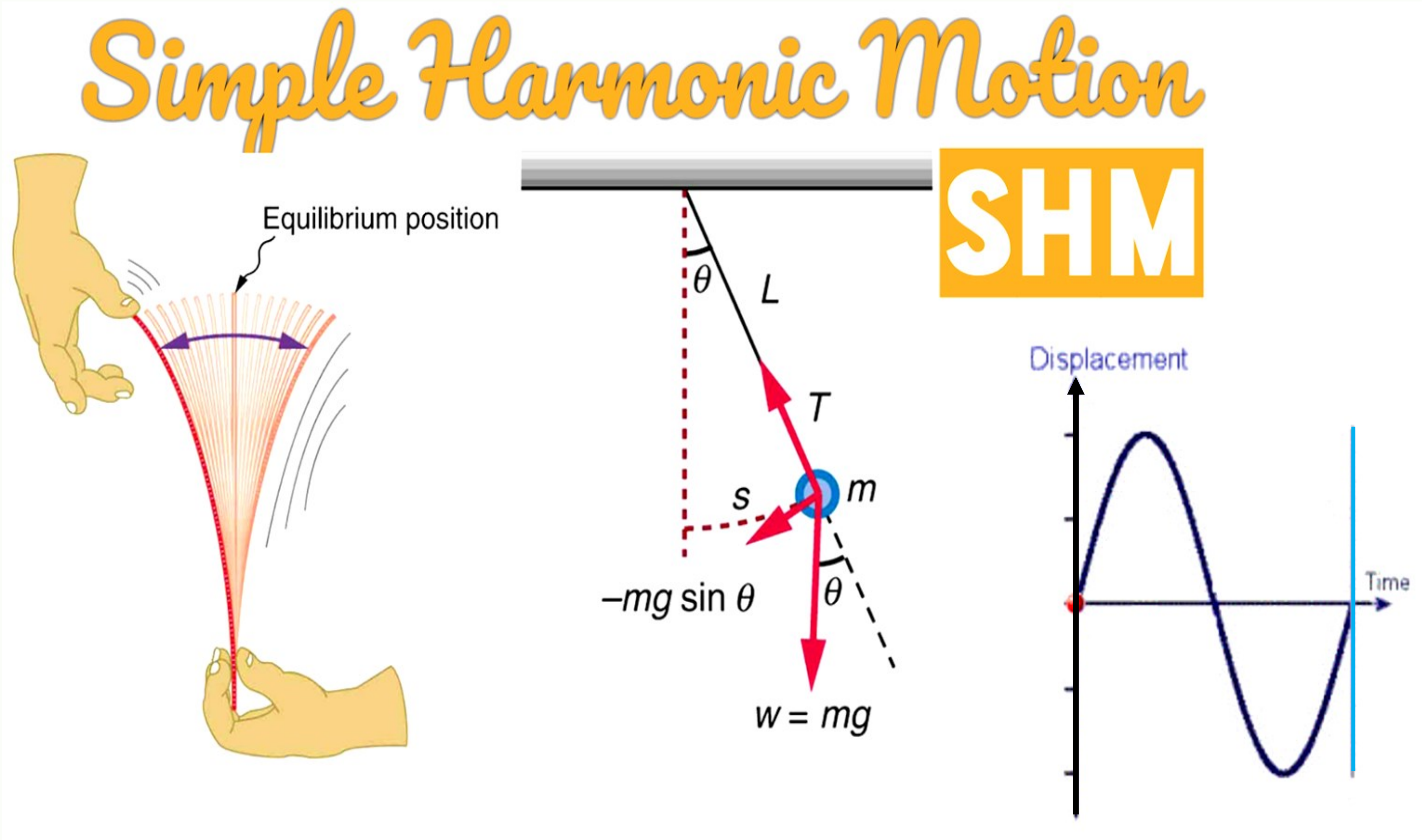
PHYSICS

for JEE, NEET & BOARD

11th



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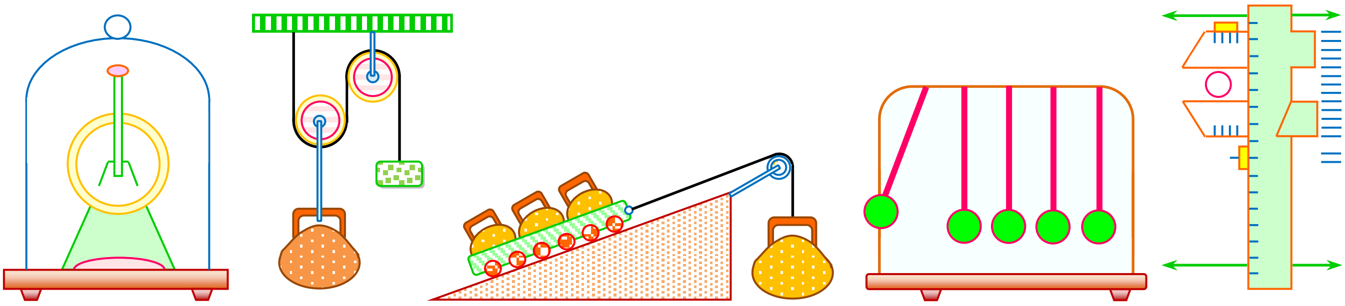

PHYSICS BOOKLET FOR JEE NEET & BOARDS

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SIMPLE HARMONIC MOTION

Periodic Motion: Motions, processes or phenomena, which repeat themselves at regular intervals, are called periodic.

Period & frequency: In a periodic motion the smallest interval of time after which the process repeats itself is called period. Usually the period is denoted by the symbol T and is measured in seconds.

The reciprocal of T gives the number of periodic motions that occur per second and is called the frequency of the periodic motion. It is represented by the symbol ν and is measured in units called hertz.

$$\nu = \frac{1}{T}$$

Oscillatory motion: If a body moves to and fro on the same path about a fixed point then its motion is called as oscillatory motion.

1 SIMPLE HARMONIC MOTION (SHM)

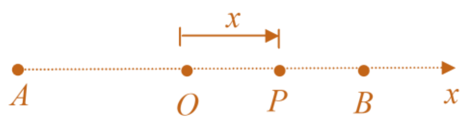
Simple harmonic motion is special type of periodic oscillatory motion in which;

(i) the particle oscillates on a straight line

(ii) the acceleration of the particle is always directed towards a fixed point on the line.

(iii) the magnitude of acceleration is proportional to the displacement of the particle from the fixed point.

This fixed point is called the centre of oscillation or the mean position. Taking this point as origin 'O' and the line of motion as the x-axis, we can write the equation of simple harmonic motion based on its definition as,



$$a = -\omega^2 x$$

Where ω^2 is a positive constant. P is the particle, which is at a distance 'x' from fixed point 'O'. a is the acceleration which is directed opposite to the displacement and towards centre of oscillation 'O'.

According to Newton's laws of motion in inertial frame of reference,

$a = F / m = -\omega^2 x$; $\quad$ m = mass of particle

$\therefore F = m\omega^2 x = -kx$ $\quad$ F = resultant force acting on the particle

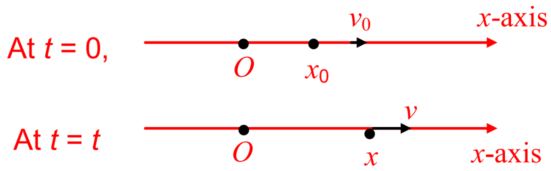
$\therefore \quad F = -kx$

i.e., the resultant force acting on the particle is proportional to displacement and directed towards mean position.

The constant $k = m\omega^2$ is called the force constant. It is to be noted that resultant force is zero at mean position so it is also the dynamic equilibrium position of the particle.

2 EQUATION OF MOTION OF A SIMPLE HARMONIC MOTION

Now we will derive the equation of motion for a particle of mass 'm' moving along x-axis under the effect of force $F = -kx$. Here k is a positive constant and x is the displacement of the particle from the assumed origin.



Suppose we start observing the motion of the particle at $t = 0$ when it is at $x = x_0$ and its velocity is $v = v_0$ as shown in the figure.

The acceleration of the particle at any instant is

$$a = F/m = \frac{-k}{m}x = -\omega^2 x \text{ ; where } \omega = \sqrt{\frac{k}{m}}$$

$$\therefore \quad \frac{dv}{dt} = -\omega^2 x \qquad \ldots (1)$$

$$\therefore \quad \frac{vdv}{dx} = -\omega^2 x$$

$$\therefore \quad \int_{v_0}^{v} vdv = \int_{x_0}^{x} \omega^2 x dx$$

where 'x' is the instantaneous position of the particle and 'v' is the velocity at that instant

$$\therefore \quad \left[\frac{v^2}{2}\right]_{v_0}^{v} = -\omega^2 \left[\frac{x^2}{2}\right]_{x_0}^{x}$$

$$v^2 - v_0^2 = -\omega^2 (x^2 - x_0^2)$$

$$v^2 = v_0^2 + \omega^2 x_0^2 - \omega^2 x^2$$

$$v = \omega \sqrt{\left(\frac{v_0}{\omega}\right)^2 + x_0^2 - x^2}$$

$\therefore$ Equation (2) can be written as

$$\therefore \quad v = \frac{dx}{dt} = \omega\sqrt{A^2 - x^2} \qquad \ldots (2)$$

where $A = \sqrt{\left(\frac{v_0}{\omega}\right)^2 + x_0^2}$

$$\therefore \quad \int_{x_0}^{x} \frac{dx}{\sqrt{A^2 - x^2}} = \int_0^t \omega dt$$

$$\left[\sin^{-1}\left(\frac{x}{A}\right)\right]_{x_0}^{x} = \omega t$$

$\therefore \quad \sin^{-1}\dfrac{x}{A} - \sin^{-1}\dfrac{x_0}{A} = \omega t$

Put $\quad \sin^{-1}\dfrac{x_0}{A} = \phi$

$\therefore \quad \sin^{-1}\dfrac{x}{A} = (\phi + \omega t)$

$\therefore \quad x = A \sin(\omega t + \phi)$ … **(3)**

and velocity; $v = \dfrac{dx}{dt} = \omega A \cos(\omega t + \phi)$ … **(4)**

2.1 AMPLITUDE

It is maximum displacement of the particle from its mean position.

Equation (3) gives the displacement of the particle. The value of 'x' is maximum when $\sin(\omega t + \phi)$ is maximum i.e., $\sin(\omega t + \phi) = \pm 1$

$\therefore \quad x_{max} = \pm A$

$\therefore$ 'A' which was used as constant while deriving the equation of motion is nothing but the amplitude of simple harmonic motion.

2.2 TIME PERIOD

Periodic functions $f(t)$ with period T are those functions of the variable 't' which have the property,

$f(t + T) = f(t)$ … **(5)**

Both $\sin(\omega t + \phi)$ and $\cos(\omega t + \phi)$ will repeat their values if the angle $(\omega t + \phi)$ increases by 2π or its multiple. As T is smallest time for repetition.

$\omega(t + T) + \phi = \omega t + \phi + 2\pi$

$\therefore \quad \omega T = 2\pi \quad$ or $\quad T = \dfrac{2\pi}{\omega}$

Since $\quad \omega = \sqrt{\dfrac{k}{m}}$

$\therefore \quad T = 2\pi\sqrt{\dfrac{m}{k}}$ … **(6)**

2.3 FREQUENCY AND ANGULAR FREQUENCY

Frequency is defined as the number of oscillations per second or simply as the reciprocal of time period.

$$\nu = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{… (7)}$$

The constant ω is called the angular frequency. The angular frequency and period in simple harmonic motion are independent of the amplitude.

2.4 PHASE

The quality $\theta = \omega t + \phi$ is called the phase. It determines the states of the particle in simple harmonic motion.

When the particle is at **mean position** $x = 0$

i.e., $A\sin(\omega t + \phi) = 0$

$\therefore$ $\omega t + \phi = n\pi; n = 0, 1, 2, 3, \ldots$

(i) consider $n = 0$; $\therefore$ $\omega t + \phi = 0$

$\therefore$ $x = 0$

and $v = \omega A \cos(\omega t + \phi) = \omega A$

i.e., the particle is crossing the mean position and is moving towards the positive direction.

(ii) consider $n = 1$

$\therefore$ $\omega t + \phi = \pi$

$\therefore$ $x = 0$

and $v = -\omega A$

i.e., again the particle is crossing the mean position but now it is moving towards the negative direction.

When the particle is at **extreme position**.

$x = x_{max}$

i.e., $A \sin(\omega t + \phi) = \pm A$

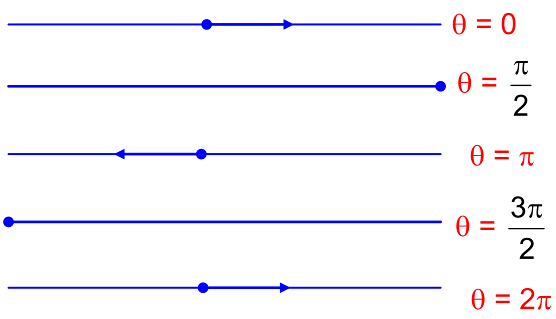
$\Rightarrow$ $(\omega t + \phi) = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

i.e., $(\omega t + \phi) = \frac{(2n+1)}{2}\pi$; $n = 0, 1, 2 \ldots$

$\therefore$ Consider $n = 1$; $\therefore$ $\omega t + \phi = \frac{3\pi}{2}$

$\therefore$ $x = -A$

and $v = 0$



i.e., the particle is at extreme left and again its velocity is zero.

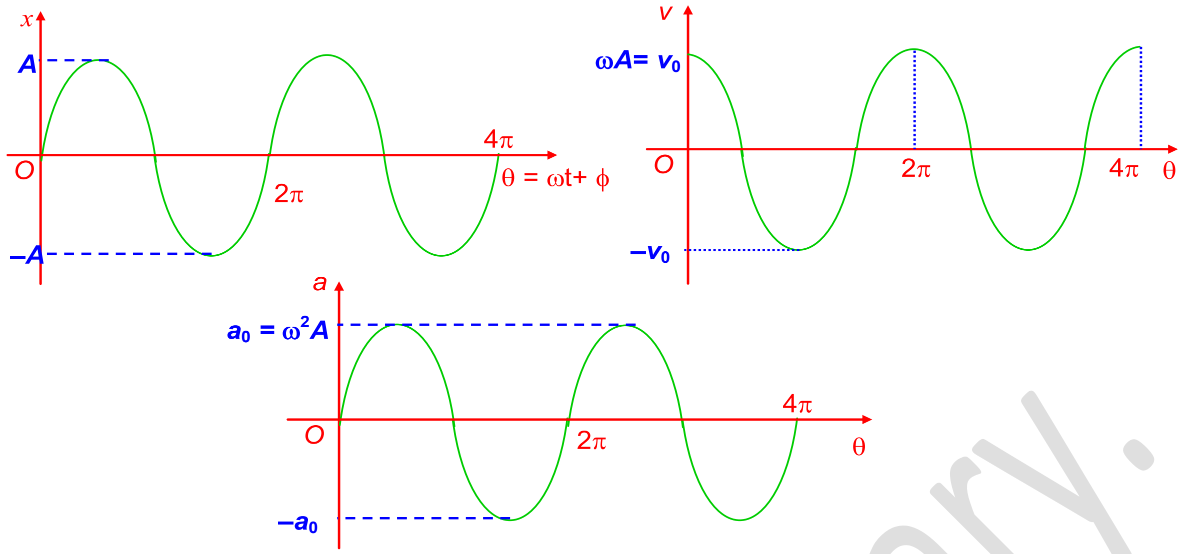
From above it is clear that as time increases the phase increases. An increase of 2π brings the particle to the same status in the motion. Thus, a phase $\omega t + \phi$ is equivalent to a phase $\omega t + \phi + 2\pi$.

Similarly acceleration of the particle is given by

$$\frac{d^2x}{dt^2} = -\omega^2 A \sin(\omega t + \phi)$$

It is zero when phase $(\omega t + \phi) = 0$ and maximum $(\omega^2 A)$ when phase $(\omega t + \phi) = \left(\frac{2n+1}{2}\right)\pi$.

Graphically the variation of position, velocity and acceleration with the phase is shown below.



2.5 PHASE CONSTANT

The constant term φ in the equation (3) is called phase constant or initial phase or epoch of the particle. This constant depends on the choice of the instant $t = 0$.

Suppose we choose $t = 0$ at an instant when the particle is passing through its mean position towards right (i.e. positive direction). Then the phase $\theta = \omega t + \phi$ has to be zero. Since $t = 0$ this means $\phi = 0$. So the equation for displacement becomes;

$$x = A \sin \omega t$$

If we choose $t = 0$ when the particle is at its extreme position in the positive direction. The phase $\theta = \frac{\pi}{2}$ at this instant and hence $\phi = \frac{\pi}{2}$. Therefore equation of displacement becomes

$$x = A \cos\omega t$$

The sine form and cosine form are basically equivalent. The value of phase constant, however, depends on the form chosen, for example,

$$x = A \sin(\omega t + \phi) = A \sin(\omega t + \pi/2 + \phi')$$

$$x = A \cos(\omega t + \phi')$$

Illustration 1

Question: **A particle executes simple harmonic motion of amplitude 5 cm and a period π sec. Find the speeds of the particle at (i) 3 cm from the mean position and (ii) at the mean position.**

Solution: Speed of the particle at a distance x from the mean position is given by

$$v = \omega\sqrt{a^2 - x^2} = \frac{2\pi}{T}\sqrt{a^2 - x^2}$$

(i) When $x = 3$ cm, $v = \frac{2\pi}{\pi}\sqrt{5^2 - 3^2}$ = **8 cm/sec**.

(ii) At the mean position x = 0,

$$v = \omega a = \frac{2\pi}{T}.a = \frac{2\pi}{\pi}\times 5 \qquad = \mathbf{10\ cm/sec}$$

Illustration 2

Question: **A particle executes S.H.M. of period π sec and amplitude 2 cm. Find the acceleration of it when it is (i) at the maximum displacement from the mean position and (ii) at 1 cm from the mean position.**

Solution: Acceleration a at displacement x is given by

$$a = \omega^2 x$$

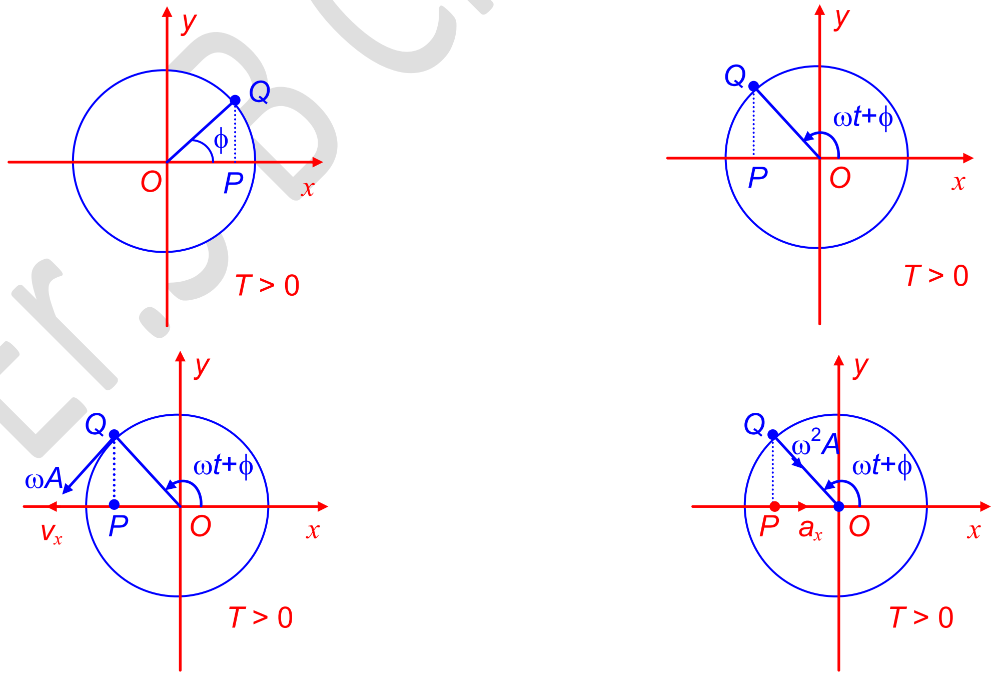
(i) When x (amplitude) = 2 cm, $a = \left(\frac{2\pi}{T}\right)^2 \times 2 = \frac{4\pi^2}{\pi^2}\times 2 = \mathbf{8\ cm/sec^2}$

(ii) When x = 1 cm, a = $\left(\frac{2\pi}{T}\right)^2 \times 1 = \frac{4\pi^2}{\pi^2}\times 1 = \mathbf{4\ cm/sec^2}$

3 RELATION BETWEEN SIMPLE HARMONIC & UNIFORM CIRCULAR MOTION

The relation we are going to discuss is useful in describing many features of simple harmonic motion. It also gives a simple geometric meaning to the angular frequency ω and phase constant ϕ.

In the figure shown, Q is the point moving on a circle of radius A with constant angular speed of ω (in rad/sec). P is the perpendicular projection of Q on the horizontal diameter, along the x-axis. Let us take Q as the reference point and the circle on which it moves the reference circle. As the reference point revolves, the projected point P moves back and forth along the horizontal diameter.



Let the angle between the radius OQ and the x-axis at the time t = 0 be called ϕ. At any later time t, the angle between OQ and the x-axis is $(\omega t + \phi)$, the point Q moving with constant angular speed ω. The x-coordinate of Q at any time is, therefore

$$x = A\cos(\omega t + \phi)$$

i.e., P moves with simple harmonic motion

Thus, when a particle moves with uniform circular motion, its projection on a diameter moves with simple harmonic motion. The angular frequency ω of simple harmonic motion is the same as the angular speed of the reference point.

The velocity of Q is $v = \omega A$. the component of v along the x-axis is

$$v_x = -v \sin(\omega t + \phi)$$

$$v_x = -\omega A \sin(\omega t + \phi)$$

Which is also the velocity of P. The acceleration of Q is centripetal and has a magnitude, $a = \omega^2 A$

The component of 'a' along the x-axis is

$$a_x = -a \cos(\omega t + \phi)$$

$$a_x = -\omega^2 A \cos(\omega t + \phi)$$

which is acceleration of P.

4 ENERGY CONSIDERATIONS IN SIMPLE HARMONIC MOTION (SHM)

Simple harmonic motion is defined by the equation

$$F = -kx$$

The work done by the force F during a displacement from x to $x + dx$ is

$$dW = Fdx = -kx\,dx$$

The work done in a displacement from $x = 0$ to x is

$$W = \int_0^x (-kx)\,dx = -\frac{1}{2}kx^2$$

Let $U(x)$ be the potential energy of the system when the displacement is x. As the change in potential energy corresponding to a force is negative of the work done by the force,

$$U(x) - U(O) = -W = \frac{1}{2}kx^2$$

Let us choose the potential energy to be zero when the particle is at the centre of oscillation $x = 0$.

Then $U(0) = 0$ and $U(x) = \frac{1}{2}kx^2$

$\because$ $k = m\omega^2$

$$\therefore \quad U(x) = \frac{1}{2}m\omega^2 x^2 \quad \ldots \textbf{(8)}$$

But $x = A\sin(\omega t + \phi)$

$$\therefore \quad U = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi) \quad \ldots \textbf{(9)}$$

kinetic energy of the particle at any instant

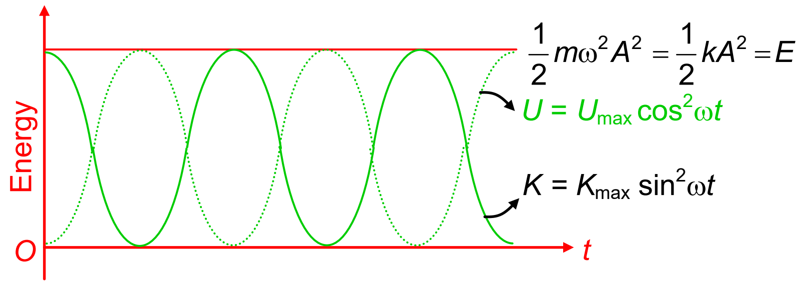
$$K = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\cos^2(\omega t + \phi) = \frac{1}{2}m\omega^2\left(A^2 - x^2\right) \quad \ldots \textbf{(10)}$$

So the total mechanical energy at time 't' is

$$E = U + K$$

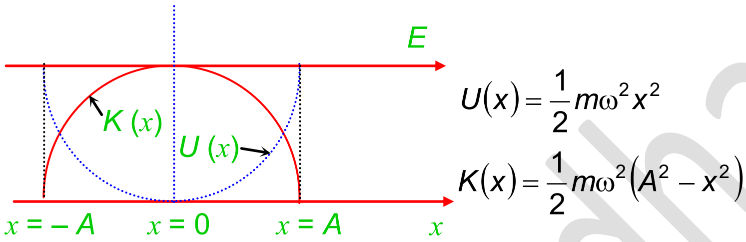
$$E = \frac{1}{2}m\omega^2 A^2 \qquad \ldots (11)$$

(i)



Potential, kinetic and total energy plotted as a function of time.

(ii)



Potential, kinetic and total energy plotted as a function of displacement from the equilibrium position.

Illustration 3

Question: **If a particle of mass 10 kg executes S.H.M. of amplitude 20 cm and period of 1 sec find (i) the total mechanical energy at any instant (ii) kinetic and potential energies when the displacement is 10 cm. ($\pi^2 = 10$)**

Solution: (i) Total mechanical energy at any instant is given by

$$E = \frac{1}{2}mA^2\omega^2$$

$$= \frac{1}{2}(10\text{ kg})\,(20 \times 10^{-2}m)^2\left(\frac{2\pi}{1}\right)^2$$

$$= \mathbf{8\ J}$$

(ii) K.E. at the instant when the displacement, x is given by

$$\text{K.E.} = \frac{1}{2}m\omega^2(A^2 - x^2)$$

when $x = 1$ cm,

$$\text{K.E.} = \frac{1}{2}(10\,\text{kg})\left(\frac{2\pi}{1}\right)^2(400\times10^{-4} - 100\times10^{-4})J$$

$$= \mathbf{6\ J}$$

P.E. at that instant = Total energy – K.E.

= **2J**

5 EXAMPLES OF SIMPLE HARMONIC MOTION

5.1 SIMPLE PENDULUM

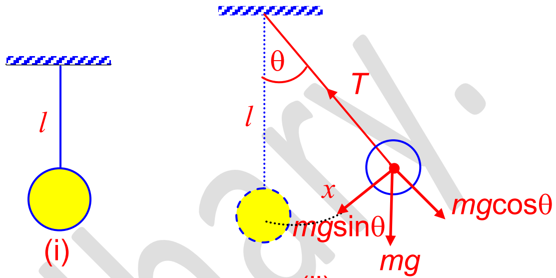
A simple pendulum consists of a heavy particle suspended from a fixed support through a light inextensible string.

The time period for simple pendulum can be found by force/ torque method and also by energy method.

(a) Force method: The mean position or the equilibrium position of the simple pendulum is when $\theta = 0$ as shown in figure (i). The length of the string is l, and mass of the bob is m.

When the bob is displaced through a distance 'x', the forces acting on it are shown in the figure (ii).

The restoring force acting on the bob to bring it to the mean position is,



(i) (ii)

$F = -mg\sin\theta$ (–ve sign indicates that force is directed away from the displacement towards the mean position).

For small angular displacements,

$$\text{Sin}\,\theta \approx \theta = \frac{x}{l}$$

$$\therefore \quad F = \frac{mgx}{l}$$

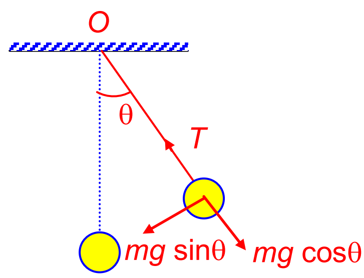
$$\therefore \quad a = -(g/l)x$$

Comparing it with equation of simple harmonic motion; $a = -\omega^2 x$

$$\omega^2 = g/l \quad \Rightarrow \omega = \sqrt{\frac{g}{l}}$$

$$\therefore \quad \text{Time period } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}} \qquad \dots \textbf{(12)}$$

(b) Torque method: Now taking moment of forces acting on the bob about O,



$$\tau = -(mg\sin\theta)\, l$$

$$I\alpha = -mg\, l\,(\theta)\ ;\ \text{since } \sin\theta \approx \theta$$

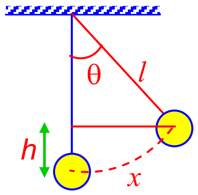
$$ml^2\alpha = mgl\,\theta$$

$$\therefore \quad \alpha = -\left(\frac{g}{l}\right)\theta$$

Comparing with simple harmonic motion equation; $\alpha = -\omega^2\theta$

$$\therefore \quad \omega = \sqrt{\frac{g}{l}} \quad \text{and} \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$$

(c) Energy method: Let the potential energy at the mean position be zero. When the bob is displaced through an angle ‘θ’, let its velocity be ‘v’.



Then potential energy at this new position

$$= mgl\,(1-\cos\theta) = U$$

Kinetic energy at this instant $= \dfrac{1}{2}mv^2 = K$

Total mechanical energy at this instant,

$$E = U + K = mgl\,(1-\cos\theta) + \frac{1}{2}mv^2$$

We know, in simple harmonic motion, E = constant.

$$\therefore \quad \frac{dE}{dt} = 0 \Rightarrow mgl\left[\sin\theta\frac{d\theta}{dt}\right] + mv\frac{dv}{dt} = 0$$

But $\quad v = l\dfrac{d\theta}{dt}$

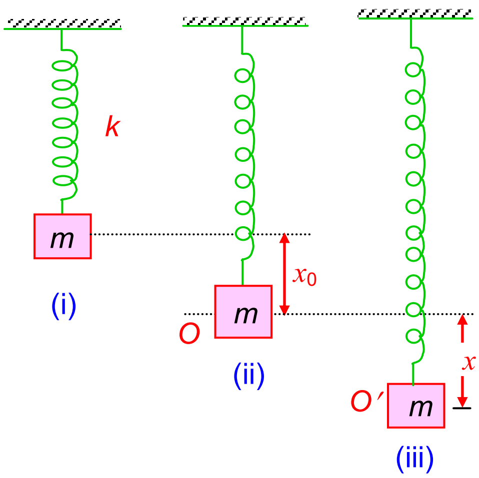
$$\therefore \quad \frac{dv}{dt} = -g\sin\theta \approx -g\theta$$

$$a = -g\frac{x}{l}$$

$$\therefore \quad \text{Time period} = T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$$

5.2 MASS-SPRING SYSTEM

Let a mass ‘m’ be attached to a massless spring of stiffness k. Because of its weight the mass will come down through a distance x_0 till it is balanced by the spring force kx_0. This position is called as mean or equilibrium position. Let the block be at rest in this position.



$$\therefore \quad mg = kx_0$$

$$\text{or} \quad x_0 = \frac{mg}{k} \qquad \dots \text{(i)}$$

Now the block is further displaced by a distance x in the downward direction as shown in figure (iii) and is left. Forces on the block at this instant,

$$F = -k(x_0 + x) + mg$$

$$\therefore \quad F = -kx \qquad \dots \text{(ii)}$$

This is the net restoring force acting on the block.

$$\therefore \quad ma = -kx$$

$$\therefore \quad a = -\left(\frac{k}{m}\right)x \qquad \dots \text{(iii)}$$

Comparing this with equation of simple harmonic motion,

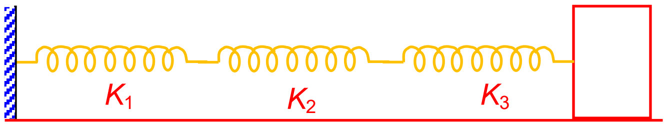
$$\omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}}$$

$$\therefore \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \qquad \dots (13)$$

5.3 COMBINATION OF SPRINGS

Series combination: The effective spring constant of all the springs connected in series (K_{eff}) is given as

$$\frac{1}{K_{eff}} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \ldots$$

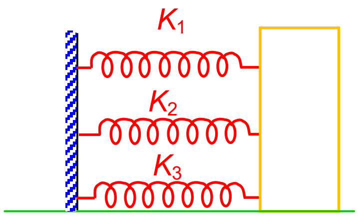


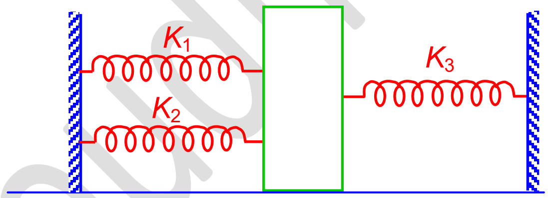
Force exerted by each spring is same.

Parallel combination:

The effective spring constant of all the springs connected in parallel combination (K_{eff}) is given as :

$$K_{eff} = K_1 + K_2 + K_3 + \ldots$$



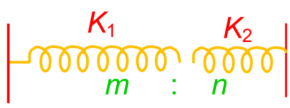


Displacement of each spring is same in parallel combination.

If the spring of spring constant K is cut into two part in a ratio of lengths of $m : n$ then two formed springs have new spring constants given by

$$K_1 = \frac{K(m+n)}{m}$$

$$K_2 = \frac{K(m+n)}{n}$$



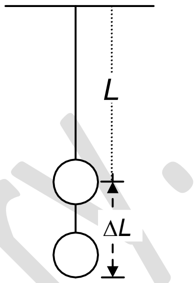
Spring constant K is inversely proportional to length of the spring i.e.

$$K \propto \frac{1}{l} \quad \text{(l = natural length of spring)}$$

Illustration 4

Question: **A point mass m is suspended at the end of a massless wire of length L and cross-section A. If Y be the Young's modulus for the wire, find the time period of oscillation for the simple harmonic motion along the vertical line. (take $\frac{AY}{mL} = \pi^2$)**

Solution: Let a force F be applied to stretch the wire by a length ΔL. If A be cross-section of the wire,



Stress = $\frac{F}{A}$

Strain = $\frac{\Delta L}{L}$

Young's modulus, $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta L/L}$ or, $F = AY\frac{\Delta L}{L}$

The restoring force due to elasticity = $-\frac{AY}{L}\Delta L$

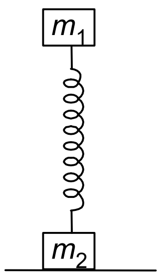
Since the force is proportional to the displacement of the suspended mass, when made to vibrate, executes simple harmonic motion.

The spring factor = $\frac{AY}{L}$

The period of oscillation $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{AY/L}} = 2\pi\sqrt{\frac{mL}{AY}}$ = **2 s**

Illustration 5

Question: **A body of mass m_1 is connected to another body of mass m_2 as shown in Figure and placed on a horizontal surface. The mass m_1 performs vertical harmonic oscillations with amplitude A = 10 cm and frequency $\omega = \pi$ rad/s.**



Neglecting the mass of the spring find the maximum and minimum values of force that the system exerts on the surface.

(Take m_1 = 6 kg and m_2 = 4 kg, π^2 = 10)

Solution: By compressing the spring let m_1 remain in equilibrium. Now the force with which the system presses the horizontal surface is $(m_1 + m_2)g$.

When m_1 performs vertical oscillations let it have an acceleration a at any instant, when moving down.

$\therefore$ the downward force due to oscillation = $m_1 a$

The maximum acceleration m_1 can have is $\omega^2 A$

where ω is the angular frequency and A the amplitude.

$\therefore \quad F_{max} = (m_1 + m_2)g + m_1\,\omega^2 A$

By similar reasoning,

$$F_{min} = (m_1 + m_2)g - m_1\,\omega^2 A$$

$$F_{max} = (6 + 4) \times 10 + 6 \times \pi^2 \times 10 \times 10^{-2} = \mathbf{106\ N}$$

$$F_{min} = 10 \times 10 - 6 \times \pi^2 \times 10 \times 10^{-2} = \mathbf{94\ N}$$

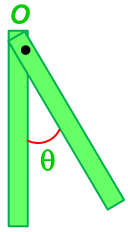
6 ANGULAR SIMPLE HARMONIC MOTION

A body free to rotate about a given axis can make angular oscillations. For example, a wooden stick nailed to a wall can oscillate about its mean position in the vertical plane.

The conditions for an angular oscillation to be angular simple harmonic motion are:

(i) When a body is displaced through an angle from the mean position ($\theta = 0$; $\tau = 0$), a resultant torque acts which is proportional to the angle displaced,

(ii) This torque is restoring in nature and it tries to bring the body towards the mean position,



If the angular displacement of the body at an instant is θ, then resultant torque on the body,

$$\tau = -k\theta \qquad \ldots \textbf{(14)}$$

If the moment of inertia is I, the angular acceleration is

$$\alpha = \frac{\tau}{I} = -\frac{k}{I}\theta$$

or, $$\frac{d^2\theta}{dt^2} = -\omega^2\theta \ ; \quad \text{where} \quad \omega = \sqrt{\frac{k}{I}} \qquad \ldots \textbf{(15)}$$

Solution of equation (13) gives,

$$\theta = \theta_0 \sin(\omega t + \phi) \qquad \ldots \textbf{(16)}$$

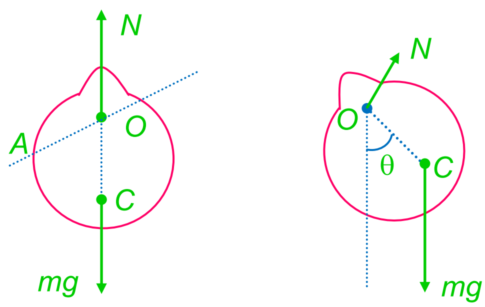
where θ_0 is the maximum angular displacement on either side.

Angular velocity at time 't' is given by

$$\Omega = \frac{d\theta}{dt} = \theta_0\omega\cos(\omega t + \phi) \qquad \ldots \textbf{(17)}$$

6.1 PHYSICAL PENDULUM

Any rigid body suspended from a fixed support constitutes a physical pendulum. For example a disc suspended through a hole in it. Figure (i) shows a physical pendulum. A rigid body is suspended through a hole at O. When the centre of mass C is vertically below O, the body may remain at rest. This is $\theta = 0$ position.



The body is rotated through an angle θ about a horizontal axis OA passing through O and perpendicular to the plane of motion.

The torque of the forces acting on the body, about the axis OA is

$\tau = mgl\sin\theta$; {where $l = OC$]

If moment of inertia of the body about OA is I, the angular acceleration becomes,

$$\alpha = \frac{\tau}{I} = -\frac{mgl}{I}\sin\theta$$

For small angular displacements $\sin\theta \approx \theta$

$$\therefore \quad \alpha = -\left(\frac{mgl}{I}\right)\theta$$

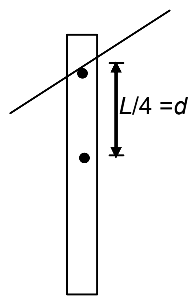
Comparing with $\alpha = -\omega^2\theta$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgl}} \qquad \dots \textbf{(18)}$$

Illustration 6

Question: **A uniform stick is suspended through a small-hole at the $L/4$ mark. Find the time period of small oscillations about the point of suspension. (take $L = \frac{12}{7}$m, $\pi = \sqrt{10}$)**

Solution: Let the mass of the stick be M. The moment of inertia of the stick about the axis of rotation through the point of suspension is



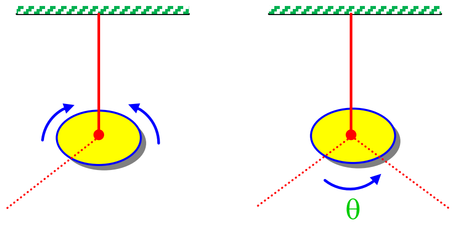
$$I = \frac{mL^2}{12} + md^2$$

Where L = 12/7m and d = 3/7 m

Time period = $T = 2\pi\sqrt{\frac{I}{mgd}}$ = **2 sec.**

6.2 TORSIONAL PENDULUM

In torsional pendulum, an extended body is suspended by a light thread or a wire. The body is rotated through an angle about the wire as the axis of rotation.



The wire remains vertical during this motion but a twist 'θ' is produced in the wire. The twisted wire exerts a restoring torque on the body, which is proportional to the angle of twist,

$\tau \propto -\theta$

$\tau = -k\theta$; k is proportionality constant and is called torsional constant of the wire.

If I be the moment of inertia of the body about the vertical axis, the angular acceleration is

$$\alpha = \frac{\tau}{I} = \frac{-k}{I}\theta = -\omega^2\theta$$

$$\therefore \quad \omega = \sqrt{\frac{k}{I}}$$

$$\therefore \quad \text{Time period } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{k}} \quad \dots \textbf{(19)}$$

Illustration 7

Question: **The moment of inertia of the disc used in a torsional pendulum about the suspension wire is 2000 kg – cm². It oscillates with a period of 2s. Another disc is placed over the first one and the time period of the system becomes 2.5 s. Find the moment of inertia of the second disc about the wire.**

Solution: Let the torsional constant of the wire be k.

$$T = 2\pi\sqrt{\frac{I}{k}} = 2\pi\sqrt{\frac{0.2}{k}} = 2 \quad \dots \text{(i)}$$

When the second disc having moment of inertia I_1 about the wire is added, the time period is,

$$2.5 = 2\pi\sqrt{\frac{0.2 + I}{k}} \quad \dots \text{(ii)}$$

from (i) & (ii) $I_1 \approx$ **1100 kg – cm²**

7 COMPOSITION OF TWO SIMPLE HARMONIC MOTIONS

If the particle is acted upon by two separate forces each of which can produce a simple harmonic motion, the resultant motion of the particle is a combination of two simple harmonic motions.

Let $\vec{r_1}$ denote the position of the particle at time *t* if the force $\vec{F_1}$ alone acts on it. Similarly, let $\vec{r_2}$ denote the position at time '*t*' if the force $\vec{F_2}$ alone acts on it.

According to Newton's second law of motion,

$$m\frac{d^2\vec{r_1}}{dt^2}=\vec{F_1} \quad \text{and,} \quad \frac{md^2\vec{r_2}}{dt^2}=\vec{F_2}$$

adding them, $$m\left(\frac{d^2\vec{r_1}}{dt^2}+\frac{d^2\vec{r_2}}{dt^2}=\vec{F_1}+\vec{F_2}\right)$$

$$\therefore \qquad m\frac{d^2}{dt^2}(\vec{r_1}+\vec{r_2})=\vec{F_1}+\vec{F_2} \qquad \ldots \text{(i)}$$

But $\vec{F_1}+\vec{F_2}$ is the resultant force acting on the particle and so the position $\vec{r}$ of the particle when both the forces act, is given by

$$m\frac{d^2\vec{r}}{dt^2}=(\vec{F_1}+\vec{F_2}) \qquad \ldots \text{(ii)}$$

Comparing equation (i) & (ii) we can show that

$$\vec{r}=\vec{r_1}+\vec{r_2} \quad \text{and} \quad \vec{v}=\vec{v_1}+\vec{v_2}$$

If these conditions are met at $t = 0$.

Thus the actual position of the particle is given by the vector sum of $\vec{r_1}$ & $\vec{r_2}$.

7.1 COMPOSITION OF TWO SIMPLE HARMONIC MOTIONS IN THE SAME DIRECTION

Let the direction be along *x*-axis and the simple harmonic motions produced by two forces $\vec{F_1}$ & $\vec{F_2}$ be;

$x_1 = A_1 \sin \omega t$

$x_2 = A_2 \sin (\omega t + \phi)$ respectively

From above discussion, the resultant position of the particle is then,

$x = x_1 + x_2 \;= A_1 \sin \omega t + A_2 \sin (\omega t + \phi)$

Put $A_1 + A_2 \cos\phi = A \cos \delta$

$A_2 \sin \phi = A \sin \delta$

$\therefore \quad A = \sqrt{(A_1 + A_2 \cos\phi)^2 + (A_2 \sin\phi)^2}$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi} \qquad \ldots \text{(i)}$$

and $x = A \cos \delta \sin \omega t + A \sin \delta \cos \omega t$

$$x = A \sin (\omega t + \delta) \qquad \ldots \textbf{(20)}$$

and, $$\tan \delta = \frac{A_2 \sin\phi}{A_1 + A_2 \cos\phi} \qquad \ldots \textbf{(21)}$$

The amplitude of resultant simple harmonic motion is given by

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi} \qquad \ldots \textbf{(22)}$$

It is maximum when $\phi = 0$

$$A_{max} = \sqrt{(A_1 + A_2)^2} = A_1 + A_2$$

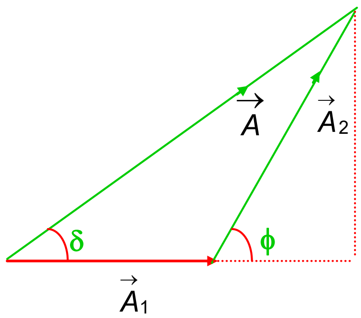
It is minimum when $\cos\phi = -1$ i.e., $\phi = \pi$

$$A_{min} = \sqrt{(A_1 - A_2)^2} = A_1 - A_2$$

7.2 VECTOR METHOD OF COMBINING TWO SIMPLE HARMONIC MOTION

$x_1 = A_1 \sin \omega t$

$x_2 = A_2 \sin (\omega t + \phi)$



We draw a vector of magnitude A_1 and another vector of magnitude A_2 making an angle ϕ with first vector as shown in the figure.

The resultant $\vec{A}$ of these two vectors will represent the resultant simple harmonic motion. From vector algebra,

$$\therefore \quad |\vec{A}| = A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi}$$

and $$\tan \delta = \frac{A_2 \sin\phi}{A_1 + A_2 \cos\phi}$$

Illustration 8

Question: **A particle is subjected to two simple harmonic motions**

$x_1 = A_1 \sin \omega t$ and $x_2 = A_2 \sin(\omega t + \pi/3)$

Find (a) the displacement at $t = 0$

(b) the maximum speed of the particle

(take $A_1 = 2\sqrt{3}\,\text{m}, A_2 = 4\sqrt{3}\,\text{m}, \ \omega = \dfrac{1}{\sqrt{21}}\,\text{rad/s}$)

Solution: (a) At $t = 0, x_1 = A_1 \sin \omega t = 0$

And $x_2 = A_2 \sin(\omega t + \pi/3) = \dfrac{A_2\sqrt{3}}{2}$

Thus resultant displacement at $t = 0$ is

$$x = x_1 + x_2 = \frac{A_2\sqrt{3}}{2} = \mathbf{6\ m}$$

(b) $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\dfrac{\pi}{3}}$

$$A = \sqrt{A_1^2 + A_2^2 + A_1A_2}$$

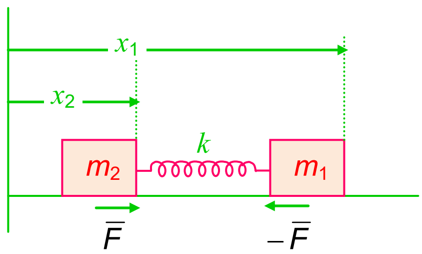
The maximum speed is

$$V_{max} = \omega A = \omega\sqrt{A_1^2 + A_2^2 + A_1A_2} = \mathbf{2\ m/s}$$

8 TWO BODY SYSTEM

In a two body oscillations, such as shown in the figure, a spring connects two objects, each of which is free to move. When the objects are displaced and released, they both oscillate.

The relative separation $x_1 - x_2$ gives the length of the spring at any time. Suppose its unstretched length is L; then $x = (x_1 - x_2) - L$ is the change in length of the spring, and $F = kx$ is the magnitude of the force exerted on each particle by the spring as shown in the figure.



Applying newton's second law separately to the two particles, taking force component along the x-axis; we get

$$\frac{m_1 d^2x_1}{dt^2} = -kx \quad \text{and} \quad \frac{m_2 d^2x_2}{dt^2} = +kx$$

We now multiply the first of these equations by m_2 and the second by m_1, and then subtract. The result is

$$m_1 m_2 \frac{d^2 x_1}{dt^2} - m_1 m_2 \frac{d^2 x_2}{dt^2} = -m_2 kx - m_1 kx$$

which can be written as

$$\frac{m_1 m_2}{(m_1 + m_2)} \frac{d^2}{dt^2}(x_1 - x_2) = -kx \qquad \ldots \text{(i)}$$

The quantity $\frac{m_1 m_2}{m_1 + m_2}$ has the dimensions of mass and is known as the reduced mass μ

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \qquad \ldots \textbf{(23)}$$

Since L is constant

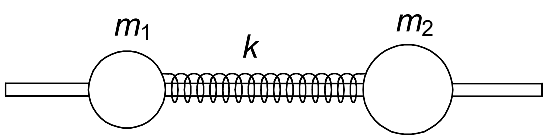
$$\frac{d}{dt}(x_1 - x_2) = \frac{d}{dt}(x + L) = \frac{dx}{dt}$$

$\therefore$ equation (i) becomes $\frac{d^2 x}{dt^2} + \frac{k}{\mu} x = 0$

$$\therefore \quad T = 2\pi \sqrt{\frac{\mu}{k}} \qquad \ldots \textbf{(24)}$$

Illustration 9

Question: **Two balls with masses m_1 = 1 kg and m_2 = 2 kg are slipped on a thin smooth horizontal rod. The balls are interconnected by a light spring of spring constant 24 N/m. The left hand ball is imparted the initial velocity v_1 = 12 cm/s. Find (a) the oscillation period (in ms) of the system, (b) the amplitude of the oscillation.**



Solution: **(a)** As discussed earlier;

$\therefore \quad \omega_o^2 = \dfrac{k}{\dfrac{m_1 m_2}{m_1 + m_2}}$ where ω_0 is the natural frequency of oscillation.

$\omega_o^2 = \dfrac{k}{\mu}$ where $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$ called reduced mass.

$\therefore \quad \omega_o = \sqrt{\dfrac{k}{\mu}}$; $\quad T = 2\pi\sqrt{\dfrac{\mu}{k}} = \mathbf{1047\ ms}$

(b) The initial velocity given to the mass m_1 is v_1.

For undamped oscillation, this initial energy will remain constant.

Hence total energy of S.H.M. of two balls is given as $E = \frac{1}{2}\mu v_1^2$

If amplitude of oscillation is a, then

$$\frac{1}{2}\mu v_1^2 = \frac{1}{2}ka^2$$

$\Rightarrow \quad v_1 = \omega_0 a$

$\Rightarrow \quad a = \frac{v_1}{\omega_o}$

So on putting the values, we get a = **2 cm**

PROFICIENCY TEST

The following questions deal with the basic concepts of this section. Answer the following briefly. Go to the next section only if your score is at least 80%. Do not consult the Study Material while attempting these questions.

1. Find the ratio of time periods of two simple pendulums whose lengths are l_1 and l_2 where $l_1 : l_2 = 1:4$

2. A mass 'm' is connected to a spring of spring constant k. Now (2/3) of the length of the spring is removed and only (1/3) of the length is holding the mass then find the ratio of time period of oscillations of first case to that of the second case.

3. The equation of a SHM of a particle is given by

$$y = 100 \sin (25\,t + \pi/6) \text{ (cm)}$$

Find the maximum acceleration and maximum velocity of the particle.

4. Write down the possible equations of simple harmonic motion for a particle oscillating along x-axis if the amplitude of its oscillations is 100 units and angular frequency is 20 rad/s, and at $t = 0$ it is at its extreme position.

5. The equations of two SHMs in the same direction are given by

$$y_1 = 50 \sin 30\pi\, t \text{ (cm)}$$

$$y_2 = 50 \sin (30\,\pi\, t + \pi/3) \text{ (cm)}$$

Find the amplitude of resultant SHM formed by their superposition. ($\sqrt{3} = 1.7$)

6. A body is oscillating about an axis through which its moment of inertia = 100 kg-m^2. When it is displaced by small angle 'θ' from its equilibrium position the torque on it is $\tau = -4\theta$. Then find the time period of oscillations of the body. ($\pi = 3.1$)

7. In a laboratory experiment with simple pendulum it was found that it took 36 s to complete 20 oscillations when the effective length was kept at 80 cm. Calculate the acceleration due to gravity from these data.

8. A particle of mass 40 g executes a simple harmonic motion of amplitude 2.0 cm. If the time period is 0.20 s, find the total mechanical energy of the system.

9. A block of mass 5 kg executes simple harmonic motion under the restoring force of a spring on a smooth horizontal surface. The amplitude and the time period of the motion are 0.1 m and 3.14 s respectively. Find the maximum force exerted by the spring on the block.

10. A particle moves in the x-y plane according to the equation $\bar{r} = (i + 2j)\,A\cos\omega t$, which of the statements about the motion of the particle given below are true

(a) it is on a straight line

(b) it is periodic

(c) it is simple harmonic motion

ANSWERS TO PROFICIENCY TEST

1. $1:2$

2. $\sqrt{3}:1$

3. $625\ ms^{-2}$, $25\ ms^{-1}$

4. $x = 100 \sin(20\,t \pm \pi/2)$

5. 85 cm

6. 31 sec

7. $975\ cm\ s^{-2}$

8. 8 mJ

9. 2 N

10. All are true

SOLVED OBJECTIVE EXAMPLES

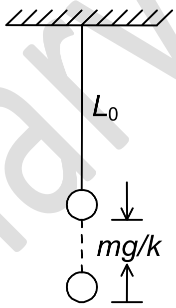
***Example* 1:**

A body of mass *m* is suspended from a rubber cord with force constant *k*. The maximum distance over which the body can be pulled down for the body's oscillation to remain harmonic is

(a) $\frac{2mg}{k}$ (b) $\frac{mg}{k}$ (c) $\frac{2k}{mg}$ (d) $\frac{mg}{2k}$

Solution:

Let L_0 be the unstretched length of the cord; when the body is suspended the extension is $\frac{mg}{k}$.



The equilibrium length is thus $L_0 + \frac{mg}{k}$. If during oscillations the amplitude is greater than $\frac{mg}{k}$, the bob will be more than $\frac{mg}{k}$ above the equilibrium position and the cord will no longer be taut.

∴ **(b)**

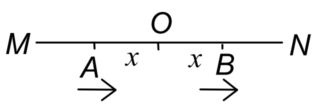
***Example* 2:**

A particle moving with S.H.M. passes through points *A* and *B* with same velocity having occupied 2 seconds in passing from *A* to *B*. After another 2 seconds it returns to *B*. The period in seconds is

(a) 2 **(b) 4** **(c) 5** **(d) 8**

Solution:

The velocity of a particle executing simple harmonic motion is given by $v = \omega\sqrt{a^2 - x^2}$ where a is the amplitude and x is the displacement from equilibrium position O.



Time taken from A to B = 2 seconds. Time taken from B to N and back to B is another 2 seconds. Time taken from B to A must be 2 seconds.

∴ time taken from A to M and back is 2 seconds. Hence the total time for one complete to and fro motion will be 8 seconds.

∴ **(d)**

***Example* 3:**

An object is attached to a vertical spring and slowly lowered to its equilibrium position. This stretches the spring by a distance *d*. If the same object is attached to the same vertical spring but permitted to fall instead the spring will be stretched through a distance

(a) $\frac{d}{2}$ (b) $\frac{3d}{2}$ (c) $\frac{d}{\sqrt{2}}$ (d) $2d$

Solution:

If the spring is loaded and lowered slowly to the equilibrium position, the object has no kinetic energy and comes to rest with an extension d to the spring, which is equal to $\frac{mg}{k}$. When it is allowed to fall, it acquires a kinetic energy as it passes the equilibrium position and will oscillate about the equilibrium position with amplitude d.

$\therefore$ **(d)**

***Example* 4:**

A block is on a horizontal surface, which makes oscillations horizontally with frequency 2 Hz. The coefficient of friction between block and surface is 0.4. What is the maximum amplitude so that the block does not slide?

(a) 2.5 cm (b) 0.8 cm (c) 0.2 cm (d) 0.4 cm

Solution:

When the displacement is maximum $F = m\omega^2 a \leq \mu mg$ for no sliding.

$$\omega^2 a = \mu g; \quad \frac{4\pi^2}{T^2} \cdot a = \mu g$$

$$a = \frac{\mu g T^2}{4\pi^2} = \frac{0.4 \times 9.8 \times \left(\frac{1}{2}\right)^2}{4\pi^2} = 0.025 \text{ m} \quad \text{or } 2.5 \text{ cm}$$

$\therefore$ **(a)**

Example 5:

A particle starts executing S.H.M. of amplitude *a* and total energy *E*. At the instant its kinetic energy is $\frac{3E}{4}$ its displacement *y* is given by

(a) $y = \frac{a}{\sqrt{2}}$ **(b) $y = \frac{a}{2}$** **(c) $y = \frac{a\sqrt{3}}{2}$** **(d) $y = a$**

Solution:

Total energy $E = \frac{1}{2}m(\omega a)^2$

Kinetic energy $= \frac{1}{2}m\omega^2\sqrt{a^2-y^2}$; Potential energy $= \frac{1}{2}m\omega^2 y^2$

Since its kinetic energy $= \frac{3}{4}E$, the potential energy $= \frac{E}{4}$

$$\therefore \quad \frac{\frac{1}{2}m\omega^2y^2}{\frac{1}{2}m\omega^2a^2} = \frac{1}{4} \Rightarrow y^2 = \frac{a^2}{4}; y = \frac{a}{2}$$

$\therefore$ **(b)**

Example 6:

An object of mass 0.2 kg executes S.H.M. along *x*-axis with a frequency of $\frac{25}{\pi}$ Hz. At the position *x* = 0.04 m the object has kinetic energy 0.5 J and potential energy 0.4 J. The amplitude of oscillation will be

(a) 0.06 m **(b) 0.04 m** **(c) 0.05 m** **(d) 0.25 m**

Solution:

Total energy $E = \frac{1}{2}m\omega^2A^2$

$$A^2 = \frac{2E}{m\omega^2} = \frac{2(\text{Kinetic energy} + \text{Potential energy})}{m(2\pi n)^2} = \frac{2(0.5+0.4)}{0.2\left(2\pi\times\frac{25}{\pi}\right)^2}$$

$$= \frac{2\times 0.9}{0.2\times 50^2} = \left(\frac{3}{50}\right)^2$$

$A = \frac{3}{50}$ m or, 0.06 m

$\therefore$ **(a)**

Example 7:

A mass M attached to a spring oscillates with a period of 2 seconds. If the mass is increased by 2 kg the period increases by one second. The initial mass M will be

(a) 1.6 kg **(b) 1 kg** **(c) 1.5 kg** **(d) 2 kg**

Solution:

Period of the mass attached to the spring is given by

$$T=2\pi\sqrt{\frac{M}{k}} \quad \therefore \quad 2=2\pi\sqrt{\frac{M}{k}} \qquad \text{... (i)}$$

In the second case, $3=2\pi\sqrt{\frac{M+2}{k}}$ ... (ii)

Dividing (i) by (ii), $\frac{2}{3}=\sqrt{\frac{M}{M+2}} \quad \therefore \frac{4}{9}=\frac{M}{M+2}$

$9M = 4M + 8$; $5M = 8$; $M =$ **1.6 kg**

$\therefore$ **(a)**

Example 8:

A long spring is stretched by 2 cm and its potential energy is V. If the spring is stretched by 10 cm the potential energy will be

(a) $\frac{V}{25}$ **(b) $\frac{V}{5}$** **(c) 5 V** **(d) 25 V**

Solution:

When the spring is stretched by 2 cm, the energy stored $= \frac{1}{2}kx^2 = \frac{1}{2}k \times 4$

$\therefore \quad V = \frac{1}{2} \times k \times 4$

When $x = 10$, Energy $= \frac{1}{2} \times k \times 100$; potential energy $y = \frac{1}{2} \times 100 \times \frac{V}{2} = 25$ V

$\therefore$ **(d)**

Example 9:

A particle moves in a straight line. If v is the velocity at a distance x from a fixed point on the line and $v^2 = a - bx^2$, where a and b are constants, then

(a) the motion continues along the positive x direction only

(b) the motion is simple harmonic

(c) the particle oscillates with a frequency equal to $\frac{\sqrt{b}}{2\pi}$

(d) the total energy of the particle is ma

Solution:

$$v^2 = a - bx^2$$

Differentiating with respect to time,

$$2v\frac{dv}{dt} = -2bx\frac{dx}{dt}$$

$$\frac{dv}{dt} = -bx$$

The particle executes S.H.M. Angular frequency = $\omega = \sqrt{b}$

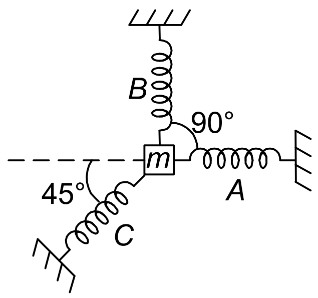
Frequency $\nu = \frac{\omega}{2\pi} = \frac{\sqrt{b}}{2\pi}$

Total energy of the particle, $E = \frac{1}{2}mv^2 = \frac{1}{2}ma$ when $x = 0$

∴ **(b) and (c)**

Example 10:

A particle of mass m is attached to three springs A, B and C of equal force constants as shown in the figure. If the particle is pushed slightly against the spring C and released, the time of oscillation will be



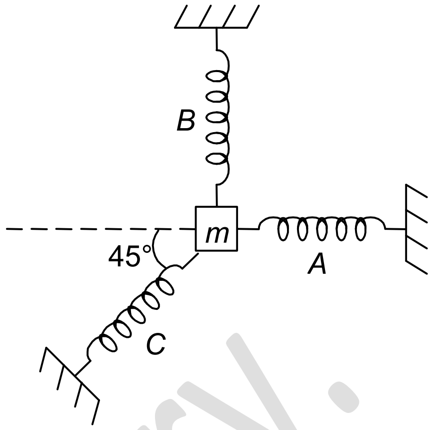
(a) $2\pi\sqrt{\frac{m}{2k}}$

(b) $2\pi\sqrt{\frac{m}{k}}$

(c) $2\pi\sqrt{\frac{m}{3k}}$

(d) $2\pi\sqrt{\frac{2m}{k}}$

Solution:



Let x be the displacement of C.

Forces acting on m in the displaced position of mass m are

(1) kx in the direction of C

(2) $kx \cos 45°$ in the direction of B

(3) $kx \cos 45°$ in the direction of A

Net force in the direction of C

$$= kx + \sqrt{\left(\frac{kx}{\sqrt{2}}\right)^2 + \left(\frac{kx}{\sqrt{2}}\right)^2} = 2kx$$

$$\frac{md^2x}{dt^2} = -2kx$$

Period of oscillation, $T = 2\pi\sqrt{\frac{m}{2k}}$

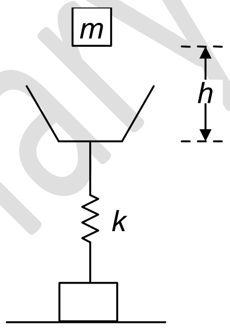
$\therefore$ **(a)**

SOLVED SUBJECTIVE EXAMPLES

***Example* 1:**

A body of mass m = 1kg falls from a height h = 1m on to the pan of a spring balance. The masses of the pan and spring are negligible. The spring constant of the spring is k= 50N/m. Having stuck to the pan the body starts performing harmonic oscillations in the vertical direction. Find the energy of oscillation.

***Solution*:**



Suppose by falling down through a height h, the mass m compresses the spring balance by a length x.

The P.E. lost by the mass = $mg\,(h + x)$

This is stored up as energy of the spring by compression

$$= \frac{1}{2}kx^2$$

$$\therefore \qquad mg(h+x) = \frac{1}{2}kx^2 \text{ or } \frac{1}{2}kx^2 - mgx - mgh = 0$$

or $$x^2 - \frac{2mgx}{k} - \frac{2mgh}{k} = 0$$

Solving this quadratic equation, we get

$$x = \frac{\frac{2mg}{k} \pm \sqrt{\left(\frac{2mg}{k}\right)^2 + \left(\frac{8mgh}{k}\right)}}{2} = \frac{mg}{k} \pm \frac{mg}{k}\sqrt{1 + \frac{2kh}{mg}}$$

In the equilibrium position, the spring will be compressed through the distance mg/k and hence the amplitude of oscillation is

$$A = \frac{mg}{k}\sqrt{1 + \frac{2kh}{mg}}$$

$$\text{Energy of oscillation} = \frac{1}{2}kA^2 = \frac{1}{2}k\left(\frac{mg}{k}\right)^2\left(1 + \frac{2kh}{mg}\right)$$

$$= mgh + \frac{(mg)^2}{2k} = \mathbf{11\ J}$$

***Example* 2:**

A particle rests on a horizontal plane, which is displaced up and down, with S.H.M. of frequency 50 Hz. If the amplitude of motion is half of the maximum amplitude of motion for the particle to remain in contact with the plane, upto what value can the frequency of vibration be increased, the particle still remaining in contact with the plane. ($\sqrt{2}=1.4$)

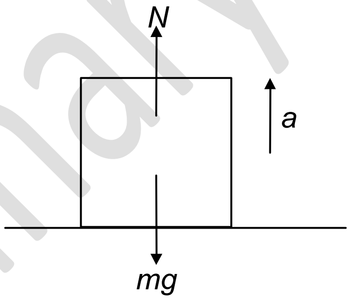
***Solution*:**

Let N be the reaction of the plane at any instant and 'a' the acceleration of the plane.

When the plane is moving up

$$N - mg = ma \quad \text{or} \quad N = ma + mg$$

Since $N \neq 0$ during the upward motion of the vibration of the plane the particle will not lose contact with the plane whatever be the value of the upward acceleration of the plane.



Now let us consider the downward motion of the particle during the vibration. The force equation becomes

$$mg - N = ma \quad \text{or,} \quad N = mg - ma = m(g - a)$$

If $a = g$, N becomes zero and at that instant the particle loses contact with the plane. Hence the condition for the particle to be in contact with the plane is that the downward acceleration, at any instant, should be smaller than g. The maximum value of the downward acceleration is

$$a = -\omega^2 A$$

where A is the amplitude of vibration.

$$\therefore \quad \omega^2 A < g \quad \text{or} \quad A < \frac{g}{\omega^2}$$

$$\text{or} \quad A < \frac{g}{4\pi^2\nu^2}$$

$$\text{Now} \quad \frac{g}{4\pi^2\nu^2} = \frac{9.8}{4\pi^2 50^2} = 9.93 \times 10^{-5}\, m$$

Hence the maximum amplitude permissible = 9.93×10^{-5} m

If the amplitude be halved, let the maximum angular frequency be ω_1.

$$\text{Now} \quad \omega_1^2 \frac{A}{2} = g$$

Since $g = \omega^2 A$, we have

$$\omega_1^2 \frac{A}{2} = \omega^2 A$$

$$\omega_1^2 = 2\omega^2$$

or $\quad 4\pi\nu_1^2 = 2 \times 4\pi\nu^2$

or $\quad \nu_1^2 = 2 \times 50^2$

or $\quad \nu_1 = 50\sqrt{2} = \mathbf{70\ Hz}$

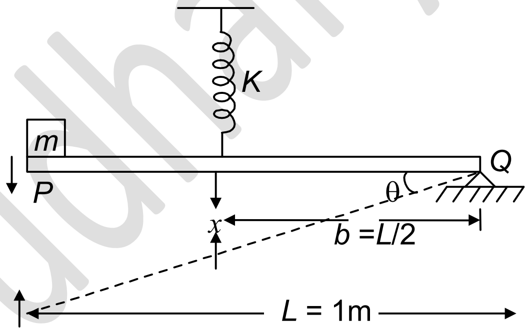
***Example* 3:**

The rod *PQ* of mass, *M*= 3 kg is attached as shown to a spring of constant *K*= $8\pi^2$ N/m. A small block of mass, m =1 kg is placed on the rod at its free end *P*. If end *P* is moved down through a small distance *x'* and released, determine the period of vibration.

***Solution*:**

Method I: Using energy equation :

moment of inertia of the system about Q is



$$I = \left[\frac{ML^2}{3} + mL^2\right]$$

where L is the length of the rod.

Elastic potential energy of the spring = $\frac{1}{2}Kx^2$

Rotational energy of the rod PQ $= \frac{1}{2}I\omega^2$

By the law of conservation of energy,

$$\frac{1}{2}I\omega^2 = \frac{1}{2}Kx^2$$

$$\frac{1}{2}\left[\frac{ML^2}{3} + mL^2\right]\frac{v^2}{L^2} = K\left(\frac{x'^2b^2}{L^2}\right); \text{ where } x = \frac{x'b}{L}$$

and x' = displacement of m

Differentiating with respect to time,

$$\left(\frac{\frac{M}{3}+m}{2}\right)L^2 \times 2v\frac{dv}{dt} = \frac{1}{2}Kb^2 \times 2x'\frac{dx'}{dt}; \text{ where } \frac{dx'}{dt} = v = \text{velocity of } m$$

Acceleration of the block, $\frac{dv}{dt} = \frac{Kb^2x'}{\left(\frac{M}{3}+m\right)L^2}$

Acceleration of the block is directly proportional to its linear displacement.

This represents an SHM with angular frequency ω given by

$$\omega^2 = \frac{Kb^2}{\left(\frac{M}{3}+m\right)L^2}$$

Period of vibration $\frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\left(\frac{M}{3}+m\right)L^2}{Kb^2}} = \mathbf{2s}$

Method II: Using torque equation, when the system is displaced through small angle θ, tension in the string, $T = Kx = K(b\theta)$, $(\because\ x = b\theta)$

$\Rightarrow$ Restoring torque, $\tau = T \times b = Kb^2\,\theta = -I\ddot{\theta}$

where $I = \frac{ML^2}{3} + mL^2$

$$\Rightarrow \quad \ddot{\theta} = -\left(\frac{Kb^2}{I}\right)\theta = -\left\{\frac{Kb^2}{\left(\frac{M}{3}+m\right)L^2}\right\}\theta$$

This is an angular SHM with angular frequency, $\omega = \sqrt{\frac{Kb^2}{\left(\frac{M}{3}+m\right)L^2}}$

$$\Rightarrow \quad \text{Period of vibration} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\left(\frac{M}{3}+m\right)L^2}{Kb^2}} = \mathbf{2s}$$

Example 4:

A simple pendulum of length L and mass m is suspended in a car that is travelling with a constant speed V around a circular track of radius R. If the pendulum makes small oscillations about its equilibrium position, what will be the time period of oscillations?

(take $\pi = \sqrt{10}$, $g^2R^2 + V^4 = 40000$, $R = 10\,\text{m}$, $L = 2\,\text{m}$**)**

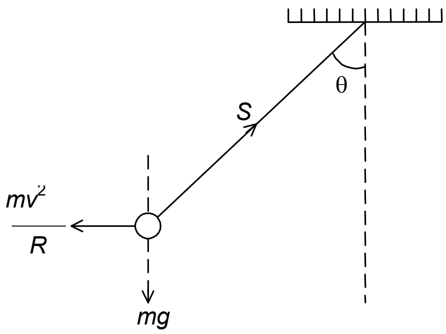
Solution:

When the car comes round a circle it is an accelerated frame of reference. A fictitious force $\frac{mV^2}{R}$ is to be introduced to the simple pendulum as a centrifugal force. If θ be the angular displacement of the pendulum in its new equilibrium position, then

$S\cos\theta = mg$

$S\sin\theta = \frac{mV^2}{R}$

where S is the tension in the string.



$$S = \sqrt{(mg)^2 + \left(\frac{mV^2}{R}\right)^2}$$

$$= m\sqrt{g^2 + \frac{V^4}{R^2}}$$

Let the pendulum be slightly displaced so that it makes an angle $(\theta + d\theta)$ with the vertical and then let go.

The restoring forces $= S\sin d\theta \approx Sd\theta = \frac{Sx}{L}$

where x is the linear displacement and L the length of the pendulum and $x = Ld\theta$.

The restoring force/unit displacement $k = \dfrac{Sx}{L}\Big/x$

$$= \frac{S}{L}$$

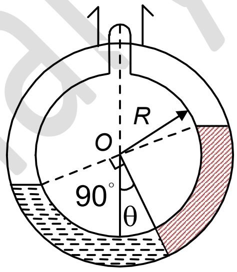
The period of oscillation of the pendulum

$$T=2\pi\sqrt{\frac{m}{k}}=2\pi\sqrt{\frac{m}{S/L}} = 2\pi\sqrt{\frac{Lm}{S}}$$

$$= 2\pi\sqrt{\frac{Lm}{m\left(g^2+\frac{V^4}{R^2}\right)^{1/2}}} = 2\pi\sqrt{\frac{L}{\left(g^2+\frac{V^4}{R^2}\right)^{1/2}}} = \mathbf{2\ s}$$

***Example* 5:**

Two non-viscous, incompressible and immiscible liquids of densities ρ and 1.5 ρ are poured into the two limbs of a circular tube of radius *R* and small cross-section kept fixed in a vertical plane as shown in the figure. Each liquid occupies one-fourth of the circumference of the tube.



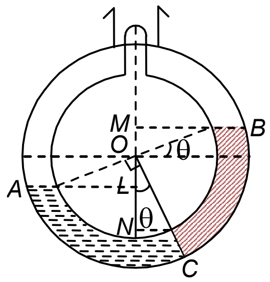
If the whole liquid is given a small displacement from its equilibrium position, show that the resulting oscillations are simple harmonic. Find the time period of these oscillations. (*R* = 2π³ cos θ, π² = 10)

***Solution*:**

Let A = cross-sectional area of tube.

Mass of liquid column $AC = \dfrac{2\pi R}{4}A\ (1.5\rho)$

Mass of liquid column $CB = \dfrac{2\pi R}{4}A\rho$



Moment of inertia of the whole liquid about $O = I = \left(\dfrac{\pi RA}{2}\right)(1.5+1)\,\rho R^2$

Let y be the small displacement from equilibrium position P towards left side.

If α is the corresponding angular displacement,

$$\alpha=\frac{y}{R} \quad \text{or} \quad y=\alpha R$$

Torque about $O = I\dfrac{d^2\alpha}{dt^2}$

$$= \frac{\pi RA}{2}\times 2.5\rho R^2\frac{d^2\alpha}{dt^2}$$

Restoring torque due to the displaced liquid = – [Ay (1.5 ρ) g + Ay ρg] × Rcos θ

where 'R cos θ' is perpendicular distance of gravitational force from axis of rotation.

∴ $T_{restoring} = -\,2.5\,A\alpha\rho g R^2\cos\theta$

$$\left(\frac{\pi RA}{2}\right)2.5\rho R^2\frac{d^2\alpha}{dt^2} = -2.5A\alpha\rho gR^2\cos\theta$$

$$\frac{d^2\alpha}{dt^2} = -2g\frac{\cos\theta\alpha}{\pi R}$$

(i.e) angular acceleration ∝ angular displacement

$\therefore \quad \omega = \sqrt{2g\cos\theta / \pi R}$

$\therefore \quad$ time period $= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\pi R}{2g\cos\theta}}$ **= 20s**

Example 6:

A particle of mass m is performing simple harmonic motion in a straight line with amplitude $r = \sqrt{2}$ cm and period *T*. Find the law of force. When at a distance *Kr* from the centre of oscillation, it collides with a stationary particle of the same mass and coalesces with it. If the law of force is the same, find the new amplitude. (k = $\sqrt{15}$ SI unit)

Solution:

Amplitude given = *r*

Period = *T*; hence angular frequency $\omega = \frac{2\pi}{T}$

$$a = -\omega^2 x$$

$$F = ma = -m\left(\frac{2\pi}{T}\right)^2 x$$ at a position x

$\therefore \quad v = \omega\sqrt{r^2 - x^2}$

speed at x = kr is $v = \omega r\sqrt{1-k^2}$

according to conservation of linear momentum during collision –

$$mv = 2mv'$$

$$v' = \frac{\omega r}{2}\sqrt{1-k^2}$$

Now the oscillating mass becomes 2m then new angular frequency ω'

$-2m\omega'^2 x = -m\omega^2 x$ (force law is same)

$$\omega' = \frac{\omega}{\sqrt{2}}$$

then new amplitude r' is

$$v' = \omega'\sqrt{r'^2 - x^2}$$

Hence $r' = r\sqrt{\frac{1}{2}(1+K^2)}$

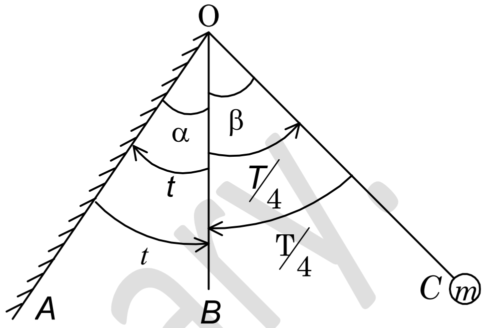
Thus the new amplitude of oscillation = $r\sqrt{\frac{1}{2}(1+K^2)}$ **= 4cm**

***Example* 7:**

A ball is suspended by a thread of length ℓ at the point O on the wall, forming a small angle α with the vertical. Then the thread with the ball was deviated through small angle β ($\beta > \alpha$) and set free. Assuming the collision of the ball against the wall to be perfectly elastic, find the oscillation period of such a pendulum. ($\frac{\alpha}{\beta} = \frac{1}{\sqrt{2}}, \sqrt{\frac{l}{g}} = \frac{2}{\pi}$)

***Solution*:**

As β is a small angle, the motion of the ball is S.H.M. After perfectly elastic collision the velocity of the ball is simply reversed. As shown in figure, the time period of one oscillation will be



$$T' = \frac{T}{4} + \frac{T}{4} + t + t = \frac{T}{2} + 2t$$

where $\quad T = 2\pi\sqrt{\frac{l}{g}} \quad$ or $\quad \frac{T}{2} = \pi\sqrt{\frac{l}{g}}$

$\theta = \theta_0 \sin \omega t$

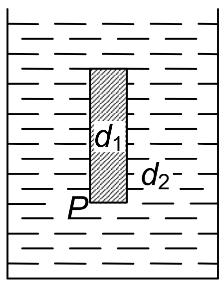
$\alpha = \beta \sin \omega t$, where t is the time taken from B to A.

$$t = \frac{1}{\omega}\sin^{-1}\left(\frac{\alpha}{\beta}\right) = \sqrt{\frac{l}{g}}\sin^{-1}\left(\frac{\alpha}{\beta}\right)$$

$$T' = \pi\sqrt{\frac{l}{g}} + 2\sqrt{\frac{l}{g}}\sin^{-1}\left(\frac{\alpha}{\beta}\right) = 2\sqrt{\frac{l}{g}}\left[\frac{\pi}{2} + \sin^{-1}\left(\frac{\alpha}{\beta}\right)\right] = 3\text{s}$$

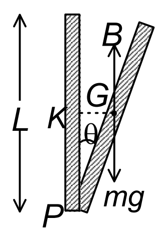
***Example* 8:**

A thin rod of length L = 15 cm and area of cross-section S is pivoted at its lowest point P inside a stationary, homogeneous and non-viscous liquid. The rod is free to rotate in a vertical plane about a horizontal axis passing through P. The density d_1 of the material of the rod is smaller than the density d_2 of the liquid. The rod is displacement by a small angle θ from its equilibrium position and then released. Show that the motion of the rod is simple harmonic and determine its angular frequency in terms of the given parameters.
(given $d_2 = 10000$ kg/m^3, $d_1 = 2000$ kg/m^3)



***Solution*:**

Let the rod be displaced through an angle θ. The different forces on the rod are shown in the figure.



The force acting upwards at the middle point G of the rod

= upward thrust – weight of rod = $B - mg$

= weight of displaced liquid – weight of rod

= $LSd_2g - LSd_1g = LSg\,(d_2 - d_1)$

Moment of the couple restoring it to the original position

$$= LSg\,(d_2 - d_1)\,KG = LSg\,(d_2 - d_1)\,\frac{L}{2}\sin\theta$$

Torque $\tau = LSg\,(d_2 - d_1)\,\frac{L}{2}\sin\theta = I\left(\frac{d^2\theta}{dt^2}\right)$

where I is moment of inertia of the rod about the axis through O.

$$\therefore \quad I\left(\frac{d^2\theta}{dt^2}\right) = -\frac{L^2Sg}{2}(d_2 - d_1)\theta \quad \text{[because } \theta \text{ is small } \sin\theta \simeq \theta]$$

So, $\quad \frac{d^2\theta}{dt^2} \propto \theta$. Hence it executes S.H.M.

$$\therefore \quad \frac{d^2\theta}{dt^2} = \frac{-L^2 Sg}{2I}(d_2 - d_1)\theta$$

But $$I = \frac{ML^2}{3}$$

$$\therefore \quad \frac{d^2\theta}{dt^2} = \frac{-L^2 Sg(d_2-d_1)\theta}{\frac{2ML^2}{3}} = -\frac{3}{2}\frac{Sg(d_2-d_1)}{M}\theta$$

$$= -\frac{3}{2}\frac{Sg(d_2-d_1)\,\theta}{LSd_1}$$

$$= -\frac{3}{2}\frac{g}{L}\left(\frac{d_2-d_1}{d_1}\right)\theta$$

But $$\frac{d^2\theta}{dt^2} = -\omega^2\theta$$

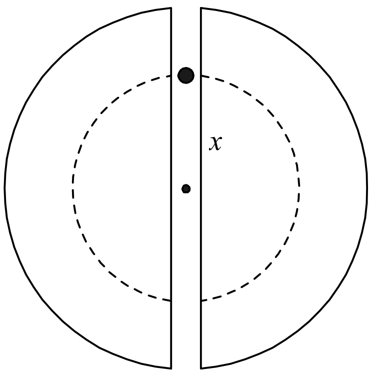
$$\omega = \sqrt{\frac{3g(d_2-d_1)}{2Ld_1}} = \mathbf{20\ rad/s}$$

***Example* 9:**

Assume that a narrow tunnel is dug between two diametrically opposite points of earth. Treat the earth as a solid sphere of uniform density. Show that if a particle is released in this tunnel it will execute S.H.M. Find the time period. (Given $R = 6400$ km, $\pi = 3.14$)

***Solution*:**

Consider the situation shown in the figure. Suppose at an instant t the particle in the tunnel is at a distance x from centre of earth. Let us draw a sphere of radius x with its centre at the centre of the earth. Only the part of the earth within the sphere will exert a net force of attraction on the particle.



Mass of this part $$M' = \frac{\frac{4}{3}\pi x^3}{\frac{4}{3}\pi R^3}M = \frac{x^3}{R^3}.M$$

$\therefore$ the force of attraction $$F = \frac{G\cdot\left(\frac{x^3}{R^3}\right)Mm}{x^2} = \frac{GMm}{R^3}.x$$

The force acts towards the centre of the earth. Thus the resultant force on the particle is opposite to the displacement from centre of earth and is proportional to it. The particle therefore executes S.H.M in the tunnel with the centre of earth as mean position.

$$\therefore \quad m.\frac{d^2x}{dt^2} = \frac{GMm}{R^3}.x \qquad \frac{d^2x}{dt^2} = \frac{GM}{R^3}.x$$

$\therefore$ comparing this equation with the standard equation of S.H.M we see $\omega^2 = \frac{GM}{R^3}$

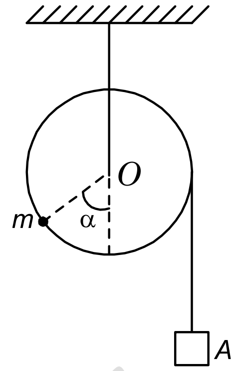
$$\therefore \quad \text{period} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R^3}{GM}}$$

But $\frac{GM}{R^2} = g$ (acceleration due to gravity)

$$\therefore \quad \text{period} = 2\pi\sqrt{\frac{R}{g}}. = \mathbf{5024\ s}$$

Example 10:

A uniform cylindrical pulley of mass M and radius R can freely rotate about the horizontal axis O. The free end of a thread tightly wound on the pulley carries a dead weight A. At a certain angle α it counter balances a point mass m fixed at the rim of the pulley. Find the frequency of small oscillations of the arrangement. (Given $\dfrac{MR+2mR(1+\sin\alpha)}{2m} = g\pi^2\cos\alpha$, **$\pi^2 = 10$)**



Solution:

Considering rotational equilibrium about O, we have

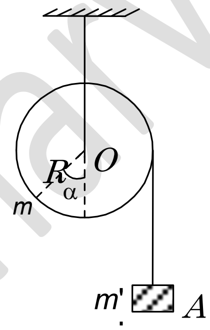
$m'gR = mgR\sin\alpha$, where m' is mass of A

$\therefore\quad m' = m\sin\alpha$

Consider a small angular displacement by θ in clockwise direction.

Then unbalanced torque τ in the clockwise direction

$= m'gR - mgR\sin(\alpha+\theta)$



Moment of inertia of the system $= \dfrac{1}{2}MR^2 + m'R^2 + mR^2$

$$\therefore\quad m'gR - mgR\sin(\alpha+\theta) = \left(\frac{1}{2}MR^2 + m'R^2 + mR^2\right)\frac{d^2\theta}{dt^2}$$

Putting $m' = m\sin\alpha$, we have

$$mgR\sin\alpha - mgR\sin(\alpha+\theta) = \left(\frac{1}{2}MR^2 + (m\sin\alpha)R^2 + mR^2\right)\frac{d^2\theta}{dt^2}$$

$$mgR\sin\alpha - mgR[\sin\alpha\cos\theta + \cos\alpha\sin\theta] = \frac{1}{2}R^2[M + 2m\sin\alpha + 2m]\frac{d^2\theta}{dt^2}$$

$$mgR\sin\alpha - mgR\sin\alpha - mgR\,\theta\cos\alpha = \frac{1}{2}R^2\,[M + 2m\sin\alpha + 2m]\frac{d^2\theta}{dt^2}$$

Because $\sin\theta = \theta$ and $\cos\theta = 1$ when θ is small

$$-2mg\,\theta\cos\alpha = [MR + 2mR\,(1+\sin\alpha)]\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} = -\frac{2mg\cos\alpha}{MR + 2mR\,(1+\sin\alpha)}\cdot\theta$$

$\therefore\quad \dfrac{d^2\theta}{dt^2}$ is proportional to θ.

The motion is simple harmonic.

$$\omega^2 = \frac{2mg\cos\alpha}{MR + 2mR\,(1+\sin\alpha)}$$

$$\therefore\quad \text{period} = \frac{2\pi}{\omega}$$

$$= 2\pi\sqrt{\frac{MR+2mR(1+\sin\alpha)}{2mg\cos\alpha}}$$

$= \mathbf{20\ s}$

MIND MAP

SHM

1. Equation of S.H.M.

(i) **Linear:** $a = -\omega^2 x$

2. Linear SHM

(i) Displacement of particle:

$x = A \sin(\omega t + \phi)$

(ii) **Velocity**

$$\frac{dx}{dt} = A\omega \cos(\omega t + \phi)$$

$$= \omega\sqrt{A^2 - x^2}$$

(iii) **Acceleration:**

$$\frac{d^2x}{dt^2} = -A\omega^2 \sin(\omega t + \phi)$$

$$= -\omega^2 x$$

3. Angular S.H.M.:

(i) **Displacement:**

$\theta = \theta_0 \sin(\omega t + \phi)$

(ii) **Angular velocity:**

$$\frac{d\theta}{dt} = \theta_0\omega \cos(\omega t + \phi)$$

(iii) **Angular acceleration:**

$$\frac{d^2\theta}{dt^2} = -\theta_0\omega^2 \sin(\omega t + \phi)$$

(iv) **Phase:** $\omega t + \phi$

(v) **Phase constant:** ϕ

4. Energy in S.H.M.

(i) $K = \frac{1}{2} m\omega^2 (A^2 - x^2)$

(ii) $U = \frac{1}{2} m\omega^2 x^2$

(iii) $E = K + U = \frac{1}{2} m\omega^2 A^2$

= constant

5. Time Period:

Pendulums:

(a) **Simple pendulum:**

$T = 2\pi\sqrt{l/g}$

(b) **Physical pendulum:**

$T = 2\pi\sqrt{I/mgl}$

(c) **Torsional pendulum:**

$T = 2\pi\sqrt{I/k}$

6. Mass-spring system:

(a) $T = 2\pi\sqrt{m/k}$

(b) **Two bodies system:**

$T = 2\pi\sqrt{\mu/k}$;

where $\mu = \frac{m_1 m_2}{m_1 + m_2}$

Combination of springs:

(a) series - $\frac{1}{K_{eff}} = \frac{1}{K_1} + \frac{1}{K_2}$

(b) parallel - $K_{eff} = K_1 + K_2$

(c) Spring cut into two parts $m : n$

$K_1 = \frac{(m+n)K}{m}$, $K_2 = \frac{(m+n)K}{n}$

7. Composition of 2 SHMs:

$x_1 = A_1 \sin\omega t$

$x_2 = A_2 \sin(\omega t + \phi)$

$x = x_1 + x_2$

$x = A \sin(\omega t + \delta)$

where, $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

and $\tan\delta = \frac{A_2 \sin\phi}{A_1 + A_2\cos\phi}$

EXERCISE – I

IIT JEE & NEET-SINGLE CHOICE CORRECT

1. A particle of mass 2 *g* executes SHM with a frequency of 5 oscillations per second. If the body has a maximum velocity of 10π cm/s, the amplitude of oscillations will be

(a) 10π cm (b) 1 cm (c) π cm (d) 5π cm

2. A mass suspended from a spring vibrates with a period of 2 seconds on the surface of the earth. If it is taken to the surface of moon and set oscillating, what would be its time period there? (*g* on moon = 1.6 m/s^2)

(a) 12 s (b) $\frac{1}{3}$ s (c) 2s (d) Zero

3. A body executes SHM given by the equation, $x = 10 \sin\left(4\pi t + \frac{\pi}{4}\right)$, where x is in m and t is in second. The frequency of SHM is

(a) π Hz (b) 4π Hz (c) 2π Hz (d) 2 Hz

4. A particle executing simple harmonic motion has amplitude 0.01 m and frequency 60 Hz. The maximum acceleration of the particle is

(a) $144\pi^2$ m/s^2 (b) $120\pi^2$ m/s^2 (c) $80\pi^2$ m/s^2 (d) $60\pi^2$ m/s^2

5. A particle is executing S H M according to the equation y = A sin$\left(7t + \frac{\pi}{4}\right)$. The time instant when it passes through $x = +\frac{A}{2}$ for the first time is (time in seconds)

(a) $\frac{\pi}{7}$ s (b) $\frac{\pi}{12}$ s (c) $\frac{5\pi}{84}$ s (d) $\frac{\pi}{84}$ s

6. For a particle executing SHM along *x*-axis force is given by:

(a) $-Akx$ (b) $A\cos kx$ (c) $A\exp(-kx)$ (d) Akx

7. A simple harmonic motion has an amplitude A and time period T. The time required by it to travel from $x = A$ to $x = A/2$ is:

(a) $T/6$ (b) $T/4$ (c) $T/3$ (d) $T/2$

8. A simple harmonic motion has an amplitude A and time period T. The maximum velocity will be

(a) $4\ AT$ (b) $2A/T$ (c) $2\pi\sqrt{A/T}$ (d) $2\pi A/T$

9. A particle is executing SHM with amplitude A. At what displacement from the mean position is the potential energy of the body one-fourth of its total energy?

(a) $A\sqrt{2}$ (b) $A/2$ (c) $A/\sqrt{2}$ (d) $A\sqrt{2/3}$

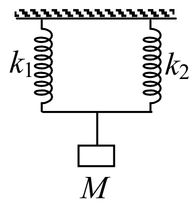
10. The particle executes SHM with a frequency f. The frequency with which its KE oscillates is

(a) $f/2$ (b) f (c) $2f$ (d) $4f$

11. A simple pendulum is made of a bob which is a hollow sphere full of sand suspended by means of a wire. If all the sand is drained out, the period of the pendulum will

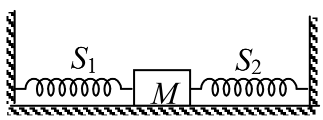
(a) increase (b) decrease (c) remains constant (d) become erratic

12. Two springs of force constant k_1 and k_2 have been arranged parallel to each other and a mass m is attached to the combination. This arrangement is equivalent to a single spring of force constant k given by



(a) $\dfrac{k_1+k_2}{k_1k_2}$ (b) $\dfrac{k_1k_2}{k_1+k_2}$ (c) k_1+k_2 (d) $\dfrac{k_1+k_2}{2}$

13. In figure, S_1 and S_2 are identical springs. The oscillation frequency of the mass M is f. If one spring is removed, the frequency will become:



(a) f (b) $2f$ (c) $\sqrt{2}f$ (d) $\dfrac{f}{\sqrt{2}}$

14. The frequency of a seconds pendulum in an elevator moving up with an acceleration of $\dfrac{1}{2}\ g$ is

(a) 0.5 Hz (b) 1 Hz (c) 0.612 Hz (d) 1.5 Hz

15. Two particles execute simple harmonic motion of same amplitude and frequency on parallel lines. They pass one another when moving in opposite directions each time their displacement is half their amplitude. What is the phase difference between them?

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{6}$ (d) π

16. A simple pendulum has a bob suspended by an inextensible thread of length 1 metre from a point *A*. As the bob reaches one extreme position, the thread is caught by a peg at a point *B* distant $\frac{1}{4}$ m from *A* and the bob begins to oscillate in the new position. The change in frequency of oscillation of the pendulum in Hz is approximately (take $g = 10$ m/s^2)

(a) $\frac{2\pi}{\sqrt{10}}\left[\sqrt{\frac{4}{3}}-1\right]$ (b) $\frac{\sqrt{10}}{2\pi}\left[\sqrt{\frac{4}{3}}-1\right]$ (c) $\frac{\pi}{\sqrt{10}}\left[\sqrt{\frac{4}{3}}-1\right]$ (d) $\frac{\sqrt{10}}{2\pi}\left[1-\sqrt{\frac{3}{4}}\right]$

17. Masses *m* and 3*m* are attached to the two ends of a spring of constant *k*. If the system vibrates freely, the period of oscillation will be

(a) $\pi\sqrt{\frac{m}{k}}$ (b) $2\pi\sqrt{\frac{3m}{2k}}$ (c) $\pi\sqrt{\frac{3m}{k}}$ (d) $2\pi\sqrt{\frac{4m}{3k}}$

18. The potential energy $U(x)$ of a particle executing SHM is given by:

(a) $U(x)=\frac{k}{2}(x-a)^2$ (b) $U(x)=k_1x+k_2x^2+k_3x^3$

(c) $U(x)=A\times\exp(-bx)$ (d) $U(x)=$ constant

19. Two SHMs are given by $y_1=a\left[\sin\left(\frac{\pi}{2}\right)t+\phi\right]$ and $y_2=b\sin\left[\left(\frac{2\pi t}{3}\right)+\phi\right]$. The phase difference between these after 1 s is:

(a) π (b) $\pi/2$ (c) $\pi/4$ (d) $\pi/6$

20. Time period of a simple pendulum of length *L* is T_1 and time period of a uniform rod of the same length *L* pivoted about one end and oscillating in a vertical plane is T_2. Amplitude of oscillations in both the case is small. Then T_1/T_2 is

(a) $\sqrt{\frac{4}{3}}$ (b) 1 (c) $\sqrt{\frac{3}{2}}$ (d) $\sqrt{\frac{1}{3}}$

21. A spring having force constant K and it is cut in two parts in 2 : 3 ratio. Then the new springs constant are

(a) $\frac{2K}{5}, \frac{3K}{5}$ (b) $\frac{5K}{2}, \frac{5K}{3}$ (c) $2K, 3K$ (d) $3K, 2K$

22. The motion of a particle is given by $x = 3\sin\omega t + 4\cos\omega t$. The motion of the particle is

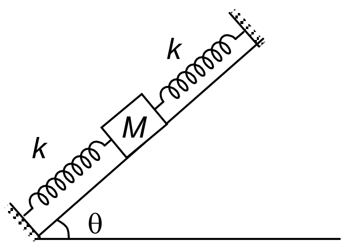
(a) not simple harmonic (b) simple harmonic with amplitude 7

(c) simple harmonic with amplitude 3.5 (d) simple harmonic with amplitude 5

23. In the spring mass system performing SHM, the average kinetic energy when the average is taken with respect to time over one period of the motion is (k : spring constant and A; amplitude)

(a) $\frac{1}{2}kA^2$ (b) $\frac{1}{6}kA^2$ (c) $\frac{1}{4}kA^2$ (d) none of these

24. On a smooth inclined plane, a body of mass M is attached between two springs. The other ends of the springs are fixed to firm supports. If each spring has force constant k, the period of oscillation of the body (assuming the springs as massless) is:



(a) $2\pi\sqrt{\frac{M}{2k}}$ (b) $2\pi\sqrt{\frac{2M}{k}}$ (c) $2\pi\frac{Mg\sin\theta}{2k}$ (d) $2\pi\sqrt{\frac{2Mg}{k}}$

25. Two bodies M and N of equal masses are suspended from two separate massless springs of spring constant k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum accelerations are equal, the ratio of the amplitude of M to that of N is

(a) $\frac{k_1}{k_2}$ (b) $\frac{k_2}{k_1}$ (c) $\sqrt{\frac{k_1}{k_2}}$ (d) $\sqrt{\frac{k_2}{k_1}}$

EXERCISE – II

IIT-JEE-SINGLE CHOICE CORRECT

1. In a simple harmonic motion the phase difference between the velocity and the displacement of the particle is

(a) $\frac{\pi}{2}$ radian (b) zero (c) π radian (d) $\frac{3\pi}{4}$ radian

2. A body executing SHM has an acceleration of 0.1 m/s^2 when the displacement is 0.05 m. The time period of oscillation is

(a) 2 s (b) $\frac{1}{2}$ s (c) $\pi\sqrt{2}s$ (d) $\pi/\sqrt{2}$ s

3. A 0.1 kg body hung on a spring causes it to elongate 2 cm. When a certain mass m is added to it and the system is set vibrating, its period is $\frac{\pi}{10}$ s. The value of m in grams is

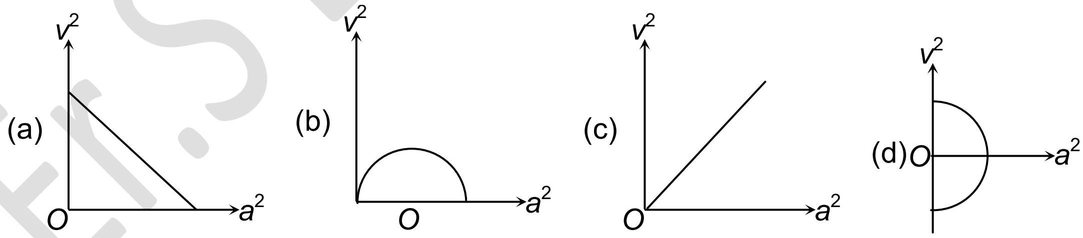
(a) 22.5 (b) 12.5 (c) 50 (d) 100

4. How much time will a seconds pendulum gain or lose in a day if its length is increased by 2%?

(a) A gain of 3 %. (b) A loss of 1%.

(c) A gain of 2% of total time in a day. (d) A loss of 2% of total time in a day.

5. A body is performing linear SHM. If the acceleration and the corresponding velocity of the body are a and v respectively, which of the following graphs is correct?



6. A body executes linear simple harmonic motion of time period 10 s and amplitude 2 cm. What would be the speed of the body 2.5 s after it passes through the mean position?

(a) 2 cm/s (b) 10 cm/s (c) 2.5 cm/s (d) zero

7. If the length of a simple pendulum increases by 3% the approximate percentage change in its time period is

(a) +1.5% (b) –2% (c) –2.5% (d) – 1.5%

8. A bottle weighing 200 gm and of area of cross-section 50 cm^2 and height 4 cm oscillates on the surface of water is in vertical position. Its frequency of oscillation (in Hz) is

(a) 1.5 per second (b) 2.5 per second (c) 3.5 per second (d) 4.5 per second

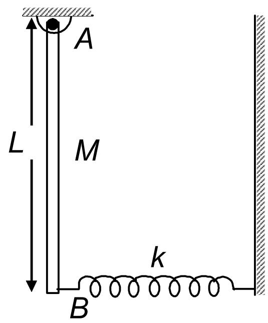
9. A simple pendulum has period of oscillation $\frac{k}{\sqrt{g}}$, where k is a constant. When this pendulum is suspended from the roof of a lift moving upward with an acceleration a, its period of oscillation will be

(a) zero (b) $k/\sqrt{g}$ (c) $\frac{k}{\sqrt{g+a}}$ (d) $\frac{k}{\sqrt{g-a}}$

10. A light spring of force constant k_1 carries a mass m. The spring is suspended from the end of another vertically suspended spring of constant k_2 and the mass is allowed to make oscillations. The time period will be

(a) $2\pi\sqrt{m\left(\frac{1}{k_1}+\frac{1}{k_2}\right)}$ (b) $2\pi\sqrt{\frac{m}{k_1+k_2}}$ (c) $2\pi\sqrt{\frac{m}{k_1-k_2}}$ (d) $2\pi\sqrt{\frac{k_1k_2}{m(k_1+k_2)}}$

11. A thin uniform rod AB of length L and mass M hangs from a smooth pivot at A and is connected at the bottom by a spring of constant k to the wall as shown in the figure. The system is set into oscillation by slightly displacing end B of the rod and releasing. The period of oscillation is

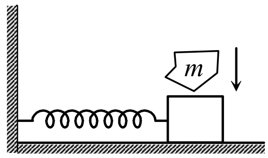


(a) $2\pi\sqrt{\frac{2\,Mg+kL}{3\,ML}}$ (b) $2\pi\sqrt{\frac{3ML}{Mg+2kL}}$ (c) $2\pi\sqrt{\frac{2\,ML}{3\,(Mg+2\,kL)}}$ (d) $2\pi\sqrt{\frac{ML}{3\,KL+2Mg}}$

12. A simple pendulum bob of mass m oscillates about its equilibrium position O, with time period T_0. At the instant of passing O during a particular cycle it picks up an identical mass m, initially at rest. The ratio of new time period T' to T_0 is

(a) 2 (b) (1/2) (c) 1 (d) $(1/\sqrt{2})$

13. Molten-wax of mass m drops on a block of mass M, which is oscillating on a frictionless table as shown, Select the incorrect option.



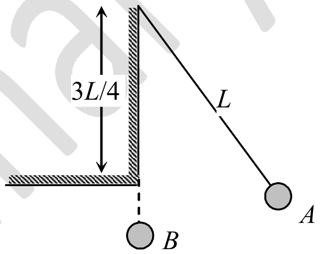
(a) If the collision takes place at extreme position, amplitude does not change

(b) If the collision takes place at mean position, amplitude decreases

(c) If the collision takes place at extreme position, time period decreases

(d) If the collision takes place at extreme position, the period increases

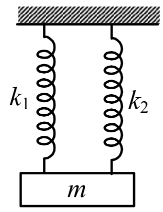
14. A pendulum has period T for small oscillations. An obstacle is placed directly beneath the pivot, so that only the lowest one quarter of the string can follow the pendulum bob when it swings in the left of its resting position as shown in the figure. The pendulum is released from rest at a certain point A. The time taken by it to return to that point is:



(a) T (b) $T/2$

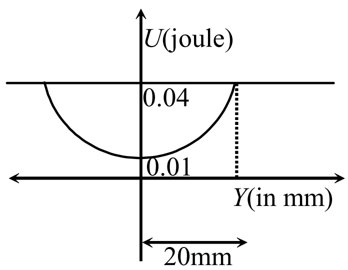
(c) $3T/4$ (d) $T/4$

15. A mass is suspended separately by two springs of spring constants k_1 and k_2 in successive order. The time periods of oscillations in the two cases are T_1 and T_2 respectively. If the same mass be suspended by connecting two springs in parallel as shown in the figure, then the time period of the oscillation is T. The correct relation is



(a) $T^2 = T_1^2 + T_2^2$ (b) $T^{-2} = T_1^{-2} + T_2^{-2}$ (c) $T^{-1} = T_1^{-1} + T_2^{-1}$ (d) $T = T_1 + T_2$

16. The variation of PE of harmonic oscillator is as shown in figure. The spring constant is



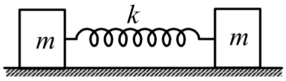
(a) 1×10^2 N/m

(b) 1.5×10^2 N/m

(c) 0.667×10^2 N/m

(d) 3×10^2 N/m

17. Consider the situation shown in figure. If the blocks are displaced slightly in opposite directions and released, they will execute simple harmonic motion. The time period is



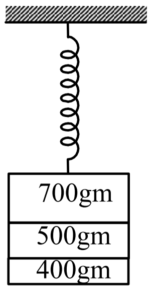
(a) $2\pi\sqrt{\frac{m}{k}}$ (b) $2\pi\sqrt{\frac{m}{2k}}$ (c) $2\pi\sqrt{\frac{m}{5k}}$ (d) $2\pi\sqrt{\frac{m}{9k}}$

18. A body is executing simple harmonic motion. At a displacement *x* its potential energy is E_1 and at a displacement *y* its potential energy is E_2. The potential energy *E* at displacement (x + y) is

(a) $\sqrt{E} = \sqrt{E_1} - \sqrt{E_2}$ (b) $\sqrt{E} = \sqrt{E_1} + \sqrt{E_2}$

(c) $E = E_1 + E_2$ (d) $E = E_1 - E_2$

19. Three masses 700g, 500g and 400g are suspended at the end of a spring as shown and are in equilibrium. When the 700g mass is removed, the system oscillates with a period of 3 seconds, when the 500 gm mass is also removed, it will oscillate with a period of



(a) 1 s (b) 2 s

(c) 3 s (d) $\sqrt{\frac{12}{5}}$ s

20. Two simple harmonic motions are represented by the equations $y_1 = 0.1\sin\left(\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1\cos\pi t$. The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is

(a) $-\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $-\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

ONE OR MORE THAN ONE CHOICE CORRECT

1. A simple harmonic oscillator has a period *T* and energy *E*. The amplitude of the oscillator is doubled. Choose the correct answer.

(a) period gets doubled (b) period remains same

(c) energy gets doubled (d) energy becomes 4 times

2. The phase (at a time t) of a particle in simple harmonic motion tells

(a) the position of the particle at time t

(b) the direction of motion of the particle at time t

(c) neither the position nor the direction of motion of the particle at time t

(d) none of these

3. For a particle executing SHM which of the following statements is wrong.

(a) The mechanical energy of the particle remains constant

(b) The restoring force is maximum at extreme position

(c) The restoring force is always directed toward a fixed point

(d) The velocity is minimum at the equilibrium position

4. Potential energy of a particle executing SHM is assumed to be zero at equilibrium position then average KE of the particle is equal to

(a) average potential energy

(b) $\frac{1}{4}ma^2\omega^2$

(c) $\frac{1}{2}mv^2{}_{rms}$

(d) $\frac{1}{2}m(\bar{v})^2$

where $\vec{v}$ is average speed over one oscillation

5. In SHM, speed of particle at displacement X_1 is v_1 and at displacement X_2 is v_2, then

(a) $\omega = \sqrt{\frac{v_1^2 - v_2^2}{X_2^2 - X_1^2}}$

(b) $T = 2\pi\sqrt{\frac{v_1^2 - v_2^2}{X_2^2 - X_1^2}}$

(c) $A = \sqrt{\frac{(v_1X_2)^2 - (v_2X_1)^2}{v_1^2 - v_2^2}}$

(d) none of these

6. Along the straight line joining two consecutive displacement nodes in a pure stationary sound wave at different points:

(a) the SHM will be in different phases

(b) the velocities are in phase

(c) the accelerations are in phase

(d) the frequencies are equal

7. For a body executing SHM with amplitude A, time period T, maximum velocity V_{max} and initial phase zero, which of the following statements are correct?

(a) At $y = A/2$, $v > v_{max}/2$

(b) $v = v_{max}/2$ for $y > A/2$

(c) At $t = T/8$, $y > A/2$

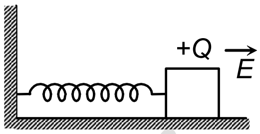
(d) At $y = A/2$, $t < T/8$

8. A linear harmonic oscillator of force constant 2×10^4 N/m and amplitude 0.1 m has a total mechanical energy of 160 J. Its

(a) maximum potential energy is 100 J (b) maximum kinetic energy is 100 J

(c) maximum potential energy is 160 J (d) minimum potential energy is 60 J

9. A spring –block system undergoes simple harmonic motion on a smooth horizontal surface. The block is now given some positive charge, and a uniform horizontal electric field to the right is switched on. As a result,



(a) the time period of oscillation will increase

(b) the time period of oscillation will decrease

(c) the time period of oscillation will remain unaffected

(d) the mean position of simple harmonic motion will shift to the right

10. A coin is placed on a horizontal platform, which undergoes horizontal simple harmonic motion about a mean position *O*. The coin does not slip on the platform. The force of friction acting on the coin is *F*

(a) *F* is always directed towards *O*

(b) *F* is directed towards *O* when the coin is moving away from *O*, and away from *O* when the coin moves towards *O*.

(c) $F = 0$ when the coin and platform come to rest momentarily at the extreme position of the harmonic motion.

(d) *F* is maximum when the coin and platform come to rest momentarily at the extreme position of the harmonic motion.

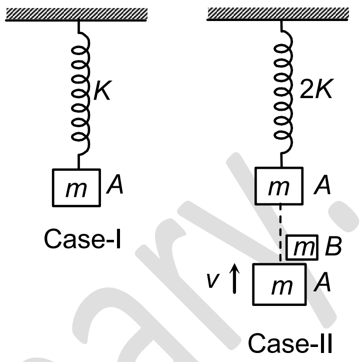
EXERCISE –III

MATCH THE FOLLOWING

Note: Each statement in column – I has one or more than one match in column –II

1. **Case: I** A block *A* of mass *m* attached to a spring of spring constant *K* as shown in the figure is released when the spring is in its unstretched length.

Case: II The block *A* is now attached to a spring of spring constant 2*K* and released from unstretched length of the spring. Another block *B* of mass *m* is placed on the block A with zero vertical velocity when the block *A* is passing the mean position with upward velocity.



Column I	Column II
I. Block *A* in case I performs	**A.** Periodic
II. Block A in case II performs	**B.** with time period $= 2\pi\sqrt{\dfrac{m}{K}}$
III. Kinetic energy of block *A* at equilibrium in case I	**C.** $\dfrac{m^2g^2}{K}$
IV. Spring potential energy at equilibrium in case II	**D.** $\dfrac{1}{2}\dfrac{m^2g^2}{K}$
	E. oscillatory

REASONING TYPE

Directions: Read the following questions and choose

(A) **If both the statements are true and statement-2** is the correct explanation of **statement-1**.
(B) **If both the statements are true but statement-2** is not the correct explanation of **statement-1**.
(C) **If statement-1** is True and **statement-2** is False.
(D) **If statement-1** is False **and statement-2** is True.

1. **Statement-1**: In SHM total mechanical energy can be negative.

Statement-2: Potential energy is always negative and if it is greater than kinetic energy total mechanical energy will be negative.

(a) (A) (b) (B) (c) (C) (d) (D)

2. **Statement-1**: On A block undergoing SHM, many forces are action. A new constant force starts acting on block, then its time period remains same although mean position may change.

Statement-2: In SHM net force acts towards mean position.

(a) (A) (b) (B) (c) (C) (d) (D)

3. **Statement-1**: In SHM it is possible to doubled maximum acceleration while keeping maximum speed constant.

Statement-2: It is possible when frequency is doubled while amplitude is halved.

(a) (A) (b) (B) (c) (C) (d) (D)

4. **Statement-1**: In an elevator a spring clock of frequency f_s and a pendulum clock of frequency f_p are kept. If the elevator accelerates upwards f_s remains same but f_p increases.

Statement-2: A constant force in spring block system doesnot change frequency while in case of pendulum it changes effective acceleration due to gravity.

(a) (A) (b) (B) (c) (C) (d) (D)

5. **Statement-1**: A particle undergoing SHM with amplitude A, if the time taken from mean position to $\frac{A}{\sqrt{2}}$ is t_1 and from $\frac{A}{\sqrt{2}}$ to A is t_2, then $t_2 = \sqrt{2}t_1$.

Statement-2: Equation of motion for the particle starting from mean position is x = ± A sin ωt and of the particle starting from extreme position is x = A cos ωt.

(a) (A) (b) (B) (c) (C) (d) (D)

LINKED COMPREHENSION TYPE

In simple harmonic motion force acting on a particle is given as $F = -4x$. The total mechanical energy of the particle is 10 J and the amplitude of oscillations is 2 m. At time $t = 0$ acceleration of the particle is –16 m/s^2. Mass of the particle is 0.5 kg

1. Potential energy of the particle at mean position is

(a) 10 J (b) 2 J (c) 6 J (d) 8 J

2. At x = +1m, potential energy of the particle is

(a) 2 J (b) 8 J (c) 4 J (d) 6 J

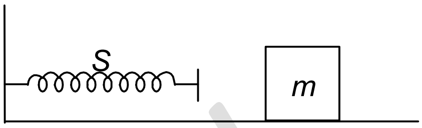
3. At x = +1m, kinetic energy of the particle is

(a) 2 J (b) 8 J (c) 4 J (d) 6 J

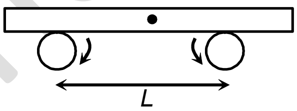
EXERCISE –IV

SUBJECTIVE PROBLEMS

1. A light spring S is attached to a wall as shown. The spring constant k is 400 N/m. The mass m = 200gm shown in the figure initially moves to the left at a speed of 8.0 m/s. It strikes the spring and becomes attached to it. (a) How far does it compress the spring? (b) The system then oscillates back and forth. What is the amplitude of oscillation? The floor can be assumed to be smooth.



2. A uniform plate of mass M stays horizontally and symmetrically on two wheels rotating in opposite directions shown in the figure. The separation between the wheels is L. The friction coefficient between each wheel and the plate is μ. Find the time period of oscillation of the plate if it is slightly displaced along its length and released.

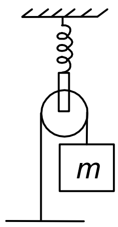


3. A particle is moving in a straight line with simple harmonic motion of amplitude $a = \sqrt{2}$ m. At a distance $S = \frac{2}{\sqrt{3}}$ m from the centre of motion, the particle receives a blow in the direction of motion, which instantaneously doubles the velocity. Find the new amplitude A'.

4. A particle moving in a straight line has velocity v given by $v^2 = \alpha - \beta x^2$, where α and β are constants and x is its distance from a fixed point in the line. Show that the motion of the particle is simple harmonic and determine its period and amplitude. ($\beta = \pi^2$, $\alpha = \pi^6$, $\pi^2 = 10$)

5. A particle moving with SHM in a straight line has a speed of v_1 = 6 m/s when x_1 = 4 m from the centre of oscillations and a speed of v_2 = 8 m/s when x_2 = 3 m from the centre. Find the amplitude A' of oscillation.

6. The spring has a force constant k. The pulley is light and smooth while the spring and the string are light. If the block of mass 'm' is slightly displaced vertically and released, find the period of vertical oscillation. (take $\frac{m}{k} = \frac{1}{\pi^2}$)

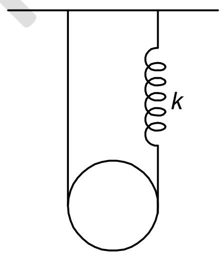


7. The vertical motion of a ship in sea is described by the equation $\frac{d^2x}{dt^2} = -4x$, where x (meter) is the vertical height of the ship above its mean position. If it oscillates through a total distance of 1 m in half oscillation, find the greatest vertical speed and the greatest vertical acceleration.

8. One end of an elastic string of natural length a and force constant $\frac{\lambda}{a}$ is fixed to a point on a smooth horizontal surface and the other end is attached to a particle of mass m lying on the surface. The particle is pulled from the fixed point to a distance 2a and released. Find the time for a complete oscillation of the particle. (Given $m = 1$ kg, $\frac{\lambda}{a} = (\pi + 2)^2$)

9. A mass m attached to a vertically hung spring (k) executes SHM of amplitude $A = \sqrt{10}$ SI unit and time period T. When it passes through its mean position another stationary mass m gently sticks to it. Find new amplitude A'. (Given $mg = 2$k)

10. The pulley of radius r shown in Figure has a moment of inertia I about its axis and mass m. Find the time period of vertical oscillations of its centre of mass. The spring has spring constant k and the string does not slip over pulley. (Given $I = (\pi^2 k - m)r^2$



ANSWERS

EXERCISE – I

IIT JEE & NEET-SINGLE CHOICE CORRECT

1. (b)	2. (c)	3. (c)	4. (a)	5. (b)
6. (a)	7. (a)	8. (d)	9. (b)	10. (c)
11. (c)	12. (c)	13. (d)	14. (c)	15. (b)
16. (b)	17. (c)	18. (a)	19. (d)	20. (c)
21. (b)	22. (d)	23. (c)	24. (a)	25. (b)

EXERCISE – II

IIT-JEE-SINGLE CHOICE CORRECT

1. (a)	2. (c)	3. (a)	4. (b)	5. (a)
6. (d)	7. (a)	8. (b)	9. (c)	10. (a)
11. (c)	12. (c)	13. (c)	14. (c)	15. (b)
16. (b)	17. (b)	18. (b)	19. (b)	20. (c)

ONE OR MORE THAN ONE CHOICE CORRECT

1. (b,d)	2. (a,b)	3. (a,b,d)	4. (a,b,c)	5. (a,c)
6. (b,c,d)	7. (a,c,d)	8. (b,c,d)	9. (c,d)	10. (a,d)

EXERCISE – III

MATCH THE FOLLOWING

1. I. A, B,E; II – A, B, E; III- D, IV- C

REASONING TYPE

1. (c)	2. (b)	3. (a)	4. (a)	5. (d)

LINKED COMPREHENSION TYPE

1. (b)	2. (c)	3. (d)

EXERCISE – IV

SUBJECTIVE PROBLEMS

1. $x = v\sqrt{m/k}$ = 179 mm, 179 mm

2. $T = 2\pi\sqrt{\frac{L}{\mu g}}$ = 10 s

3. $A' = \sqrt{4A^2 - 3s^2}$ = 2 m

4. $T = 2\pi\sqrt{\frac{1}{\beta}}, \quad A = \sqrt{\frac{\alpha}{\beta}}$ = 2 s, 10 m

5. $A = \sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}}$ = 5 m

6. $T = 4\pi\sqrt{\frac{m}{k}}$ = 4 s

7. $v_{max} = a\omega, \quad \alpha_{max} = a\omega^2$ = 1 m/s, 2 m/s^2

8. $T = 2(\pi + 2)\sqrt{\frac{ma}{\lambda}}$ = 2 s

9. $A' = \sqrt{\frac{A^2}{2} + \left(\frac{mg}{k}\right)^2}$ = 3 SI unit

10. $T = 2\pi\sqrt{\frac{\left(\frac{I}{r^2} + m\right)}{4k}}$ = 10 s

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 1

Q.1 The acceleration of a particle executing S.H.M. is
(1) Always directed towards the equilibrium position
(2) Always towards the one end
(3) Continuously changing in direction
(4) Maximum at the mean position

Q.2 A particle executing S.H.M. completes a distance (taking friction as negligible) in one complete time period, equal to:
(1) Four times the amplitude
(2) Two times the amplitude
(3) One times the amplitude
(4) Eight times the amplitude

Q.3 A particle of mass m is executing S.H.M. If amplitude is a and frequency n, the value of its force constant will be:
(1) mn^2 (2) $4mn^2a^2$
(3) ma^2 (4) $4\pi^2mn^2$

Q.4 The mass of particle executing S.H.M. is 1 gm. If its periodic time is π seconds, the value of force constant is:
(1) 4 dynes/cm (2) 4N/cm
(3) 4N/m (4) 4 dynes/m

Q.5 The equation of motion of a particle executing S.H.M. is :
(1) $\frac{d^2x}{dt^2} = -\frac{k}{m}x$ (2) $\frac{d^2x}{dt^2} = +\omega^2 x$
(3) $\frac{d^2x}{dt^2} = -\omega^2 x^2$ (4) $\frac{d^2x}{dt^2} = -kmx$

Q.6 The equation of motion of a particle executing SHM is $\left(\frac{d^2x}{dt^2}\right)$ + kx = 0. The time period of the particle will be:
(1) $2\pi/\sqrt{k}$ (2) $2\pi/k$
(3) $2\pi k$ (4) $2\pi\sqrt{k}$

Q.7 The phase of a particle in S.H.M. is $\pi/2$, then:
(1) Its velocity will be maximum
(2) Its acceleration will be minimum
(3) Restoring force on it will be minimum
(4) Its displacement will be maximum.

Q.8 The displacement of a particle in S.H.M. is indicated by equation $y = 10 \sin(20t + \pi/3)$ where y is in meters. The value of time period of vibration will be (in seconds):
(1) $10/\pi$ (2) $\pi/10$
(3) $2\pi/10$ (4) $10/2\pi$

Q.9 In the above question, the value of maximum velocity of the particle will be:
(1) 100 m/sec (2) 150 m/sec
(3) 200 m/sec (4) 400 m/sec

Q.10 In the above question, the value of phase constant will be :
(1) Zero (2) 45º
(3) 60º (4)30º

Q.11 The phase of a particle in SHM at time t is $\pi/6$. The following inference is drawn from this:
(1) The particle is at x = a/2 and moving in +X-direction
(2) The particle is at x = a/2 and moving in –X-direction
(3) The particle is at x = – a/2 and moving in +X-direction
(4) The particle is at x = – a/2 and moving in –X-direction

Q.12 The value of phase at maximum distance from the mean position of a particle in S.H.M. is:
(1) $\pi/2$ (2) π
(3) Zero (4) 2π

Q.13 Two particles execute S.H.M. along the same line at the same frequency. They move in opposite direction at the mean position. The phase difference will be:
(1) 2π (2) $2\pi/3$
(3) π (4) $\pi/2$

Q.14 The time period of an oscillator is 8 sec. The phase difference from t = 2 sec to t = 4 sec will be:
(1) π (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{4}$ (4) 2π

Q.15 The displacement from mean position of a particle in SHM at 3 seconds is $\sqrt{3}/2$ of the amplitude. Its time period will be:
(1) 18 sec (2) $6\sqrt{3}$ sec
(3) 9 sec (4) $3\sqrt{3}$ sec

Q.16 A particle executes SHM of type x = asinωt. It takes time t_1 from x = 0 to x = $\frac{a}{2}$ and t_2 from x = $\frac{a}{2}$ to x = a. The ratio of $t_1 : t_2$ will be:
(1) 1 : 1 (2) 1 : 2
(3) 1 : 3 (4) 2 :1

Q.17 The time taken by a particle in SHM for maximum displacement is:
(1) T/8 (2) T/6
(3) T/2 (4) T/4

Q.18 A particle executes SHM with periodic time of 6 seconds. The time taken for traversing a distance of half the amplitude from mean position is:
(1) 3 sec (2) 2 sec
(3) 1 sec (4) 1/2 sec

Q.19 The phase difference between the displacement and acceleration of particle executing S.H.M. in radian is:

(1) $\pi/4$ (2) $\pi/2$

(3) π (4) 2π

Q.20 The phase difference in radians between displacement and velocity in S.H.M. is

(1) $\pi/4$ (2) $\pi/2$

(3) π (4) 2π

Q.21 If the maximum velocity of a particle in SHM is v_0 then its velocity at half the amplitude from position of rest will be:

(1) $v_0/2$ (2) v_0

(3) $v_0\sqrt{3/2}$ (4) $v_0\sqrt{3}/2$

Q.22 At a particular position the velocity of a particle in SHM with amplitude a is $\sqrt{3}/2$ that at its mean position. In this position, its displacement is:

(1) a/2 (2) $\sqrt{3}a/2$

(3) $a\sqrt{2}$ (4) $\sqrt{2}a$

Q.23 The acceleration of a particle in SHM at 5 cms from its mean position is 20 cm/sec^2. The value of angular velocity in radian/sec will be:

(1) 2 (2) 4 (3) 10 (4) 14

Q.24 The amplitude of a particle in SHM is 5 cms and its time period is π. At a displacement of 3 cms from its mean position the velocity in cms/sec wil be

(1) 8 (2) 12 (3) 2 (4) 16

Q.25 The maximum velocity and acceleration of a particle in S.H.M. are 100 cm/sec and 157 cm/sec^2 respectively. The time period in seconds will be:

(1) 4 (2) 1.57

(3) 0.25 (4) 1

Q.26 If the displacement, velocity and acceleration of a particle in SHM are 1 cm, 1cm/sec, 1 cm/sec^2 respectively, its time period will be (in seconds):

(1) π (2) 0.5π

(3) 2π (4) 1.5π

Q.27 The particle is executing S.H.M. on a line 4 cms long. If its velocity at mean position is 12 cm/sec, its frequency in Hertz will be:

(1) $2\pi/3$ (2) $3/2\pi$

(3) $\pi/3$ (4) $3/\pi$

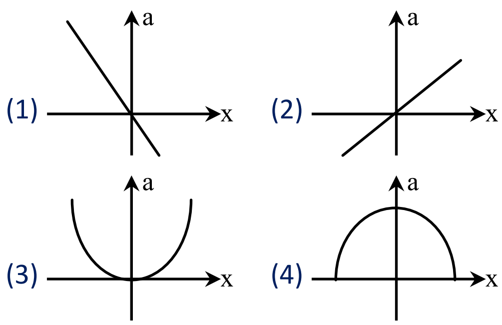
Q.28 If the amplitude of a simple pendulum is doubled, how many times will the value of its maximum velocity be that of the maximum velocity in initial case:

(1) 1/2 (2) 2
(3) 4 (4) 1/4

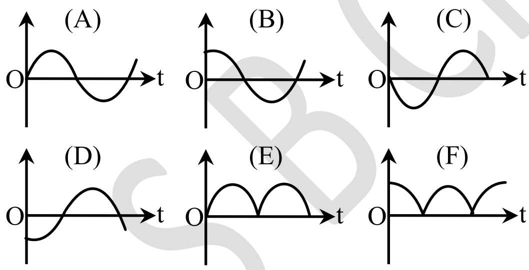
Q.29 Which of the following statement is incorrect for an object executing S.H.M.

(1) The value of acceleration is maximum at the extreme points
(2) The total work done for completing one oscillation is zero
(3) The energy changes from one form to another
(4) The velocity at the mean position is zero

Q.30 The variation of acceleration (a) and displacement (x) of the particle executing SHM is indicated by the following curve:



Q.31 The displacement of a particle in S.H.M. is x = a sinωt. Which of the following graph between displacement and time is correct?



(1) A (2) B (3) C (4) D

Q.32 In question 31, which of the graph between velocity and time is correct ?

(1) A (2) B (3) C (4) D

Q.33 In question 31, which of the graph between kinetic energy and time is correct ?

(1) A (2) B (3) E (4) F

Q.34 In question 31, which of the graph between potential energy and time is correct ?

(1) A (2) B (3) E (4) F

Q.35 In question 31, which of the graph between acceleration and time is correct ?

(1) A (2) B (3) C (4) D

Q.36 In question 31, if the displacement of a particle executing SHM is x = a cosωt, which of the graph between displacement and time is correct ?

(1) A (2) B (3) C (4) D

Q.37 In question 36 which of the graph between velocity and time is correct ?

(1) A (2) B (3) C (4) D

Q.38 In question 36 which of the graph between acceleration and time is correct ?

(1) A (2) B (3) C (4) D

Q.39 In question 36 which of the graph between K.E. and time is correct ?

(1) A (2) B (3) E (4) F

Q.40 In question 36 which of the graph between P.E. and time is correct ?

(1) A (2) B (3) E (4) F

Q.41 The energy at the mean position of a pendulum will be:

(1) Zero
(2) Partial P.E. and partial K.E.
(3) Totally K.E.
(4) Totally P.E.

Q.42 The total energy of a particle executing SHM is directly proportional to the square of the following quantity.

(1) Acceleration (2) Amplitude
(3) Time period (4) Mass

Q.43 The total energy of a vibrating particle in SHM is E. If its amplitude and time period are doubled, its total energy will be:

(1) 16E (2) 8E (3) 4E (4) E

Q.44 The total vibrational energy of a particle in S.H.M. is E. Its kinetic energy at half the amplitude from mean position will be:

(1) E/2 (2) E/3
(3) 3/4 (4) 3E/4

Q.45 If total energy of a particle in SHM is E, then the potential energy of the particle at half the amplitude will be-

(1) E/2 (2) E/4 (3) 3E/4 (4) E/8

Q.46 A particle executes SHM on a line 8 cm long. Its K.E. and P.E. will be equal when its distance from the mean position is:

(1) 4 cm (2) 2 cm

(3) $2\sqrt{2}$ cm (4) $\sqrt{2}$ cm

Q.47 The energy of a simple harmonic oscillator in the state of real is 3 joules. If its mean K.E. is 4 joules, its total energy will be:

(1) 7J (2) 8J (3) 10 J (4) 11 J

Q.48 The total energy of a harmonic oscillator of mass 2kg is 9 joules. If its energy at rest is 5 joules, its K.E. at the mean position will be:

(1) 9J (2) 14J (3) 4J (4) 11J

Q.49 The average P.E. of a body executing S.H.M. is:

(1) $\frac{1}{2}ka^2$ (2) $\frac{1}{4}ka^2$

(3) ka^2 (4) Zero

Q.50 The value of total mechanical energy of a particle in S.H.M. is:

(1) Always constant

(2) Depend on time

(3) $\frac{1}{2}kA^2\cos^2(\omega t+\phi)$

(4) $\frac{1}{2}mA^2\cos^2(\omega t+\phi)$

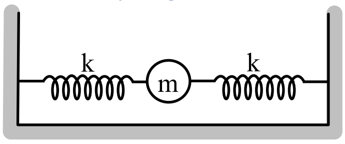
IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 2

Q.1 The maximum K.E. of a oscillating spring is 5 joules and its amplitude 10 cms. The force constant of the spring is:
(1) 100 Newton/m (2) 1000 Newton-m
(3) 1000 Newton/m (4) 1000 watts

Q.2 A particle executes SHM with a frequency f. The frequency of its P.E. will be:
(1) f/2 (2) f
(3) 2f (4) 4f

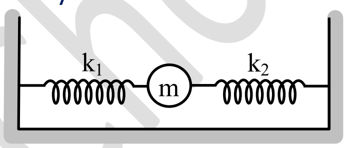
Q.3 The force acting on a 4 gm mass in the potential field $U = 8x^2$ at $x = -2$ cm is:
(1) 8 dyne (2) 4 dyne
(3) 16 dyne (4) 32 dyne

Q.4 On suspending a mass m from a spring of force constant k, frequency of vibration f is obtained. If a second spring as shown in the figure, is arranged then the frequency will be:



(1) $f\sqrt{2}$ (2) $f/\sqrt{2}$
(3) 2f (4) f

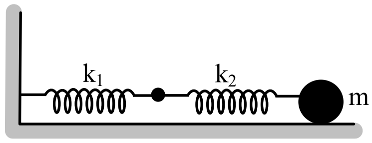
Q.5 In the adjoining figure the frequency of oscillation for a mass m will be proportional to:



(1) k_1k_2 (2) $k_1 + k_2$
(3) $\sqrt{k_1 + k_2}$ (4) $\sqrt{1/(k_1 + k_2)}$

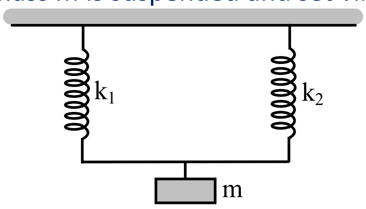
Q.6 An object of mass m is suspended from a spring and it executes S.H.M. with frequency ν. If the mass is increased 4 times, the new frequency will be:
(1) 2ν (2) ν/2 (3) ν (4) ν/4

Q.7 A shown in the figure, two light springs of force constant k_1 and k_2 oscillate a block of mass m. Its effective force constant will be:



(1) k_1k_2 (2) $k_1 + k_2$
(3) $\frac{1}{k_1} + \frac{1}{k_2}$ (4) $\frac{k_1k_2}{k_1 + k_2}$

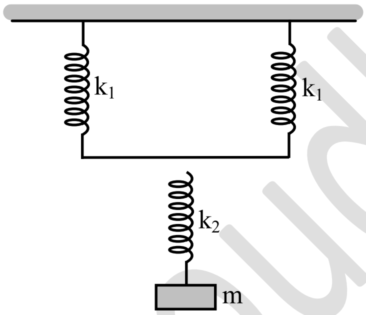
Q.8 The spring constant of two springs of same length are k_1 and k_2 as shown in figure. If an object of mass m is suspended and set vibration, the time period will be:



(1) $2\pi\sqrt{\frac{mk_1}{k_2}}$ (2) $2\pi\sqrt{\frac{m}{k_1k_2}}$

(3) $2\pi\sqrt{\frac{m}{k_1-k_2}}$ (4) $2\pi\sqrt{m/(k_1+k_2)}$

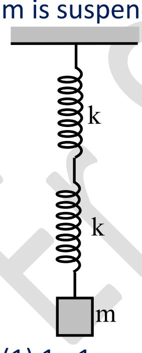
Q.9 The total spring constant of the system as shown in the figure will be:

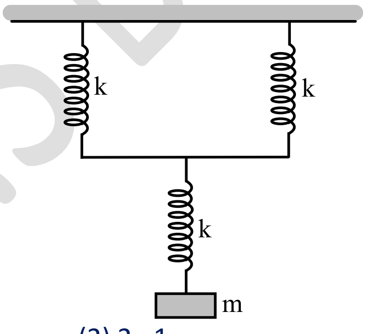


(1) $\frac{k_1}{2}+k_2$ (2) $\left[\frac{1}{2k_1}+\frac{1}{k_2}\right]^{-1}$

(3) $\frac{1}{2k_1}+\frac{1}{k_2}$ (4) $\left[\frac{2}{k_1}+\frac{1}{k_2}\right]^{-1}$

Q.10 Some springs are combined in series and parallel arrangement as shown in the figure and a mass m is suspended from them. The ratio of their frequencies will be:





(1) 1 : 1 (2) 2 : 1

(3) $\sqrt{3}:2$ (4) 4 : 1

Q.11 The force constant of a spring is k. The amount of work done in expanding it from ℓ_1 to ℓ_2 will be:

(1) $k(\ell_2-\ell_1)$ (2) $k\left(\frac{\ell_1+\ell_2}{2}\right)$

(3) $k(\ell_2^2-\ell_1^2)$ (4) $\frac{k}{2}(\ell_2^2-\ell_1^2)$

Q.12 A spring is made to oscillate after suspending a mass m from one of its ends. The time period obtained is 2 seconds. On increasing the mass by 2 kg, the period of oscillation is increased by 1 second. The initial mass m will be:
(1) 2 kg (2) 1 kg
(3) 0.5 kg (4) 1.6 kg

Q.13 The force constant of spring A is greater than that of spring B. If their lengths are elongated by same amount, which of the following statement is correct ?
(1) The work done on A will be greater than that on B
(2) The work done on B will be greater than that on A
(3) Work done on both the springs will be equal, if their initial lengths are same
(4) Work done on both of them will be equal

Q.14 The time period of a spring pendulum on earth is T. If it is taken on the moon, and made to oscillate, the period of vibration will be :
(1) Less than T (2) Equal to T
(3) More than T (4) None of these

Q.15 The length of a spring becomes 10 cm on suspending a mass of 20 kg in a vertical plane and 12 cms on suspending 32 kg. What should be the weight suspended from it so as to cause the length to be 15 cms ($g = 10\ m/sec^2$):
(1) 40 kg (2) 50 kg
(3) 60 kg (4) 80 kg

Q.16 On loading a spring with bob, its period of oscillation in a vertical plane is T. If this spring pendulum is tied with one end to the a friction less table and made to oscillate in a horizontal plane, its period of oscillation will be-
(1) T
(2) 2T
(3) T/2
(4) will not execute S.H.M.

Q.17 In a winding (spring) watch, the energy is stored in the form of :
(1) Kinetic energy (2) Potential energy
(3) Electrical energy (4) None of these

Q.18 A and B are two similar springs, of which A is more rigid than B i.e. $k_A > k_B$. These are pulled through the same length. The work done in these cases is:
(1) More in spring A
(2) More in spring B
(3) Equal in spring A and B
(4) No definite information can be furnish in this connection

Q.19 In the previous question, on pulling the springs with equal force, the work done in spring A is:
(1) More than spring B
(2) Less than spring B
(3) Equal to spring B
(4) Nothing certain can be stated

Q.20 In an artificial satellite, the object used is:
(1) Spring watch
(2) Pendulum watch
(3) Watches of both spring and pendulum
(4) None of these

Q.21 The length of a spring is ℓ and its spring constant is k. It is cut into two parts of lengths ℓ_1 and ℓ_2 and $\ell_1 = n\ell_2$. The spring constant k_1 of the part ℓ_1 will be:
(1) k(1 + 1/n) (2) k(1 – 1/n)
(3) k(1 + 1/2n) (4) k(1 – 1/2n)

Q.22 An object of 4 kg mass, moving at 6m/sec velocity strikes a spring & compresses it by a distance x. If the force constant of the spring is 900 N/m. What is the value of x:
(1) 4 cm (2) 40 cm
(3) 20 cm (4) None of these

Q.23 The time period of an oscillating body executing SHM is 0.05 sec and its amplitude is 40 cm. The maximum velocity of particle is:
(1) $16\pi\ ms^{-1}$ (2) $2\pi ms^{-1}$
(3) $3.1\ ms^{-1}$ (4) $4\pi\ ms^{-1}$

Q.24 The mass of a bob, suspended in a simple pendulum is halved from the initial mass, its time period will:
(1) Be less (2) Be more
(3) Remain unchanged (4) None of these

Q.25 The length of a simple pendulum is $39.2/\pi^2$ m. If g = 9.8 m/sec^2, the value of time period is:
(1) 4 sec (2) 8 sec
(3) 2 sec (4) 3 sec

Q.26 The length of a simple pendulum is increased four times of its initial value, its time period with respect to its previous value will:
(1) Become twice (2) Not be different
(3) Be halved (4) Be $\sqrt{2}$ times

Q.27 Water is filled in a hollow metallic sphere and it is suspended from a long string. A fine hole is made at the bottom of the sphere through which water tickles. The sphere is set into oscillations. Its period of oscillation will:
(1) Remain constant
(2) Decrease continuously
(3) Increase continuously
(4) First increase then decrease

Q.28 The time taken for a second pendulum from one extreme point to another is:
(1) 1 sec. (2) 2 sec.
(3) 1/2 sec. (4) 4 sec.

Q.29 The length of a seconds pendulum is (approximately):
(1) 1 m (2) 1 cm
(3) 2 m (4) 2 cm

Q.30 The acceleration due to gravity at height R above the surface of the earth is g/4. The periodic time of a simple pendulum in an artificial satellite at this height will be:
(1) $T = 2\pi\sqrt{2\ell/g}$ (2) $T = 2\pi\sqrt{\ell/2g}$
(3) Zero (4) Infinity

Q.31 In an artificial satellite, the use of a pendulum watch is discarded, because :
(1) The satellite is in a constant state of motion
(2) The value of g becomes zero in the earth satellite
(3) The periodic time of the pendulum watch is reduced

(4) None of these

Q.32 An oscillating pendulum stops, because its energy
(1) Changes into kinetic energy
(2) Change into potential energy
(3) Change into heat energy
(4) Is destroyed

Q.33 The length of a simple pendulum is made equal to the radius of the earth. Its period of oscillation will be:
(1) 84.6 min. (2) 59.8 min.
(3) 42.3 min. (4) 21.15 min.

Q.34 The maximum time period of oscillation of a simple pendulum is:
(1) Infinity (2) 24 hours
(3) 12 hours (4) 1½ hours

Q.35 In a simple oscillating pendulum, the work done by the string in one oscillation will be:
(1) Equal to the total energy of the pendulum (2) Equal to the K.E. of the pendulum
(3) Equal to the P.E. of the pendulum
(4) Zero

Q.36 The distance between the point of suspension and the centre of gravity of a compound pendulum is ℓ and the radius of gyration about the horizontal axis through the centre of gravity is k, then its time period will be:
(1) $2\pi\sqrt{\frac{\ell+k}{g}}$ (2) $2\pi\sqrt{\frac{\ell^2+k^2}{\ell g}}$
(3) $2\pi\sqrt{\frac{\ell+k^2}{g}}$ (4) $2\pi\sqrt{\frac{2k}{\ell g}}$

Q.37 In the compound pendulum, the minimum period of oscillation will be:
(1) $2\pi\sqrt{\frac{k}{g}}$ (2) $2\pi\sqrt{\frac{\ell}{g}}$
(3) $2\pi\sqrt{\frac{2\ell}{g}}$ (4) $2\pi\sqrt{\frac{2k}{g}}$

Q.38 The distance of point of a compound pendulum from its centre of gravity is ℓ, the time period of oscillation relative to this point is T. If g = π^2, the relation between ℓ and T will be:
(1) $\ell^2-\left(\frac{T^2}{4}\right)\ell+k^2=0$
(2) $\ell^2+\left(\frac{T^2}{4}\right)\ell+k^2=0$
(3) $\ell^2-\left(\frac{T^2}{4}\right)\ell-k^2=0$
(4) $\ell^2+\left(\frac{T^2}{4}\right)\ell-k^2=0$

Q.39 The distance between the points of suspension and centre of oscillation for a compound pendulum is:

(1) $\frac{k^2-\ell^2}{2}$ (2) $\frac{k^2+\ell^2}{\ell}$

(3) $\frac{k^2+\ell^2}{2\ell}$ (4) $k^2+\frac{\ell^2}{k}$

Q.40 For a compound pendulum, the maximum time period is:

(1) $2\pi\sqrt{\frac{L}{g}}$ (2) $2\pi\sqrt{\frac{2k}{g}}$

(3) Zero (4) Infinite

Q.41 If the distance between the centre of gravity and point of suspension of a compound pendulum is ℓ and the radius of gyration about the axis passing through its centre of gravity is k, its time period will be infinite if:

(1) $\ell = 0$ (2) $\ell = \infty$

(3) $\ell = k$ (4) $\ell = 2k$

Q.42 A ring of radius R is suspended from its circumference and is made to oscillate about a horizontal axis in a vertical plane. The length equivalent to a simple pendulum will be:

(1) 2R (2) R

(3) $\frac{3R}{2}$ (4) $\frac{R}{2}$

Q.43 A disc of radius R is suspended from its circumference and made to oscillate. Its time period will be:

(1) $2\pi\sqrt{\frac{3R}{2g}}$ (2) $2\pi\sqrt{\frac{4R}{g}}$

(3) $2\pi\sqrt{\frac{R}{g}}$ (4) $2\pi\sqrt{\frac{2R}{g}}$

Q.44 In the above question, its length equivalent to a simple pendulum will be:

(1) $\frac{R}{2}$ (2) R

(3) $\frac{3R}{2}$ (4) 2R

Q.45 Holes are drilled along the diameter of a disc of radius R. On oscillating it through the holes along a horizontal axis the minimum time period will be:

(1) $2\pi\sqrt{\frac{1.5R}{g}}$ (2) $2\pi\sqrt{\frac{1.414R}{g}}$

(3) $2\pi\sqrt{\frac{R}{g}}$ (4) $2\pi\sqrt{\frac{2R}{g}}$

Q.46 Holes are drilled along the diameter of a disc. For a minimum time period, the disc should be suspend from the centre of gravity at a distance of:

(1) R (2) $\frac{R}{2}$

(3) $\frac{R}{\sqrt{2}}$ (4) Zero

Q.47 The formula for time period of a compound pendulum is:

(1) $T = 2\pi\sqrt{\frac{I}{g}}$ (2) $T = 2\pi\sqrt{\frac{I}{mg\ell}}$

(3) $T = 2\pi\sqrt{\frac{m}{k}}$ (4) $T = 2\pi\sqrt{\frac{k}{m}}$

Q.48 The mass of a solid body is 200 gm and oscillates about a horizontal axis at a distance of 20 cms from the centre of gravity. If length equivalent to simple pendulum is 40 cms, its M.I. through the centre of gravity will be:

(1) 8×10^4 gm– cm^2 (2) 8×10^4 kg– cm^2

(3) 8×10^4 gm–m^2 (4)8×10^4 kg–cm^2

Q.49 A thin rod of length 1 m is suspended from its end and is made to oscillate in a vertical plane. The distance between the point of suspension and centre of oscillation will be:

(1) $\frac{1}{2}$m (2) $\frac{3}{4}$m (3)1m (4) $\frac{2}{3}$ m

Q.50 If a disc is made to oscillate from a point $\frac{R}{4}$ away from centre and the axis of oscillations is perpendicular to the plane of disc, then the length of equivalent simple pendulum will be:

(1) $\frac{3R}{4}$ (2) $\frac{5R}{4}$ (3) $\frac{7R}{4}$ (4) $\frac{9R}{4}$

Q.51 A rod of length L is suspended from its one end and is oscillating. Its time period will be:

(1) $2\pi\sqrt{\frac{L}{g}}$ (2) $2\pi\sqrt{\frac{2L}{g}}$

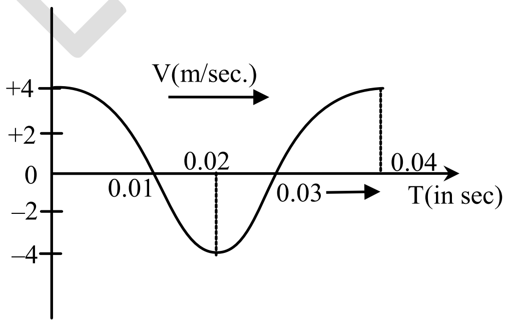
(3) $2\pi\sqrt{\frac{L}{2g}}$ (4) $2\pi\sqrt{\frac{2L}{3g}}$

Q.52 A body of mass 5 gm is executing S.H.M. about a point with amplitude 10 cm. Its maximum velocity is 100 cm/sec. Its velocity will be 50 cm/sec at a distance :

(1) 5 cm (2) $5\sqrt{2}$ cm

(3) $5\sqrt{3}$ cm (4) $10\sqrt{2}$ cm

Q.53 The velocity-time diagram of a harmonic oscillator is shown in the adjoining figure. The frequency of oscillation is:



(1) 25 Hz (2) 50 Hz

(3) 12.25 Hz (4) 33.3 Hz

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 3

Q.1 When the displacement is half of the amplitude, then what fraction of total energy of a simple harmonic oscillator is kinetic ?
(1) 3/4 th (2) 2/7 th
(3) 5/7 th (4) 2/9 th

Q.2 A particle starts S.H.M. from the mean position. Its amplitude is A and time periods is T. At the time when its speed is half of the maximum speed, its displacement y is:
(1) $\frac{A}{2}$ (2) $\frac{A}{\sqrt{2}}$
(3) $\frac{A\sqrt{3}}{2}$ (4) $\frac{2A}{\sqrt{3}}$

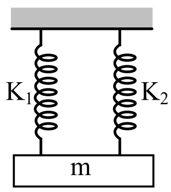
Q.3 If the metal bob of a simple pendulum is replaced by wooden bob, then its time period will:
(1) Increase
(2) Decrease
(3) Remain the same
(4) First increase then decrease

Q.4 A particle executes simple harmonic motion with an angular velocity and maximum acceleration of 3.5 rad/sec. and 7.5 m/s^2 respectively. Amplitude of the oscillations is:
(1) 0.28 m (2) 0.36 m
(3) 0.53 m (4) 0.61 m

Q.5 A lift is ascending with acceleration g/3. What will be the time period of a simple pendulum suspended from its ceiling if its time period in stationary lift is T ?
(1) $\frac{T}{2}$ (2) $\sqrt{3}\frac{T}{2}$
(3) $\sqrt{3}\frac{T}{4}$ (4) $\frac{T}{4}$

Q.6 The total energy of a particle performing SHM depends on:
(1) K,m (2) K, a
(3) K, a, x (4) K, x

Q.7 A mass is suspended separately by two different springs in successive order then time period is $T_1$and T_2 respectively. If it is connected by both spring as shown in figure then time period is T_0, the correct relation is:



(1) $T_0^2 = T_1^2 + T_2^2$ (2) $T_0^{-2} = T_1^{-2} + T_2^{-2}$
(3) $T_0^{-1} = T_1^{-1} + T_2^{-1}$ (4) $T_0 = T_1 + T_2$

Q.8 When an oscillator completes 100 oscillation its amplitude reduced to $\frac{1}{3}$ of initial value. What will be its amplitude, when it completes 200 oscillation ?

(1) $\frac{1}{8}$ (2) $\frac{2}{3}$ (3) $\frac{1}{6}$ (4) $\frac{1}{9}$

Q.9 Displacement between max. P.E. position and max. K.E. position for a particle executing simple harmonic motion is:

(1) $\pm \frac{a}{2}$ (2) + a

(3) ± a (4) – 1

Q.10 When a long spring is stretched by 2 cm, its potential energy is U. If the spring is stretched by 10 cm., the potential energy stored in it will be

(1) U/5 (2) 5 U

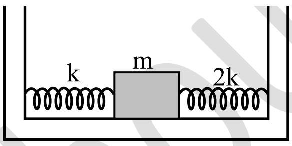
(3) 10 U (4) 25 U

Q.11 The time period of a mass suspended from a spring is T. If the spring is cut into four equal parts and the same mass is suspended from one of the parts, then the new time period will be

(1) $\frac{T}{4}$ (2) T

(3) $\frac{T}{2}$ (4) 2T

Q.12 Two springs of force constant k and 2k are connected to a mass as shown below. The frequency of oscillation of the mass is:



(1) $(1/2\pi)\sqrt{(k/m)}$

(2) $(1/2\pi)\sqrt{(2k/m)}$

(3) $(1/2\pi)\sqrt{(3k/m)}$

(4) $(1/2\pi)\sqrt{(m/k)}$

Q.13 Which one of the following statements is true for the speed 'v' and the acceleration 'a' of a particle executing simple harmonic motion ?

(1) Value of 'a' is zero, whatever may be the value of 'v'

(2) When 'v' is zero, 'a' is zero

(3)When 'v' is maximum, 'v' is zero

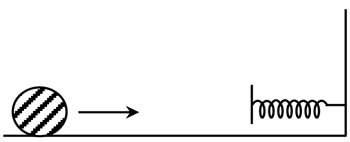
(4) When 'v' is maximum, 'a' is maximum

Q.14 Two springs of spring constants k_1 and k_2 are joined in series. The effective spring constant of the combination is given by :

(1) $\frac{(k_1+k_2)}{2}$ (2) $k_1 + k_2$

(3) $\frac{k_1k_2}{(k_1+k_2)}$ (4) $\sqrt{k_1k_2}$

Q.15 A mass of 0.5 kg moving withy a speed of 1.5 m/s on a horizontal smooth surface, collides with a nearly weightless spring of force constant k = 50 N/m. The maximum compression of the spring would be:



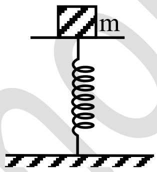
(1) 0.12 m (2) 1.5 m
(3) 0.5 m (4) 0.15 m

Q.16 For a particle executing simple harmonic motion which of the following statement is not correct:
(1) The total energy of particle always remains to same
(2) The restoring force is always directed towards a fix point.
(3) The restoring force is maximum at the extreme positions.
(4) The acceleration of particle is maximum at the equilibrium positions.

Q.17 A particle executing simple harmonic motion of amplitude 5 cm has maximum speed of 31.4 cm/s. The frequency of oscillation is:
(1) 1 Hz (2) 3 Hz
(3) 2 Hz (4) 4 Hz

Q.18 The potential energy of a long spring when stretched by 2 cm is U. If the spring is stretched by 8 cm the potential energy stored in it is:
(1) 4 U (2) 8 U (3) 16 U (4) $\frac{U}{4}$

Q.19 A mass of 2.0 kg is put on a flat pan attached to a vertical spring fixed on the ground as shown in the figure. The mass of the spring and the pan is negligible. When pressed slightly and released the mass executes a simple harmonic motion. The spring constant is 200N/m. What should be the minimum amplitude of the motion so that the mass gets detached from the pan ?(Take g = 10 m/s^2)



(1) 4.0 cm
(2) 8.0 cm
(3) 10.0 cm
(4) Any value less than 12.0 cm

Q.20 The phase difference between the instantaneous velocity and acceleration of a particle executing simple harmonic motion is-
(1) Zero (2) 0.5 π
(3) π (4) 0.707π

Q.21 The particle executing simple harmonic motion has a kinetic energy $K_0 \cos^2 \omega t$. The maximum values of the potential energy and the total energy are respectively:
(1) K_o and K_o (2) 0 and $2K_0$
(3) $\frac{K_o}{2}$ and K_o (4) K_o and $2K_o$

Q.22 A particle executes simple harmonic oscillation with an amplitude a. The period of oscillation is T. The minimum time taken by the particle to travel half of the amplitude from the equilibrium positions is-
(1) T/2 (2) T/4
(3) T/8 (4) T/12

Q.23 A simple pendulum performs simple harmonic motion about x = 0 with an amplitude a and time period T. The speed of the pendulum at x = a/2 will be:-
(1) $\frac{\pi a\sqrt{3}}{T}$ (2) $\frac{\pi a\sqrt{3}}{2T}$

(3) $\frac{\pi a}{T}$ (4) $\frac{3\pi^2 a}{T}$

Q.24 Which one of the following equations of motion represents simple harmonic motion-
(1) Acceleration = kx
(2) Acceleration = $-k_0x + k_1 x^2$
(3) Acceleration = $-k(x + a)$
(4) Acceleration = $k(x + a)$
Where k, k_0, k_1 and a are all positive

Q.25 The displacement of a particle along the x-axis is given by $x = a \sin^2\omega t$. The motion of the particle corresponds to –
(1) simple harmonic motion of frequency ω/π
(2) simple harmonic motion of frequency $3\omega/2\pi$
(3) non simple harmonic motion
(4) simple harmonic motion of frequency $\omega/2\pi$

Q.26 The period of oscillation of a mass M suspended from a spring of negligible mass is T. If along with it another mass M is also suspended, the period of oscillation will now be–
(1) T (2) $T/\sqrt{2}$
(3) 2T (4) $\sqrt{2}$ T

Q.27 A particle moves in x – y plane according to rule $x = a \sin \omega t$ and $y = a \cos \omega t$. The particle follows-
(1) a circular path
(2) a parabolic path
(3) a straight line path inclined equally to x and y-axes
(4) an elliptical path

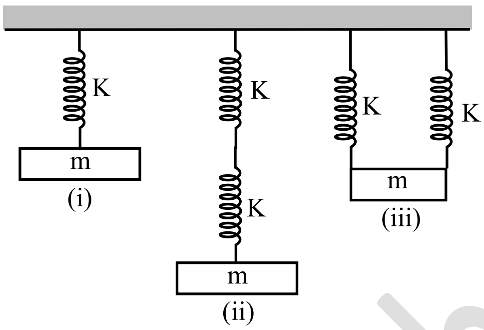
Q.28 Two particles are oscillating along two close parallel straight lines side by side, with the same frequency and amplitudes. They pass each other, moving in opposite directions when their displacement is half of the amplitude. The mean positions of the two particles lie on a straight line perpendicular to the paths of the two particles. The phase difference is :
(1) $\pi/6$ (2) 0
(3) $2\pi/3$ (4) π

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 4

Q.1 The amplitude of a particle executing S.H.M. with frequency of 60 Hz is 0.01 m. The maximum value of the acceleration of the particle is:
(1) $144\pi^2$ m/sec^2 (2) 144 m/sec^2
(3) $\frac{144}{\pi^2}$ m/sec^2 (4) $288\pi^2$ m/sec^2

Q.2 A particle executing S.H.M. of amplitude 4 cm and T = 4 sec. The time taken by it to move from positive extreme position to half the amplitude is-
(1) 1 sec (2) 1/3 sec
(3) 2/3 sec (4) $\sqrt{3/2}$ sec

Q.3 Five identical springs are used in the following three configurations. The time periods of vertical oscillations in configurations (i), (ii) and (iii) are in the ratio:



(1) $1:\sqrt{2}:\frac{1}{\sqrt{2}}$ (2) $2:\sqrt{2}:\frac{1}{\sqrt{2}}$
(3) $\frac{1}{\sqrt{2}}:2:1$ (4) $2:\frac{1}{\sqrt{2}}:1$

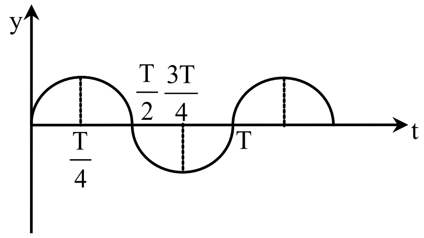
Q.4 A simple harmonic oscillator has a period of 0.01 sec. and an amplitude of 0.2 m. The magnitude of the velocity in m sec^{-1} at the mean position will be -
(1) 20π (2) 100 (3) 40π (4) 100π

Q.5 A simple pendulum is suspended from the roof of a trolley which moves in a horizontal direction with an acceleration a, then the time period is given by $T = 2\pi\sqrt{\frac{\ell}{g'}}$ where g' =
(1) g (2) g – a
(3) g + a (4) $\sqrt{g^2 + a^2}$

Q.6 A simple pendulum performs simple harmonic motion about X = 0 with an amplitude A and time period T. The speed of the pendulum at X = $\frac{A}{2}$ will be:
(1) $\frac{\pi A\sqrt{3}}{T}$ (2) $\frac{\pi A}{T}$
(3) $\frac{\pi A\sqrt{3}}{2T}$ (4) $\frac{3\pi^2 A}{T}$

Q.7 Displacement of a particle is x = 3 sin 2t + 4 cos 2t, the amplitude and the max. velocity will be:
(1) 5, 10 (2) 3, 2
(3) 4, 2 (4) 3, 8

Q.8 The graph shows the variation of displacement of a particle executing S.H.M. with time. We inference from this graph that:



(1) The force is zero at time 3T/4
(2) The velocity is maximum at time T/2
(3) The acceleration is maximum at time T
(4) The P.E. is equal to half of total energy at time T/2

Q.9 A body is executing simple harmonic motion with an angular frequency 2 rad/s. The velocity of the body at 20mm displacement, when the amplitude of motion is 60 mm, is :
(1) 40 mm/s (2) 60 mm/s
(3) 113 mm/s (4) 120 mm/s

Q.10 A clock S works on the oscillations of a spring. Another clock P works on the oscillations of pendulum both the clocks keep correct time on the earth. What will happen if they are taken to the moon ?
(1) Only the clock P will keep correct time
(2) Only clock S will keep correct time
(3) Both of them will keep correct time
(4) None of them will keep correct time

Q.11 The potential energy of a particle executing S.H.M. at a distance x from mean position is :
(1) $\frac{1}{2}m\omega^2x^2$ (2) 0
(3) $m^2\omega x$ (4) $\frac{1}{2}m^2\omega^2x^2$

Q.12 A spring of force constant k is cut into two pieces such that one piece is double the length of the other. Then the long piece will have a force constant of:
(1) (2/3)k (2) (3/2) k
(3) 3k (4) 6k

Q.13 The spring constant of two springs are K_1 and K_2 respectively spring are stretch up to that limit when potential energy of both becomes equal. The ratio of applied force (F_1 and F_2) on them will be:
(1) $K_1 : K_2$ (2) $K_2 : K_1$
(3) $\sqrt{K_1} : \sqrt{K_2}$ (4) $\sqrt{K_2} : \sqrt{K_1}$

Q.14 A particle is describing SHM with amplitude 'a'. When the potential energy of particle is one fourth of the maximum energy during oscillation, then its displacement from mean position will be:
(1) $\frac{a}{4}$ (2) $\frac{a}{3}$ (3) $\frac{a}{2}$ (4) $\frac{2a}{3}$

Q.15 Force constant of a spring is K. One fourth part is detach then force constant of remaining spring will be:

(1) $\frac{3}{4}$K (2) $\frac{4}{3}$K (3) K (4) 4K

Q.16 A particle is executing S.H.M. of frequency 300 Hz and with amplitude 0.1 cm. Its maximum velocity will be :
(1) 60 π cm/s (2) 0.6 π cm/s
(3) 0.50 π cm/s (4) 0.05 π cm/s

Q.17 Which of the following equation does not represent a simple harmonic motion?
(1) y = a sin ωt
(2) y = b cos ωt
(3) y = a sin ωt + b cos ωt
(4) y = a tan ωt

Q.18 A child swinging on a swing in sitting position, stands up, then the period of the swing will be:
(1) Increase
(2) Decrease
(3) Remain same
(4) Increase if child is long and decrease if child is short

Q.19 The equation of a simple harmonic motion is x = 0.34 cos (3000t + 0.74). Where x and t are in mm and sec. respectively. The frequency of the motion is:
(1) 3000 (2) 3000/2π
(3) 0.74 /2π (4) 3000/π

Q.20 The spring constant of a spring is K. When it is divided into n equal parts, then what is the spring constant of one piece:
(1) nK (2) K/n
(3) $\frac{nK}{(n+1)}$ (4) $\frac{(n+1)K}{n}$

Q.21 If amplitude of the particle which is executing S.H.M., is doubled, then which quantity will become double ?
(1) Frequency (2) Time period
(3) Energy (4) Max. velocity

Q.22 Mass 'm' is suspended from a spring of force constant K. Spring is cut into two equal parts and same mass is suspended from it, then new frequency will be:
(1) 2v (2) $\sqrt{2}$v
(3) v (4) $\frac{v}{2}$

Q.23 A simple pendulum is suspended from the ceiling of a vehicle, its time period is T. Vehicle is moving with constant velocity, then time period of simple pendulum will be:
(1) Less than T (2) Equal to T
(3) More than T (4) Cannot predict

Q.24 The ratio of K.E. of the particle at mean position to the point when distance is half of amplitude will be:
(1) $\frac{1}{3}$ (2) $\frac{2}{3}$
(3) $\frac{4}{3}$ (4) $\frac{3}{2}$

Q.25 A particle is executing S.H.M., If its P.E. & K.E. is equal then the ratio of displacement & amplitude will be :

(1) $\frac{1}{\sqrt{2}}$ (2) $\sqrt{2}$ (3) $\frac{1}{2}$ (4) $\frac{3}{2}$

Q.26 In SHM velocity is maximum:

(1) At extreme position
(2) When displacement is half of amplitude
(3) At the central position
(4) When displacement is $\frac{1}{\sqrt{2}}$ of amplitude

Q.27 Which of the following is constant during SHM:

(1) Velocity (2) Acceleration
(3) Total energy (4) Phase

Q.28 Time period of a compound pendulum is T, if its mass is doubled, then its times period will be:

(1) Unchanged (2) 2 times
(3) $\sqrt{2}$ times (4) 4 times

Q.29 The maximum velocity of simple harmonic motion represented by y = 3sin $\left(100t+\frac{\pi}{6}\right)$ is given by

(1) 300 (2) $\frac{3\pi}{6}$
(3)100 (4) $\frac{\pi}{6}$

Q.30 If < E > and < V > denotes the average kinetic and average potential energies respectively of mass describing a simple harmonic motion over one period then the correct relation is:

(1) < E > = < V > (2) < E > = 2 < V >
(3) < E > = – 2 < V > (4) < E > = – < V >

Q.31 The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 m is 4.4 m/s. The period of oscillation is:

(1) 100 S (2) 0.01 S
(3) 10 S (4) 0.1 S

Q.32 A ring is suspended from its one end and oscillating then its time period for small oscillations will be

(1) $2\pi\sqrt{\frac{R}{g}}$ (2) $2\pi\sqrt{\frac{2R}{g}}$
(3) $2\pi\sqrt{\frac{R}{2g}}$ (4) $2\pi\sqrt{\frac{3R}{2g}}$

Q.33 In figure-1 if the time period is T then calculate time period for figure-2 in which two springs are connected in series with same mass M:-

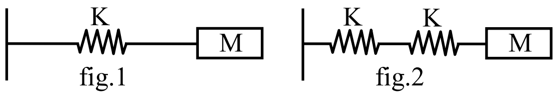


fig.1 fig.2

(1) $\sqrt{2}T$ (2) $\frac{T}{\sqrt{2}}$

(3) 2T (4) $\frac{T}{2}$

Q.34 A mass of 10g is connected to a massless spring then time period of small oscillation is 10 seconds. If 10 g mass is replaced by 40 g mass in same spring, then its time period will be:-
(1) 5s (2) 10s
(3) 20s (4) 40s

Q.35 If a rod of length L is hung by its end on a nail and allowed to oscillate, find the equivalent length of simple pendulum for same time period.
(1) $\frac{L}{3}$ (2) 2L (3) $\frac{2L}{3}$ (4) $\frac{L}{6}$

Q.36 If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then, which of the following does not change with time ?
(1) $a^2T^2 + 4\pi^2v^2$ (2) aT / x
(3) $aT + 2\pi v$ (4) aT/v

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 5

These questions consists of two statements each, printed as Assertion and Reason. While answering these questions you are required to choose any one of the following four responses.

(A) If both Assertion & Reason are true & the Reason is a correct explanation of the Assertion.

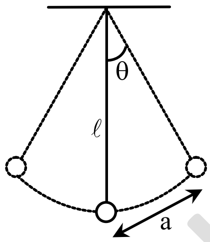
(B) If both Assertion and Reason are true but Reason is not a correct explanation of the Assertion.

(C) If Assertion is true but the Reason is false.

(D) If Assertion & Reason both are false.

Q.1 **Assertion :** The motion of a simple pendulum is simple harmonic only for a << ℓ.

Reason : Motion of a simple pendulum is SHM for small angular displacement.



(1) A (2) B (3) C (4) D

Q.2 **Assertion :** Pendulum clocks go slow in summer and fast in winter.

Reason : The length of the pendulum used in clock increases in summer.

(1) A (2) B (3) C (4) D

Q.3 **Assertion :** A simple pendulum is mounted on a truck which move with constant velocity. The time period of pendulum will increases.

Reason : The effective length of pendulum will decrease.

(1) A (2) B (3) C (4) D

Q.4 **Assertion :** SHM is not a periodic motion.

Reason : Periodic motion does not repeat its position after certain interval of time.

(1) A (2) B (3) C (4) D

Q.5 **Assertion :** When a particle is at extreme position performing SHM, its momentum is equal to zero.

Reason : At extreme position the velocity of particle performing SHM is equal to zero.

(1) A (2) B (3) C (4) D

Q.6 **Assertion :** In compound pendulum, if suspension point and centre of oscillation are mutually interchange, then no change in time period is obtained.

Reason : Length of equivalent simple pendulum remains same in both the case.

(1) A (2) B (3) C (4) D

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 1 (ANSWERS)

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	1	1	4	1	1	1	4	2	3	3	1	1	3	2	1	2	4	4	3	2
Q.No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	4	1	1	1	1	3	4	2	4	1	1	2	4	3	3	2	3	4	3	4
Q.No.	41	42	43	44	45	46	47	48	49	50										
Ans.	3	2	4	4	2	3	4	3	2	1										

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 2 (ANSWERS)

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	3	3	4	1	3	2	4	4	2	3	4	4	1	2	2	1	2	1	2	1
Q.No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	1	2	1	3	1	1	4	1	1	4	2	3	2	4	4	2	4	1	2	4
Q.No.	41	42	43	44	45	46	47	48	49	50	51	52	53							
Ans.	1	1	1	3	2	3	2	1	4	4	4	3	1							

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 3 (ANSWERS)

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	1	3	3	4	2	2	2	4	3	4	3	3	3	3	4	4	1	3	3	2
Q.No.	21	22	23	24	25	26	27	28												
Ans.	1	4	1	3	3	4	1	3												

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 4 (ANSWERS)

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	1	3	1	3	4	1	1	2	3	2	1	2	3	3	2	1	4	2	2	1
Q.No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36				
Ans.	4	2	2	3	1	3	3	1	1	1	3	2	1	3	3	1,2				

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 5 (ANSWERS)

Q.No.	1	2	3	4	5	6
Ans.	1	1	4	4	1	1