03$ m = **3 cm**

***Example* 12:**

A ring is cut from a platinum tube of 8.5 cm internal and 8.7cm external diameter. It is supported horizontally from a pan of a balance so that it comes in contact with the water in a glass vessel. What is the surface tension of water if an extra 3.97 g weight is required to pull it away from water? ($g = 980$ cm/s²)



***Solution*:**

The ring is in contact with water along its inner and outer circumference; so when pulled out the total force on it due to surface tension will be

$$F = T(2\pi r_1 + 2\pi r_2)$$

So $\quad T = \dfrac{mg}{2\pi(r_1 + r_2)}$ (as $F = mg$)

i.e., $\quad T = \dfrac{3.97\times 980}{2\times 3.14\times\left(\frac{8.5}{2} + \frac{8.7}{2}\right)}$ = **36 dyne/cm**

***Example* 13:**

The lower end of a capillary tube of diameter 2.00 mm is dropped 8.00 cm below the surface of water in a beaker. What is the excess pressure required in the tube to blow a bubble at its end in water? [Surface tension of water = 73 × 10^{-3} N/m, density of water = 10^3 kg/m^3, 1 atmosphere = 1.01 × 10^5 Pa and g = 9.8 m/s^2].

***Solution*:**

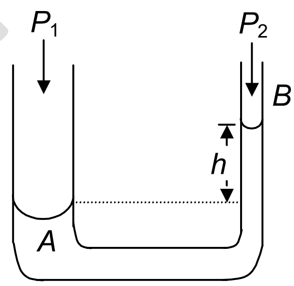
As the bubble is in water, it has only one surface.

So, $p = p_{in} - p_{out} = \dfrac{2T}{r} = \dfrac{2 \times 7.3 \times 10^{-2}}{10^{-3}}$ = **146 Pa**

***Example* 14:**

The limbs of a manometer consist of uniform capillary tubes of radii 1.4 × 10^{-3} m and 7.2 × 10^{-4} m. Find out the correct pressure difference if the level of the liquid (density 10^3 kg/m^3, surface tension 72 × 10^{-3} N/m) in narrower tube stands 0.2 m above that in the broader tube.

***Solution*:**



If p_1 and p_2 are the pressures in the broader and narrower tubes of radii r_1 and r_2 respectively, the pressure just below the meniscus in the respective tubes will be

$$p_1 - \frac{2T}{r_1} \text{ and } p_2 - \frac{2T}{r_2}$$

So that $\left[p_1 - \dfrac{2T}{r_1}\right] - \left[p_2 - \dfrac{2T}{r_2}\right] = h\rho g$

or $\quad p_1 - p_2 = h\rho g - 2T\left[\dfrac{1}{r_2} - \dfrac{1}{r_1}\right]$

Assuming the angle of contact to be zero, i.e., radius of meniscus equal to that of capillary,

$$p_1 - p_2 = 0.2 \times 10^3 \times 9.8 - 2 \times 72 \times 10^{-3}\left[\frac{1}{7.2 \times 10^{-4}} - \frac{1}{14 \times 10^{-4}}\right]$$

$p_1 - p_2 = 1960 - 97 =$ **1863 Pa**

***Example* 15:**

A boat of area 100 m^2 floating on the surface of a river is made to move horizontally with a speed of 20 m/s by applying a tangential force. If the river is 1 m deep and the water in contact with the bed is stationary, find the tangential force needed to keep the boat moving with same velocity. Viscosity of water is 0.01 poise.

***Solution*:**

As velocity changes from 2 m/s at the surface to zero at the bed which is at a depth of 1 m,

Velocity gradient = $\dfrac{dv}{dy} = \dfrac{2-0}{1} = 2s^{-1}$

Now from Newton's law of viscous force,

$|F| = \eta A \dfrac{dv}{dy} = (10^{-2} \times 10^{-1}) \times 100 \times 20 =$ **2 N**

So to keep the boat moving at same velocity, force equal to viscous force, i.e., 2 N must be applied.

***Example* 16:**

Spherical particles of pollen are shaken up in water and allowed to settle. The depth of the water is 2 × 10⁻² m. What is the diameter (in nm) of largest particles remaining in suspension one hour later?
Density of pollen = 1.8 × 10³ kg m⁻³
Viscosity of water = 1 × 10⁻² poise and
Density of water = 1 × 10³ kg/m³

Solution:

For pollen particles not reaching the bottom in 1 hour,

$$v \le \frac{2\times10^{-2}}{60\times60} = \frac{10^{-4}}{18}\ \text{m/s}$$

Due to viscosity effects, the particles will move with terminal velocity *v* given by

$$\frac{4}{3}\pi r^3\rho g = \frac{4}{3}\pi r^3\sigma g + 6\pi\eta r v$$

$$r^2 = \frac{9\eta v}{2g(\rho-\sigma)}$$

Substituting, $\eta = 1\times10^{-2}$ poise $= 10^{-3}\ pl$, $\rho = 1.8\times10^3$ kg/m³, $\sigma = 1\times10^3$ kg/m³, $g = 10$ m/s² and

$$v = \frac{10^{-4}}{18}\ \text{m/s},$$

we have $$r^2 = \frac{9\times10^{-3}}{2}\times\frac{1}{10\,(1.8-1)\times10^3}$$

$\therefore$ $r = 1.77\times10^{-6}$m

and diameter = $2r$ = **3540 nm**

Example 17:

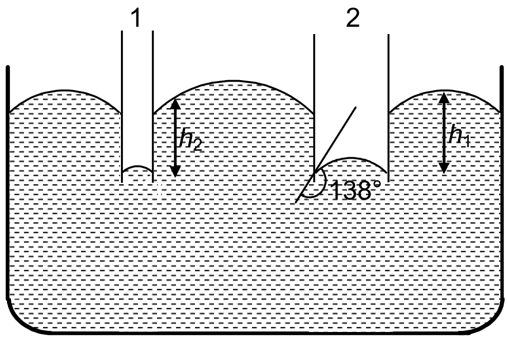
Find the difference in height of mercury columns in two communicating vertical capillaries whose diameters are d_1 = 0.50 mm and d_2 = 1.00 mm, if the contact angle θ = 138°.

Solution:

We have for tube number 1 and 2

$$h_1\rho g = \frac{4T}{d_1}|\cos\theta|$$

$$h_2\rho g = \frac{4T}{d_2}|\cos\theta|$$



$$\rho g(h_1-h_2) = 4T|\cos\theta|\left(\frac{1}{d_1}-\frac{1}{d_2}\right)$$

or $$h_1-h_2 = 4T|\cos\theta|\left(\frac{d_2-d_1}{\rho g d_1 d_2}\right)$$

$$= \frac{4\times(490\times10^{-3})|\cos138^\circ|(1-0.5)10^{-3}}{13.6\times10^{+3}\times9.8\times0.5\times10^{-6}} = \mathbf{11\ mm}$$

Example 18:

A glass rod of diameter d_1 = 1.5 mm is inserted symmetrically into a glass capillary tube with inside diameter d_2 = 2.0 m. Then the whole arrangement is vertically oriented and brought in contact with the surface of water. To what height will the water rise in the capillary?

Solution:

Let *h* be the height of water inside the capillary. Total upward force tending to pull water is

$$T(2\pi r_1 + 2\pi r_2)$$

this supports the weight of the liquid.

$$h(\pi r_2^2 - \pi r_1^2)\rho g$$

Hence, $$h=\frac{2T(r_1+r_2)}{(r_2^2-r_1^2)\rho g}=\frac{2T}{(r_2-r_1)\rho g}=\frac{4T}{(d_2-d_1)\rho g}$$

$$=\frac{4\times73\times10^{-3}}{10^3\times9.8(2-1.5)10^{-3}}=\mathbf{6\ cm}$$

***Example* 19:**

An oil drop falls through air with a terminal velocity of 5 × 10^{-4} m/s. Calculate
(a) radius of the drop (in nm)
(b) the terminal velocity of a drop of half of this radius. (in μm/s)

Solution:

We know that, terminal velocity

$$v=\frac{2}{9}\times\frac{r^2(\rho-\sigma)g}{\eta}=\frac{2}{9}\frac{r^2\rho g}{\eta} \qquad (\text{if } \rho >> \sigma)$$

or $$r=\left[\frac{9}{2}\times\frac{v\eta}{\rho g}\right]^{1/2}$$

Substituting the given values, we have

$$r=\left[\frac{9}{2}\times\frac{(5\times10^{-4})(1.8\times10^{-5})}{900\times9.8}\right]^{1/2}=2.14\times10^{-6}\ \text{m}=\mathbf{2140\ nm}$$

It is obvious that the terminal velocity is directly proportional to r^2. When the radius of the drop is half (i.e., r′ = r/2), let the terminal velocity be v'. Then

$$\frac{v'}{v}=\frac{r'^2}{r^2} \qquad \text{or} \qquad v'=\frac{r'^2}{r^2}\times v$$

or $$v'=\frac{(r/2)^2}{r^2}\times(2.14\times10^{-6})=1.25\times10^{-4}\ \text{m/s}=\mathbf{125\ \mu\ m/s}$$

***Example* 20:**

The diameter of a gas bubble formed at the bottom of a pond is d = 4.0 μm. When the bubble rises to the surface its diameter increases n = 1.1 times, find how deep is the pond at that spot. The atmospheric pressure is standard, the gas expansion is assumed isothermal.
(Surface tension = 0.07 N/m)

Solution:

Apply Boyle's law for the bubble of the bottom and at the top surface.

$$\left(p_0+\frac{4T}{nd}\right)\frac{4\pi}{3}\left(\frac{nd}{2}\right)^3=\left(p_0+h\rho g+\frac{4T}{d}\right)\left(\frac{4\pi}{3}\left(\frac{d}{2}\right)^3\right)$$

or $$p_0+h\rho g+\frac{4T}{d}=\left(p_0+\frac{4T}{nd}\right)n^3$$

or $$h=\frac{1}{\rho g}\left[p_0(n^3-1)+\frac{4T}{d}(n^2-1)\right]=\mathbf{5m}$$

MIND MAP

1. Stress = $\dfrac{F}{A}$

Normal stress = $\dfrac{F_n}{A}$

Tangential stress = $\dfrac{F_t}{A}$

2. Strain

Longitudinal strain = $\dfrac{\Delta L}{L}$

Volume strain = $\dfrac{\Delta V}{V}$

Shearing strain = $\dfrac{x}{L}$

3. **Hook's Law:** within elastic limit stress $\propto$ strain

(i) Young's modulus (Y)

$\dfrac{F}{A} = Y\dfrac{l}{L}$

(ii) Bulk modulus (B)

$\dfrac{F}{A} = -B\dfrac{\Delta V}{V}$

(iii) Modulus of rigidity (η)

$\dfrac{F}{A} = \eta\dfrac{x}{L}$

4. Energy stored in a deformed body per unit volume

$= \dfrac{1}{2} \times$ stress $\times$ strain

FLUID & SOLID

Fluid Statics

1. Variation of pressure in a liquid $\rightarrow$ Pressure varies along the depth of a liquid as ρgh.

2. Force of Buoyancy

= Weight of displaced liquid = $V\rho g$

3. Center of Buoyancy is a point from which force of buoyancy acts. It is center of gravity of displaced liquid.

Fluid Dynamics

1. Continuity equation $A_1V_1 = A_2V_2$

2. Bernoullis theorem $P + \rho gh + \dfrac{1}{2}\rho v^2$ = constant

3. Velocity of efflux

$= \sqrt{\dfrac{2gh}{1-\dfrac{A_2^2}{A_1^2}}} \approx \sqrt{2gh}$

MIND MAP

1. $T = \frac{F}{L}$

$T = \frac{W}{\Delta A}$

Angle of contact:

(i) Acute: Adhesion > cohesion; Meniscus is concave
(ii) Right angle: Adhesion = cohesion; Meniscus is plane
(iii) Obtuse: Adhesion < cohesion; Meniscus is convex.

2. Capillarity:

(i) $h = \frac{2T\cos\theta}{R\rho g} = \frac{2T}{r\rho g}$

(ii) hr = constant

3. Excess pressure:

(i) Inside a drop

$p = \frac{2T}{r} = p_i - p_o$

(ii) Inside a bubble

$p = \frac{4T}{r} = p_i - p_o$

SURFACE TENSION

&

VISCOSITY

4. Newton's Law

$F = -\eta A \frac{dv}{dx}$

5. Stokes Law:

(i) $F = 6\pi\eta rv$
(ii) Terminal velocity

$v_T = \frac{2}{9} r^2 \frac{(\rho - \sigma)}{\eta} g$

EXERCISE - I

NEET-SINGLE CHOICE CORRECT

1. The breaking stress of a wire depends on
(a) material of the wire (b) length of the wire
(c) radius of the wire (d) none of these

2. A wire can be broken by applying a load of 20 kg wt. The force required to break the wire of twice the diameter is
(a) 20 kg wt (b) 5 kg wt (c) 80 kg wt (d) 160 kg wt

3. An elongation of 0.1% in a wire of cross-sectional area 10^{-6} m^2 causes a tension of 100 N. The Young's modulus is
(a) 10^{12} N/m^2 (b) 10^{11} N/m^2 (c) 10^{10} N/m^2 (d) 10^{2} N/m^2

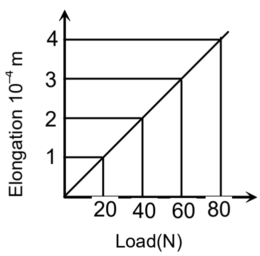
4. A vessel of 1×10^{-3} m^3 volume, contains oil. When a pressure of 1.2×10^5 N/m^2 is applied on it, then volume decreases by 0.3×10^{-6} m^3. The bulk modulus of oil is
(a) 1×10^6 N/m^2 (b) 2×10^7 N/m^2 (c) 4×10^8 N/m^2 (d) 6×10^{10} N/m^2

5. A body of mass 10 kg is attached to a wire of length 0.3m. The maximum angular velocity with which it can be rotated in a horizontal circle is: (Breaking stress of wire = 4.8×10^7 N/m^2 and area of cross-section of a wire = 10^{-6} m^2)
(a) 4 rad/s (b) 8 rad/s (c) 1 rad/s (d) 2 rad/s

6. A wire can sustain the weight of 20 kg before breaking. If the wire is cut into two equal parts, each part can sustain a weight of
(a) 10 kg (b) 20 kg (c) 40 kg (d) 80 kg

7. The length of a metal wire is l_1 when the tension in it is T_1 and is l_2 when the tension is T_2. The natural length of the wire is
(a) $\dfrac{l_1+l_2}{2}$ (b) $\sqrt{l_1 l_2}$ (c) $\dfrac{l_1T_2-l_2T_1}{T_2-T_1}$ (d) $\dfrac{l_1T_2+l_2T_1}{T_2+T_1}$

8. A massless rod of length 1 m is hanged from the ceiling at one end and the other end is connected to the weight W. Cross-sectional area of the wire is 10^{-6} m^2. With reference to the above graph, Young's modulus of the material of the wire (in N/m^2) is



(a) 2×10^{11} (b) 2×10^{-5}
(c) 5×10^4 (d) 5×10^{10}

9. A body of mass 0.5 kg which just floats in a liquid is attached to a thread which just floats in a liquid. The tension in the thread is

(a) 0.5 kg wt
(b) more than 0.5 kg wt
(c) less than 0.5 kg wt
(d) zero

10. A barometer kept in an elevator accelerating upward reads 76 cm. The air pressure in the elevator is

(a) 76 cm
(b) < 76 cm
(c) > 76 cm
(d) zero

11. The volume of a liquid flowing per sec out of an orifice at the bottom of a tank does not depend upon

(a) the height of the liquid above the orifice
(b) the acceleration due to gravity
(c) the density of the liquid
(d) the area of the orifice

12. The pressure just below the meniscus of water

(a) is greater than just above it
(b) is lesser than just above it
(c) is same as just above it
(d) is always equal to atmospheric pressure

13. A tank is filled with water and an orifice is made in the wall so that the horizontal range x of water rushing out is maximum. If H is the height of water in the tank, then

(a) $x = H$
(b) $x = 2H$
(c) $2x = H$
(d) $4x = H$

14. A sphere of wax of relative density 0.9 and volume 18 cm^3 has some iron nails pierced into it. It floats on water just completely immersed. The mass of iron nails into it is

(a) 0.9 g
(b) 1.8 g
(c) 2.7 g
(d) 3.6 g

15. A cubical block of wood of specific gravity 0.5 and chunk of concrete of specific gravity 2.5 are fastened together. The ratio of the mass of wood to the mass of concrete which makes the combination to float with its entire volume submerged under water is

(a) $\frac{3}{5}$
(b) $\frac{4}{5}$
(c) $\frac{3}{7}$
(d) $\frac{2}{7}$

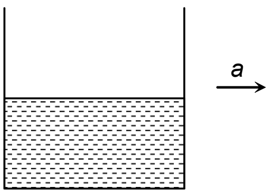
16. A rectangular block of wood measuring 3 m × 1 m × 1 m of relative density 0.75 floats in water with its long axis horizontal and four faces vertical. The work done in lifting the block vertically out of water is

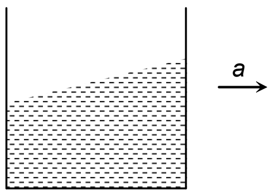
(a) 3000 J
(b) 8269 J
(c) 9500 J
(d) 4280 J

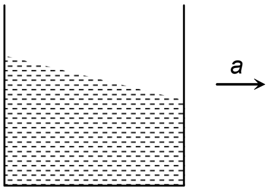
17. A boat with scrap iron is floating in a lake. If the scrap iron is thrown in the lake the water level will

(a) Go up
(b) Go down
(c) Remain unchanged
(d) None of these

18. A vessel containing water is given a constant acceleration 'a' to the right along a horizontal path. Which one of the diagrams represents the surface of the liquid?

(a) 

(b) 

(c) 

(d) none of these

19. The gate of a canal is 5 m wide, and the water levels on either side of it are 20 m and 12m. The resultant thrust on the gate (g = 10 m/s^2) is

(a) 64×10^5 N (b) 64×10^3 N (c) 64×10^4 N (d) 64×10^7 N

20. Air is pushed into a soap bubble of radius r to double its radius. If the surface tension of the soap solution is S, the work done in the process is

(a) $8\pi r^2 S$ (b) $12\pi r^2 S$ (c) $16\pi r^2 S$ (d) $24\pi r^2 S$

21. A glass capillary tube of internal radius r = 0.25 mm is immersed in liquid. The liquid level rises to a height of 2cm in the tube. At what angle does the liquid meet the tube? Surface tension of liquid = 0.07 N m^{-1}

(a) $\sin^{-1}(0.5)$ (b) $\cos^{-1}(0.5)$ (c) $\cos^{-1}(0.35)$ (d) $\sin^{-1}(0.35)$

22. Rain drops fall from a height under gravity; we observe that

(a) their velocities go on increasing until they hit the ground but the velocity with which the drops hit the ground depends on the radius of the rain drop

(b) their velocities go on increasing until they hit the ground, velocity being independent of the radius of the drop

(c) they fall with a terminal velocity which is independent of the radius of the rain drop

(d) they fall with a terminal velocity which depends upon the radius of the rain drop

23. The terminal velocity of a rain drop of radius 0.01 mm and density 1g/cm^3 falling in air of density 1.2 kg/m^3 and viscosity 1.8×10^{-5} N-s/m^2 and will be

(a) 1.5 cm/s (b) 1.2 cm/s (c) 1.0 cm/s (d) 0.9 cm/s

24. The velocity of a small ball of mass m and density d_1 when dropped in a container filled with glycerine becomes constant after some time. What is the viscous force acting on the ball if density of glycerine is d_2?

(a) $mg\left[1-\frac{d_1}{d_2}\right]$ (b) $mg\left[1-\frac{d_2}{d_1}\right]$ (c) $mg\left[1+\frac{d_2}{d_1}\right]$ (d) $mg\left[1+\frac{d_1}{d_2}\right]$

25. What is the velocity V of a metallic ball of radius R falling in a tank of liquid at the instant? When its acceleration is one half that of a freely falling body? (The densities of metal and of liquid are ρ and σ respectively and the viscosity of liquid is η)

(a) $\frac{gr^2}{9\eta}[\rho-2\sigma]$ (b) $\frac{gr^2}{9\eta}[2\rho-\sigma]$ (c) $\frac{gr^2}{9\eta}[\rho-\sigma]$ (d) $\frac{2r^2 g}{9\eta}[\rho-\sigma]$

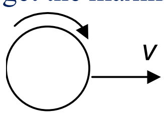
EXERCISE - II

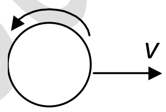
IIT-JEE- SINGLE CHOICE CORRECT

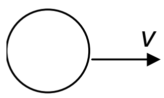
1. Two blocks *A* and *B* made of iron and aluminium respectively have exactly the same weight. They are completely immersed in water and weighed. If the densities of iron and aluminium are 8000 kg/m^3 and 2700 kg/m^3, then

(a) *A* will weigh more than *B*
(b) *B* will weigh more than *A*
(c) *A* and *B* will weigh the same as before
(d) data insufficient

2. To get the maximum flight, a ball must be thrown as

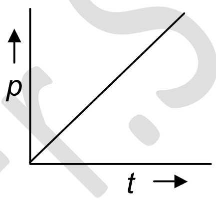
(a) 

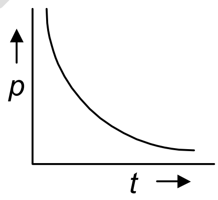
(b) 

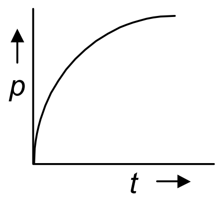
(c) 

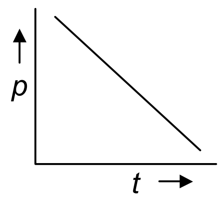
(d) any of (a), (b) and (c)

3. A soap bubble is blown slowly at the end of a tube by a pump supplying air at a constant rate. Which one of the following graphs represents the correct variation of the excess of pressure inside the bubble with time

(a) 

(b) 

(c) 

(d) 

4. A solid uniform ball having volume *V* and density ρ floats at the interface of two immiscible liquids as shown in figure. The densities of the upper and the lower liquids are ρ_1 and ρ_2 respectively, such that $\rho_1 < \rho < \rho_2$. The fraction of the volume of the ball in the lower liquid is

(a) $\frac{\rho - \rho_2}{\rho_1 - \rho_2}$
(b) $\frac{\rho_1}{\rho_1 - \rho_2}$
(c) $\frac{\rho - \rho_1}{\rho_2 - \rho_1}$
(d) $\frac{\rho_1 - \rho_2}{\rho_2}$

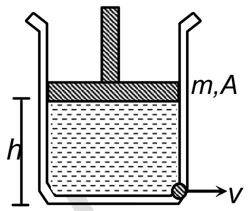
5. The Young's modulus of brass and steel are respectively 10×10^{10} N/m^2 and 20×10^{10} N/m^2. A brass wire and a steel wire of the same length are extended by 1 mm under the same force, the radii of brass and steel wires are R_B and R_S respectively. Then

(a) $R_S = \sqrt{2} R_B$
(b) $R_S = \frac{R_B}{\sqrt{2}}$
(c) $R_S = 4R_B$
(d) $R_S = \frac{R_B}{4}$

6. A wire of length *L* and cross-sectional area *A* is made of a material of Young's modulus *Y*. If the wire is stretched by an amount *x*, the work done is

(a) $\frac{YAx^2}{2L}$ (b) $\frac{YAx}{2L^2}$ (c) $\frac{YAx}{2L}$ (d) $\frac{YAx^2}{L}$

7. A cylindrical vessel contains a liquid of density ρ upto a height *h*. The liquid is closed by a piston of mass *m* and area of cross section *A*. There is a small hole at the bottom of the vessel. The speed *v* with which the liquid comes out of the hole is



(a) $\sqrt{2gh}$ (b) $\sqrt{2\left(gh+\frac{mg}{\rho A}\right)}$ (c) $\sqrt{2\left(gh+\frac{mg}{A}\right)}$ (d) $\sqrt{2gh+\frac{mg}{A\rho}}$

8. When a certain weight is suspended from a long uniform wire, its length increases by one cm. If the same weight is suspended from another wire of the same material and length but having a diameter half of the first one, the increase in length will be

(a) 0.5 cm (b) 2 cm (c) 4 cm (d) 8 cm

9. A uniformly tapering vessel of height *h* whose lower and upper radii are *r* and *R* is completely filled with a liquid of density □. The force that acts on the base of the vessel due to the liquid is

(a) $\pi R^2 h\rho g$ (b) $\pi r^2 h\rho g$ (c) $\pi\left(\frac{R+r}{2}\right)^2 h\rho g$ (d) $\frac{1}{3}\pi(R^2-r^2)h\rho g$

10. Two parallel glass plates having separation '*d*' are dipped in water. Some water rise up in the gap between the plates. The surface tension of water is *S*, atmospheric pressure = P_0, pressure of water just below the water surface in the region between the plates is *P*, find the relation between them

(a) $P=P_0-\frac{2S}{d}$ (b) $P=P_0+\frac{2S}{d}$ (c) $P=P_0-\frac{4S}{d}$ (d) $P=P_0+\frac{4S}{d}$

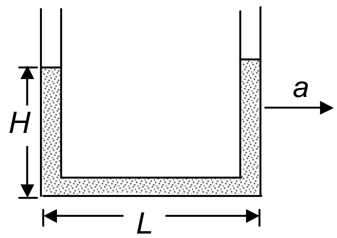
11. When equal volumes of two substances are mixed, the specific gravity of mixture is 4. When equal weights of the same substances are mixed, the specific gravity of the mixture is 3. The specific gravity of the two substances would be

(a) 6 and 2 (b) 3 and 4 (c) 2.5 and 3.5 (d) 5 and 3

12. One end of a uniform wire of length *L* and of weight *W* is attached rigidly to a point in the roof and a weight W_1 is suspended from its lower end. If *S* is the area of cross-section of the wire, the stress in the wire at a height (3*L*/4) from its lower end is

(a) W_1/S (b) $[W_1+(W/4)]/S$ (c) $[W_1+(3W/4)]/S$ (d) $[W_1+W]/S$

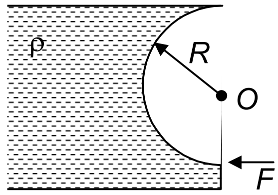
13. A liquid stands at the same level in the U-tube when at rest. If area of cross-section of both the limbs are equal, the difference in heights *h* of the liquid in the two limbs of *U*-tube, when the system is given an acceleration *a* in horizontal direction as shown, is



(a) $\frac{gL^2}{aH}$ (b) $\frac{La}{g}$

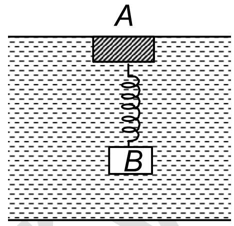
(c) $\frac{L^2a}{Hg}$ (d) $\frac{Hg}{a}$

14. The figure shows a semi-cylindrical massless gate (of width R) pivoted at the point O holding a stationary liquid of density ρ. A horizontal force F is applied at its lowest position to keep it stationary. The magnitude of the force is



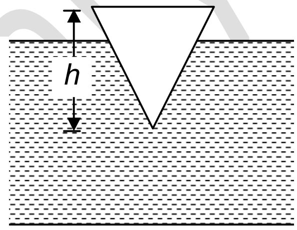
(a) $\frac{9}{2}\rho g R^3$ (b) $\frac{3}{2}\rho g R^3$
(c) $\rho g R^3$ (d) zero

15. A block A of mass 10 kg, connected to another hollow block B of same size and negligible mass, by a spring of spring constant 500 N/m, floats in water as shown in the figure. The compression in the spring is
($\rho_{water} = 1 \times 10^3$ kg/m^3, $g = 10$ m/s^2)



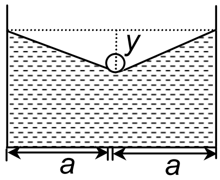
(a) 10 cm (b) 20 cm
(c) 50 cm (d) 100 cm

16. A conical block, floats in water with 90% height immersed in it. Height h of the block is equal to the diameter of the block i.e., 20 cm. The mass to be kept on the block, so that the block just floats at the surface of water, is



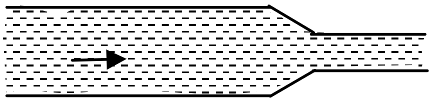
(a) 568 g (b) 980 g
(c) 112 g (d) 196 g

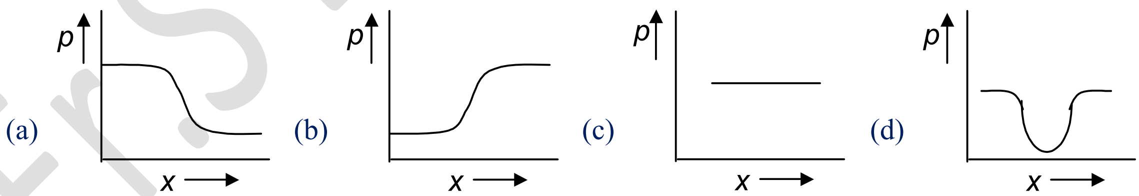
17. A uniform wire having mass per unit length λ is placed over a liquid surface. The wire causes the liquid to depress by $y(y << a)$ as shown in figure, then the value of surface tension of liquid is (neglect end effect).



(a) $\frac{a\lambda g}{4y}$ (b) $\frac{agy}{2\lambda}$ (c) $\frac{agy}{\lambda}$ (d) $\frac{a\lambda g}{2y}$

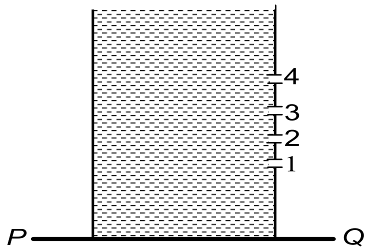
18. Water flows through a frictionless duct with a cross-section varying as shown in figure. Pressure p at points along the axis is represented by





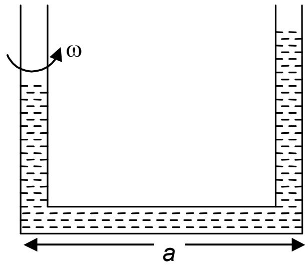
(a) (b) (c) (d)

19. A cylindrical vessel of 90 cm height is kept filled upto brim. It has four holes 1,2,3,4 which are respectively at heights of 20 cm, 30 cm, 45 cm and 50 cm from the horizontal floor PQ. The water falling at the maximum horizontal distance from the vessel comes from



(a) hole number 4 (b) hole number 3
(c) hole number 2 (d) hole number 1

20. A manometer shown in the figure contains a liquid of density ρ. Find the difference in the levels when the manometer rotates with a constant angular velocity ω about one of its vertical limbs.



(a) $\frac{\omega^2 a^2}{g}$ (b) $\frac{\omega^2 a^2}{4g}$ (c) $\frac{\omega^2 a^2}{2g}$ (d) none of these

ONE OR MORE THAN ONE CHOICE CORRECT

1. A metal wire of length *L*, area of cross-section *A* and young's modulus *Y* is stretched by a variable force *F* such that *F* is always slightly greater than the elastic forces of resistance in the wire. When the elongation of the wire is *l*,

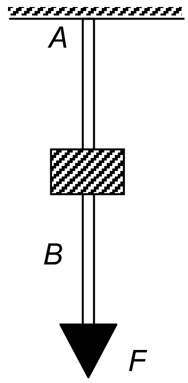
(a) the work done by *F* is $\frac{YAl^2}{L}$

(b) the work done by *F* is $\frac{YAl^2}{2L}$

(c) the elastic potential energy stored in the wire is $\frac{YAl^2}{2L}$

(d) heat is produced during the elongation.

2. The wires *A* and *B* shown in the figure are made of the same material, and have radii r_A and r_B respectively. The block between them has a mass *m*. When the force *F* is *mg*/3, one of the wires breaks. Then



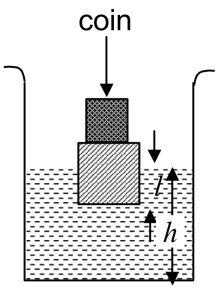
(a) A will break before *B* if $r_A = r_B$

(b) A will break before *B* if $r_A < 2r_B$

(c) Either *A* or *B* may break if $r_A = 2r_B$

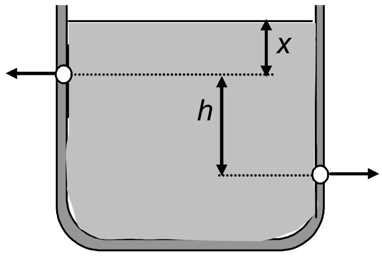
(d) The lengths of *A* and *B* must be known to predict which wire will break

3. A wooden block, with a coin placed on its top, floats in water as shown in figure. The distances *l* and *h* are shown there. After some time, the coin falls into water, then



(a) *l* decreases (b) *l* increases
(c) *h* increases (d) *h* decreases

4. There are two identical small holes on the opposite sides of a tank containing a liquid. The tank is open at the top. The difference in height between the two holes is h. As the liquid comes out of the two holes,



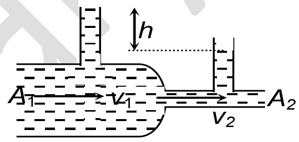
(a) the tank will experience a net horizontal force proportional to $\sqrt{h}$

(b) the tank will experience a net horizontal force proportional to h

(c) mass of liquid discharged per second = $\alpha v \rho\, h^{3/2}$

(d) mass of liquid discharged per second can not be determined from the given data.

5. A non-viscous liquid flows though a horizontal tube. The velocities of the liquid in the two sections, which have areas of cross section A_1 and A_2 are v_1 and v_2 respectively. The difference in the levels of the liquid in the two vertical tubes is h.



(a) the volume of the liquid flowing through the tube in unit time is A_1v_1
(b) $v_2 - v_1 = \sqrt{2gh}$
(c) $v_2^2 - v_1^2 = 2gh$
(d) the energy per unit mass of the liquid is the same in both sections of the tube.

6. A glass capillary of length 0.11 m is sealed at the upper end and having internal diameter 2×10^{-5}m. The capillary is immersed vertically into a liquid very slowly of surface tension 5.0×10^{-2} N/m in such a way that the liquid level inside and outside the capillary becomes same. (atmospheric pressure $P_0 = 1.01 \times 10^5$ N/m, angle of contact is zero)
(a) the pressure of gas inside the tube after immersing into the liquid is 1.0606×10^5 N/m
(b) the length by which the capillary has to be immersed so that the liquid level inside and outside the capillary becomes same is 0.01 m
(c) the radius of meniscus inside tube is 1.00×10^{-5} m
(d) the radius of meniscus inside tube is 2.00×10^{-5} m

7. Choose the correct alternatives
(a) If the radius of a soap bubble A is four times that of another soap bubble B, then the ratio of excess pressure (P_B/P_A) will be 4 : 1
(b) If two small drops of mercury, each of radius R coalesce to form a single large drop, the ratio of the total surface energy before and after change will be $2^{1/3}$: 1
(c) The energy required to blow a bubble of radius 4 cm and 3 cm in the same liquid is in the ratio of 16 : 9
(d) Two soap bubbles are blown. In the first bubble excess pressure in 4 times that of the second soap bubble. The ratio of radii of first to second soap bubble is 1 : 4

8. When an air bubble rises from the bottom of a deep lake to a point just below the water surface, the pressure of air inside the bubble
(a) is less than the pressure outside it
(b) is greater than the pressure outside it
(c) decreases as the bubble moves up
(d) increases as the bubble moves up

9. A small sphere of mass m is dropped from a great height. After it has fallen 100m it has attained its terminal velocity and continues to fall at that speed. The work done by air friction against the sphere during the first 100 m of fall is

(a) greater than the work done by air friction in the second 100 m
(b) less than the work done by air friction in the second 100 m
(c) equal to 100 mg
(d) less than 100 mg

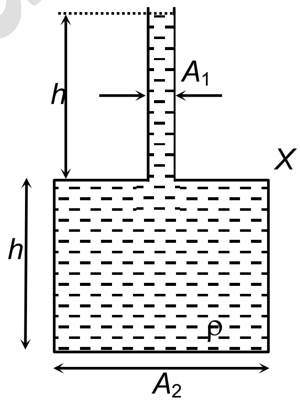
10. A tube of length l and radius R carries a steady flow of fluid whose density is ρ and viscosity η. The velocity v of flow is given by $v = v_0\left(\frac{R^2 - r^2}{R^2}\right)$, where r is the distance of the flowing fluid from the axis. Then
(a) volume of fluid, flowing across the section of the tube, in unit time is $2\pi v_0 \frac{R^2}{4}$.
(b) kinetic energy of the fluid within the volume of the tube is $\pi\rho l v_0^2 \frac{R^2}{6}$
(c) the frictional force exerted on the tube by the fluid is $4\pi\eta l v_0$
(d) the difference of pressures at the ends of the tube is $\frac{4\eta l v_0}{R^2}$

EXERCISE - III

MATCH THE FOLLOWING

Note: Each statement in column – I has one or more than one match in column –II.

1. The vessel shown in the figure has two sections of area of cross-section A_1 and A_2. A liquid of density ρ fills both the sections, up to a height h in each. Neglect atmospheric pressure. (Assume $A_2 > 3A_1$)



Column-I		Column-II	
I.	The net force exerted by liquid on side walls is	**A.**	greater than $2h\rho g A_1$
II.	The force exerted by the liquid on the base of the vessel is	**B.**	zero
III.	The weight of the liquid is	**C.**	$h\rho g (A_1 + A_2)$
IV.	The walls of the vessel at the level X exert a downward force is	**D.**	Less then $2h\rho g A_2$
		E.	$h \rho g (A_2 - A_1)$ on the liquid

REASONING TYPE

Directions: Read the following questions and choose
(A) **If both the statements are true and statement-2** is the correct explanation of **statement-1**.
(B) **If both the statements are true but statement-2** is not the correct explanation of **statement-1**.
(C) **If statement-1** is True and **statement-2** is False.

(D) If statement-1 is False **and statement-2** is True.

1. **Statement-1:** The impurities always decrease the surface tension of a liquid.
Statement-2: The change in surface tension of the liquid depends upon the degree of contamination of the impurity.
(a) (A) (b) (B) (c) (C) (d) (D)

2. **Statement-1:** The viscosity of liquid increases rapidly with rise of temperature.
Statement-2: Viscosity of a liquid is the property of liquid by virtue of which it opposes the relative motion amongst its different layer's.
(a) (A) (b) (B) (c) (C) (d) (D)

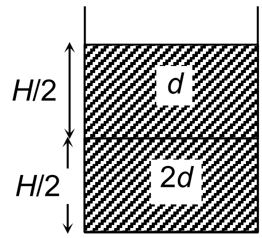
3. **Statement-1:** For a floating body to be in stable equilibrium, its centre of buoyancy must be located above the centre of gravity.
Statement-2: The torque formed by the weight of the body and the upthrust will restore the body back to its normal position, after the body is disturbed.
(a) (A) (b) (B) (c) (C) (d) (D)

4. **Statement-1:** All the raindrops hit the surface of the earth with the same constant velocity.
Statement-2: An object falling through a viscous medium eventually attains a terminal velocity.
(a) (A) (b) (B) (c) (C) (d) (D)

5. **Statement-1:** Water flows faster than honey.
Statement-2: The co-efficient of viscosity of water is less than honey.
(a) (A) (b) (B) (c) (C) (d) (D)

LINKED COMPREHENSION TYPE

A container of large uniform cross section area A resting on a horizontal surface, holds two immiscible, non viscous and incompressible liquids of densities d and $2d$ each of height $H/2$ as shown in figure. The lower density liquid is open to atmosphere having pressure P_0. A homogeneous solid cylinder of length $L(L < H/2)$, cross section area $A/5$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $(L/4)$ in the denser liquid.



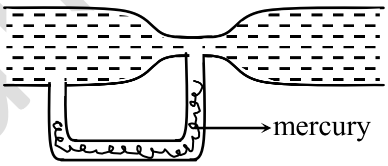
1. What is the density of cylinder?
(a) $\frac{3d}{2}$ (b) $3d$ (c) $\frac{5d}{4}$ (d) $\frac{2d}{3}$

2. Total pressure at the bottom of container is
(a) $P_0 + \left(\frac{6H+L}{4}\right)dg$ (b) $P_0 + \left(\frac{3H+L}{4}\right)dg$
(c) $P_0 + \left(\frac{3H+2L}{4}\right)dg$ (d) $P_0 + \left(\frac{2H+3L}{6}\right)dg$

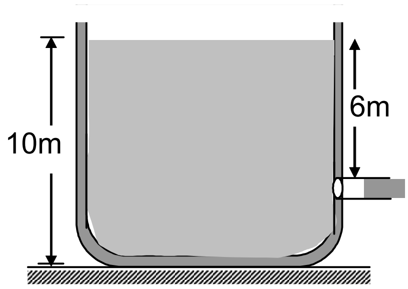
3. The net horizontal force exerted by upper liquid on the cylinder is
(a) zero (b) $\frac{3L}{24}\frac{A}{5}dg$ (c) $Aldg$ (d) $\frac{1}{4}\frac{A}{5}\partial g$

EXERCISE - IV

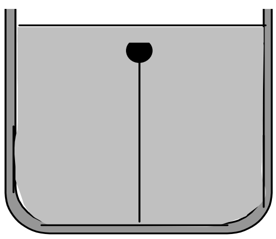
1. Water flows through the tube as shown in the figure. The area of cross section of the wide and the narrow portions of the tube are 5 cm^2 and 2 cm^2 respectively. The rate of flow of water through the tube is 500 cm^3/s. Find the difference in mercury levels in the U-tube.



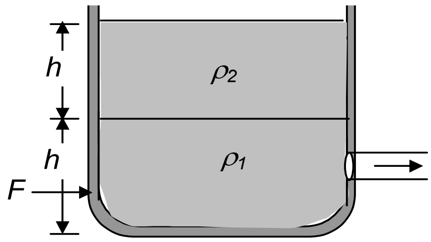
2. A fresh water on a reservoir is 10 m deep. A horizontal pipe 4.0 cm in diameter passes though the reservoir 6.0 m below the water surface as shown in figure. A plug secures the pipe opening.
(a) Find the friction force between the plug and pipe wall.
(b) The plug is removed. What volume of water flows out of the pipe in 1 hr? Assume area of reservoir to be too large. ($g = 10$ m/s^2)



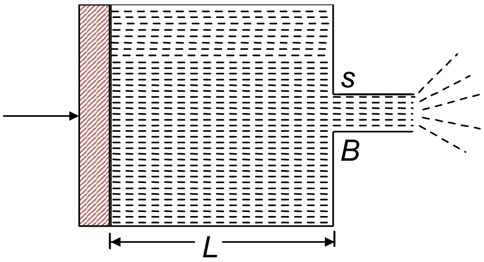
3. A solid sphere of mass m = 2 kg and specific gravity s = 0.5 is held stationary relative to a tank filled with water as shown in figure. The tank is accelerating vertically upward with acceleration $a = 2$ ms^{-2}.
(a) Calculate tension in the thread connected between the sphere and the bottom of the tank.
(b) If the thread snaps, calculate acceleration of sphere with respect to the tank.
(density of water is $\rho = 1000$ kg m^{-3}) ($g = 10$ m/s^2)



4. A cylindrical tank having cross sectional area A = 5.0 m^2 is filled with two liquids of density ρ_1 = 900 kg m^{-3} and ρ_2 = 600 kg m^{-3} to height h = 60 cm each as shown in figure. A small hole having area a = 5 cm^2 is made in right vertical wall at a height y = 20 cm from the bottom. Calculate:
(a) velocity of efflux
(b) horizontal force F to keep the cylinder in static equilibrium, if it is placed on a smooth horizontal plane.
(c) minimum and maximum values of F to keep the cylinder initially in static equilibrium, if coefficient of friction between the cylinder and the plane is μ = 0.001.(g = 10 m/s^2)

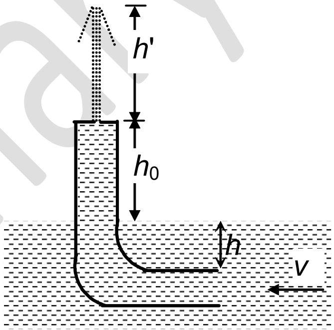


5. What work should be done in order to squeeze all water from a horizontally located cylinder as shown in the figure during the time t by means of a constant force acting on the piston? Volume of water in the cylinder is V cross-sectional area of the orifice is s , with s being considerably less than the piston area. The friction and viscosity are negligibly small. Density of water is ρ.

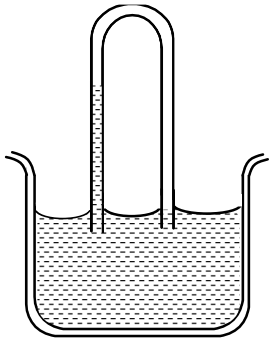


6. A cylindrical vessel of radius r with water in it is rotated about its vertical axis with a constant angular velocity ω. (a) Show that the shape of the free surface of the liquid is parabolic. (b) What will be the pressure at a point situated at the bottom situated at a distance $\frac{r}{2}$ from the axis if the pressure at the centre of bottom is P_0?

7. A bent tube is lowered into a stream of water as shown in figure. The velocity of stream V is 2.5 m/s. The closed upper end of the tube is at a height h_0 = 12 cm from the surface of water in the stream and has an orifice. To what height h' will the water jet from the orifice spurt? (g = 9.8 m/s^2)

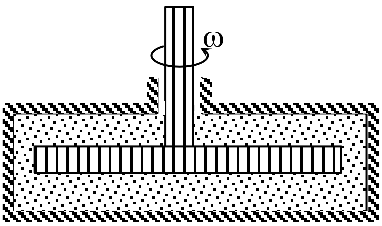


8. A glass U-tube is inverted with open ends of the straight limbs, of diameters 0.5 mm and 1.0 mm below the surface of water in a beaker. The air pressure in the upper part is increased until the meniscus in one limb is level with the water outside. Find the height of water (in mm)in the other limb. Density of water is 10^3 kg/m^3 and surface tension of water is 7.5×10^{-2} N/m. Take contact angle θ = 0^0.



9. Two glass discs of radius R were wetted with water and put together so that the thickness of the water layer between them as h. Assuming the wetting to be complete, find the force that has to be applied at right angles to the plates in order to pull them apart (surface tension of water is T).

10. A thin horizontal disc of radius R = 10 cm is located with in a cylindrical cavity filled with oil whose viscosity η = 0.08 P. The clearance between the disc and the horizontal planes of the cavity is equal to h = 1.0 mm. Find the power developed by the viscous forces acting on the disc when it rotates with the angular velocity ω = 60 rad/s. The end effects are to be neglected.



ANSWERS

EXERCISE - I

NEET-SINGLE CHOICE CORRECT

1. (a)	2. (c)	3. (b)	4. (c)	5. (a)
6. (b)	7. (c)	8. (a)	9. (d)	10. (a)
11. (c)	12. (b)	13. (a)	14. (b)	15. (a)
16. (b)	17. (b)	18. (c)	19. (a)	20. (d)
21. (c)	22. (d)	23. (b)	24. (b)	25. (a)

EXERCISE - II

IIT-JEE-SINGLE CHOICE CORRECT

1. (a)	2. (b)	3. (b)	4. (c)	5. (b)
6. (a)	7. (b)	8. (c)	9. (b)	10. (a)
11. (a)	12. (c)	13. (b)	14. (d)	15. (a)
16. (a)	17. (d)	18. (a)	19. (b)	20. (c)

ONE OR MORE THAN ONE CHOICE CORRECT

1. (b,c,d)	2. (a,b,c)	3. (a,d)	4. (b,d)	5. (a,c,d)
6. (b,c)	7. (a,b,c,d)	8. (b,c)	9. (b,d)	10.(a,b,c,d)

EXERCISE - III

MATCH THE FOLLOWING

1. I – B, II – A, III – A, C, D, IV – A, D, E

REASONING TYPE

1. (a)	2. (d)	3. (a)	4. (a)	5. (a)

LINKED COMPREHENSION TYPE

1. (c)	2. (a)	3. (a)

EXERCISE - IV

SUBJECTIVE PROBLEMS

1. 1.97 cm

2. (a) 74 N (b) 49 m^3

3. (a) 24 N (b) 12 m/s^2

4. (a) 4 m/s (b) 7.2 N (c) 0, 52.2 N

5. $\frac{1}{2}\rho\left(\frac{V^3}{s^2t^2}\right)$

6. $P_0 + \frac{\rho\omega^2 r^2}{8}$

7. 0.2 m

8. 31 mm

9. $F = 2\pi R^2\, T/h$

10. 9 W

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 1

Q.1 The Young's modulus of a rubber string 8 cm long and density 1.5 kg/m^3 is 5×10^8 N/m^2, is suspended on the ceiling in a room. The increase in length due to its down weight will be -

(1) 9.6×10^{-5} m (2) 9.6×10^{-11} m

(3) 9.6×10^{-3} m (4) 9.6 m

Q.2 The bulk modulus of a metal is 10^{10} N/m^2 and poisson's ratio 0.20. If average distance between the molecules is 3Å, then the inter atomic force constant-

(1) 5.4 N/m (2) 7.5 N/m

(3) 75 N/m (4) 30 N/m

Q.3 For a constant hydraulic stress on an object, the fractional change in the object's volume $\left(\frac{\Delta V}{V}\right)$ and its bulk modulus (B) are related as -

(1) $\frac{\Delta V}{V} \propto B$ (2) $\frac{\Delta V}{V} \propto \frac{1}{B}$

(3) $\frac{\Delta V}{V} \propto B^2$ (4) $\frac{\Delta V}{V} \propto B^{-2}$

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM - 2

Q.1 When a certain weight is suspended from a long uniform wire, its length increases by one cm. If the same weight is suspended from another wire of the same material and length but having a diameter half of the first one then the increase in length will be -
(1) 0.5 cm (2) 2 cm (3) 4 cm (4) 8 cm

Q.2 The area of cross-section of a wire of length 1.1 meter is $1mm^2$. It is loaded with 1 kg. If Young's modulus of copper is 1.1×10^{11} N/m^2, then the increase in length will be (If g = 10 m/s^2) -
(1) 0.01 mm (2) 0.075 mm
(3) 0.1 mm (4) 0.15 mm

Q.3 Two wires of same diameter of the same material having the length ℓ and 2ℓ. If the force F is applied on each, the ratio of the work done in the two wires will be -
(1) 1 : 2 (2) 1 : 4
(3) 2 : 1 (4) 1 : 1

Q.4 On increasing the length by 0.5 mm of a steel wire of length 2 m and area of cross-section 2 mm^2, the force required is - [Y for steel = 2.2×10^{11}N/m^2]
(1) 1.1×10^5 N (2) 1.1×10^4 N
(3) 1.1×10^3 N (4) 1.1×10^2 N

Q.5 In CGS system, the Young's modulus of a steel wire is 2×10^{12} dyne/cm^2. To double the length of a wire of unit cross-section area, the force required is -
(1) 4×10^6 dynes (2) 2×10^{12} dynes
(3) 2×10^{14} dynes (4) 2×10^8 dynes

Q.6 The material which practically does not show elastic after effect is -
(1) Copper (2) Rubber
(3) Steel (4) Quartz

Q.7 A force F is needed to break a copper wire having radius R. The force needed to break a copper wire of radius 2R will be-
(1) $\frac{F}{2}$ (2) 2F (3) 4F (4) $\frac{F}{4}$

Q.8 The ratio of the adiabatic to isothermal elasticities of a triatomic gas is -
(1) $\frac{3}{4}$ (2) $\frac{4}{3}$ (3) 1 (4) $\frac{5}{3}$

Q.9 A brass rod of cross-sectional area 1 cm^2 and length 0.2 m is compressed lengthwise by a weight of 5 kg. If Young's modulus of elasticity of brass is 1×10^{11} N/m^2 and g = 10 m/sec^2, then increase in the energy of the rod will be-
(1) 10^{-5} Joule (2) 2.5×10^{-5} Joule
(3) 5×10^{-5} Joule (4) 2.5×10^{-4} Joule

Q.10 A wire of length L and radius r is rigidly fixed at one end. On stretching the other end of the wire with a force F, the increase in its length is ℓ. If another wire of same material but of length 2L and radius 2r is stretched with a force of 2F, then increase in its length will be-
(1) ℓ (2) 2ℓ (3) $\frac{\ell}{2}$ (4) $\frac{\ell}{4}$

Q.11 In steel, the Young's modulus and the strain at the breaking point are 2×10^{11} N/m^2 and 0.15 respectively. The stress at the breaking point for steel is therefore -
(1) 1.33×10^{11} Nm^{-2} (2) 1.33×10^{12} Nm^{-2}
(3) 7.5×10^{-13} Nm^{-2} (4) 3×10^{10} Nm^{-2}

Q.12 Which of the following statements is correct:
(1) Hooke's law is applicable only within elastic limit
(2) The adiabatic and isothermal elastic constants of a gas are equal
(3) Young's modulus is dimensionless
(4) Stress multiplied by strain is equal to the stored energy

Q.13 Which one of the following substance possesses the highest elasticity -
(1) Rubber (2) Glass
(3) Steel (4) Copper

Q.14 Two wires of the same material have lengths in the ratio 1 : 2 and their radii are in the ratio $1:\sqrt{2}$. If they are stretched by applying equal forces, the increases in their lengths will be in the ratio -
(1) 2 : 1 (2) $\sqrt{2}:2$ (3) 1 : 1 (4) 1 : 2

Q.15 When a weight of 10 kg is suspended from a copper wire of length 3 meters and diameter 0.4 mm, its length increases by 2.4 cm. If the diameter of the wire is doubled, then the extension in its length will be -
(1) 9.6 cm (2) 4.8 cm
(3) 1.2 cm (4) 0.6 cm

Q.16 When a force is applied on a wire of uniform cross-sectional area $3 \times 10^{-6} m^2$ and length 4m, the increase in length is 1 mm. Energy stored in it will be - ($Y = 2 \times 10^{11} N/m^2$)
(1) 6250 Joule (2) 0.177 Joule
(3) 0.075 Joule (4) 0.150 Joule

Q.17 A force of 10^3 newton stretches the length of a hanging wire by 1 millimetre. The force required to stretch a wire of same material and length but having four times the diameter by 1 millimetre is -
(1) 4×10^3 newton (2) 16×10^3 newton
(3) $\frac{1}{4}\times10^3$ newton (4) $\frac{1}{16}\times10^3$ newton

Q.18 A wire of cross-sectional area 3 mm^2 is first stretched between two fixed points at a temperature of 20ºC. Determine the tension when the temperature falls to 10ºC. Coefficient of linear expansion $\alpha = 10^{-5}$ ºC^{-1} and $Y = 2 \times 10^{11}$ N/m^2 .
(1) 20 N (2) 30 N (3) 60 N (4) 120 N

Q.19 For steel, the breaking stress is 6×10^6 N/m^2 and the density is 8×10^3 kg/m^3**.** The maximum length of steel wire, which can be suspended without breaking under its own weight is -[g = 10 m/s^2]
(1) 140 m (2) 120 m (3) 75 m (4) 200 m

Q.20 The unit of modulus of rigidity are -
(1) N-m^2 (2) N/m (3) N-m (4) N/m^2

Q.21 A 2m long rod of radius 1 cm which is fixed from one end is give a twist of 0.8 radians. The shear strain developed will be-
(1) 0.002 (2) 0.004
(3) 0.008 (4) 0.016

Q.22 A ball falling in a lake of depth 200m shows 0.1% decrease in its volume at the bottom. What is the bulk modulus of the material of the ball-
(1) 19.6×10^8 N/m^2 (2) 19.6×10^{-10} N/m^2
(3) 19.6×10^{10} N/m^2 (4) 19.6×10^{-8} N/m^2

Q.23 An iron wire of length 4m and diameter 2 mm is loaded with a weight of 8 kg. If the Young's modulus 'Y' for iron is 2×10^{11} N/m^2 then the increase in the length of the wire is -
(1) 0.2 mm (2) 0.5 mm
(3) 2 mm (4) 1 mm

Q.24 The dimensional formula for Young's modulus-
(1) $M^{-1}LT^2$ (2) $ML^{-1}T^{-2}$
(3) $ML^{-1}T^{-1}$ (4) $ML^{-2}T^{-1}$

Q.25 A fixed volume of iron is drawn into a wire of length ℓ. The extension produced in this wire by a constant force F is proportional to-
(1) $\frac{1}{\ell^2}$ (2) $\frac{1}{\ell}$ (3) ℓ^2 (4) ℓ

Q.26 A mass of 0.5 kg is suspended from wire, then length of wire increases by 3 mm then work done -
(1) 4.5×10^{-3} Joule (2) 7.3×10^{-3} Joule
(3) 9.3×10^{-2} Joule (4) 2.5×10^{-3} Joule

Q.26 A mass of 0.5 kg is suspended from wire, then length of wire increase by 3 mm then work done-
(1) 4.5×10^{-3} Joule (2) 7.3×10^{-3} Joule
(3) 9.3×10^{-3} Joule (4) $2.5 \times$ Joule

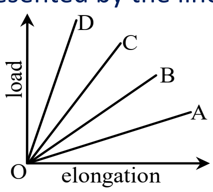
Q.27 If the strain in a wire is not more than 1/1000 and $Y = 2 \times 10^{11}$ N/m^2. Diameter of wire is 1mm. The maximum weight hung from the wire is -
(1) 110 N (2) 125 N
(3) 157 N (4) 168 N

Q.28 When a tension F is applied in uniform wire of length ℓ and radius r, the elongation produced is e. When tension 2F is applied, the elongation produced in another uniform wire of length 2ℓ and radius 2r made of same material is -
(1) 0.5 e (2) 1.0 e
(3) 1.5 e (4) 2.0 e

Q.29 How much force is required to produce an increase of 0.2% in the length of a brass wire of diameter 0.6 mm ? [Young's modulus for brass = 0.9×10^{11} N/m^2]
(1) Nearly 17 N (2) Nearly 34 N
(3) Nearly 51 N (4) Nearly 68 N

Q.30 If the interatomic spacing in a steel wire is 2.8×10^{-10}m. and $Y_{steel} = 2 \times 10^{11}$ N/m^2, then force constant in N/m is-
(1) 5.6 (2) 56
(3) 0.56 (4) 560

Q.31 The load versus elongation graph for four wires of the same material and same length is shown in the figure. The thinnest wire is represented by the line -



(1) OA (2) OB
(3) OC (4) OD

Q.32 The mean distance between the atoms of iron is 3×10^{-10}m and inter atomic force constant for iron is 7 N/m. The young's modulus of elasticity for iron is:
(1) 2.33×10^5 N/m^2 (2) 23.3×10^{10} N/m^2
(3) 233×10^{10} N/m^2 (4) 2.33×10^{10} N/m^2

Q.33 Cross section area of a steel wire ($Y = 2.0 \times 10^{11}$ N/m^2) is 0.1 cm^2. The required force, to stretched its length double will be-
(1) 2×10^{12} N (2) 2×10^{11} N
(3) 2×10^{10} N (4) 2×10^6 N

Q.34 For a given material, the Young's modulus is 2.4 times that of rigidity modulus. Its Poisson's ratio is -
(1) 2.4 (2) 1.2
(3) 0.4 (4) 0.2

Q.35 The diameter of a brass rod is 4 mm and Young's modulus of brass is 9×10^{10} N/m^2. The force required to stretch by 0.1% of its length is-
(1) 360πN (2) 36 N
(3) $144\pi \times 10^3$ N (4) $36\pi \times 10^5$ N

Q.36 Poisson's ratio can not have the value-
(1) 0.1 (2) 0.7
(3) 0.2 (4) 0.5

Q.37 There is no change in the volume of a wire due to change in its length on stretching. The poisson's ratio of the material of the wire is-
(1) + 0.50 (2) – 0.50
(3) + 0.25 (4) –0.25

Q.38 Two wires of the same length and material but different radii r_1 and r_2 are suspended from a rigid support, both carry the same load at the lower end. The ratio of the stress developed in the second wire to that developed in the first wire is-
(1) $\frac{r_1}{r_2}$ (2) $\frac{r_1^2}{r_2^2}$
(3) $\left(\frac{r_1}{r_2}\right)^{5/2}$ (4) $\left(\frac{r_1}{r_2}\right)^{1/2}$

Q.39 An increases in pressure required to decreases the 200 litres volume of a liquid by 0.004% in container is - (Bulk modulus of the liquid = 2100 MPa)
(1) 188 kPa (2) 8.4 kPa
(3) 18.8 kPa (4) 84 kPa

Q.40 If 'S' is stress and 'Y' is Young's modulus of material of a wire, the energy stored in the wire per unit volume is -
(1) $\frac{S}{2Y}$ (2) $\frac{2S}{S^2}$ (3) $\frac{S^2}{2Y}$ (4) $2S^2Y$

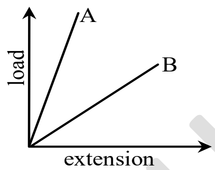
Q.41 A wire 3m long and having cross-sectional area of 0.42 m^2 is found to stretch by 0.2 m under a tension of 1000N. The value of young's modulus for the material of the wire is -
(1) 4.6×10^4 N/m^2 (2) 3.6×10^4 N/m^2
(3) 4×10^4 N/m^2 (4) 6.6×10^{-4} N/m^2

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM - 3

Q.1 If the density of the material increase, the value of Young's modulus -
(1) increases
(2) decreases
(3) first increases, then decreases
(4) first decreases, then increases

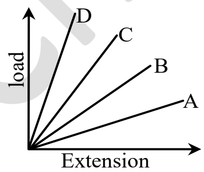
Q.2 The lower surface of a cube is fixed. On its upper surface, force is applied at an angle of 30º from its surface. The change will be in its -
(1) shape (2) size
(3) volume (4) both shape and size

Q.3 The dimensions of two wires A and B are the same. But their materials are different. Their load-extension graphs are shown. If Y_A and Y_B are the values of Young's modulus of elasticity of A and B respectively then -



(1) $Y_A > Y_B$ (2) $Y_A < Y_B$
(3) $Y_A = Y_B$ (4) $Y_B = 2Y_A$

Q.4 Figure shows the load-extension curves for four wires A, B, C and D. The dimensions of all the four wires are identical but materials of wires are different. Which wire has highest value of Young's modulus of elasticity ?



(1) A (2) B
(3) C (4) D

Q.5 Two wires of the same material and length but diameters in the ratio 1 : 2 are stretched by the same force. The potential energy per unit volume for the two wires when stretched will be in the ratio -
(1) 16 : 1 (2) 4 : 1
(3) 2 : 1 (4) 1 : 1

Q.6 One end of uniform wire of length L and of weight W is attached rigidly to a point in the roof and a weight W_1 is suspended from its lower end. If s is the area of cross-section of the wire, the stress in the wire at a height (L/4) from its lower end is -

(1) $\frac{W_1}{s}$ (2) $\frac{\left[W_1 + \frac{W}{4}\right]}{s}$

(3) $\frac{\left[W_1 + \frac{3W}{4}\right]}{s}$ (4) $\frac{W_1 + W}{4}$

Q.7 A weight is suspended from a long metal wire. If the wire suddenly breaks, its temperature -

(1) rises (2) falls
(3) remains unchanged (4) attains a value 0 K

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 4

These questions consists of two statements each, printed as Assertion and Reason. While answering these questions you are required to choose any one of the following four responses.
(A) If both Assertion & Reason are true & the Reason is a correct explanation of the Assertion.
(B) If both Assertion and Reason are true but Reason is not a correct explanation of the Assertion.
(C) If Assertion is true but the Reason is false.
(D) If Assertion & Reason both are false.

Q.1 **Assertion :** Elastic restoring forces may be conservative.
Reason : The value of strain for same stress are different while increasing the load and while decreasing the load.
(1) A (2) B (3) C (4) D

Q.2 **Assertion :** Work is required to be done to stretch a wire. This work is stored in the wire in the form of elastic potential energy.
Reason : Work is required to be done against the intermolecular forces of attraction.
(1) A (2) B (3) C (4) D

Q.3 **Assertion :** The bridges are declared unsafe after a long use.
Reason : Elastic strength of bridges losses with time.
(1) A (2) B (3) C (4) D

Q.4 **Assertion :** Young's modulus for a perfectly plastic body is zero.
Reason : For a perfectly plastic body, restoring force is zero.
(1) A (2) B (3) C (4) D

Q.5 **Assertion :** Identical springs of steel and copper are equally stretched. More work will be done on the steel spring.
Reason : Steel is more elastic than copper.
(1) A (2) B (3) C (4) D

Q.6 **Assertion :** Stress is the internal force per unit area of a body.
Reason : Rubber is more elastic than steel.
(1) A (2) B (3) C (4) D

Q.7 **Assertion :** Rubber is more elastic than glass.
Reason : The rubber has higher modulus of elasticity than glass.
(1) A (2) B (3) C (4) D

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 5

Q.1 The surface tension of which of the following liquid is maximum

(1) H_2O (2) C_6H_6

(3) CH_3OH (4) C_2H_5OH

Q.2 Two small drops of mercury. each of radius R. coalesce to form a single large drop. The ratio of the total surface energies before and after the change is:-

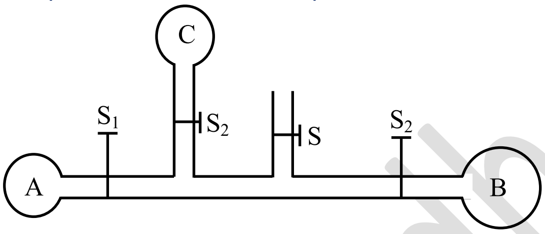
(1) $1 : 2^{1/3}$ (2) $2^{1/3} : 1$

(3) $2 : 1$ (4) $1 : 2$

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 6

Q.1 The surface tension of a liquid at its boiling point:
(1) Becomes zero
(2) Becomes infinity
(3) is equal to the value at room temperature
(4) is half to the value at the room temperature

Q.2 The adjoining diagram shows three soap bubbles A, B and C prepared by blowing the capillary tube A, B and C prepared by blowing the capillary tube fitted with stop cocks S, S_1,S_2 and S_3 With stop cock S closed and stop clocks S_1, S_2 and S_3 opened



(1) B will start collapsing with volumes of A and C increasing
(2) C will start collapsing with volumes of A and B increasing
(3) C and A will both start collapsing with the volume of B increasing
(4) Volumes of A, B and C will become equal at equilibrium

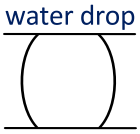
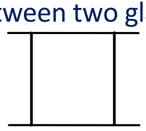
Q.3 Pressures inside two soap bubbles are 1.01 and 1.02 atmospheres. Ratio between their volumes is
(1) 102 : 101 (2) $(102)^3 : (101)^3$
(3) 8 : 1 (4) 2 : 1

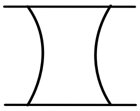
Q.4 The height of which water rises in a capillary will be-
(1) Maximum at 4ºC (2) Maximum at 0ºC
(3) Minimum at 0ºC (4) Minimum at 4ºC

Q.5 When a capillary tube of glass dipped in mercury then-
(1) Mercury level rises in tube
(2) Mercury rises in tube and come out
(3) Mercury level in tube descendes
(4) Level of mercury neither ascends or descends

Q.6 Two soap bubbles each of radius r are touching each other. The radius of curvature of the common surface will be-
(1) Infinite (2) 2r (3) r (4) $\frac{r}{2}$

Q.7 The lower end of a capillary tube touches a liquid whose angle of contact is 90º. The liquid
(1) will neither rise nor will fall inside the tube.
(2) will rise inside the tube.
(3) will rise to the top of the tube
(4) will be depressed inside the tube

Q.8 If a water drop is kept between two glass plates, then its shape is:
(1)  (2) 

(3)    (4) None of these

Q.9 Water rises in a capillary upto a height h. If now this capillary is tilted by an angle of 45º, then the length of the water column in the capillary becomes-

(1) 2h   (2) $\frac{h}{2}$

(3) $\frac{h}{\sqrt{2}}$ (4) $h\sqrt{2}$

Q.10 If the surface tension of water is 0.06 N m^{-1}, then the capillary rise in a tube of diameter 1 mm is (θ = 0º)

(1) 1.22 cm   (2) 2.44 cm

(3) 3.12 cm   (4) 3.86 cm

Q.11 The radii of two soap bubbles are in the ratio 2 : 1. the excess pressures will be in the ratio-

(1) 1 : 2   (2) 2 : 1   (3) 1 : 4   (4) 4 : 1

Q.12 A big drop of radius R is formed by 1000 small droplets of water of radius r. The radius of each small drop is

(1) $\frac{R}{1000}$ (2) $\frac{R}{500}$ (3) $\frac{R}{100}$ (4) $\frac{R}{10}$

Q.13 At which temperature, surface tension of water will be minimum-

(1) 4ºC   (2) 25ºC   (3) 50ºC   (4) 75ºC

Q.14 A liquid drop of diameter D breaks into 27 tiny drops. The resultant change in energy is-

(1) $2\pi TD^2$ (2) $4\pi TD^2$

(3) πTD^2 (4) None of these

Q.15 There are two liquid drops of different radii. The excess pressure inside over the outside is:

(1) More in the big drop
(2) More in the small drop
(3) Equal in both drops
(4) There is no excess pressure inside the drops

Q.16 If a capillary of radius r is dipped in water, the height of water that rises in it is h and its mass is M. If the radius of the capillary is doubled the mass of water that rises in the capillary will be

(1) 4M   (2) 2M   (3) M   (4) $\frac{M}{2}$

Q.17 A soap bubble in vacuum has a radius of 3 cm and another soap bubble in vacuum has a radius of 4 cm. If the two bubbles coalesce under isothermal condition, then the radius of the new bubble is:

(1) 2.3 cm   (2) 4.5 cm   (3) 5 cm   (4) 7 cm

Q.18 The spherical shape of rain-drop is due to

(1) Density of the liquid
(2) Surface tension
(3) Atmospheric pressure
(4) Gravity

Q.19 In a capillary tube, water rises by 1.2 mm. The height of water that will rise in another capillary tube having half the radius of the first, is:
(1) 1.2 mm (2) 2.4 mm
(3) 0.6 mm (4) 0.4 mm

Q.20 Water rises to a height h in a capillary at the surface of earth. On the surface of the moon the height of water column in the same capillary will be:
(1) 6h (2) 1/6 h
(3) h (4) Zero

Q.21 Shape of meniscus for a liquid of zero angle of contact is-
(1) plane (2) parabolic
(3) hemi-spherical (4) cylindrical

Q.22 Due to capillary action a liquid will rise in a tube if angle of contact is
(1) acute (2) obtuse
(3) 90º (4) 180º

Q.23 If the difference between pressure inside and outside of a soap bubble is 6 mm of water and its radius is 8 mm. What is the surface tension in dynes per cm.
(1) 116 (2) 256
(3) 378 (4) 450

Q.24 Two droplets merge with each other and from a large droplet. In this process:
(1) Energy is liberated
(2) Energy is absorbed
(3) Neither liberated nor absorbed
(4) Some mass is converted into energy

Q.25 Two capillary tubes of same diameter are put vertically one each in two liquids whose relative densities are 0.8 and 0.6 and surface tension are 60 dyne/cm and 50 dyne/cm respectively. Ratio of heights of liquids in the two tubes h_1/h_2 is:
(1) $\frac{10}{9}$ (2) $\frac{3}{10}$ (3) $\frac{10}{3}$ (4) $\frac{9}{10}$

Q.26 The property utilized in the manufacture of lead shots is:
(1) Specific weight of liquid lead
(2) Specific gravity of liquid lead
(3) Compressibility of liquid lead
(4) Surface tension of liquid lead

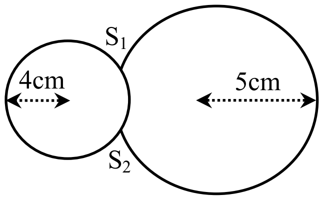
Q.27 An air bubble is lying just below the surface of water. The surface tension of water is $70 \times 10^{-3} Nm^{-1}$ and atmospheric pressure is $1.013 \times 10^5 Nm^{-2}$. If the radius of bubble is 1mm then the pressure inside the bubble will be
(1) 1.0270×10^5 Pa (2) 1.0160×10^5 Pa
(3) 1.0144×10^5 Pa (4) 1.0131×10^5 Pa

Q.28 Surface tension of a liquid is 5 N/m. If its thin film is made in a ring of area 0.02 m^2, then its surface energy will be-
(1) 5×10^{-2} Joule (2) 2.5×10^{-2} Joule
(3) 3×10^{-1} Joule (4) 2×10^{-1} Joule

Q.29 If one end of capillary tube is dipped into water then water rises up to 3 cm. If the surface tension of water is 75×10^{-3} N/m then the diameter of capillary rube will be-
(1) 0.1 mm (2) 0.5 mm
(3) 1 mm (4) 2 mm

Q.30 If the surface tension of a liquid is T and its surface area is increased by A, then the surface energy of that surface will be increased by-
(1) AT (2) A/T (3) A^2T (4) A^2T^2

Q.31 Two soap bubbles of radii r_1 and r_2 equal to 4cm and 5cm are touching each other over a common surface S_1S_2 (shown in figure). Its radius will be-



(1) 4 cm (2) 20 cm (3) 5 cm (4) 4.5 cm

Q.32 The radius of a soap bubbles is r. The surface tension of soap solution is T. Keeping temperature constant, the radius of the soap bubble is doubled, the energy necessary for this will be-
(1) $24\pi r^2T$ (2) $8\pi r^2T$
(3) $12\pi r^2T$ (4) $16\pi r^2T$

Q.33 A liquid does not wet the sides of a solid, if the angle of contact is
(1) Zero
(2) Obtuse (more than 90º)
(3) Acute (less than 90º)
(4) 90º

Q.34 The excess of pressure inside a soap bubble than that of the outer pressure is
(1) $\frac{2T}{r}$ (2) $\frac{4T}{r}$
(3) $\frac{T}{2r}$ (4) $\frac{T}{r}$

Q.35 In a capillary tube experiment, a vertical 30 cm long capillary tube is dipped in water. The water rises up to a height of 10 cm due to capillary action. If this experiment is conducted in a freely falling elevator, the length of the water column becomes:
(1) 10 cm (2) 20 cm
(3) 30 cm (4) Zero

Q.36 Radius of a capillary is 2×10^{-3} m. A liquid of weight 6.2×10^{-4} N may remain in the capillary. Then the surface tension of liquid will be:
(1) 5×10^{-3} N/m (2) 5×10^{-2} N/m
(3) 5 N/m (4) 50 N/m

Q.37 A capillary tube of radius r can support a liquid of weight 6.28×10^{-4} N. If the surface tension of the liquid is 5×10^{-2} N/m. The radius of capillary must be-
(1) 2×10^{-3} m (2) 2×10^{-4} m
(3) 1.5×10^{-3} m (4) 12.5×10^{-4} m

Q.38 Water rise in a capillary upto an extension height such that upward force of surface tension balances the force of 75×10^{-4} N. Due to weight of water. If surface tension of water is 6×10^{-2} N/m. The internal circumference of the capillary must be-
(1) 12.5×10^{-2} m (2) 6.5×10^{-2} m
(3) 0.5×10^{-2} m (4) 1.25×10^{-2} m

Q.39 Which one of the following will make its way most easily through the tiny space between the fiber of the clothing–
(1) Glycerene at 20ºC
(2) Water at 20ºC
(3) Soap water at 20º C
(4) Water at 100ºC

Q.40 Inside a drop excess pressure is maximum in
(1) .200 μm diameter (2) 20.0 μm diameter
(3) 200 μm diameter (4) 2.0 μm diameter

Q.41 When charge is given to a soap bubble, it shows:-
(1) An increase in size
(2) Sometimes an increase and sometimes a decrease in size
(3) No change in size
(4) None of these

Q.42 The diameter of one drop of water is 0.2 cm. The work done in breaking one drop into 1000 equal droplets will be:-
(surface tension of water = 7×10^{-2} N/m)
(1) 7.9×10^{-6} J (2) 5.92×10^{-6} J
(3) 2.92×10^{-6} J (4) 1.92×10^{-6} J

Q.43 Calculate the force required to separate the glass plate of area 10^{-2} m^2 with a film of water 0.05 mm thick [surface tension of water is 70×10^{-3} N/m]
(1) 25 N (2) 20 N (3) 14 N (4) 28 N

Q.44 If two bubble of radii 0.03 cm and 0.04 cm come in contact with each other then the radius of curvature of the common surface 'r' is given by.
(1) 0.03 (2) 0.06 (3) 0.12 (4) 0.24

Q.45 Work done in forming a soap bubble of radius R is W. Then work done is forming a soap bubble of radius '2R' will be:
(1) 2W (2) 4W (3) W/2 (4) W/4

Q.46 In a U-tube diameter of two limbs are 0.5 cm and 1 cm respectively and tube has filled with water (T = 72 dyne/cm) then liquid level difference between two limbs will be-
(1) 0.5 cm (2) 0.25 cm
(3) 0.293 cm (4) None of these

Q.47 At which angle liquid will not wet solid.
(1) Zero (2) acute (3) 45º (4) obtuse

Q.48 Internal radius of a capillary tube is $\frac{1}{28}$ cm and surface tension of water 70 dyne/cm, if angle of contact is zero, then water will rise up in the tube up to height.
(1) 4 cm (2) 2 cm (3) 14 cm (4) 18 cm

Q.49 Diameter of two limbs of a U-tube are 2 mm and 5mm, and surface tension is 70 dyne/cm, if water density is 1000 kg/m^2 and g = 10 m/s^2, then water level difference in both the tubes will be:
(1) 8.4 m (2) 8.4 cm (3) 84 cm (4) 0.84 cm

Q.50 At critical temperature, the surface tension of a liquid becomes:
(1) unity (2) infinity (3) zero (4) negative

Q.51 Surface tension arises due to:
(1) tension in the liquid
(2) frictional molecular force
(3) adhesive molecular force
(4) cohesive molecular force

Q.52 Area of liquid film is 6 × 10 cm^2 and surface tension is T = 20 dyne/cm, what is the work done to change area up to 12 × 10 cm^2 :
(1) 120 joule (2) 120 erg
(3) 1200 joule (4) 2400 erg

Q.53 Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (surface tension of soap solution = 0.03 Nm^{-1}) :
(1) 4π mJ (2) 0.2π mJ
(3) 2π mJ (4) 0.4π mJ

Q.54 The excess pressure inside a soap bubble A is twice that in another soap bubble B. The ratio of volumes of A and B is
(1) 1 : 2 (2) 1 : 4
(3) 1 : 8 (4) 1 : 16

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 7

Q.1 The excess pressure inside an air bubble of radius r just below the surface of water is p_1. The excess pressure inside a drop of the same radius just outside the surface is p_2. If T is surface tension, then

(1) $p_1 = 2p_2$ (2) $p_1 = p_2$
(3) $p_2 = 2p_1$ (4) $p_2 = 0, p_1 \neq 0$

Q.2 A water drop is divided into 8 equal droplets. The pressure difference between inner and outer sides of the big drop-

(1) will be the same as for smaller droplet
(2) will be half of that for smaller droplet
(3) will be one-forth of that for smaller droplet
(4) will be twice of that for smaller droplet.

Q.3 A false statement is:

(1) Angle of contact $\theta < 90^\circ$, if cohesive force < adhesive force × $\sqrt{2}$
(2) Angle of contact $\theta > 90^\circ$, if cohesive force > adhesive force × $\sqrt{2}$
(3) Angle of contact $\theta = 90^\circ$, if cohesive force = adhesive force × $\sqrt{2}$
(4) If the radius of capillary is reduced to half, the rise of liquid column becomes four times

Q.4 On dipping a capillary of radius 'r' in water, water rises upto a height H and potential energy of water is u_1. If a capillary of radius 2r is dipped in water, then the potential energy is u_2. The ratio $\frac{u_1}{u_2}$ is

(1) 2 : 1 (2) 1 : 2
(3) 4 : 1 (4) 1 : 1

Q.5 Spiders and insects move and run about on the surface of water without sinking because:

(1) Elastic membrane is formed on water due to property of surface tension
(2) Spiders and insects are lighter
(3) Spiders and insects swim on water
(4) Spiders and insects experience up-thrust

Q.6 In a surface tension experiment with a capillary tube water rises up to 0.1m. If the same experiment is repeated on an artificial satellite which is revolving round the earth, water will rise in the capillary tube up to a height of

(1) 0.1 m
(2) 0.98 m
(3) 9.8 m
(4) full length of capillary tube

Q.7 Water rises upto a height h in a capillary on the surface of earth in stationary condition. Value of h increases if this tube is taken:

(1) On sun

(2) On poles

(3) In a lift going upward with acceleration

(4) In a lift going downward with acceleration

Q.8 On dipping a capillary of length 20 cm. in a liquid, the liquid rises in it upto a height of 8 cm. If the whole arrangement is placed in a lift falling freely, then the height of liquid column in capillary will be:

(1) 6 cm (2) 8 cm

(3) 16 cm (4) 20 cm

Q.9 If more air is pushed in a soap bubble, the pressure in it :

(1) decreases (2) increase

(3) remains same (4) becomes zero

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 1 (ANSWERS)

Q.No.	1	2	3
Ans.	2	1	2

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 2 (ANSWERS)

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	3	3	1	4	2	4	3	2	2	1	4	1	3	3	4	3	2	3	3	4
Q.No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	2	1	2	2	3	2	3	2	3	2	1	4	4	4	1	2	1	2	4	3
Q.No.	41																			
Ans.	2																			

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 3 (ANSWERS)

Q.No.	1	2	3	4	5	6	7
Ans.	1	4	1	4	1	2	1

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 4 (ANSWERS)

Q.No.	1	2	3	4	5	6	7
Ans.	2	1	1	1	1	3	4

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 5 (ANSWERS)

Q.No.	1	2
Ans.	1	2

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 6 (ANSWERS)

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	1	3	3	4	3	1	1	3	4	2	1	4	4	1	2	2	3	2	2	1
Q.No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	3	1	1	1	4	4	3	4	3	1	2	1	2	2	3	2	1	1	4	1
Q.No.	41	42	43	44	45	46	47	48	49	50	51	52	53	54						
Ans.	1	1	4	3	2	3	4	1	4	3	4	4	2	3						

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 7 (ANSWERS)

Q.No.	1	2	3	4	5	6	7	8	9
Ans.	2	2	4	4	1	4	4	4	1